

TREE CONTROLLED GRAMMARS

Part Two: Controlling Paths

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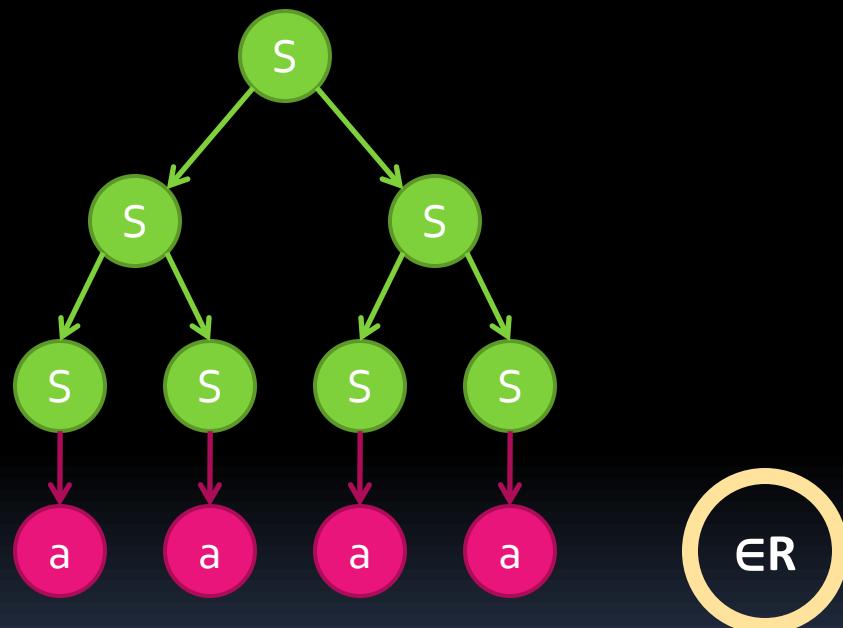
TC Grammar vs. PCTC Grammar

TC (PCTC) Grammar is a pair (G, R) ($(G, R)_{PC}$), where:

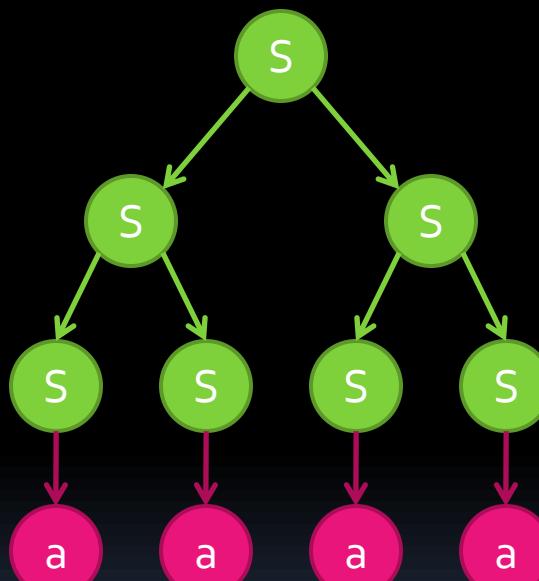
- $G = \{N, T, P, S\}$ is a context-free grammar
- $R \subseteq (NUT)^*$ is a regular
- Language of TC Grammar
 $L(G, R) = \{x \in L(G) \mid$ there exists a derivation tree of x such that each word obtained by concatenating all symbols at any level (except the last one) from left to right is in $R\}$
- Language of PCTC Grammar
 $L(G, R)_{PC} = \{x \in L(G) \mid$ there exists a derivation tree of x such that each word obtained by concatenating all symbols in any path from top to down is in $R\}$

TC vs. PCTC - Illustration

Level Controlling

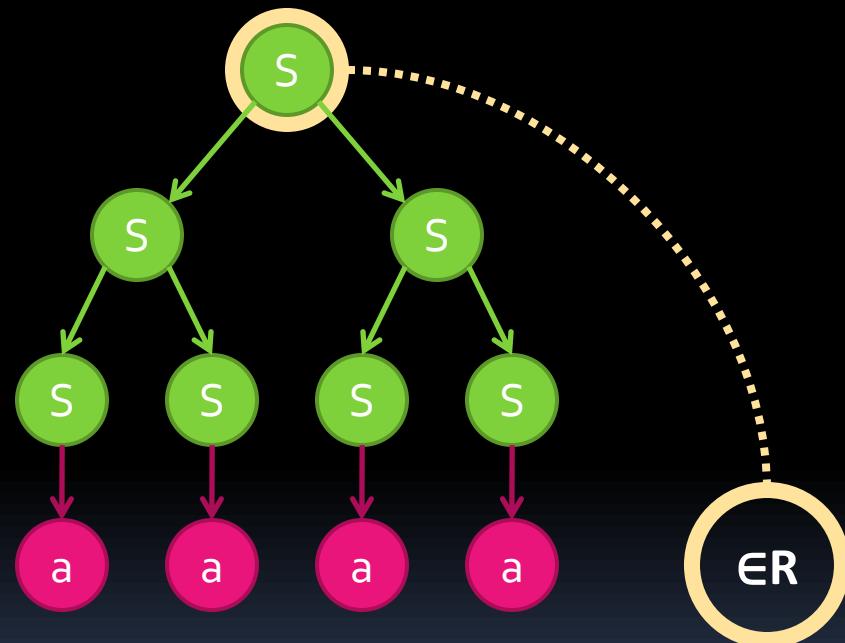


Path Controlling

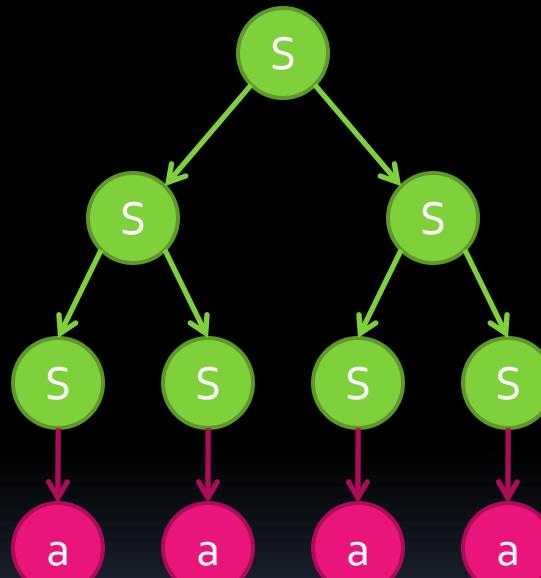


TC vs. PCTC - Illustration

Level Controlling

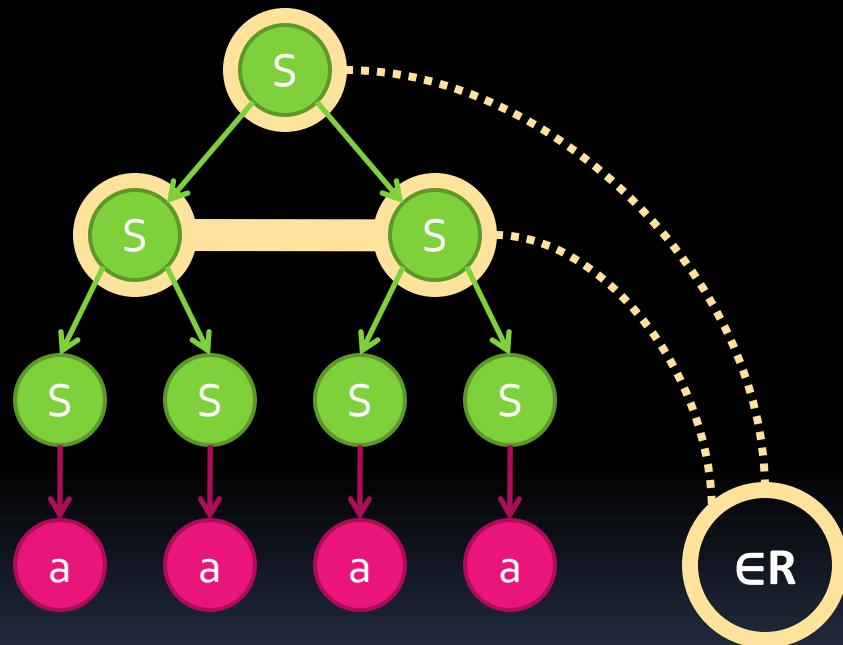


Path Controlling

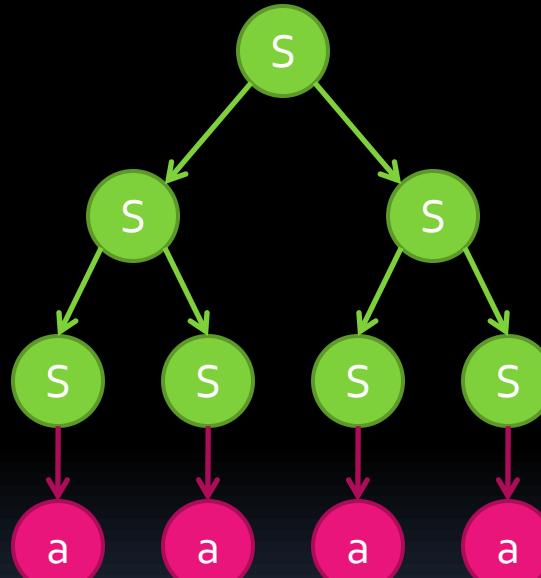


TC vs. PCTC - Illustration

Level Controlling

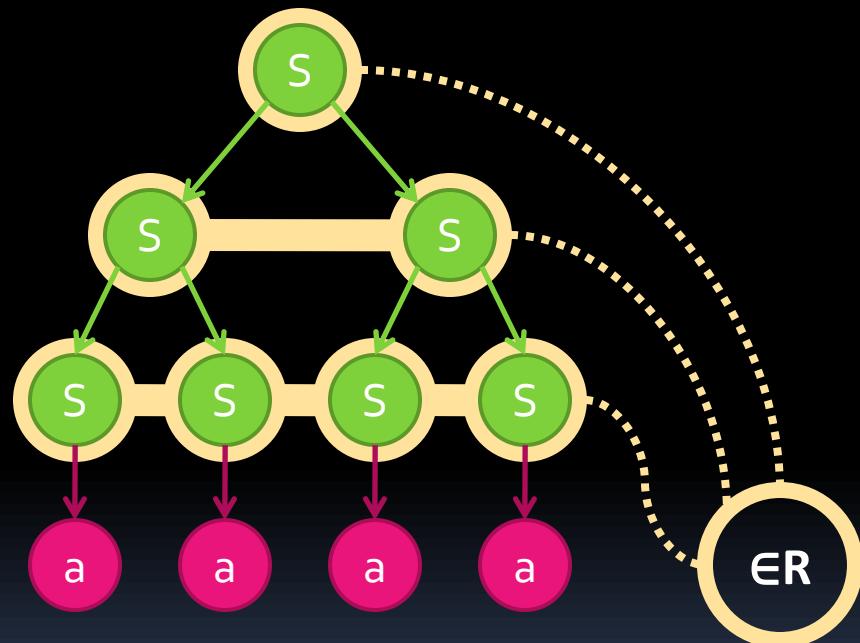


Path Controlling

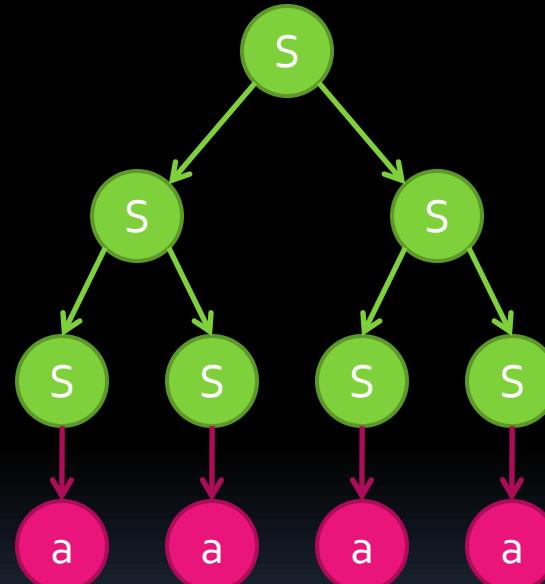


TC vs. PCTC - Illustration

Level Controlling

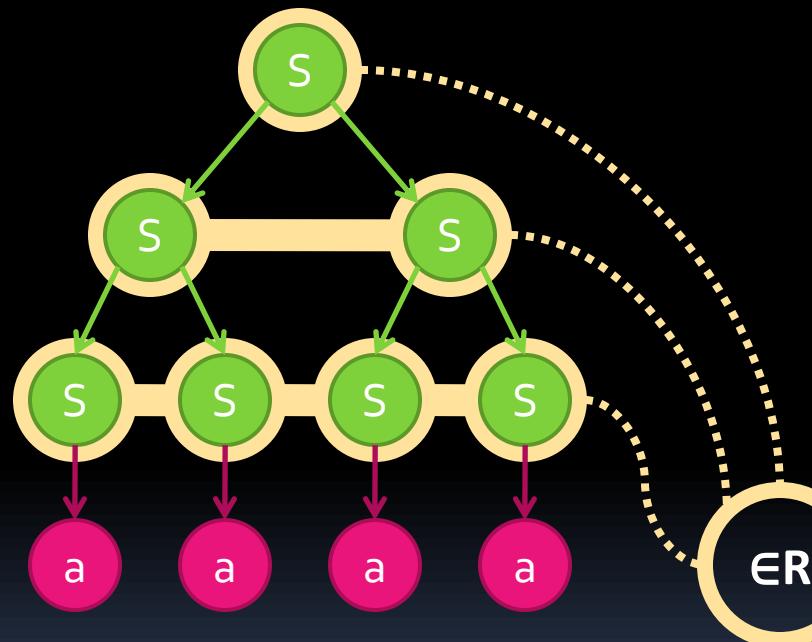


Path Controlling

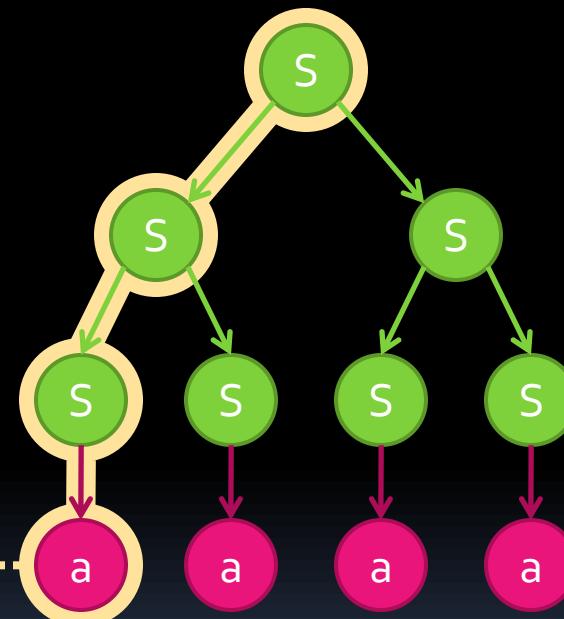


TC vs. PCTC - Illustration

Level Controlling

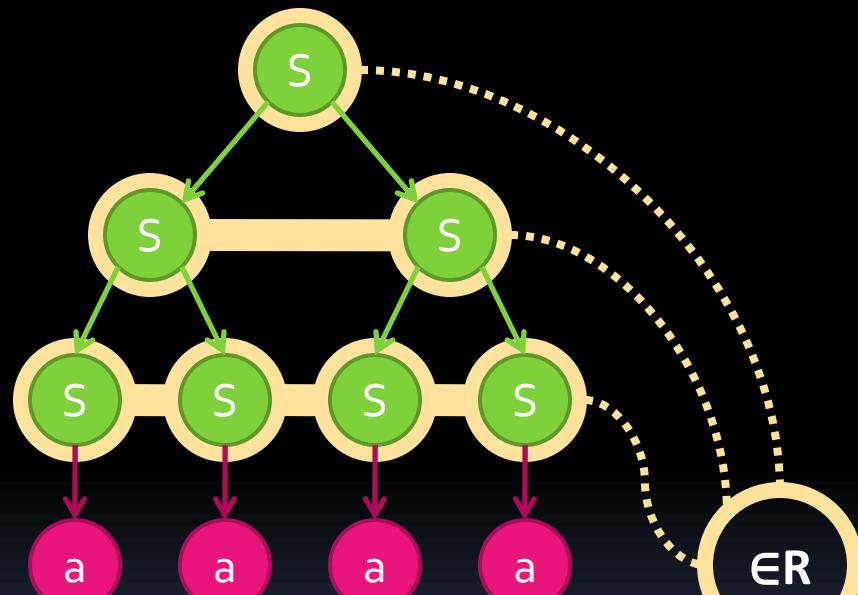


Path Controlling

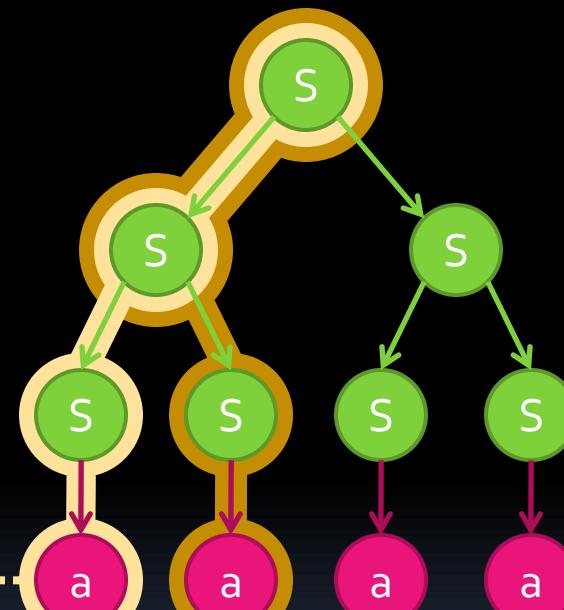


TC vs. PCTC - Illustration

Level Controlling

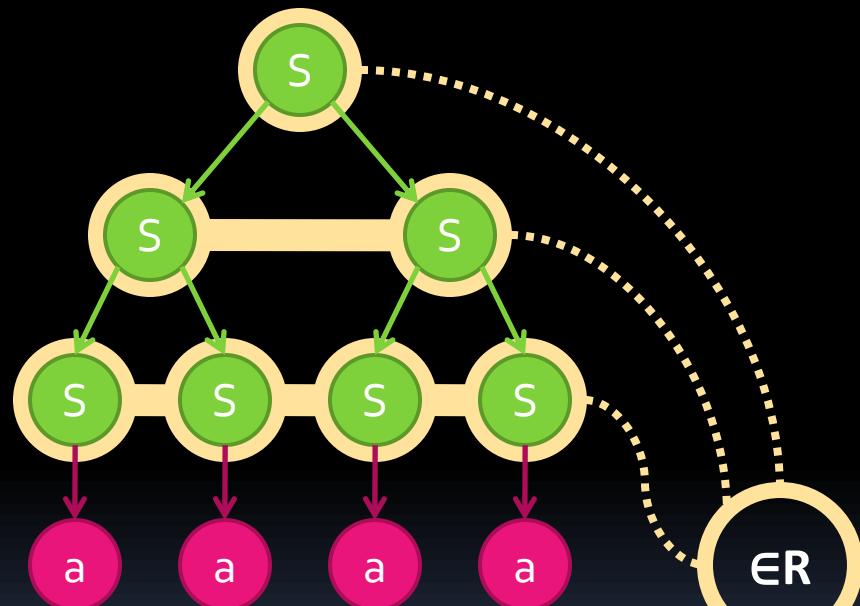


Path Controlling

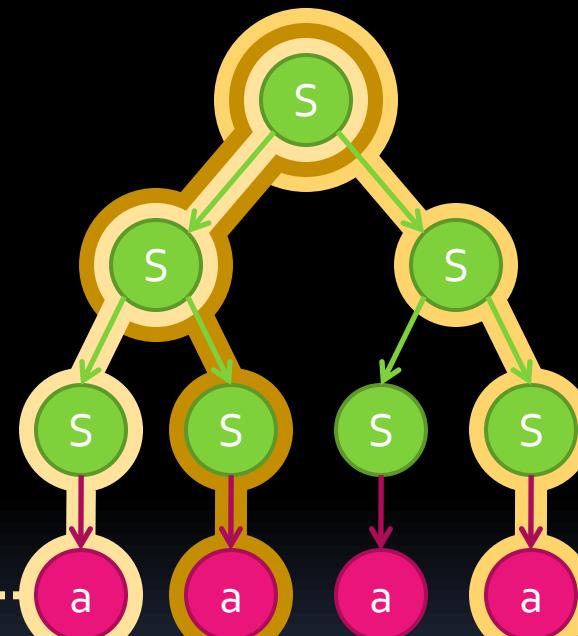


TC vs. PCTC - Illustration

Level Controlling

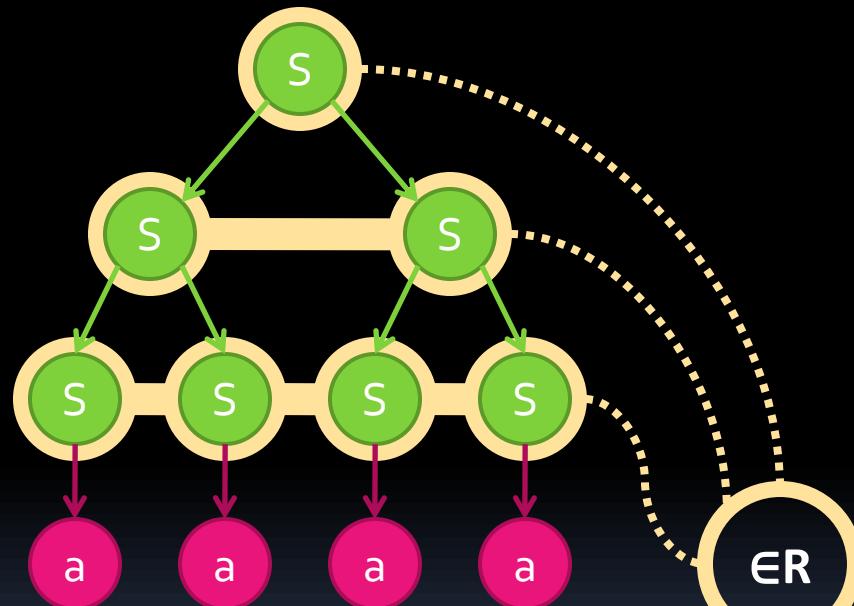


Path Controlling

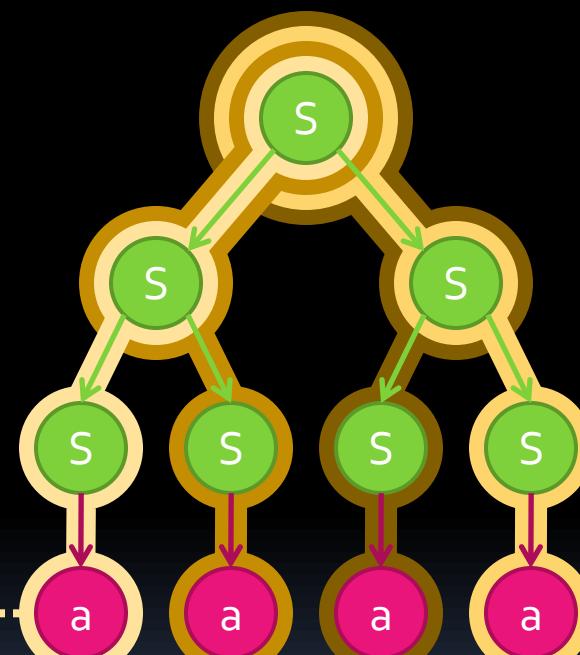


TC vs. PCTC - Illustration

Level Controlling



Path Controlling



What do we get by controlling paths?

Controlling all the paths
of derivation tree by **regular**
don't increase
generative power of CFG.

$$L(G, R)_{PC} = L(H) \in \mathcal{L}(CF)$$

Why do we get what we get?

- It's possible to transform such PCTC grammar $(G, R)_{PC}$ to CF grammar H , so that $L(G, R)_{PC} = L(H)$
 - What does it mean?
 - H is CFG \Rightarrow
 - $L(H) \in \mathcal{L}(CF) \Rightarrow$
 - $L(H) = L(G, R)_{PC} \Rightarrow$
- $\Rightarrow L(G, R)_{PC} \in \mathcal{L}(CF)$

Algorithm from $(G, R)_{PC}$ to (H)

In $(G, R)_{PC}$, the R is regular \Rightarrow

- there exists FA $M = (Q, \Sigma, R, s, F)$, such $L(M) = R$

$\forall P \in P_G: A \rightarrow B_1 B_2 \dots B_n$

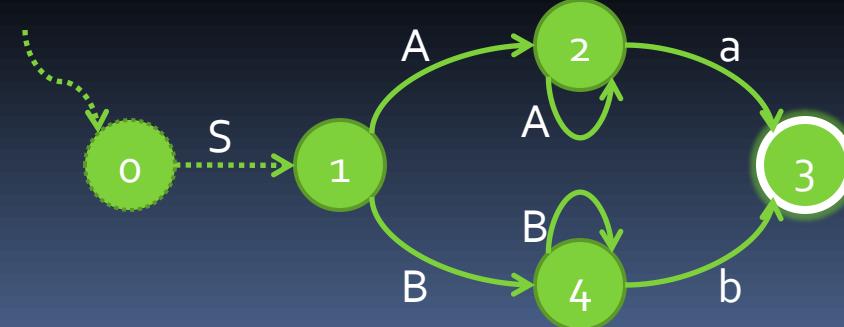
- If $\forall i, i=1, 2 \dots n: q_A B_i \rightarrow q_{B_i A} \in R_M$ then
 - $\langle A, q_A \rangle \rightarrow \langle B_1, q_{B_1 A} \rangle \langle B_2, q_{B_2 A} \rangle \dots \langle B_n, q_{B_n A} \rangle$ to P_H
 - If it holds that $B_i \in T_G$, and $q_{B_i A} \in F_M$ then
 - $\langle B_i, q_{B_i A} \rangle \rightarrow B_i$ to P_H

Example from $(G, R)_{PC}$ to (H)

Consider $(G, R)_{PC}$, $G = (N, T, P, S)$ where

- $N = \{S, A, B, C\}$
- $T = \{a, b, c\}$
- $P = \{S \rightarrow AB, A \rightarrow Aa \mid a, B \rightarrow Bb \mid b \mid BC, C \rightarrow Cc \mid c\}$
- $R = \{SA^+a, SB^+b\}$
 - Note that in R there are no symbols C or c

$KA M, L(M) = R$



Example from $(G, R)_{PC}$ to (H)

$S \rightarrow AB$

Algorithm

$\forall P \in P_G : A \rightarrow B_1 B_2 \dots B_n$

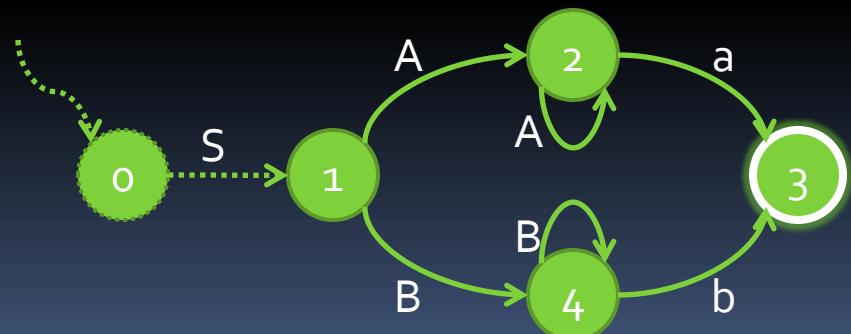
$\forall i, i=1, 2, \dots, n : q_A B_i \rightarrow q_{B_i A} \in R_M$ then

$\langle A, q_A \rangle \rightarrow \langle B_1, q_{B_1 A} \rangle < B_2, q_{B_2 A} \rangle \dots < B_n, q_{B_n A} \rangle$ to P_H

If it holds that $B_i \in T_G$ then

$q_{B_i A} \in F_M$

$\langle B_i, q_{B_i A} \rangle \rightarrow B_i$ to P_H



Example from $(G, R)_{PC}$ to (H)

$S \rightarrow AB$

${}_1A \rightarrow {}_2 \in R_M$

Algorithm

$\forall P \in P_G: A \rightarrow B_1 B_2 \dots B_n$

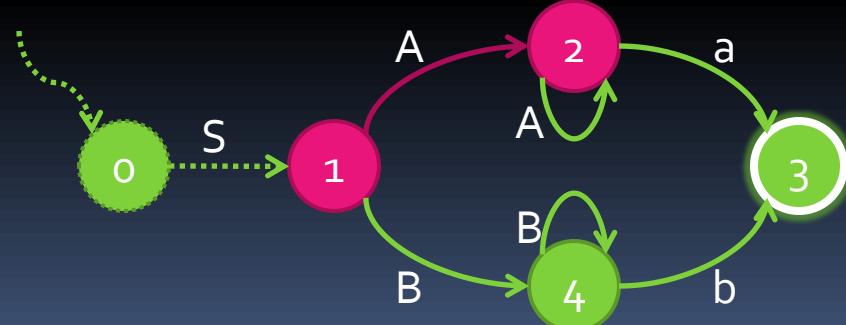
$\forall i, i=1, 2, \dots, n: q_A B_i \rightarrow q_{B_i A} \in R_M$ then

$\langle A, q_A \rangle \rightarrow \langle B_1, q_{B_1 A} \rangle \langle B_2, q_{B_2 A} \rangle \dots \langle B_n, q_{B_n A} \rangle$ to P_H

If it holds that $B_i \in T_G$ then

$q_{B_i A} \in F_M$

$\langle B_i, q_{B_i A} \rangle \rightarrow B_i$ to P_H



Example from $(G, R)_{PC}$ to (H)

$S \rightarrow AB$

${}_1A \rightarrow {}_2 \in R_M$

${}_1B \rightarrow {}_4 \in R_M$

Algorithm

$\forall P \in P_G: A \rightarrow B_1 B_2 \dots B_n$

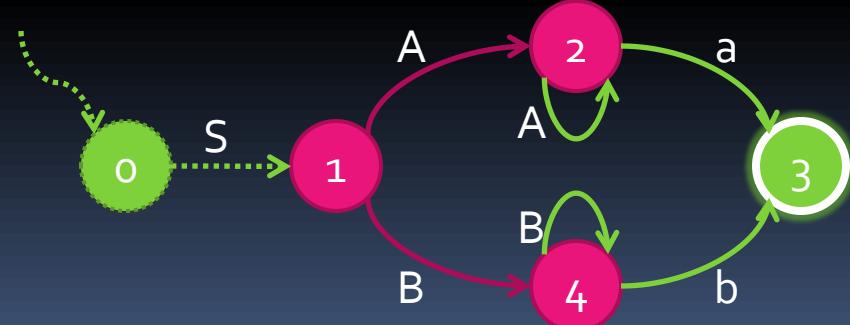
$\forall i, i=1, 2, \dots, n: q_A B_i \rightarrow q_{B_i A} \in R_M$ then

$\langle A, q_A \rangle \rightarrow \langle B_1, q_{B_1 A} \rangle \langle B_2, q_{B_2 A} \rangle \dots \langle B_n, q_{B_n A} \rangle$ to P_H

If it holds that $B_i \in T_G$ then

$q_{B_i A} \in F_M$

$\langle B_i, q_{B_i A} \rangle \rightarrow B_i$ to P_H



Example from $(G, R)_{PC}$ to (H)

$S \rightarrow AB$

${}_1A \rightarrow {}_2 \in R_M$

${}_1B \rightarrow {}_4 \in R_M$

Algorithm

$\forall P \in P_G: A \rightarrow B_1 B_2 \dots B_n$

$\forall i, i=1, 2, \dots, n: q_A B_i \rightarrow q_{B_i A} \in R_M$ then

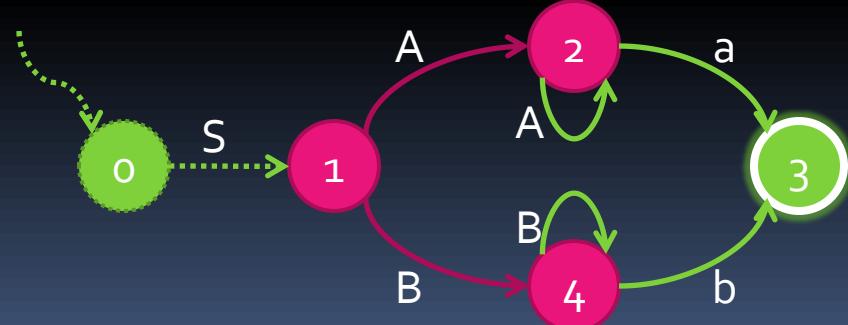
$\langle A, q_A \rangle \rightarrow \langle B_1, q_{B_1 A} \rangle \langle B_2, q_{B_2 A} \rangle \dots \langle B_n, q_{B_n A} \rangle$ to P_H

If it holds that $B_i \in T_G$ then

$q_{B_i A} \in F_M$

$\langle B_i, q_{B_i A} \rangle \rightarrow B_i$ to P_H

$\Rightarrow \langle S, {}_1 \rangle \rightarrow \langle A, {}_2 \rangle \langle B, {}_4 \rangle \in P_H$



Example from $(G, R)_{PC}$ to (H)

$S \rightarrow AB$

${}_1A \rightarrow {}_2 \in R_M$

${}_1B \rightarrow {}_4 \in R_M$

$\Rightarrow <{}S, {}_1> \rightarrow <{}A, {}_2><{}B, {}_4> \in P_H$

$A \rightarrow Aa$

Algorithm

$\forall P \in P_G : A \rightarrow B_1 B_2 \dots B_n$

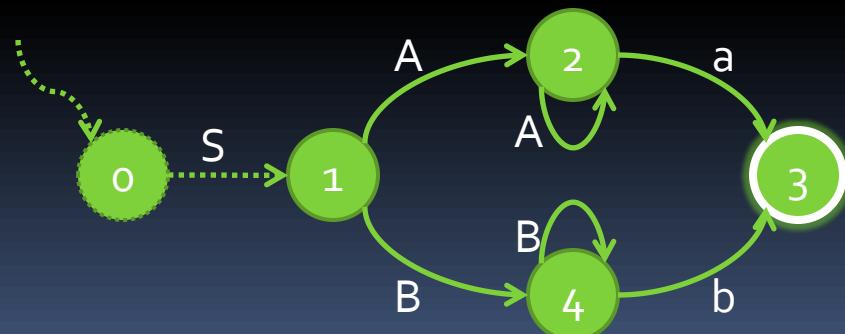
$\forall i, i=1, 2, \dots, n : q_A B_i \rightarrow q_{B_i A} \in R_M$ then

$<{}A, q_A> \rightarrow <{}B_1, q_{B_1 A}> <{}B_2, q_{B_2 A}> \dots <{}B_n, q_{B_n A}>$ to P_H

If it holds that $B_i \in T_G$ then

$q_{B_i A} \in F_M$

$<{}B_i, q_{B_i A}> \rightarrow B_i$ to P_H



Example from $(G, R)_{PC}$ to (H)

$S \rightarrow AB$

$1A \rightarrow 2 \in R_M$

$1B \rightarrow 4 \in R_M$

$\Rightarrow <S, 1> \rightarrow <A, 2><B, 4> \in P_H$

$A \rightarrow Aa$

$2A \rightarrow 2 \in R_M$

Algorithm

$\forall P \in P_G: A \rightarrow B_1 B_2 \dots B_n$

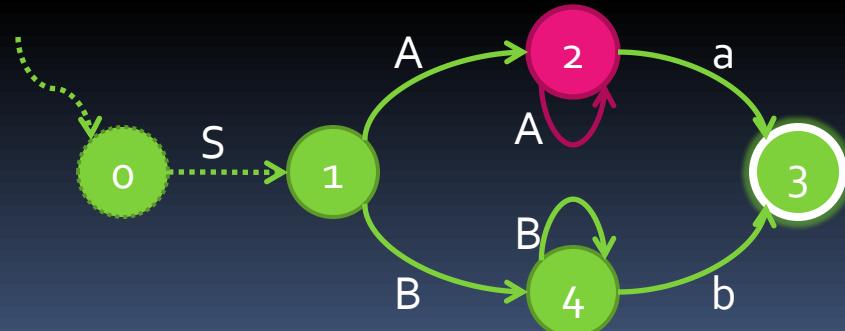
$\forall i, i=1, 2, \dots, n: q_A B_i \rightarrow q_{B_i A} \in R_M$ then

$<A, q_A> \rightarrow <B_1, q_{B_1 A}> <B_2, q_{B_2 A}> \dots <B_n, q_{B_n A}>$ to P_H

If it holds that $B_i \in T_G$ then

$q_{B_i A} \in F_M$

$<B_i, q_{B_i A}> \rightarrow B_i$ to P_H



Example from $(G, R)_{PC}$ to (H)

$S \rightarrow AB$

${}_1A \rightarrow {}_2 \in R_M$

${}_1B \rightarrow {}_4 \in R_M$

$\Rightarrow <{}S, {}_1> \rightarrow <{}A, {}_2><{}B, {}_4> \in P_H$

$A \rightarrow Aa$

${}_2A \rightarrow {}_2 \in R_M$

${}_2a \rightarrow {}_3 \in R_M$

Algorithm

$\forall P \in P_G: A \rightarrow B_1 B_2 \dots B_n$

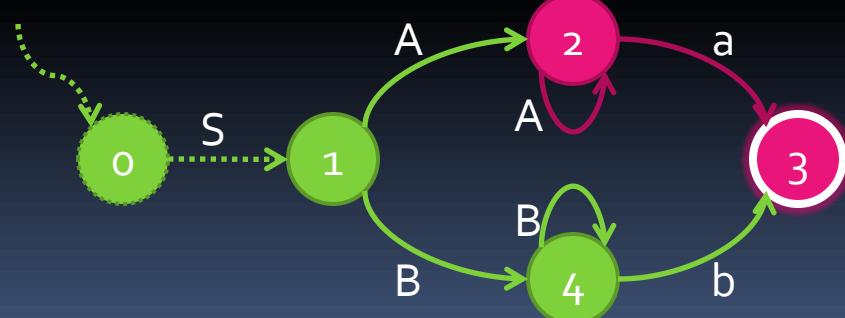
$\forall i, i=1, 2, \dots, n: q_A B_i \rightarrow q_{B_i A} \in R_M$ then

$<{}A, q_A> \rightarrow <{}B_1, q_{B_1 A}> <{}B_2, q_{B_2 A}> \dots <{}B_n, q_{B_n A}>$ to P_H

If it holds that $B_i \in T_G$ then

$q_{B_i A} \in F_M$

$<{}B_i, q_{B_i A}> \rightarrow B_i$ to P_H



Example from $(G, R)_{PC}$ to (H)

$S \rightarrow AB$

$1A \rightarrow 2 \in R_M$

$1B \rightarrow 4 \in R_M$

$\Rightarrow <S, 1> \rightarrow <A, 2><B, 4> \in P_H$

$A \rightarrow Aa$

$2A \rightarrow 2 \in R_M$

$2a \rightarrow 3 \in R_M$

$\Rightarrow <A, 2> \rightarrow <A, 2><a, 3> \in P_H$

Algorithm

$\forall P \in P_G: A \rightarrow B_1 B_2 \dots B_n$

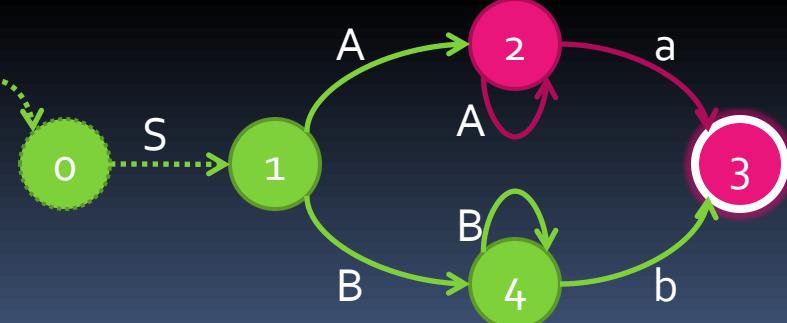
$\forall i, i=1, 2, \dots, n: q_A B_i \rightarrow q_{B_i A} \in R_M$ then

$<A, q_A> \rightarrow <B_1, q_{B_1 A}> <B_2, q_{B_2 A}> \dots <B_n, q_{B_n A}>$ to P_H

If it holds that $B_i \in T_G$ then

$q_{B_i A} \in F_M$

$<B_i, q_{B_i A}> \rightarrow B_i$ to P_H



Example from $(G, R)_{PC}$ to (H)

$S \rightarrow AB$

$1A \rightarrow 2 \in R_M$

$1B \rightarrow 4 \in R_M$

$\Rightarrow <S, 1> \rightarrow <A, 2><B, 4> \in P_H$

$A \rightarrow Aa$

$2A \rightarrow 2 \in R_M$

$2a \rightarrow 3 \in R_M$

$\Rightarrow <A, 2> \rightarrow <A, 2><a, 3> \in P_H$

$3 \in F_M \Rightarrow <a, 3> \rightarrow a \in P_H$

Algorithm

$\forall P \in P_G: A \rightarrow B_1 B_2 \dots B_n$

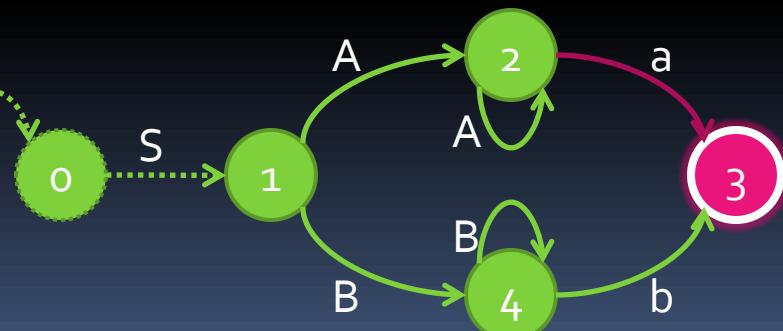
$\forall i, i=1, 2, \dots, n: q_A B_i \rightarrow q_{B_i A} \in R_M$ then

$<A, q_A> \rightarrow <B_1, q_{B_1 A}> <B_2, q_{B_2 A}> \dots <B_n, q_{B_n A}>$ to P_H

If it holds that $B_i \in T_G$ then

$q_{B_i A} \in F_M$

$<B_i, q_{B_i A}> \rightarrow B_i$ to P_H



Example from $(G, R)_{PC}$ to (H)

$S \rightarrow AB$

$1A \rightarrow 2 \in R_M$

$1B \rightarrow 4 \in R_M$

$\Rightarrow \langle S, 1 \rangle \rightarrow \langle A, 2 \rangle \langle B, 4 \rangle \in P_H$

$A \rightarrow Aa$

$2A \rightarrow 2 \in R_M$

$2a \rightarrow 3 \in R_M$

$\Rightarrow \langle A, 2 \rangle \rightarrow \langle A, 2 \rangle \langle a, 3 \rangle \in P_H$

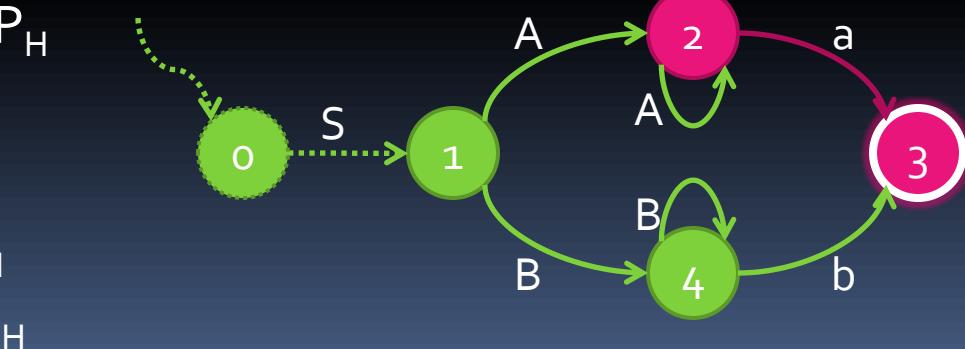
$3 \in F_M \Rightarrow \langle a, 3 \rangle \rightarrow a \in P_H$

$A \rightarrow a$

$2a \rightarrow 3 \in R_M$

$\Rightarrow \langle A, 2 \rangle \rightarrow \langle a, 3 \rangle \in P_H$

$3 \in F_M \Rightarrow \langle a, 3 \rangle \rightarrow a \in P_H$



Example from $(G, R)_{PC}$ to (H)

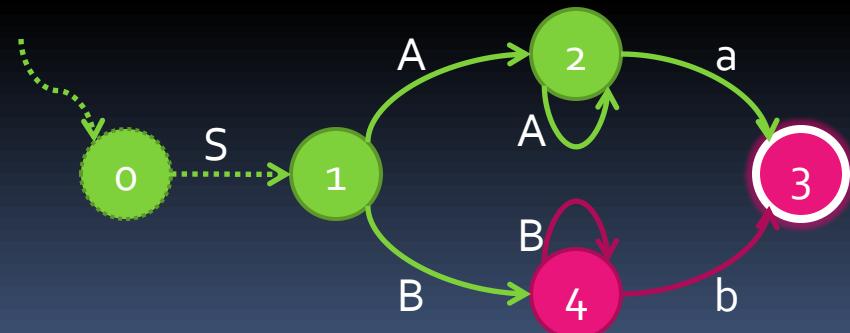
$B \rightarrow Bb$

$$4B \rightarrow 4 \in R_M$$

$$4b \rightarrow 3 \in R_M$$

$$\Rightarrow \langle B, 4 \rangle \rightarrow \langle B, 4 \rangle \langle b, 3 \rangle \in P_H$$

$$3 \in F_M \Rightarrow \langle b, 3 \rangle \rightarrow b \in P_H$$



Example from $(G, R)_{PC}$ to (H)

$B \rightarrow Bb$

$${}_4B \rightarrow {}_4 \in R_M$$

$${}_4b \rightarrow {}_3 \in R_M$$

$$\Rightarrow \langle B, {}_4 \rangle \rightarrow \langle B, {}_4 \rangle \langle b, {}_3 \rangle \in P_H$$

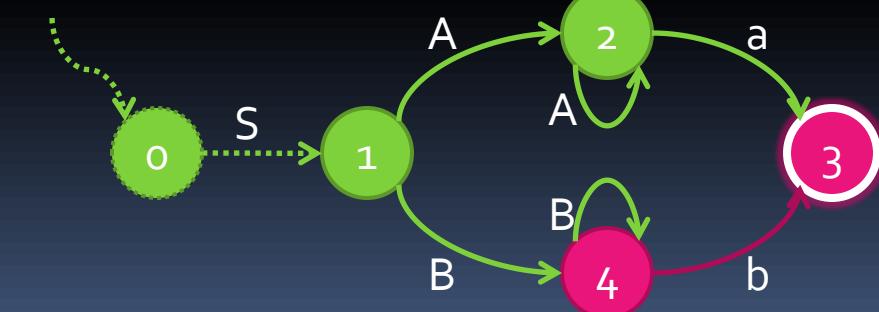
$${}_3 \in F_M \Rightarrow \langle b, {}_3 \rangle \rightarrow b \in P_H$$

$B \rightarrow b$

$${}_4b \rightarrow {}_3 \in R_M$$

$$\Rightarrow \langle B, {}_4 \rangle \rightarrow \langle b, {}_3 \rangle \in P_H$$

$${}_3 \in F_M \Rightarrow \langle b, {}_3 \rangle \rightarrow b \in P_H$$



Example from $(G, R)_{PC}$ to (H)

$B \rightarrow Bb$

$4B \rightarrow 4 \in R_M$

$4b \rightarrow 3 \in R_M$

$\Rightarrow \langle B, 4 \rangle \rightarrow \langle B, 4 \rangle \langle b, 3 \rangle \in P_H$

$3 \in F_M \Rightarrow \langle b, 3 \rangle \rightarrow b \in P_H$

$B \rightarrow b$

$4b \rightarrow 3 \in R_M$

$\Rightarrow \langle B, 4 \rangle \rightarrow \langle b, 3 \rangle \in P_H$

$3 \in F_M \Rightarrow \langle b, 3 \rangle \rightarrow b \in P_H$

$B \rightarrow BC, C \rightarrow Cc, C \rightarrow c$

no rules in FA M \Rightarrow no new rules to P_H

Algorithm

$\forall P \in P_G: A \rightarrow B_1 B_2 \dots B_n$

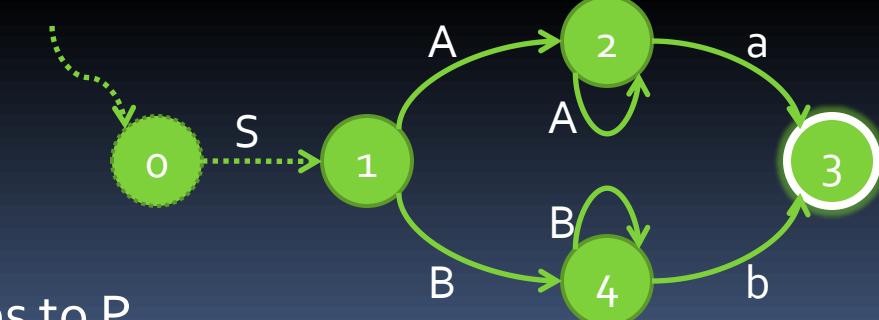
$\forall i, i=1, 2, \dots, n: q_A B_i \rightarrow q_{B_i A} \in R_M$ then

$\langle A, q_A \rangle \rightarrow \langle B_1, q_{B_1 A} \rangle \langle B_2, q_{B_2 A} \rangle \dots \langle B_n, q_{B_n A} \rangle$ to P_H

If it holds that $B_i \in T_G$ then

$q_{B_i A} \in F_M$

$\langle B_i, q_{B_i A} \rangle \rightarrow B_i$ to P_H



Summary from $(G, R)_{PC}$ to (H)

$(G, R)_{PC}$, $G = (N, T, P, S)$ where

- $N = \{S, A, B, C\}$
- $T = \{a, b, c\}$
- P
 - $S \rightarrow AB,$
 - $A \rightarrow Aa \mid a$
 - $B \rightarrow Bb \mid b \mid BC$
 - $C \rightarrow Cc \mid c$
- $R = \{SA^+a, SB^+b\}$

$H = (N, T, P, S)$, where

- $N = \{<S,1>, <A,2>, <B,4>, <a,3>, <b,3>\}$
- $T = \{a, b\}$
- P
 - $<S,1> \rightarrow <A,2><B,4>$
 - $<A,2> \rightarrow <A,2><a,3> \mid <a,3> \quad <a,3> \rightarrow a$
 - $<B,4> \rightarrow <B,4><b,3> \mid <b,3> \quad <b,3> \rightarrow b$
- $S = <S,1>$

Summary from $(G, R)_{PC}$ to (H)

$(G, R)_{PC}$, $G = (N, T, P, S)$ where

- $N = \{S, A, B, C\}$
- $T = \{a, b, c\}$
- P
 - $S \rightarrow AB,$
 - $A \rightarrow Aa \mid a$
 - $B \rightarrow Bb \mid b \mid BC$
 - $C \rightarrow Cc \mid c$
- $R = \{SA^+a, SB^+b\}$

Possible derivation in $(G, R)_{PC}$

- $S \Rightarrow AB \Rightarrow AaB \Rightarrow aaB$
 $\Rightarrow aaBb \Rightarrow aaBbb$
 $\Rightarrow aabbb$

$H = (N, T, P, S)$, where

- $N = \{<S, 1>, <A, 2>, <B, 4>, <a, 3>, <b, 3>\}$
- $T = \{a, b\}$
- P
 - $<S, 1> \rightarrow <A, 2><B, 4>$
 - $<A, 2> \rightarrow <A, 2><a, 3> \mid <a, 3>$ $<a, 3> \rightarrow a$
 - $<B, 4> \rightarrow <B, 4><b, 3> \mid <b, 3>$ $<b, 3> \rightarrow b$
- $S = <S, 1>$

Summary from $(G, R)_{PC}$ to H

$(G, R)_{PC}$, $G = (N, T, P, S)$ where

- $N = \{S, A, B, C\}$
- $T = \{a, b, c\}$
- P
 - $S \rightarrow AB,$
 - $A \rightarrow Aa \mid a$
 - $B \rightarrow Bb \mid b \mid BC$
 - $C \rightarrow Cc \mid c$
- $R = \{SA^+a, SB^+b\}$

Possible derivation in $(G, R)_{PC}$

- $S \Rightarrow AB \Rightarrow AaB \Rightarrow aaB$
 $\Rightarrow aaBb \Rightarrow aaBbb$
 $\Rightarrow aabbb$

$H = (N, T, P, S)$, where

- $N = \{<S,1>, <A,2>, <B,4>, <a,3>, <b,3>\}$
- $T = \{a, b\}$
- P
 - $<S,1> \rightarrow <A,2><B,4>$
 - $<A,2> \rightarrow <A,2><a,3> \mid <a,3>$ $<a,3> \rightarrow a$
 - $<B,4> \rightarrow <B,4><b,3> \mid <b,3>$ $<b,3> \rightarrow b$
- $S = <S,1>$

Possible derivation in H

- $<S,1> \Rightarrow <A,2><B,4> \Rightarrow <A,2><a,3><B,4>$
 $\Rightarrow <A,2>a<B,4> \Rightarrow <a,3>a<B,4>$
 $\Rightarrow aa<B,4> \Rightarrow aa<B,4><b,3>$
 $\Rightarrow aa<B,4>b \Rightarrow aa<B,4><b,3>b$
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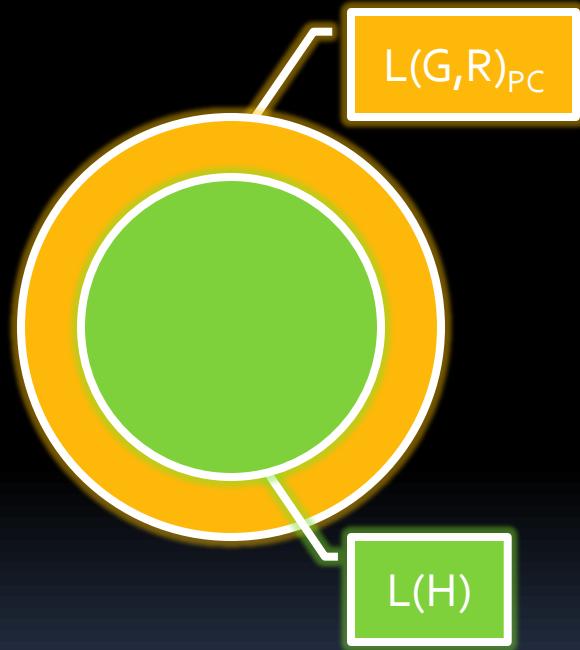
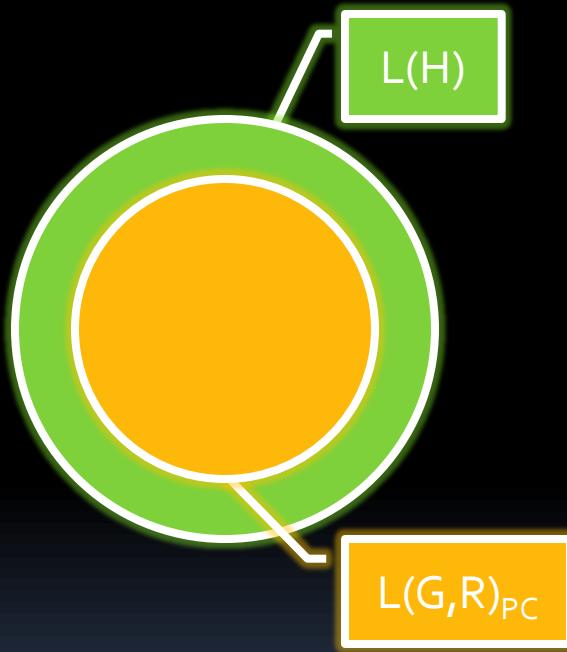
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Proof that $L(G, R)_{PC} = L(H)$

$L(G, R)_{PC} \subseteq L(H)$

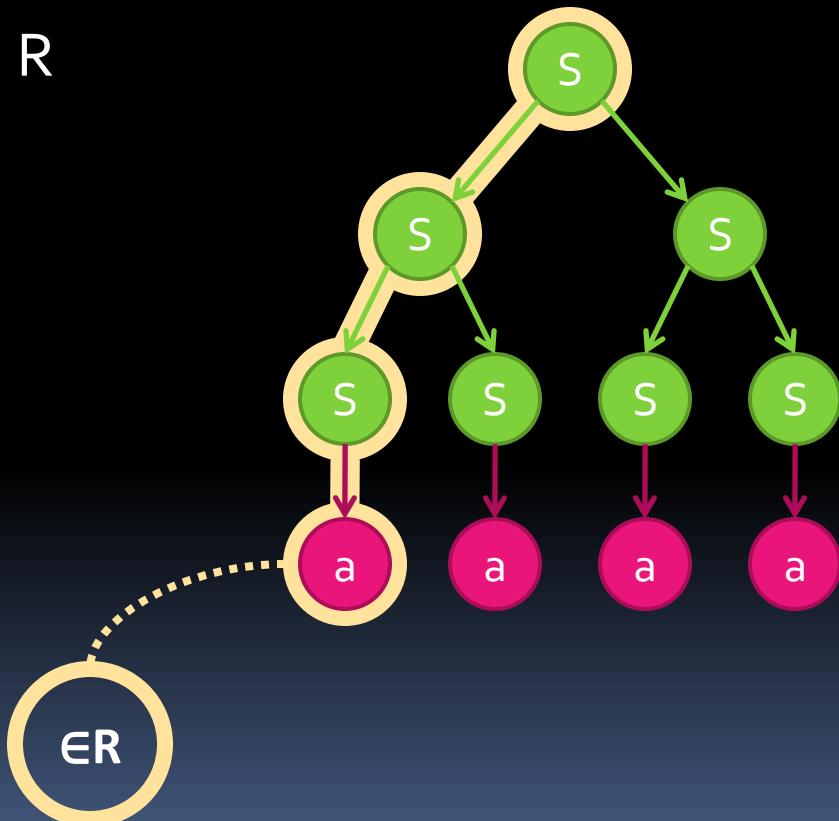
$L(H) \subseteq L(G, R)_{PC}$



...by prof. Meduna

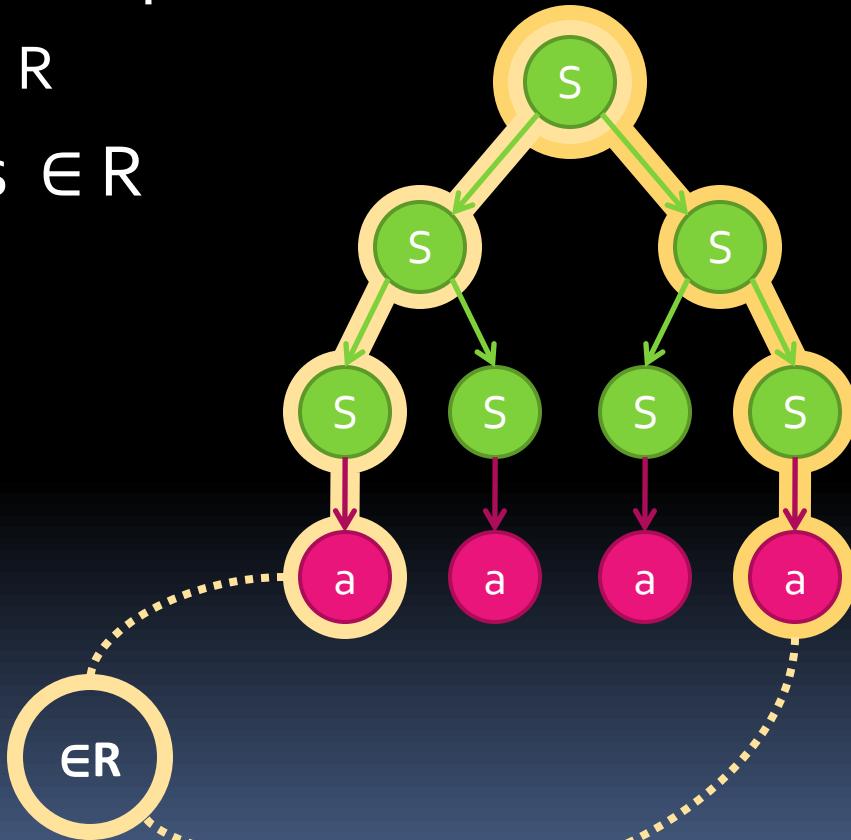
Less restrictive conditions

- What happens if we require that
 - At least **one** path $\in R$



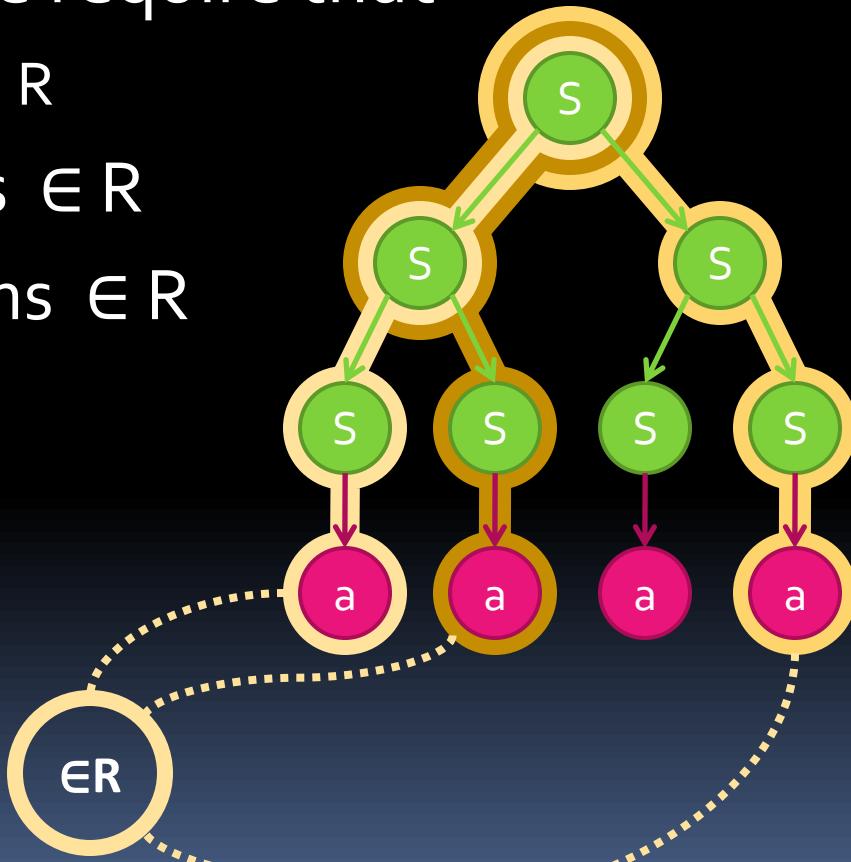
Less restrictive conditions

- What happens if we require that
 - At least **one** path $\in R$
 - At least **two** paths $\in R$



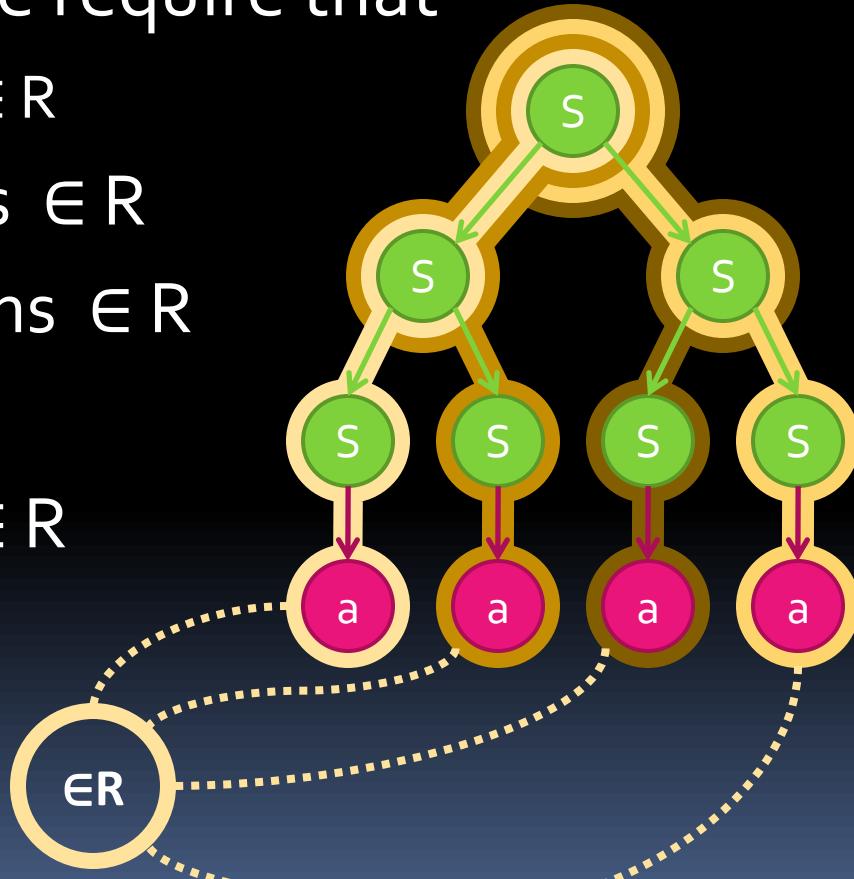
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- What happens if we require that
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 - At least **three** paths $\in R$



Less restrictive conditions

- What happens if we require that
 - At least **one** path $\in R$
 - At least **two** paths $\in R$
 - At least **three** paths $\in R$
 - ...
 - At least **n** paths $\in R$



Less restrictive conditions

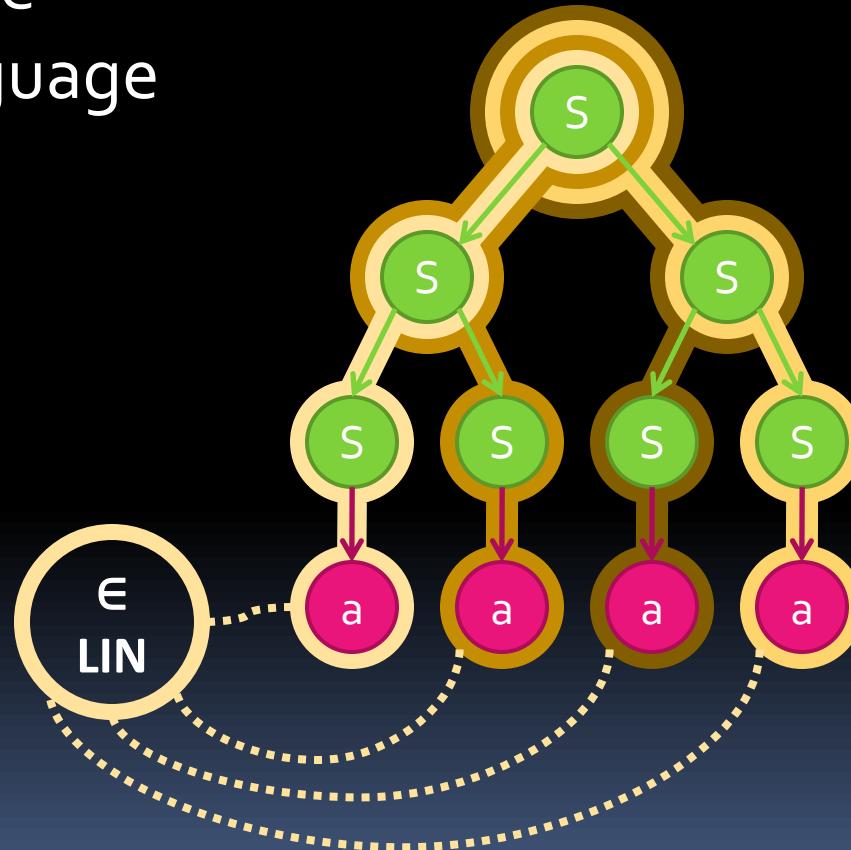
It can be shown, that if G is a Context Free Grammar, then every path in it's derivation tree is Regular

Proof

- Let $G = (N, T, P, S)$ be a CFG.
- We can construct regular grammar
 $G' = (\{S'\} \cup \{[A] \mid A \in N\}, N \cup T, S', P')$
 $P' = \{S' \rightarrow S[S]\} \cup$
 $\{[A] \rightarrow B[B] \mid A \rightarrow uBv \in P, u, v \in (N \cup T)^*, A, B \in N\} \cup$
 $\{[A] \rightarrow a \mid A \rightarrow uav \in P, u, v \in (N \cup T)^*, a \in T\}$
- $L(G')$ contains every possible path in G
- Hence every path in G is regular

Controlling Paths by Linear

What happens if we
allow Control Language
to be Linear?



What do we get?

By **regular** controlling the path(s)
of derivation tree we
don't increase
generative power of CFG.

By **linear** controlling the path(s)
of derivation tree we
significantly increase
generative power of CFG.

Example I: $\{a^n b^n c^n d^n \mid n \geq 1\}$

$G = (\{S, B, D\}, \{a, b, c, d\}, P, S)$

$P = \{ \quad S \rightarrow aSd \mid aBd,$
 $B \rightarrow bBc \mid D$
 $D \rightarrow bc \}$

$R_L = \{ \quad S^+a \cup S^+d \cup$
 $S^+B^+b \cup S^+B^+c \cup$
 $S^nB^nD^+b \cup S^nB^nD^+c \mid n \geq 1 \}$

$L = \{a^n b^n c^n d^n \mid n \geq 1\}$

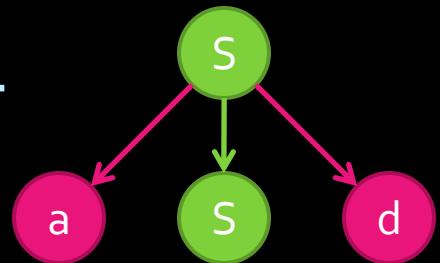
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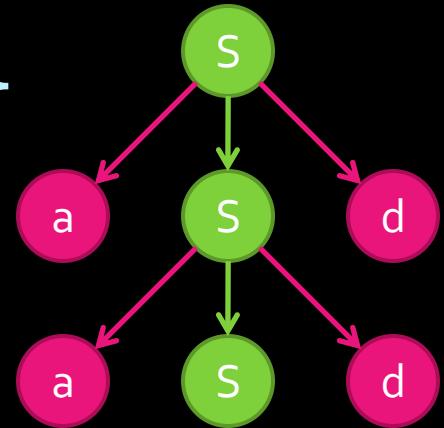
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$$L = \{a^n b^n c^n d^n \mid n \geq 1\}$$



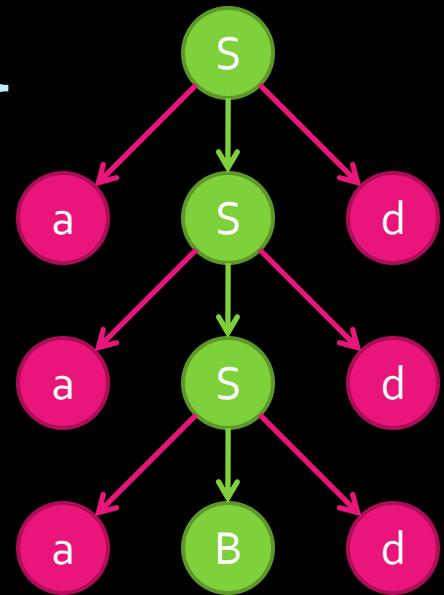
Example I: $\{a^n b^n c^n d^n \mid n \geq 1\}$

$$G = (\{S, B, D\}, \{a, b, c, d\}, P, S)$$

$$\begin{aligned} P = \{ & \quad S \rightarrow aSd \mid aBd, \\ & \quad B \rightarrow bBc \mid D \\ & \quad D \rightarrow bc \} \end{aligned}$$

$$\begin{aligned} R_L = \{ & \quad S^+ a \cup S^+ d \cup \\ & \quad S^+ B^+ b \cup S^+ B^+ c \cup \\ & \quad S^n B^n D b \cup S^n B^n D c \mid n \geq 1 \} \end{aligned}$$

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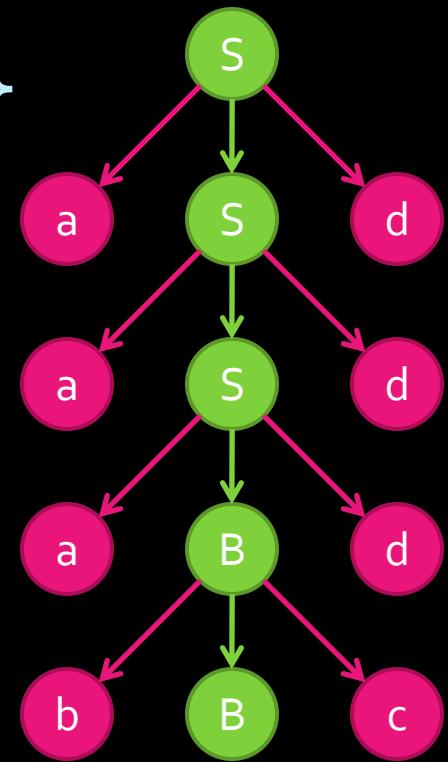
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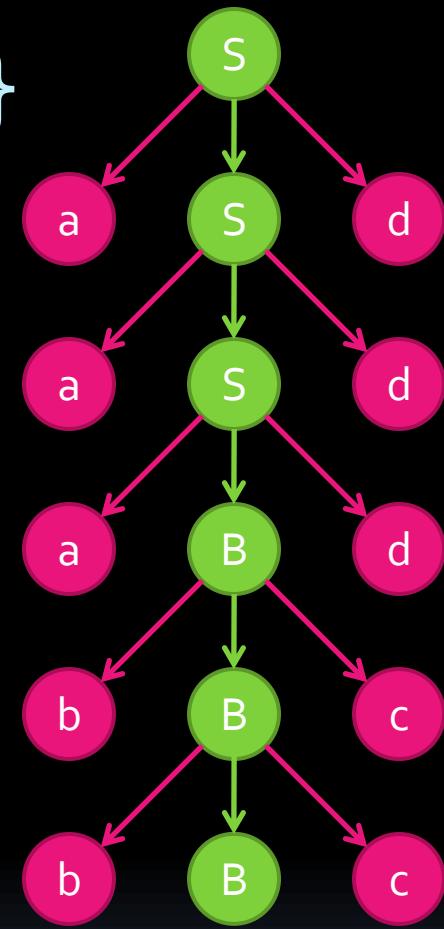
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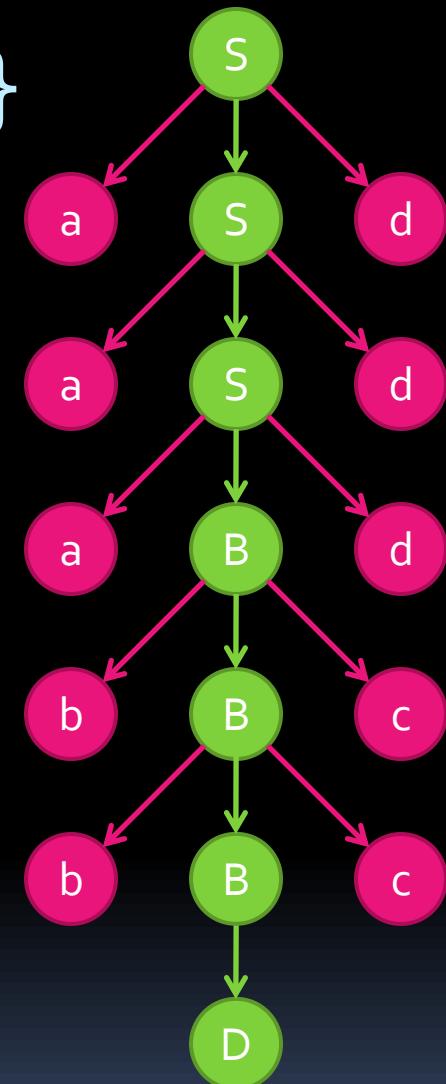
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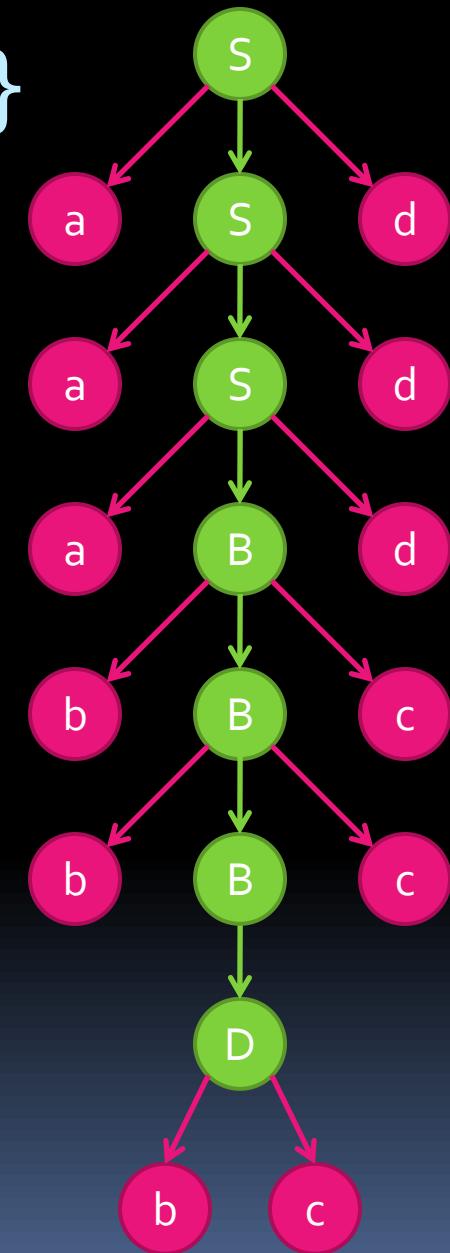
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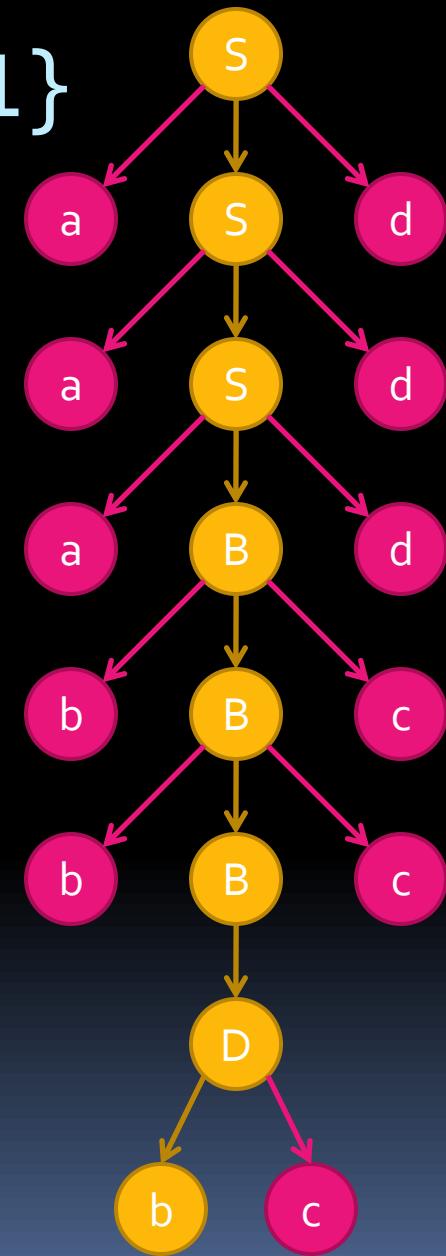
$$G = (\{S, B, D\}, \{a, b, c, d\}, P, S)$$

$$\begin{aligned} P = \{ & \quad S \rightarrow aSd \mid aBd, \\ & \quad B \rightarrow bBc \mid D \\ & \quad D \rightarrow bc \} \end{aligned}$$

$$R_L = \{S^n B^n D b \mid n \geq 1\}$$

$$L = \{a^n b^n c^n d^n \mid n \geq 1\}$$

Note: In this case it's sufficient to control only one path.



Example III: $\{a^{2^n} \mid n \geq 1\}$

$G = (\{S, A\}, \{a\}, P, S)$

$P = \{ \quad S \rightarrow S \mid A,$
 $A \rightarrow AA \mid a \}$

$R_L = \{S^n A^n a \mid n \geq 1\}$

$L = \{a^{2^n} \mid n \geq 1\}$



Example III: $\{a^{2^n} \mid n \geq 1\}$

$G = (\{S, A\}, \{a\}, P, S)$

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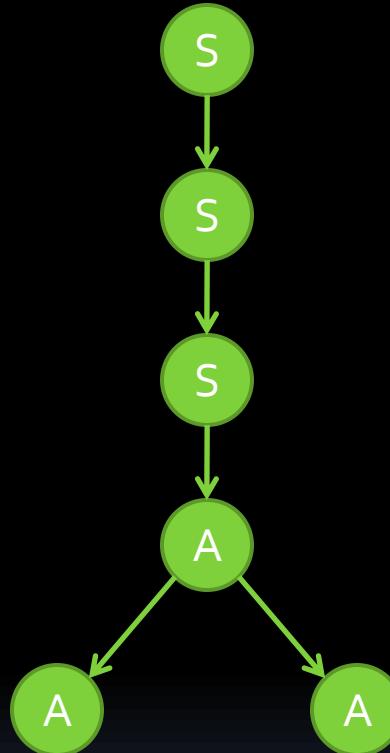
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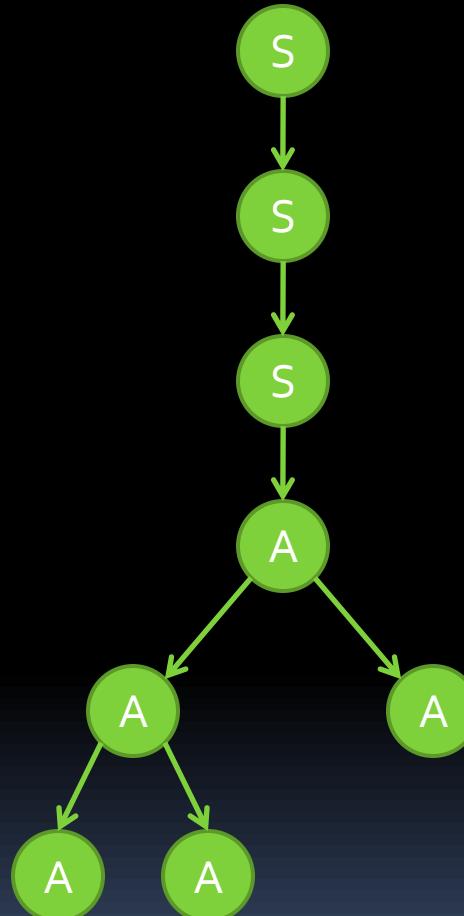
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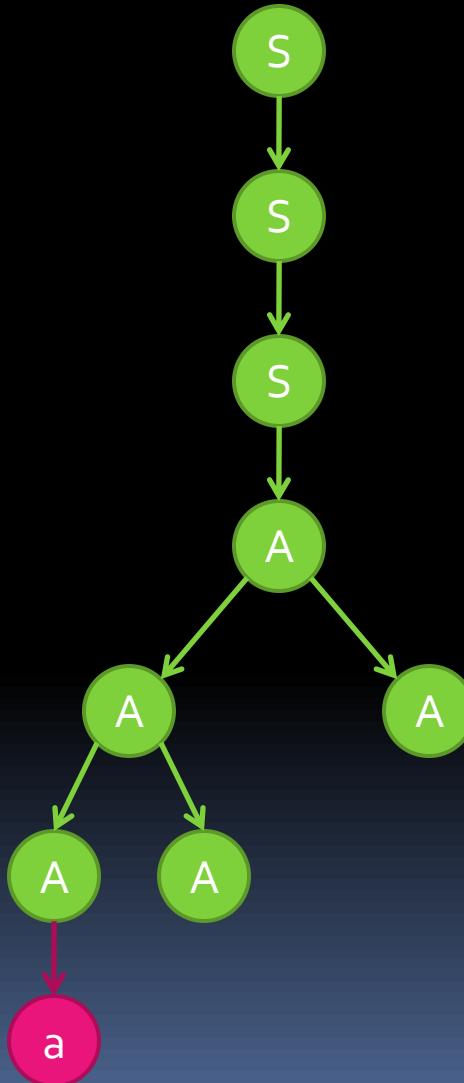
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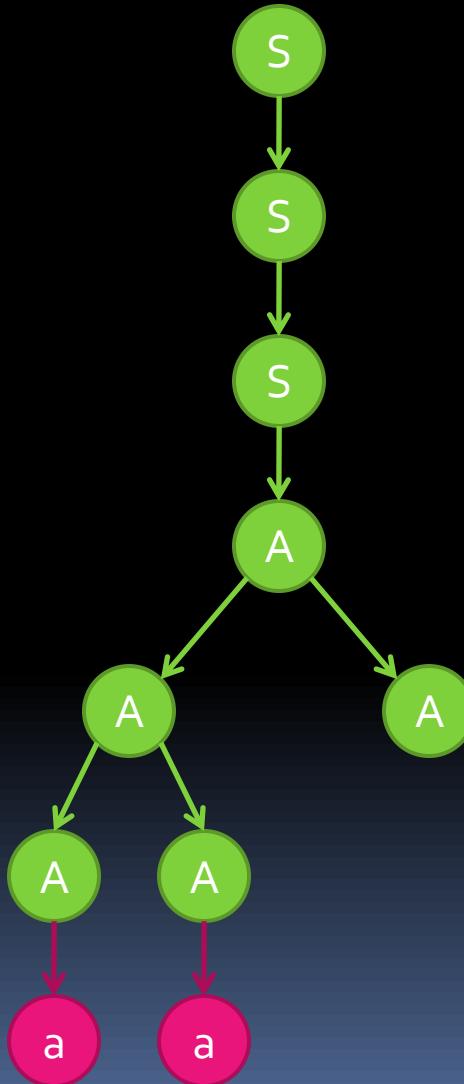
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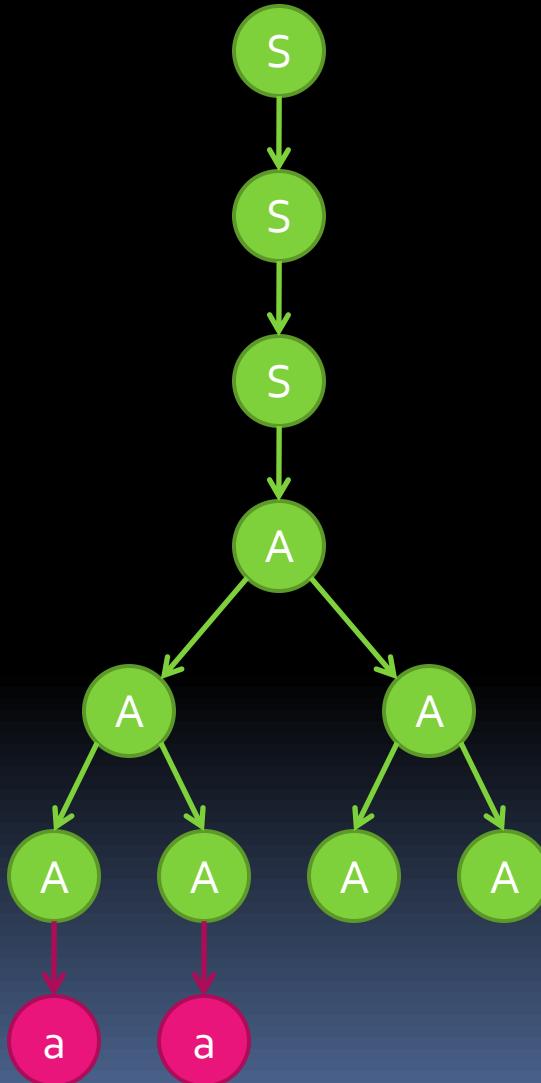
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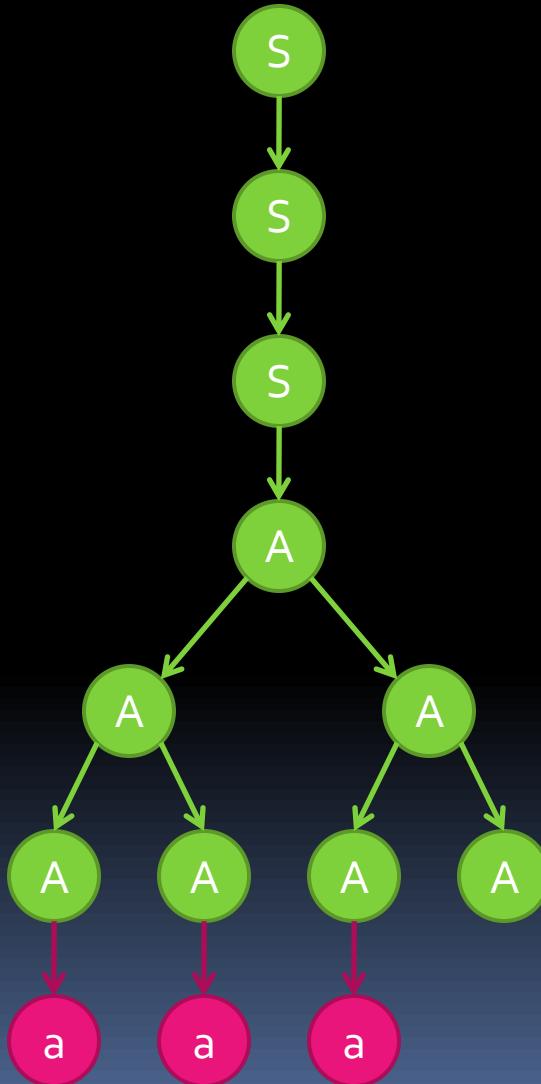
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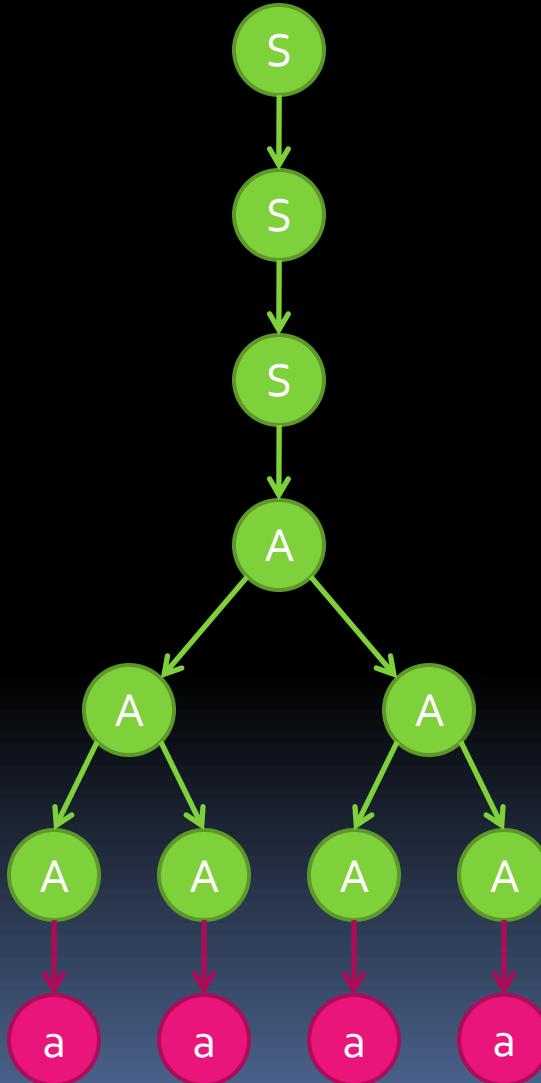
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Example III: ~~$\{a^{2^n} \mid n \geq 1\}$~~

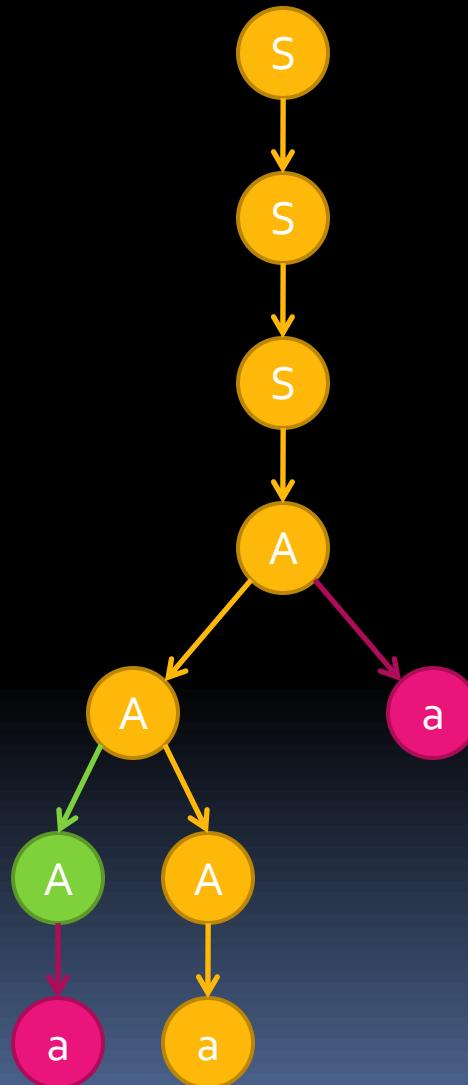
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$$R_L = \{S^n A^n a \mid n \geq 1\}$$

$$L \neq \{a^{2^n} \mid n \geq 1\}$$

Note: In this case it's NOT sufficient to control only one path.



Example IV: $\{a^n b^n c^n d^n e^n f^n \mid n \geq 1\}$

$G = (\{S, X, U, Y, V\}, \{a, b, c, d\}, P, S)$

$P = \{ \begin{array}{l} S \rightarrow aSf \mid aXYf, \\ X \rightarrow bXc \mid U \\ U \rightarrow bc \\ Y \rightarrow dYe \mid V \\ V \rightarrow de \end{array} \}$

$R_L = \{S^n X^n U^b \cup S^n Y^n V^d \mid n \geq 1\}$



$L = \{a^n b^n c^n d^n e^n f^n \mid n \geq 1\}$

Note: In this case it's sufficient
to control only two paths.

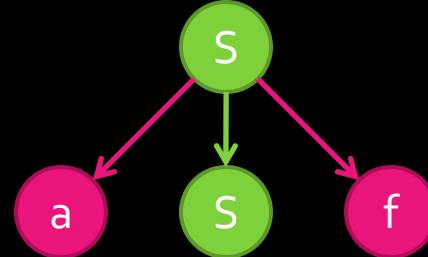
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$$G = (\{S, X, U, Y, V\}, \{a, b, c, d\}, P, S)$$

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$$R_L = \{S^n X^n U^b \cup S^n Y^n V^d \mid n \geq 1\}$$

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Note: In this case it's sufficient to control only two paths.

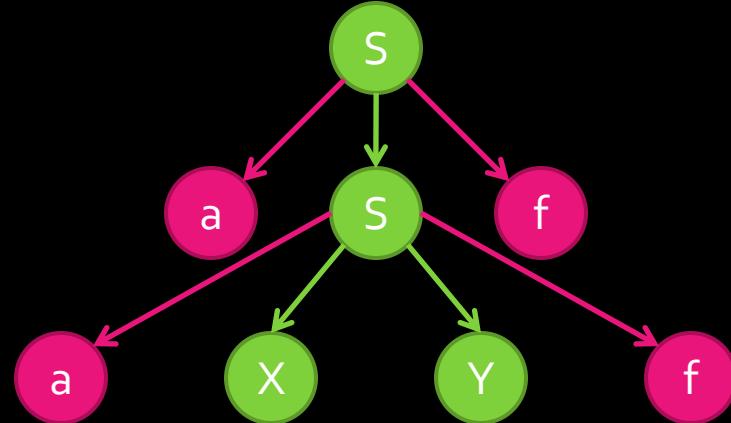
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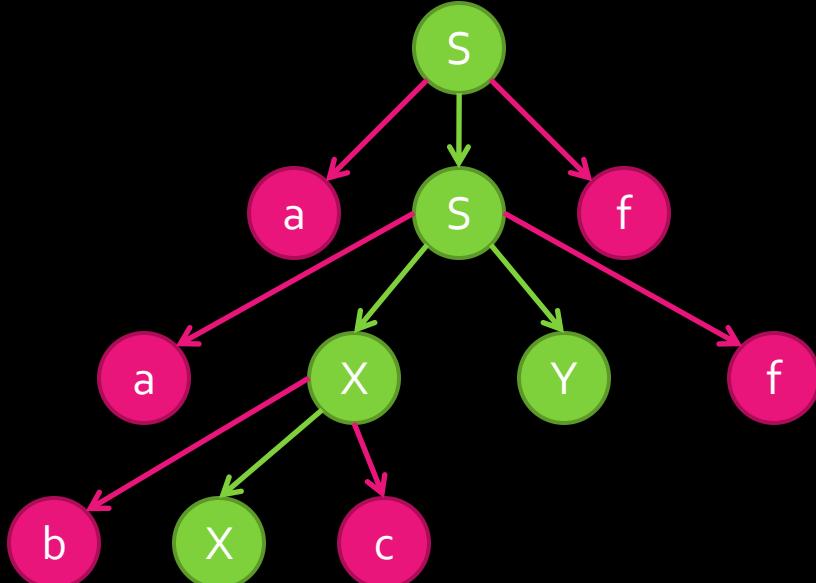
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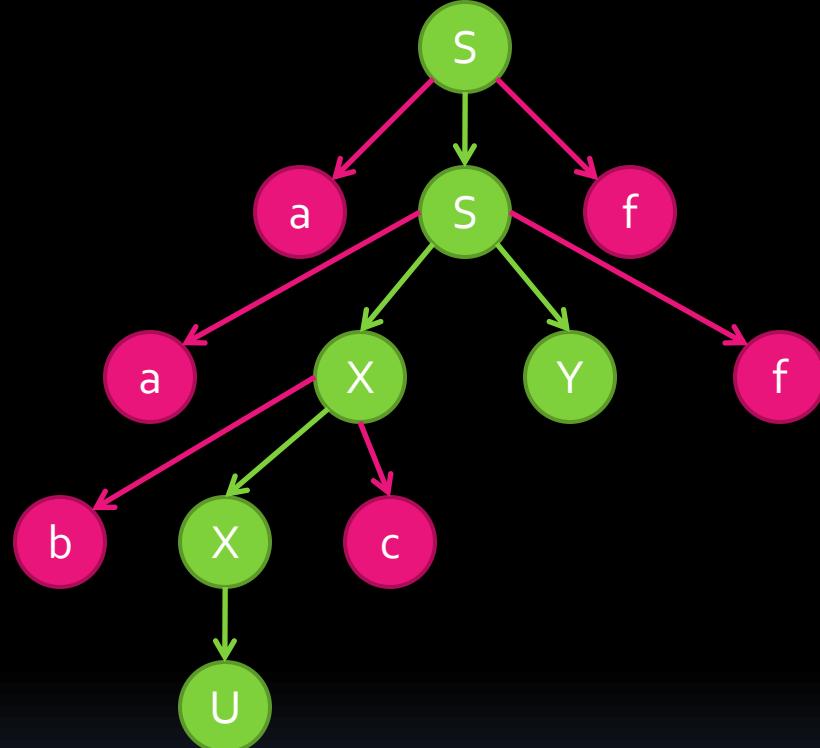
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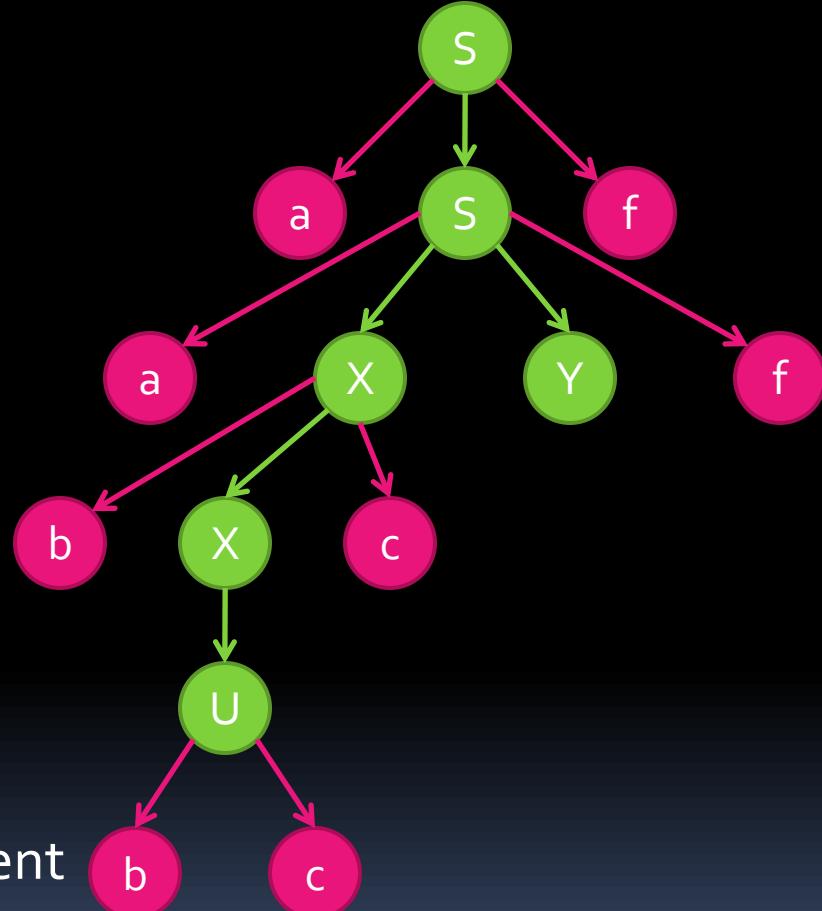
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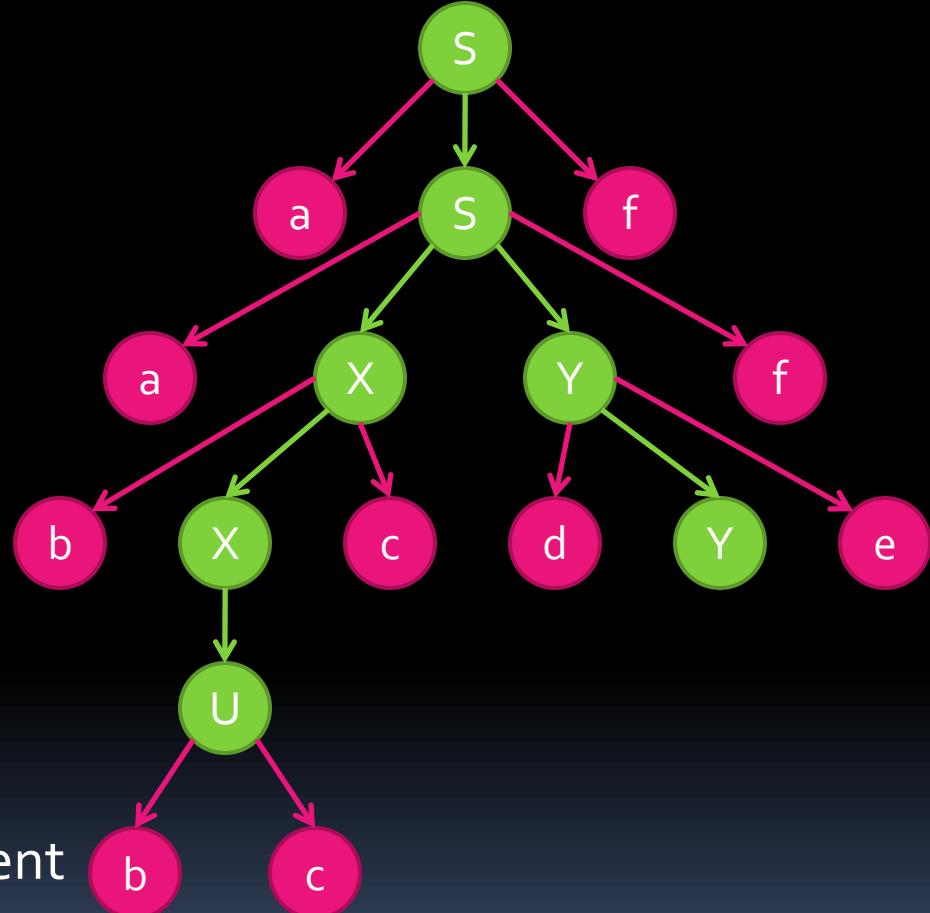
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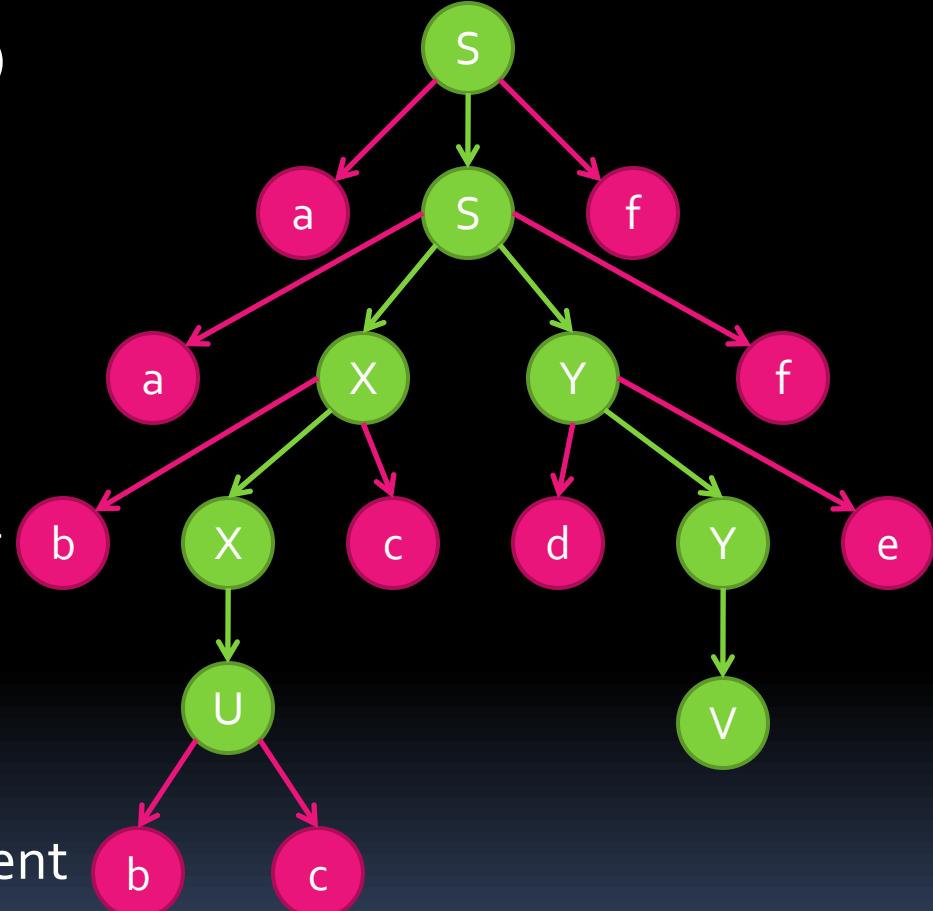
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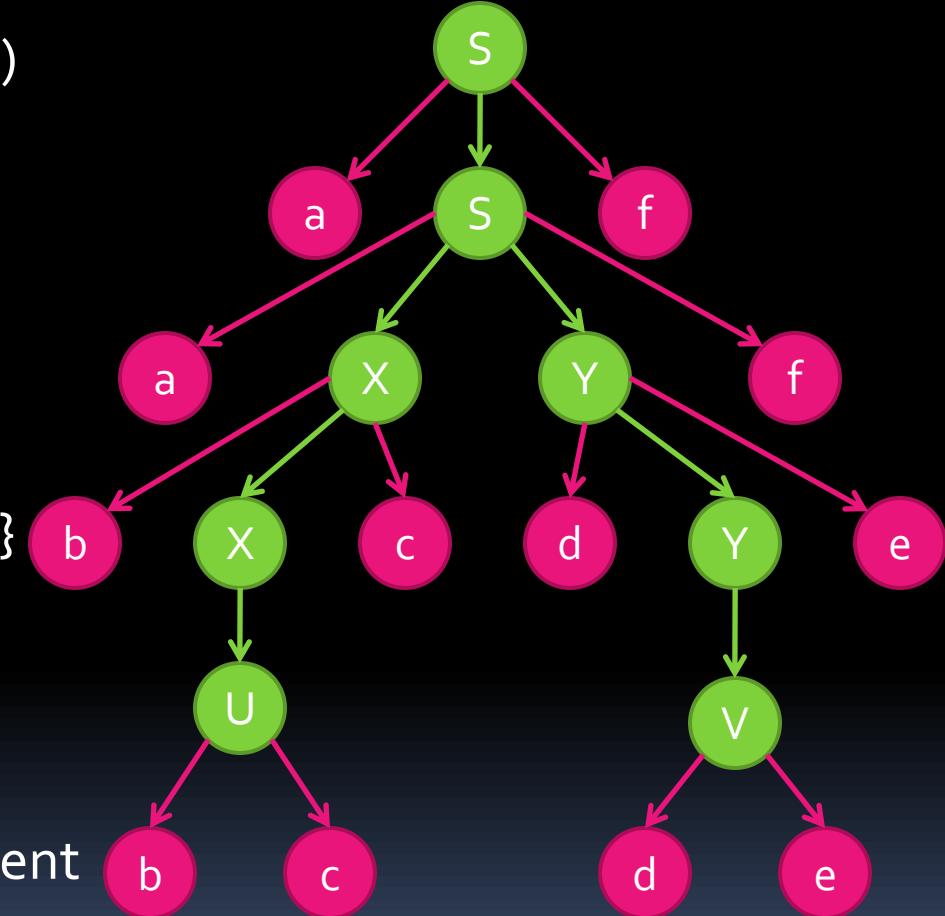
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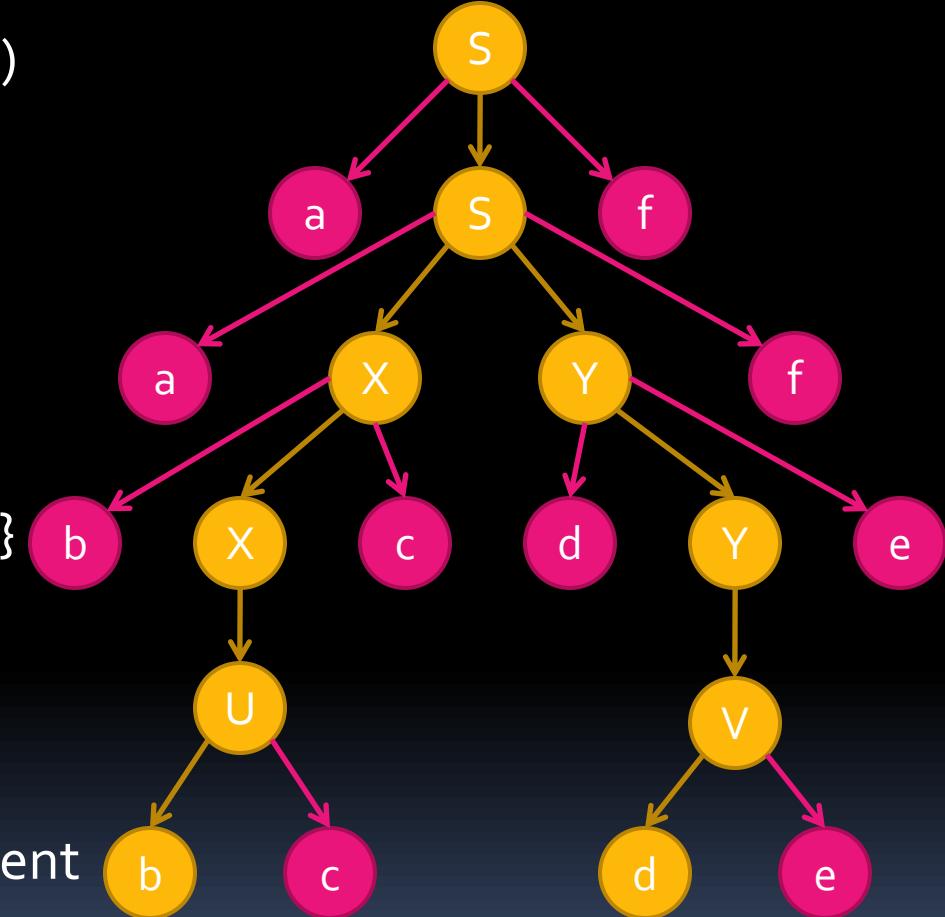
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Note: In this case it's sufficient to control only two paths.



Controlling 1...n Paths

By controlling

- a) at least one
- b) at least two
- c) at least n

path(s) in derivation tree of x , $x \in L(G)$, we can count

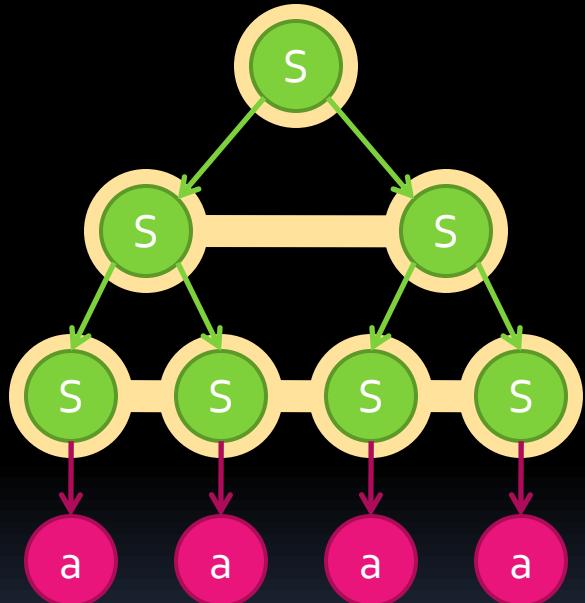
- a) up to four (for proof see [2])
- b) up to six
- c) up to $(2n+2)$ (theorem, not proved yet)

which implies (theorem, not proved yet)

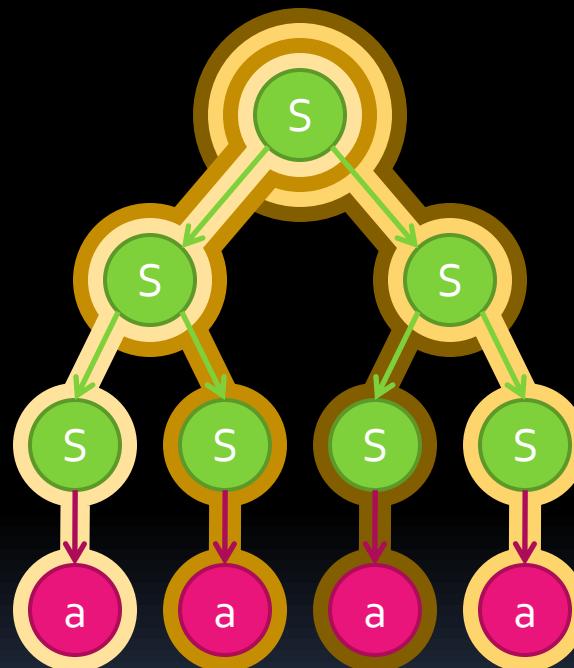
- All languages generated by $PCTC_n$ can be generated by $PCTC_{n+1}$
- There exists languages, which can be generated by $PCTC_{n+1}$ and can't be generated by $PCTC_n$

Concatenating Levels (Paths)

Concatenating Levels

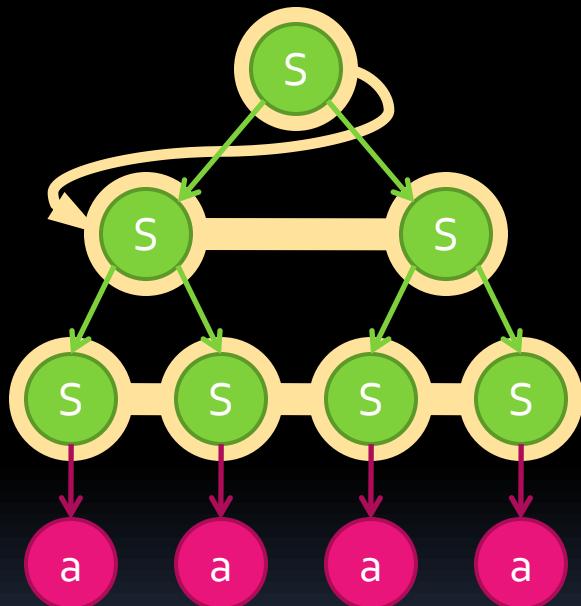


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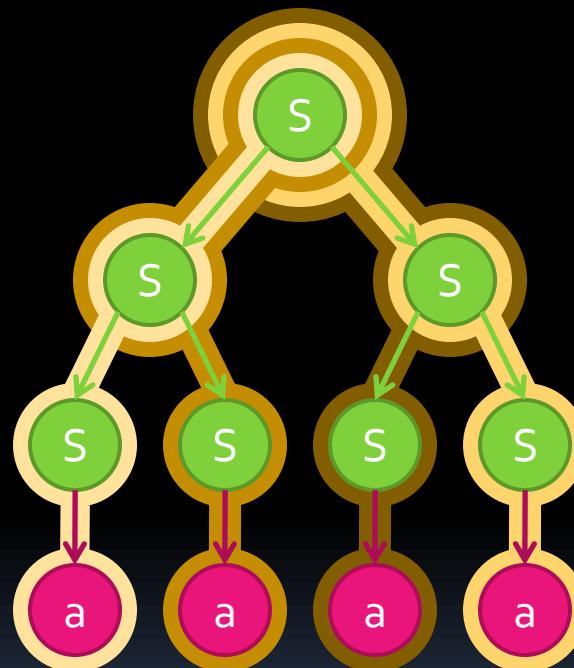


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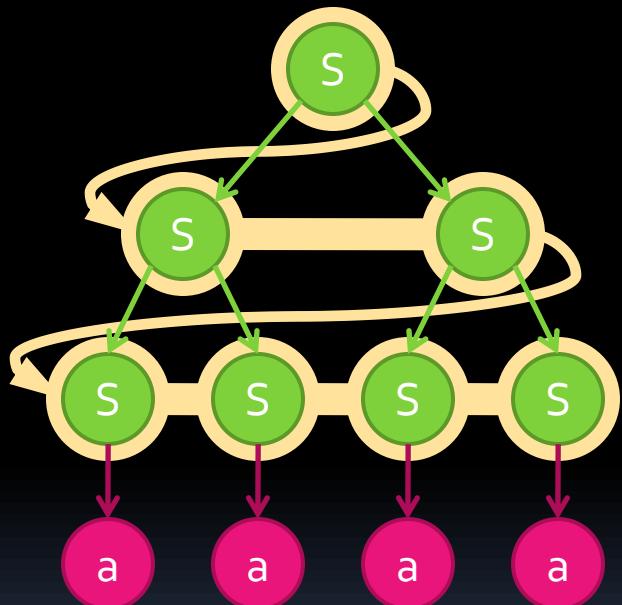
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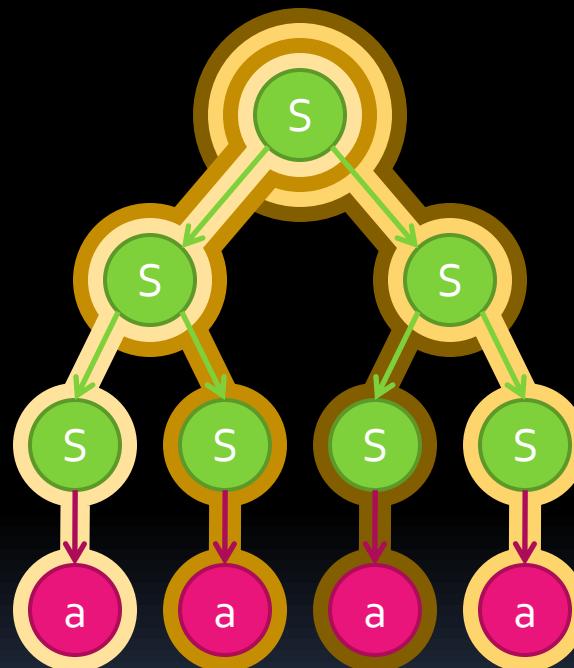
εR

Concatenating Levels (Paths)

Concatenating Levels



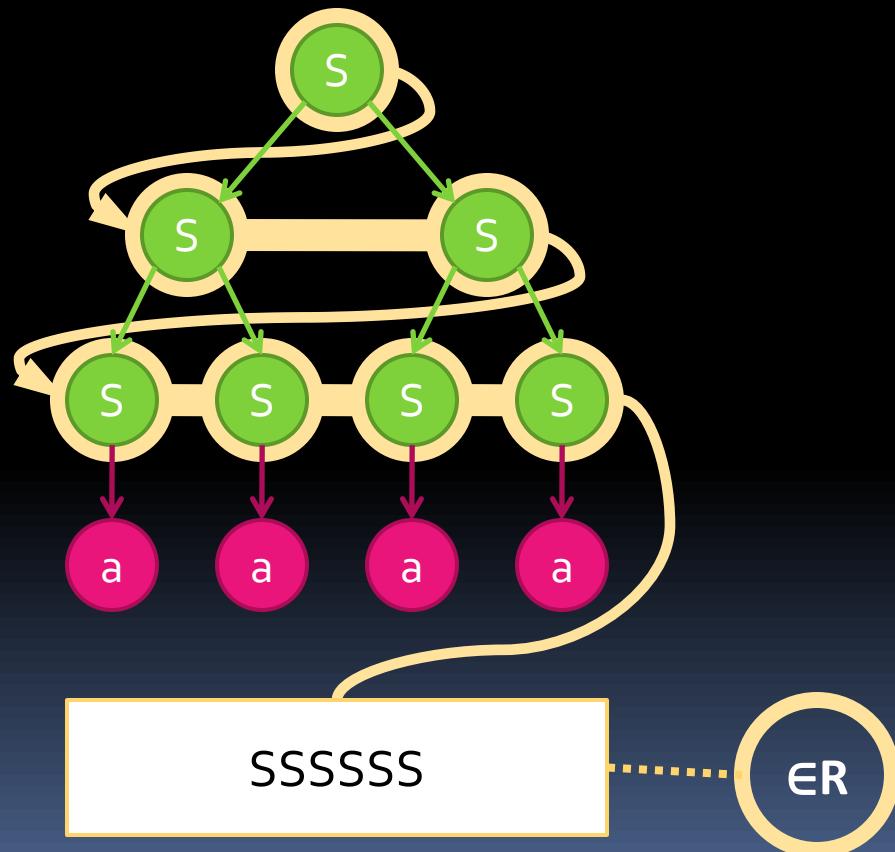
Concatenating Paths



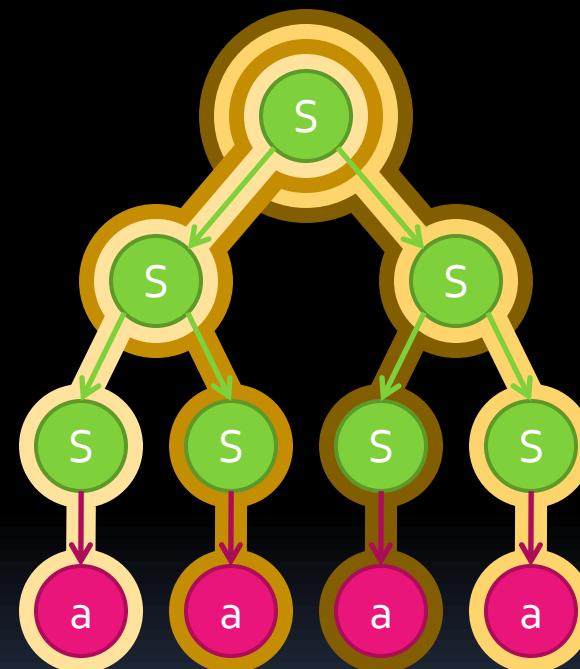
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Concatenating Levels

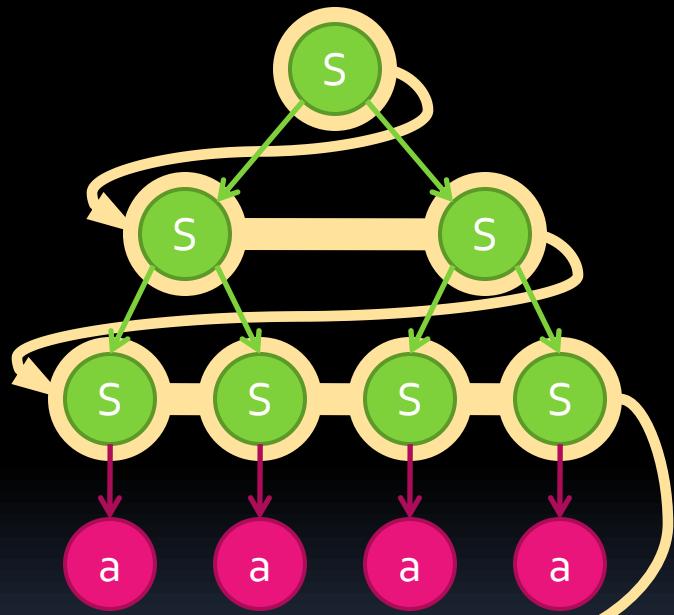


Concatenating Paths



Concatenating Levels (Paths)

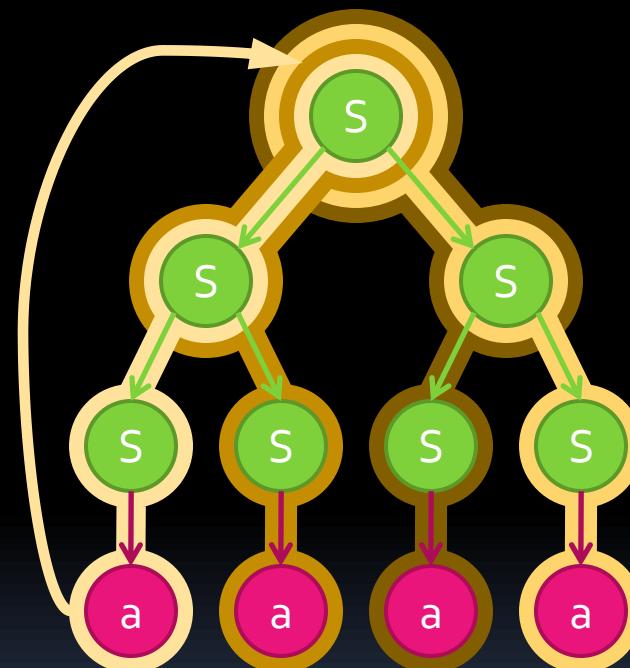
Concatenating Levels



SSSSSS

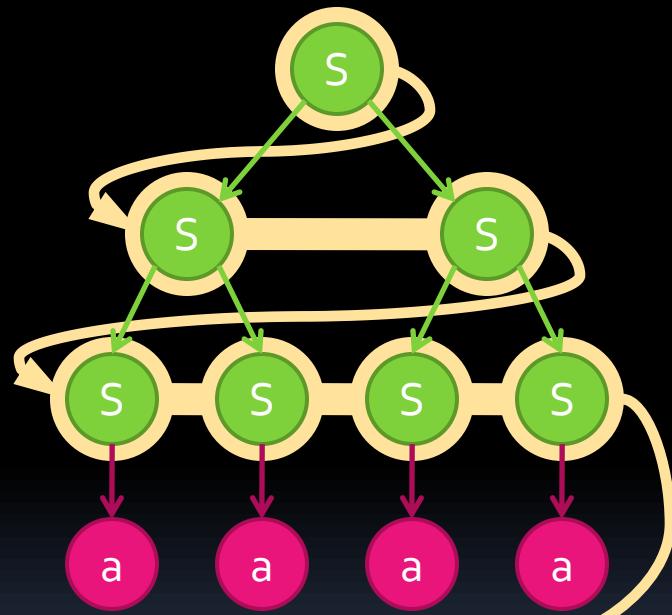
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Concatenating Paths



Concatenating Levels (Paths)

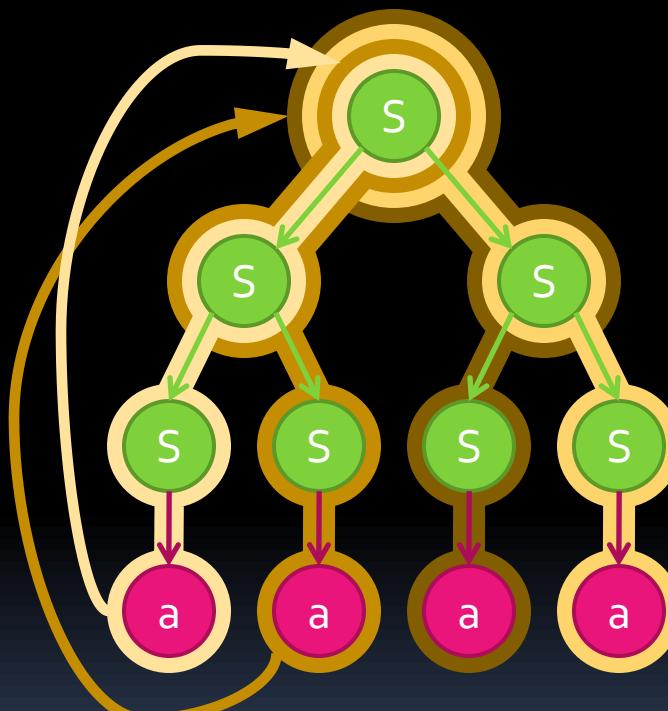
Concatenating Levels



SSSSSS

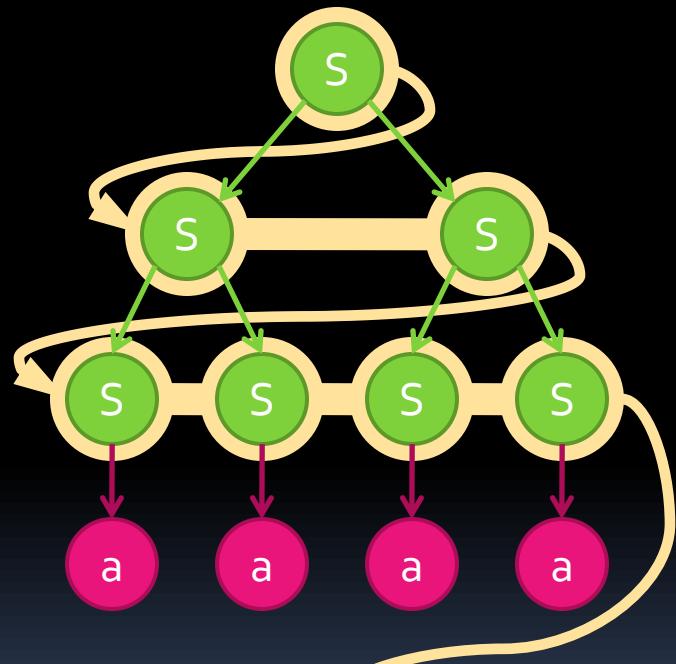
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Concatenating Paths



Concatenating Levels (Paths)

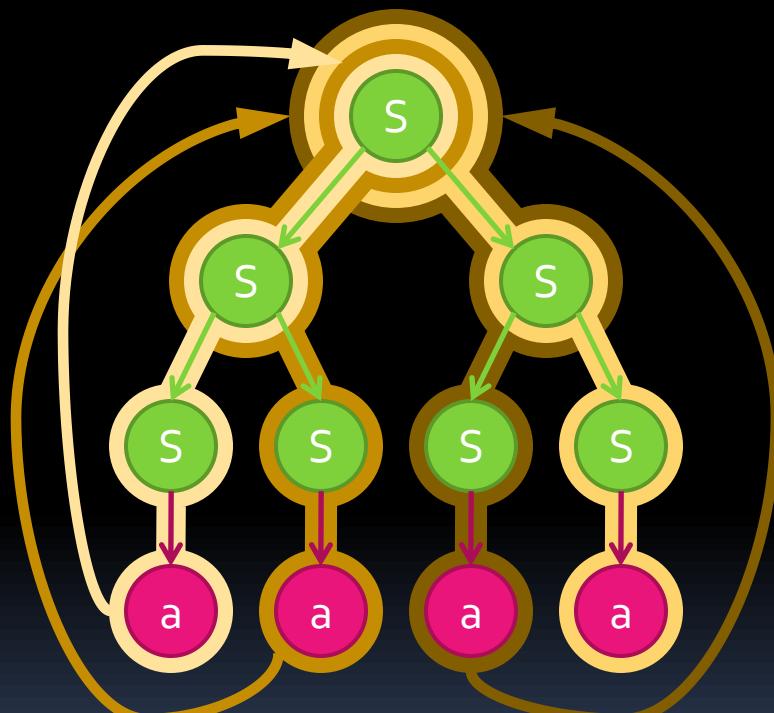
Concatenating Levels



SSSSSS

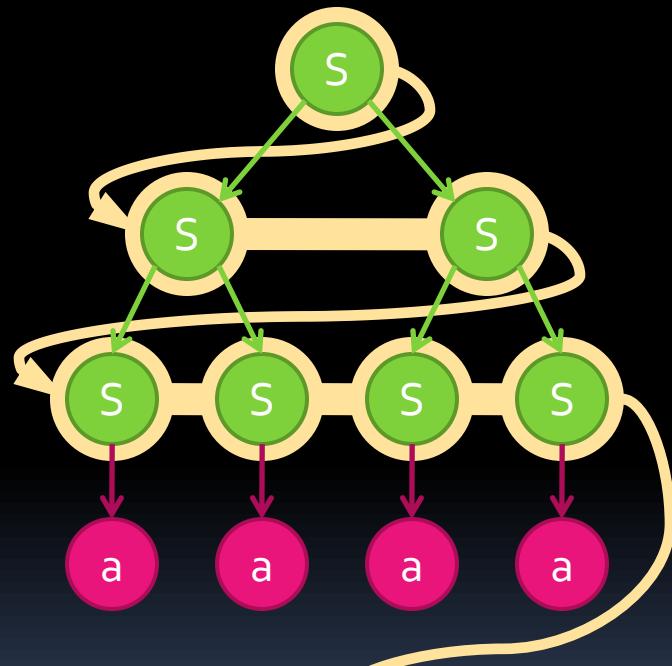


Concatenating Paths



Concatenating Levels (Paths)

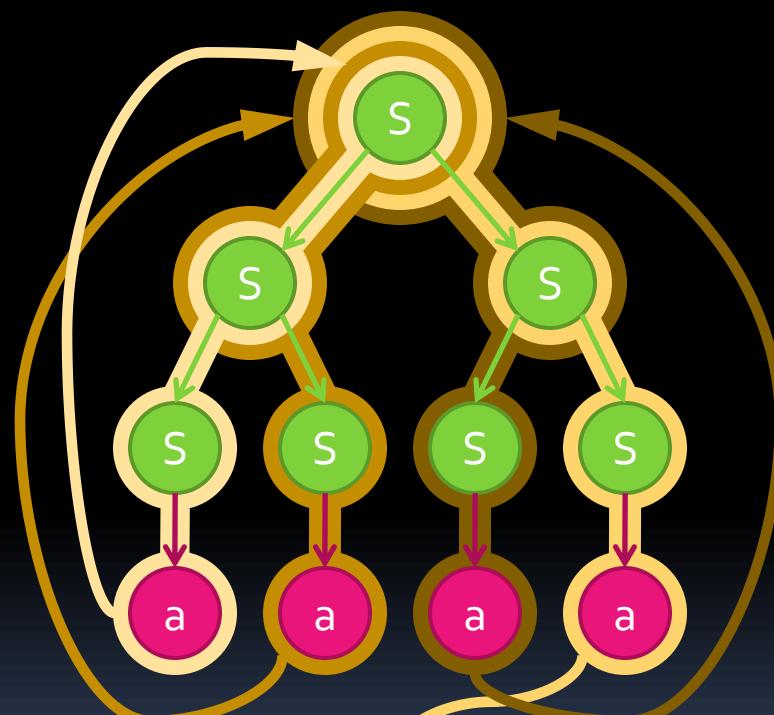
Concatenating Levels



SSSSSS

ϵR

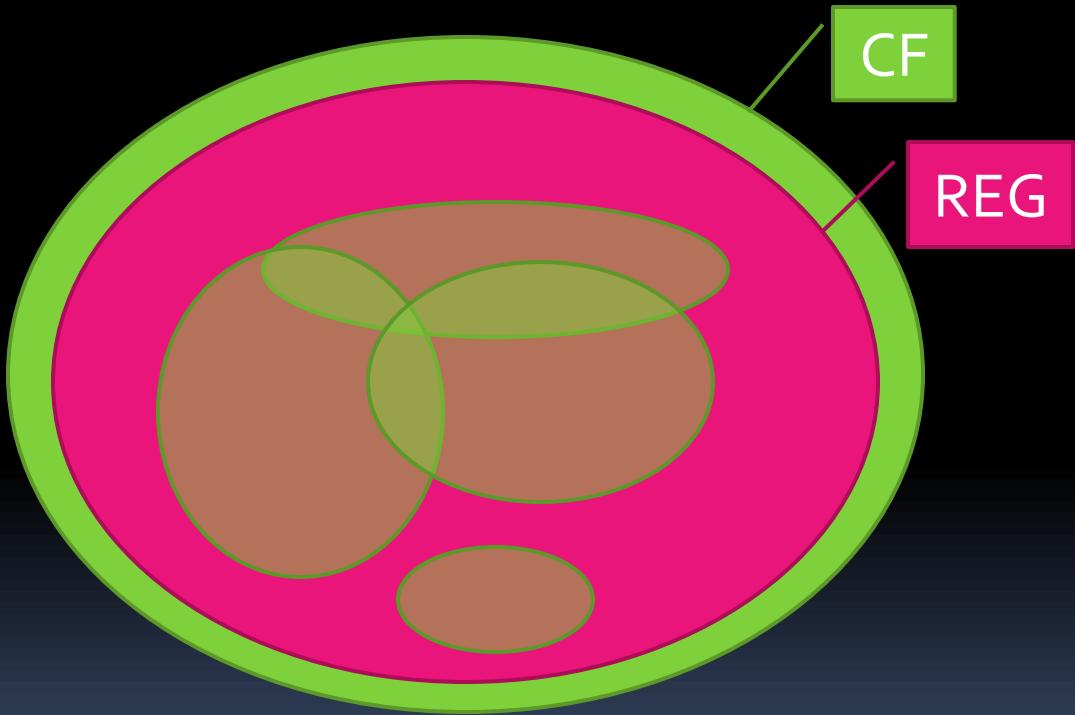
Concatenating Paths



SSSaSSSaSSSaSSSa

Sub-Regularly Controlling

- What happens if we limit regular expressions
 - $(*, .)$ – SRE
 - $(*, U)$ – SRE
 - $(., U)$ – SRE
 - $(., *)$ – SRE
 - $(.)$ – SRE
 - $(*)$ – SRE
 - (U) – SRE
- and use them to control PC(TC) Grammars?



References

1. K. Culik and H. A. Maurer:
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Computing,
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2. S. Marcus, C. Martín-Vide,
V. Mitrana, Gh. Paun:
*A New–Old Class of Linguistically Motivated
Regulated Grammars*,
Language and Computers, Computational
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pp. 111-125(15), 2000.
3. ...our own research, an article will come.



Questions



**THANK YOU
FOR YOUR ATTENTION**