Camera Pose Estimation from Lines using Plücker Coordinates

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Appendix A

Algorithm 1 Camera pose estimation from line correspondences

Input: 3D lines $\mathbf{L}_i$, image lines $\mathbf{l}_i$ and their correspondences $\mathbf{L}_i \leftrightarrow \mathbf{l}_i$ $(i = 1 \dots n, n \geq 9)$

1. Construct $\mathbf{M}$ from $\mathbf{L}_i$ and $\mathbf{l}_i$ according to Appendix B.
2. $\mathbf{QDT}^\top \leftarrow \mathrm{SVD}(\mathbf{M})$
3. Solve $\mathbf{M}\hat{\mathbf{p}} = \boldsymbol{\varepsilon}$ for $\hat{\mathbf{p}}$ in the least squares sense by minimizing $||\boldsymbol{\varepsilon}||$.
4. Construct the 3×6 matrix $\hat{\mathbf{P}}$ from the 18-vector $\hat{\mathbf{p}}$ in column-major order.
5. $[\hat{\mathbf{P}}_1 \;\; \hat{\mathbf{P}}_2] \leftarrow \hat{\mathbf{P}}$
6. $s \leftarrow 1/\sqrt[3]{\det \hat{\mathbf{P}}_1}$
7. $\hat{\mathbf{P}}_{2s} \leftarrow s\hat{\mathbf{P}}_2$
8. $\mathbf{U\Sigma V}^\top \leftarrow \mathrm{SVD}(\hat{\mathbf{P}}_{2s})$
9. $\mathbf{Z} \leftarrow \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{W} \leftarrow \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \sigma \leftarrow (\Sigma_{1,1} + \Sigma_{2,2})/2$
10. Compute 2 candidate solutions (A), (B):

    (A): $d_\mathrm{A} \leftarrow \det(\mathbf{UW}\;\mathbf{V}^\top)\,, \quad \hat{\mathbf{R}}_\mathrm{A} \leftarrow \mathbf{UW}\;\mathrm{diag}(1\,1\,d_\mathrm{A})\mathbf{V}^\top\,, \quad [\hat{\mathbf{t}}]_{\times\mathrm{A}} \leftarrow \sigma\mathbf{VZ}\;\mathbf{V}^\top$

    (B): $d_\mathrm{B} \leftarrow \det(\mathbf{UW}^\top\mathbf{V}^\top)\,, \quad \hat{\mathbf{R}}_\mathrm{B} \leftarrow \mathbf{UW}^\top\mathrm{diag}(1\,1\,d_\mathrm{B})\mathbf{V}^\top\,, \quad [\hat{\mathbf{t}}]_{\times\mathrm{B}} \leftarrow \sigma\mathbf{VZ}^\top\mathbf{V}^\top$
11. Convert the antisymmetric matrices $[\hat{\mathbf{t}}]_{\times\mathrm{A}}$, $[\hat{\mathbf{t}}]_{\times\mathrm{B}}$ into vectors $\hat{\mathbf{t}}_\mathrm{A}$, $\hat{\mathbf{t}}_\mathrm{B}$.
12. Find out which solution is physically plausible, i.e. an arbitrary point $\mathbf{x}^W$ in the visible part of the scene must lie in front of the camera.

$$test_\mathrm{A} \leftarrow \hat{\mathbf{R}}_\mathrm{A}^{(3)} \cdot \frac{\mathbf{x}^W - \hat{\mathbf{t}}_\mathrm{A}}{||\mathbf{x}^W - \hat{\mathbf{t}}_\mathrm{A}||}, \quad test_\mathrm{B} \leftarrow \hat{\mathbf{R}}_\mathrm{B}^{(3)} \cdot \frac{\mathbf{x}^W - \hat{\mathbf{t}}_\mathrm{B}}{||\mathbf{x}^W - \hat{\mathbf{t}}_\mathrm{B}||}$$

13. If $(test_\mathrm{A} \leq test_\mathrm{B})$ then $\hat{\mathbf{R}} \leftarrow \hat{\mathbf{R}}_\mathrm{A}\,, \hat{\mathbf{t}} \leftarrow \hat{\mathbf{t}}_\mathrm{A}$ else $\hat{\mathbf{R}} \leftarrow \hat{\mathbf{R}}_\mathrm{B}\,, \hat{\mathbf{t}} \leftarrow \hat{\mathbf{t}}_\mathrm{B}$

Output: Estimated camera pose $(\hat{\mathbf{R}}, \hat{\mathbf{t}})$

Appendix B

Construction of the measurement matrix M. Given n line correspondences $\mathbf{L}_i \leftrightarrow \mathbf{l}_i$, $(i = 1 \ldots n, n \geq 9)$, the $2n \times 18$ measurement matrix $\mathbf{M}$ is constructed as

$$
\begin{array}{ll}
\mathbf{m}^{(2i-1)} & = [\begin{array}{ccccccccccc} l_{iw}L_{i1} & 0 & -l_{ix}L_{i1} & l_{iw}L_{i2} & 0 & -l_{ix}L_{i2} & \ldots & l_{iw}L_{i6} & 0 & -l_{ix}L_{i6} \end{array}] \\
\mathbf{m}^{(2i)} & = [\begin{array}{ccccccccccc} 0 & l_{iw}L_{i1} & -l_{iy}L_{i1} & 0 & l_{iw}L_{i2} & -l_{iy}L_{i2} & \ldots & 0 & l_{iw}L_{i6} & -l_{iy}L_{i6} \end{array}]
\end{array} ,
$$

where $\mathbf{m}^{(i)}$ denotes the i-th row of $\mathbf{M}$, $(l_{ix}\ l_{iy}\ l_{iw})^\top$ are the homogenous coordinates of a 2D line $\mathbf{l}_i$ in the nornmalized image plane and $(L_{i1}\ L_{i2}\ L_{i3}\ L_{i4}\ L_{i5}\ L_{i6})^\top$ are the Plücker coordinates of a corresponding 3D line $\mathbf{L}_i$.