Perfect hashing using graph theory

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Motivation

- Dictionary problem
  - Old and important
  - How to efficiently access data in dictionary
  - Sorted lists, hash tables , etc.
- Hash Table
  - Collision solving
  - Memory overhead

Hash function

- Lets U be universe of all possible keys
- Let S be set of all actual keys
- Let M be output interval:

$$S \subset U$$
$$|S| \ll |U|$$
$$|M| \geqslant |S| \; yet \; |M| \ll |U|$$

- Hash function h is mapping: U → M
- Synonyms: $h(x_1) = h(x_2) \wedge x_1 \neq x_2$
- Collision for two synonyms: $x_1 \in S \wedge x_2 \in S$

Good hash function

- Uniformly distributed output
  - Birthday paradox

$$|S| = 1,25\sqrt{|M|}$$

Then probability of collision is about 50%

- Fast computation
- Application specific need
  - Cryptographic properties
  - Order preserving

Perfect hash

- Lets have set K_m such that

$$m \in M$$
$$K_m = \{x \mid x \in S \wedge h(x) = m\}$$

than hash function is l-perfect, if

$$\forall m \in M : |K_m| \leqslant l$$

- Perfect Hash is 1-perfect hash
- Minimal Perfect Hash is Perfect Hash with

$$|M| = |S|$$

Needed Graph Theory

- Graph is ordered pair G(V,E) where V is set of vertices and E is set of edges.
  - Edge e is n-tuple $e=(v_1, v_2, \ldots, v_n) \wedge \forall i \leqslant n : v_i \in V$
  - Edge of graph is couple
  - Edge of hypergraph (r-graph) is r-tuple
- Degree of Vertex $Deg(V) = \left| \left\{ x \middle| x \in E \wedge \exists i : x_i = V \right\} \right|$
- Random graph is Graph, created from graph with empty set of edges by adding new edges connecting randomly chosen vertices.

Needed Graph Theory 2

- Loop in graph is the edge e:

$$\exists i \leqslant r \wedge \exists j \leqslant r : v_i = v_j$$

- Acyclic graph is loopless graph containing edge with at least one vertices with degree 1 and after its removing, the remaining graph is still acyclic. Graph without edges is acyclic.
- Probability that random graph or hypergraph is acyclic depends on the rate between |V| and |E|.

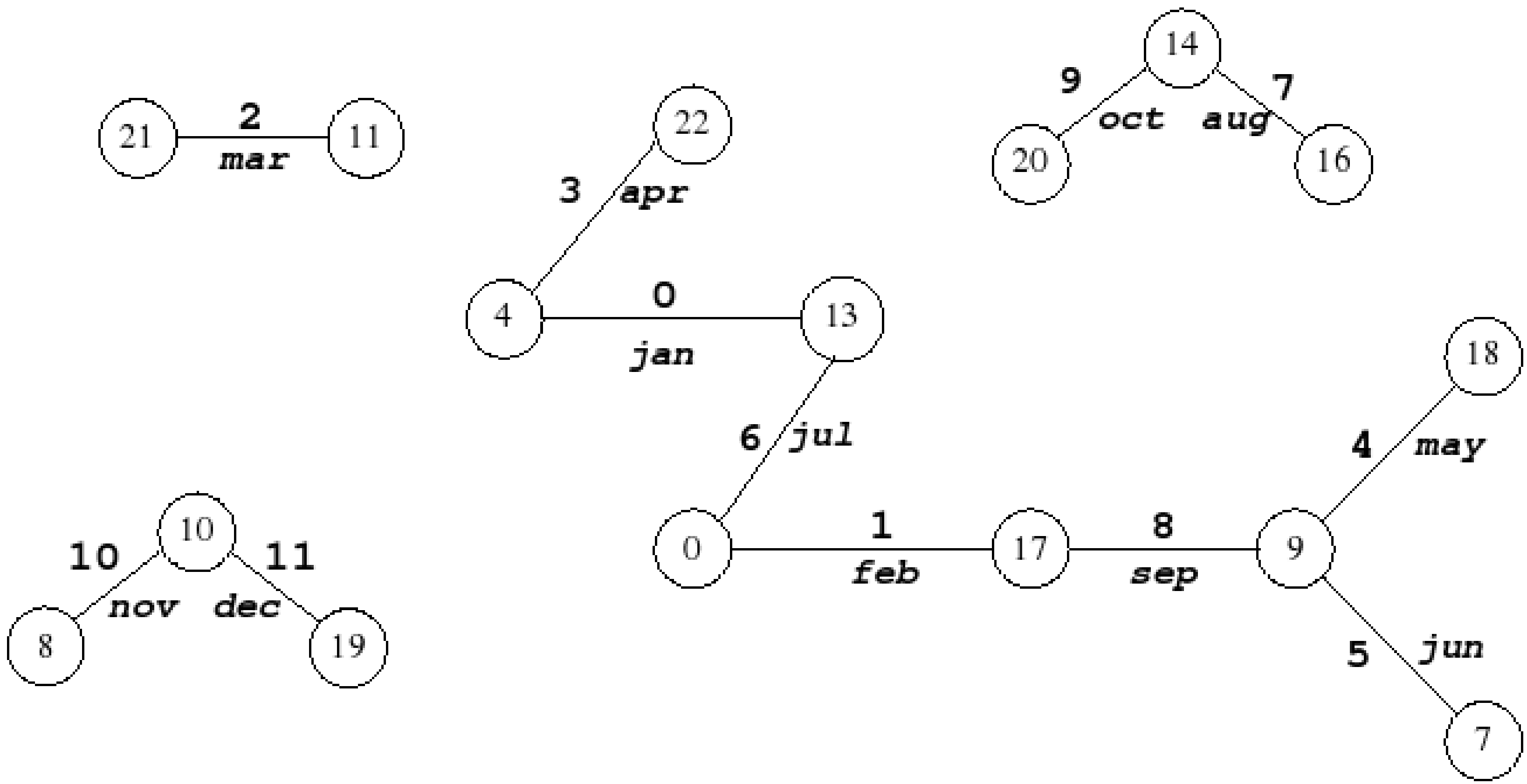
Hash functions & Random graphs

- Lets have r **randomly** chosen hash functions h_i and set of the keys S.
  - Create the set V $|V|=|M|$
  - Set of edges $E=\{(h_1(x), h_2(x), \ldots, h_r(x)) | \forall x \in S\}$
- Such graph is random because hash functions are random

Graph and Information

- We can create a mapping from E to any interval just by storing pairs <Edge, Number>
  - This mapping could be perfect hash, but
  - How efficiently found the right edge?
- Add information to the vertices
  - Fast to retrieve
- Acyclic graph can have simple function (plus) to compute label of edge from the labeled vertices

Illustration



	0	4	7	8	9	10	11	13	14	16	17	18	19	20	21	22
g	0	6	10	0	7	10	0	6	0	7	1	9	1	9	2	9

Algorithm CHM for Perfect Hashing

- Presented by Czech, Havas and Majevski
- Uses two random hash functions
- Random graph formed by hashes and set S
  - Has to be acyclic graph (loopless, not multigraph)
- If random graph doesn't meet criteria, hashes are generated again.
- To found graph in sufficiently small number of steps, graph should have $|V| \geqslant 2{,}09|E|$
- CHM is order preserving
- CHM requires about $|S|\log(|S|)$ bits

BMZ algorithm

- Why graph has to be acyclic?
  - For fast computation of values of vertices
  - To generate order preserving hash function
- BMZ does not require acyclic graph
  - Maximal half of the graph can be in the cycles
  - “right” graph can be found in reasonable time:

$$|V| \geqslant 0{,}93|E|$$

- Only difference against CHM is in labeling vertices in the cycles

BDZ algorithm

- Uses r hash functions (best results for r = 3)
  - Each hash maps keys into subset of V
    - No loops can exists
- Generated r-graph has to be acyclic
- Values of edges do not represent the output of PHF, but points to the right hash
  - Only linear space requirements

Literature

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