

Two Power-Decreasing Derivation Restrictions in Grammar Systems

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1 Preliminary Definitions

2 Two Derivation Restrictions

3 Grammar Systems

Phrase structure grammar

Phrase structure grammar is a quadruple $G = (N, T, P, S)$, where:

- N and T are alphabets,
- $S \in N$ is the start symbol and
- P is a finite set of productions of the form $\alpha \rightarrow \beta$, where:
 - $\alpha \in N^+$
 - $\beta \in (N \cup T)^*$

Derivation

If $\alpha \rightarrow \beta \in P$, $u = x\alpha y$, $v = x\beta y$ and $x, y \in (T \cup N)^*$, then
 $u \Rightarrow v[\alpha \rightarrow \beta]$ or simply $u \Rightarrow v$.

$N_{left}(P)$

$$N_{left}(P) = \{\alpha \mid \alpha \in N^+, \beta \in (N \cup T)^*, \alpha \rightarrow \beta \in P\}$$

Example

Let $P = \{S \rightarrow BA, BA \rightarrow AAc, AA \rightarrow aAC, AC \rightarrow ac, AC \rightarrow acBA\}$
then $N_{left}(P) = \{S, BA, AA, AC\}$

 $occur(\omega, W)$

If V is total alphabet, $W \subseteq V$ and $\omega \in V^*$, then $occur(\omega, W)$ denotes
the number of occurrences of symbols of W in ω

Example

Let $W = \{a, B, C, S\}$ and $\omega = aacaaBAcc$, then $occur(\omega, W) = 5$

Derivation Restriction

Let:

- $G = (N, T, P, S)$ be a phrase structure grammar,
- $V = N \cup T$ be the total alphabet of G ,
- $I \geq 1$.

If there is $\alpha \rightarrow \beta \in P$, $u = x_0\alpha x_1$, and $v = x_0\beta x_1$, where:

- $x_0 \in T^*N^*$,
- $x_1 \in V^*$,
- $\text{occur}(x_0\alpha, N) \leq I$,

then $u \not\Rightarrow v[\alpha \rightarrow \beta]$ in G or simply $u \not\Rightarrow v$

$\not\Rightarrow^*$ and $\not\Rightarrow^+$ denote transitive-reflexive and transitive closure of $\not\Rightarrow$, respectively.

Example

Let $I = 3$ and $G = (\{S, A, B, C, D\}, \{a, b\}, P, S)$, where $P = \{$

- 1: $S \rightarrow ABB,$
 - 2: $AB \rightarrow aC,$
 - 3: $AB \rightarrow aAC,$
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$W(m)$

Let $m \geq 1$. $W(m)$ denotes the set of all strings $x \in V^*$, where

- $x \in (T^*N^*)^m T^*$

Example

- $aaBadCcb \in W(3)$,
- $BBCD \in W(3)$,
- $BBaBAASaaCaC \notin W(3)$

Product

Let $u, v \in V^*$ and $u \Rightarrow v$. If $u, v \in W(m)$, then $u \underset{m}{\circ} \Rightarrow v$.

Example

Let $G = (\{S, A, B\}, \{a, b, c\}, P, S)$, where $P = \{$

1: $S \rightarrow AB,$

2: $A \rightarrow aAb,$

3: $A \rightarrow ab,$

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$\}$ and $\forall x \in V^* \mid S \xrightarrow{*} x : x \in W(1).$

$S \xrightarrow{*} AB[1] \xrightarrow{*} abB[3] \xrightarrow{*} abcB[4] \xrightarrow{*+} abc^+[4^*5]$

$S \xrightarrow{*} AB[1] \xrightarrow{*} Ac[5] \xrightarrow{*} aAbc[2] \xrightarrow{*+} a^n b^n c [2^{n-1} 3] \mid n \geq 2$

$L(G)_{W(1)} = \{abc^n, a^n b^n c \mid n \geq 1\}.$

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$$L(G)_{W(1)} = \{abc^n, a^n b^n c \mid n \geq 1\}.$$

$W(m, h)$

Let $m, h \geq 1$. $W(m, h)$ denotes the set of all strings $x \in V^*$, where

- $x \in (T^*N^*)^m T^*$
- $(y \in \text{sub}(x) \wedge |y| > h) \text{ implies } \text{alph}(y) \cap T \neq \emptyset$

Example

- $aaBadCcb \in W(3, 1)$,
- $BBCD \notin W(3, 1)$,
- $BBaBAASaaCaC \notin W(3, 1)$
- $BaAaaCaC \notin W(3, 1)$

Product

Let $u, v \in V^*$ and $u \Rightarrow v$. If $u, v \in W(m, h)$, then $u \xrightarrow[m]{h} v$.

Example

Let $G = (\{S, A, B\}, \{a, b, c, d\}, P, S)$, where $P = \{$

1: $S \rightarrow AS,$

2: $S \rightarrow BS,$

3: $S \rightarrow d,$

4: $A \rightarrow aAb,$

5: $A \rightarrow ab,$

6: $B \rightarrow cB,$

7: $B \rightarrow c,$

} and $\forall x \in V^* \mid S \xrightarrow{*} x : x \in W(1, 2).$

$$S_m^h \Rightarrow AS[1]_m^h \Rightarrow Ad[3]_m^h \Rightarrow^+ a^n b^n d [4^{n-1} 5] \mid n \geq 1$$

$$S_m^h \Rightarrow AS[1]_m^h \Rightarrow abS[5]_m^h \Rightarrow^+ ab(ab+c)^*d[\pi]$$

$$L(G)_{W(1,2)} = \{a^n b^n d, (ab+c)^* d \mid n \geq 1\}$$

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$L(G)_{W(1,2)} = \{a^n b^n d, (ab + c)^* d \mid n \geq 1\}$

Example

Let $G = (\{S, A, B\}, \{a, b, c, d\}, P, S)$, where $P = \{$

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Basic Idea

We can derivate strings with more than one grammar. Each grammar make sub-part of target string.

We have two basic type of grammar systems:

- Cooperating Distributed(CD) Grammar Systems
 - Compute in sequential mode
 - Compute step doing only one component of the grammar system
- Parallel Communicating(PC) Grammar Systems
 - Compute in parallel mode
 - At the same time more than one component of the grammar system can compute

CD Grammar System

CD grammar system is $(n + 3)$ -tuple

$$\Gamma = (N, T, S, P_1, \dots, P_n),$$

where:

- N, T are alphabets, where $T \cap N \neq \emptyset$
- $S \in N$ is start nonterminal,
- P_1, P_2, \dots, P_n are sets of rules.

Derivation modes

Let $\Gamma = (N, T, S, P_1, \dots, P_n)$ is CD grammar system.

- For all $i = 1, 2, \dots, n$, *terminating derivation* by the i -th component, denoted $\Rightarrow_{P_i}^t$, defined by

$$x \Rightarrow_{P_i}^t y \text{ iff } x \Rightarrow_{P_i}^* y \text{ there is no } z \in \Sigma^* \text{ with } y \Rightarrow_{P_i} z.$$

- For all $i = 1, 2, \dots, n$, *k -step derivation* by the i -th component, denoted $\Rightarrow_{P_i}^{=k}$, defined by

$$x \Rightarrow_{P_i}^{=k} y \text{ iff there are } x_1, \dots, x_{k+1} \in (N \cup T)^* \text{ such that}$$

$$x = x_1, y = x_{k+1}, \text{ and } \forall j = 1, \dots, k, x_j \Rightarrow_{P_i} x_{j+1}.$$

- For all $i = 1, 2, \dots, n$, *the at most k -step derivation* by the i -th component, denoted $\Rightarrow_{P_i}^{\leq k}$, defined by

$$x \Rightarrow_{P_i}^{\leq k} y \text{ iff } x \Rightarrow_{P_i}^{=k'} y \text{ for some } k' \leq k.$$

Derivation modes D

Let $D = \{*, t\} \cup \{\leq k, = k, \geq k \mid k \in \mathbb{N}^+\}$

CD grammar system language 1/2

Let $\Gamma = (N, T, S, P_1, \dots, P_n)$ is CD grammar system and let $f \in D$ is a derivation mode.

$$L_f(\Gamma) = \{\omega \in T^* \mid S \xrightarrow{f}_{P_{i_1}} \omega_1 \xrightarrow{f}_{P_{i_2}} \dots \xrightarrow{f}_{P_{i_m}} \omega_m = \omega, m \geq 1, 1 \leq j \leq m, \\ 1 \leq i_j \leq n\}$$

Example

$\Gamma = (\{S, A', A, B', B\}, \{a, b, c\}, S, P_1, P_2),$

$P_1 = \{S \rightarrow S, S \rightarrow AB, A' \rightarrow A, B' \rightarrow B\},$

$P_2 = \{A \rightarrow aA'b, B \rightarrow cB', A \rightarrow ab, B \rightarrow c\}$

$L_f(\Gamma) = \{a^n b^n c^m \mid m, n \geq 1\}, f \in \{=1, \geq 1, t\},$

$L_{=2}(\Gamma) = \{a^n b^n c^n \mid n \geq 1\},$

$L_{\geq 3}(\Gamma) = \emptyset, k \geq 3$

CD grammar system language 2/2

For a CD grammar system $\Gamma = (N, T, S, P_1, \dots, P_n)$ and a control language L , we set

$$\text{1-left } \mathcal{L}_t^L(\Gamma) = \{ w \in T^* | S \xrightarrow{I}^{t, i_1} w_1 \xrightarrow{I}^{t, i_2} \dots \xrightarrow{I}^{t, i_p} w_p = w, \\ p \geq 1, 1 \leq i_j \leq n, 1 \leq j \leq p, i_1 i_2 \dots i_p \in L \}$$

$$\text{nonter } \mathcal{L}_t^L(\Gamma, m, h) = \{ w \in T^* | S \xrightarrow{m}^{h, i_1} w_1 \xrightarrow{m}^{h, i_2} \dots \xrightarrow{m}^{h, i_p} w_p = w, \\ p \geq 1, 1 \leq i_j \leq n, 1 \leq j \leq p, i_1 i_2 \dots i_p \in L \}$$

$$\text{nonter } \mathcal{L}_t^L(\Gamma, m) = \{ w \in T^* | S \xrightarrow{m}^{t, i_1} w_1 \xrightarrow{m}^{t, i_2} \dots \xrightarrow{m}^{t, i_p} w_p = w, \\ p \geq 1, 1 \leq i_j \leq n, 1 \leq j \leq p, i_1 i_2 \dots i_p \in L \}.$$

Language Families

Let GSs denote the family of all CD grammar systems. Let $l, m, h \geq 1$. Define the following language families:

$${}_{\text{1-left}} GS_t^{REG} = \{{}_{\text{1-left}} \mathcal{L}_t^L(\Gamma) : \Gamma \in GSs, L \in REG\}$$

$${}_{\text{nonter}} GS_t^{REG}(m, h) = \{{}_{\text{nonter}} \mathcal{L}_t^L(\Gamma, m, h) : \Gamma \in GSs, L \in REG\}$$

$${}_{\text{nonter}} GS_t^{REG}(m) = \{{}_{\text{nonter}} \mathcal{L}_t^L(\Gamma, m) : \Gamma \in GSs, L \in REG\}$$

Results

1. $CF = {}_{\text{1-left}} GS_t^{REG}$,
2. $RE = {}_{\text{nonter}} GS_t^{REG}(m)$,
3. $SG_m \supseteq {}_{\text{nonter}} GS_t^{REG}(m, h)$.