Cellular automata

Modern Theoretical Computer Science

Pavol Korček

Department of Computer Systems

ikorcek@fit.vutbr.cz

BRNO UNIVERSITY OF TECHNOLOGY

FIT FACULTY OF INFORMATION TECHNOLOGY

Summary of today's Cellular Automata (CA) presentation

- Short overview of history
  How did it begin?
- Basic definition
  What is CA?
- Review of some kinds
  Which other types exist?
- Game where nobody plays
  Game of Life.
- Next intention
  What's next?
- My work
  What is it finally for?

How did it begin?

- 1948 – John von Neumann – self-replicating systems (kinematics, cellular, excitation-threshold-fatigue, model continua and probabilistic model)
  - Inspired with McCulloch, Pitts (artificial neural networks) and, of course, Turing
  - Black-box problem
  - S. Ulam: “Something like chessboard with individual cells” (each cell is one finite machine) – true beginning of cellular automata
  - Simplification of real life
  - First cellular automaton based on self-replicating system (named by A. Burks) with more than 200 000 cells (factory, duplicator, computer and tape)
  - Proof of existence: first published by Thatcher in 1964 (50 pages)
  - Non-existence of parallel computers stopped next development on CA
  - The Game of Life by J. H. Conway in 1970 (2D CA)

What is CA?

- Mathematical model, discrete in space and time
- Structure of cells in N dimensions
  - *regular* (tape, grid, loop, anuloid …)
  - *irregular* (bird flocks …)
- Each cell takes one of finite number of states (e.g. 0 – dead cell, 1 – alive cell), which is computed by local neighborhood function
  - *uniform* (same function for every cell)
  - *non-uniform* (function might be different for every cell)
- Various neighborhoods and boundary conditions
- But these properties should be always true:
  - parallelism
  - locality
  - (and sometimes) homogenity/uniformity

    => This is emergent system

Basic formal definition*

One-dimensional binary non-uniform cellular automaton with the finite number of cells is 8-tuple:

$$A = (d, Q, N, R, z, b_1, b_2, w_0), \qquad \text{where}$$

$d = 1$	*dimension*
$Q = \{0, 1\}$	*set of cell states*
N	*set of sets with (local) neighborhood*
z	*number of cells*
b_1, b_2	*boundary values*
w_0	*initial configuration*

*** Designed with inspiration in [4]**

Neighborhood and transition function

$mapping\ R : C \to (Q^N \to Q)$ *assigns to each cell in* $C = \{1, 2, \ldots, z\}$ *a local transition function* $\delta_1, \ldots, \delta_z$, *where* $\delta_i : Q^N \to Q$ *and* $1 \le i \le z$.

If only single neighborhood $N = \{-1, 0, 1\}$ *is considered, then global transition function* $G : Q^C \to Q^C$ *is defined as:*

$$G(c_i) = \begin{cases} \delta_i(b_1, c_1, c_2) & \text{for } i = 1, \\ \delta_i(c_{(i-1)}, c_i, c_{(i+1)}) & \text{for } i = 2 \ldots z-1, \\ \delta_i(c_{(z-1)}, c_z, b_2) & \text{for } i = z, \end{cases} \quad \text{and } \delta_i \text{ is defined by } R$$

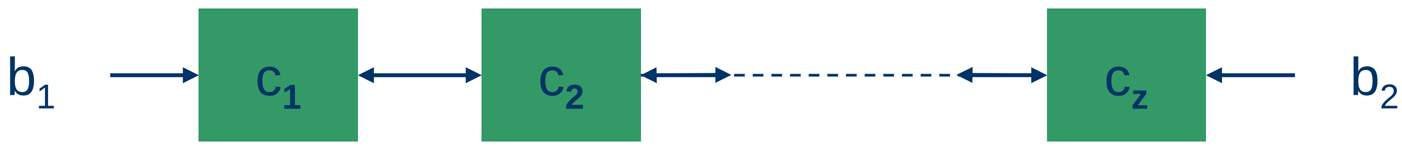
Computation of CA

G is used to define a sequence of configurations $w_0, w_1, w_2, \ldots$

such that $w_j = G(w_{(j-1)})$, *for* $j > 0$.

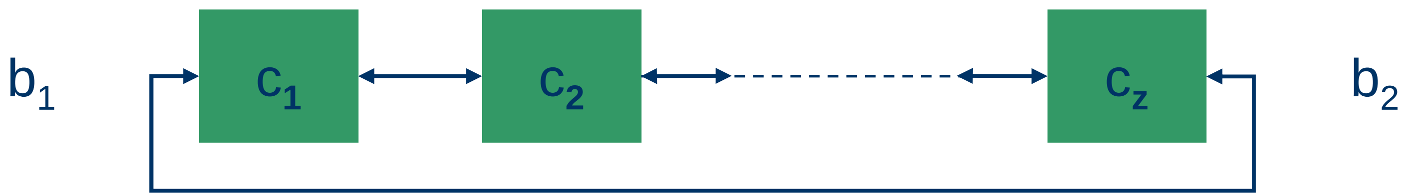
This sequence represents a computation of automaton A.

Other types: boundary conditions in 1D CA



b_1 = constant value
b_2 = constant value

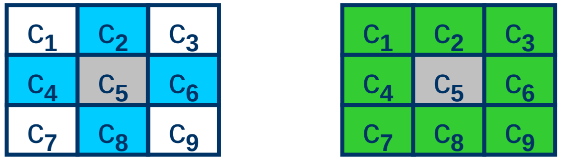
a) *Constant boundary conditions*



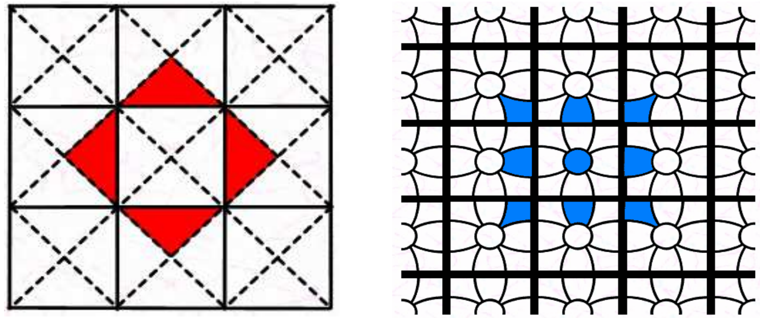
$b_1 = c_z$
$b_2 = c_1$

b) *Cyclic boundary conditions*

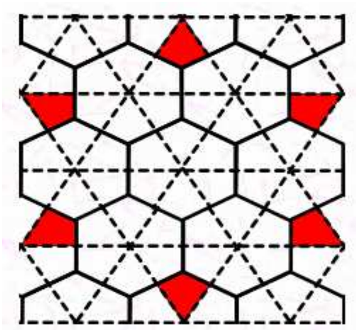
Other types: neighborhoods in 2D CA [8]



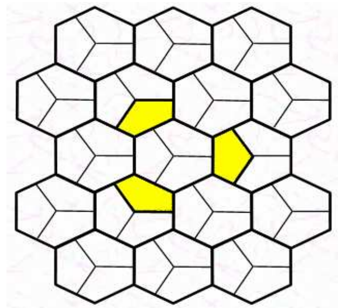
1) Traditional von Neumann and Moore neighborhood



2) Isometric von Neumann and Moore neighborhood



3) Central distant sector neighborhood

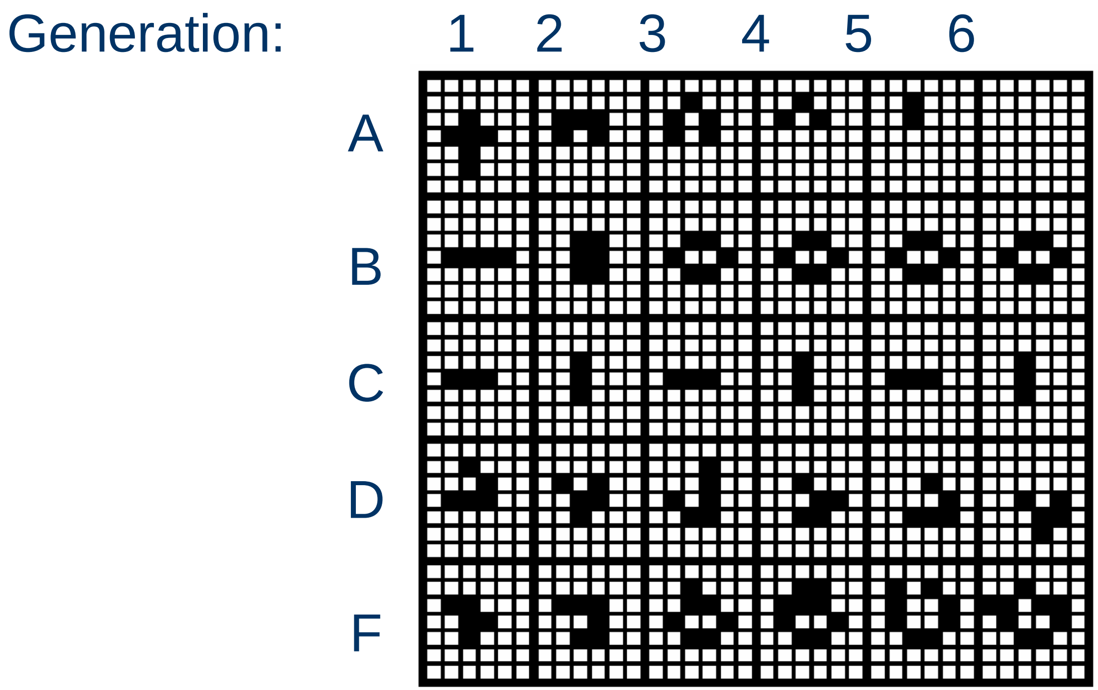


4) Triumphant neighborhood

Game of Life

- 1970 - Mathematic J. H. Conway
  - 2D CA, not so big
  - each cell has only 2 states (0,1)
  - Moore neighborhood
  - (Biological) transition rules defined as
    - **birth** – if 3 alive cells exist in surroundings
    - **survivance** – 2 or 3 alive cells, actual cell survives
    - **death** – 0,1,4,5,6,7 alive cells, actual cell dies

Game of Life: possible structures



Next (and previous) intention on cellular automata (1/3)

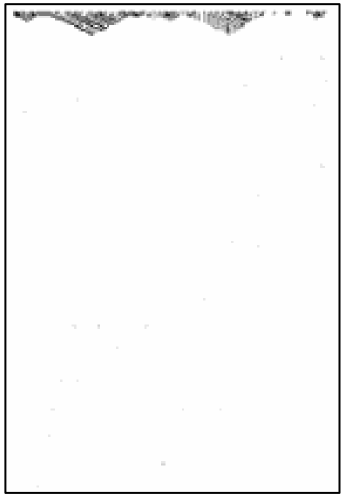
- Majority, reversibility, synchronization problem, chess table problem, adder…
- *1968 – E. F. Codd*
  - His cellular automaton based on signal paths, insulators and surrounding, is theoretically able to emulate Turing machine
- *1984 – Ch. Langton*
  - Self-reproducing 2D cellular automaton based on idea of E. F. Codd

Next (and previous) intention on cellular automata (2/3)

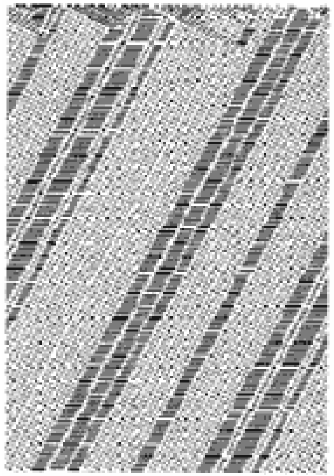
- S. Wolfram
  - **Wolfram's classification scheme** based on 1D CA rules
    - CLASS 1 – *homogenous* (e.g. 1D CA: 0, 4, 16, 32, 36, 48, 54, 60 and 62).
    - CLASS 2 – *regular* (e.g. 1D CA: 8, 24, 40, 56 and 58).
    - CLASS 3 – *chaotic* (e.g. 1D CA: 2, 10, 12, 14, 18, 22, 26, 28, 30, 34, 38, 42, 44, 46 and 50).
    - CLASS 4 – *complex* (e.g. 1D CA: 52 and 110).

Next (and previous) intention on cellular automata (3/3)

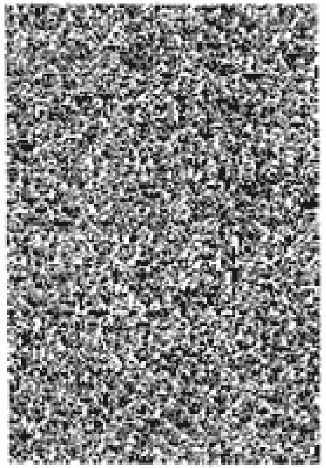
- Wolfram's classification scheme



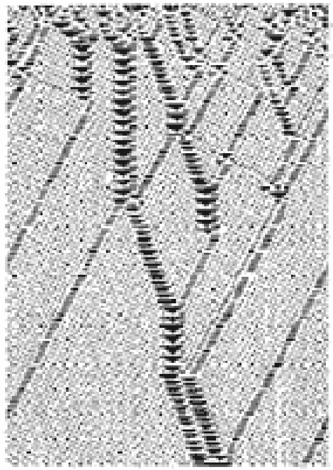
Class 1
Disappears or become static or homogenous



Class 2
Fixed finite size with indefinably repeating structures



Class 3
“Chaotic”, no regularity

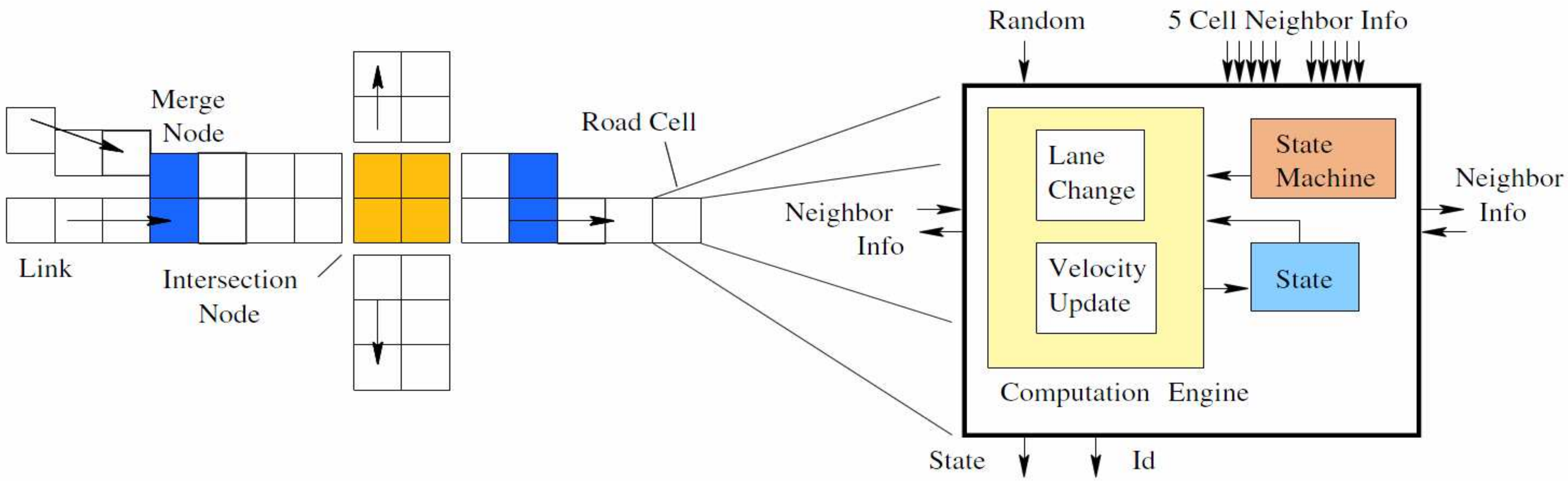


Class 4
Complex patterns grow and contract irregularly

CA in my work:
Acceleration of urban traffic simulation

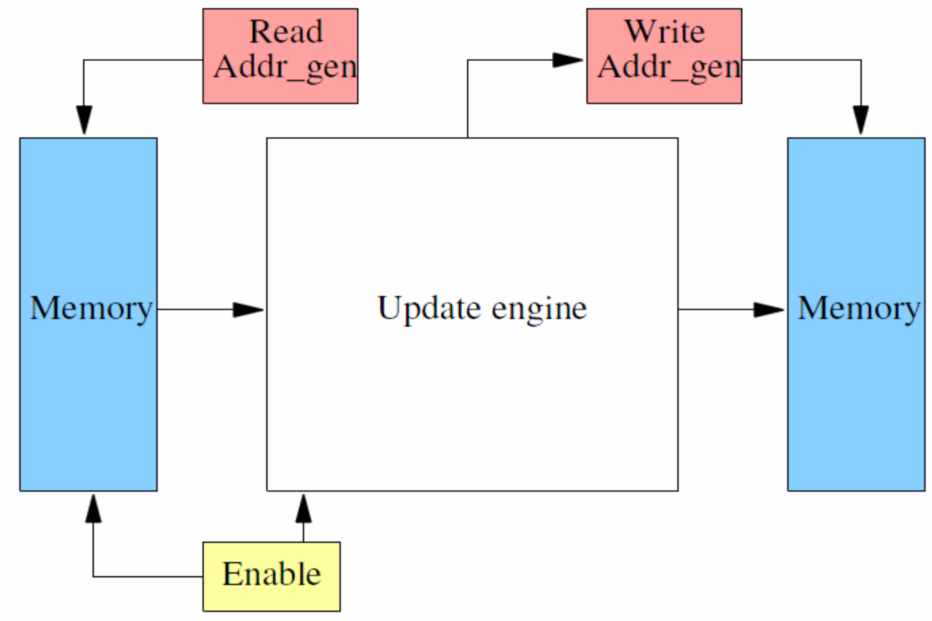
- Traffic simulation used in real applications
- Microscopic simulation model based on CA
- Effective HW implementation in FPGA (or other platform, e.g. GPGPU, MultiCORE) = *really fast real-time simulation*

CA in my work: *Basic idea*



Road network and Cell design [6]

CA in my work:
Final (multiplatform) implementation



Structure of straightforward streaming implementation [6]

Literature

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[4] – Sipper, M.: *Evolution of Parallel Cellular Machines: The Cellular Programming Approach.* 1997. Springer-Verlag, Berlin.

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[7] – Wolfram, S.: *A New Kind of Science*, Wolfram Media, Inc. 2002. 1197 pages, ISBN 1-57955-008-8.

[8] – Web pages: *http://cell-auto.com/*