Digital Images and Formal Languages

Jakub Žák

TID 2009

FIT BUT Brno

Outline

I. motivation

II. image representation

III. grayscale images and WFA

IV. conclusion

Motivation

- how to describe various types of images in efficient way?
- how to realize operations like zoom, filtering, compression...?

- could formal languages help?

Image representation- basics

- raster/vector graphics vs. formal languages
- languages over n-letter alphabet Σ -> rational numbers
- 2D points- 2 coordinates- n^2-letter alphabet
- sets of points-> images
- regular sets- black&white images

Image representation- addressing

- resolution- $2^n x 2^n, n \geq 1$
- alphabet- $\Sigma = \{0,1,2,3\}$
- $2^n x 2^n$=> addressing quadrants
- each quadrant -> single symbol of Σ
- subquadrants inductively => whole address- n symbols long

Image representation- example

- *Example 1:*

  for n=1 we obtain 4 pixel image

1	3
0	2

- *Example 2:*

  for n=2 is it 16 pixel image

11	13	31	33
10	12	30	32
01	03	21	23
00	02	20	22

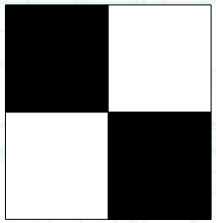
Image representation- black & white

- resolution- $2^m x 2^m, m \geq 1$
- specify image
  - Boolean function- $\Sigma^m \rightarrow \{0,1\}$
  - black pixels- $L \subseteq \Sigma^m$
- multiresolution images- specified simultaneously for all possible resolutions
- specify multires. image
  - black pixels- $L \subseteq \Sigma^*$, where $\Sigma = \{0,1,2,3\}$

Image representation- example

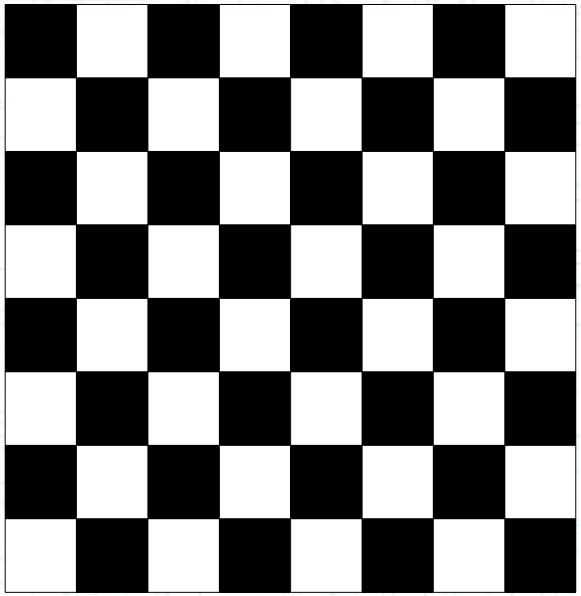
- *Example 3:*

  lets have $2^m x 2^m$ picture defined by regular set $\{1,2\}\Sigma^{m-1}$



- *Example 4:*

  generally multiresolution image of(8x8) chess board by regular set $\Sigma^2\{1,2\}\Sigma^*$



- ***they both look the same for all resolutions***

Image representation- fractals

- *Example 5:*

  now we have regular set $\{1,2\}^* 0$

- we have clearly addressed infinitely many squares ilustrated at the top picture

- *Example 6:*

  $\{1,2\}^* 0 \Sigma^*$ (center picture)

- *Example 7:*

  $\{1,2,3\}^* \{1,2\}^* 0 \Sigma^*$ (bottom picture)

Grayscale images and WFA

- pixel values- real numbers(scaled to 0 - $2^k - 1$)
- resolution $2^m x 2^m$
  - $f : \Sigma^m \rightarrow \Re$
- multiresolution
  - $g : \Sigma^* \rightarrow \Re$
- average preserving(ap) function- same spot, same color for all resolutions

$$f(w) = 1/4 * [f(w0) + f(w1) + f(w3) + f(w4)], \forall w \in \Sigma$$

Grayscale images and WFA

- an m-state *weighted finite automata(WFA) A over alphabet* Σ *is defined by*

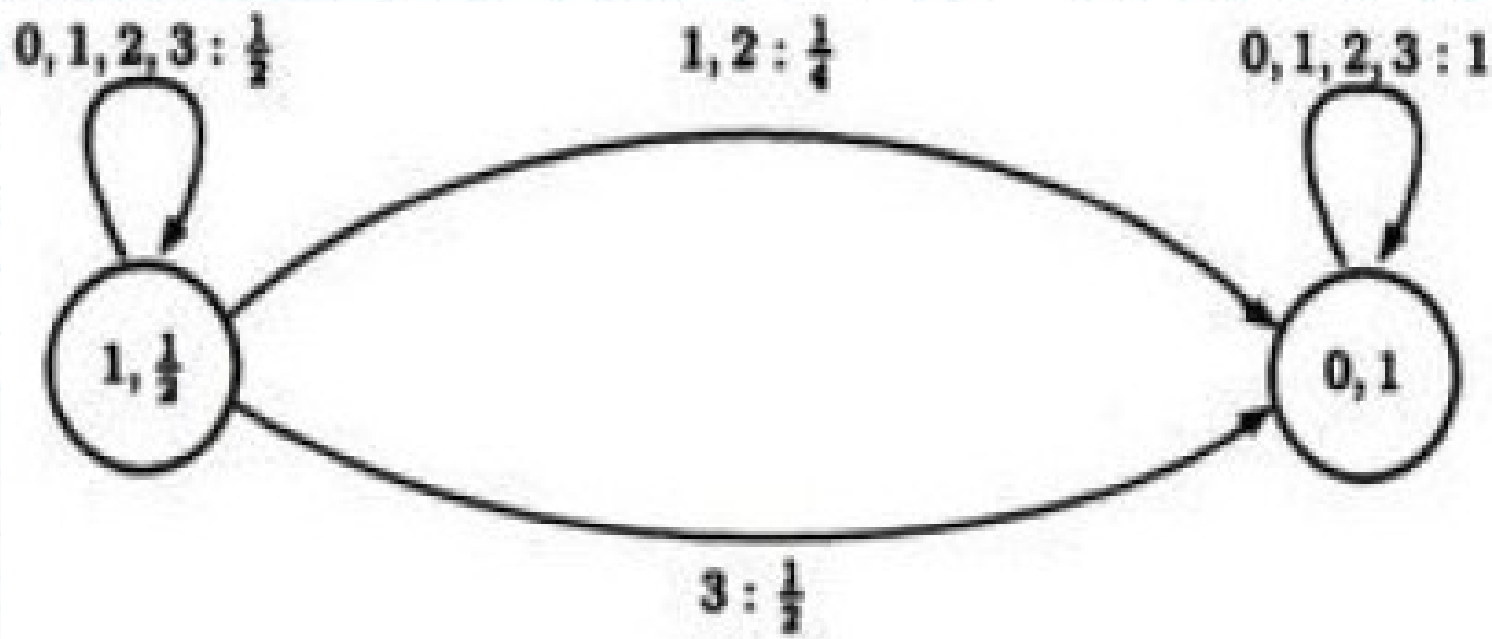
1) a row vector $I^A \in \Re^{1xm}$ (initial distribution)
2) a column vector $F^A \in \Re^{mx1}$ (final distribution)
3) weight matrices $W_a^A \in \Re^{mxm}, \forall a \in \Sigma$

- the WFA A defines a multiresolution function f_A over Σ by $f_A(a_1 a_2 \ldots a_k) = I^A * W_{a_1}^A * W_{a_2}^A * \ldots * W_{a_k}^A * F^A$, where $a_1 \ldots a_k$ is the pixel address

Grayscale images and WFA

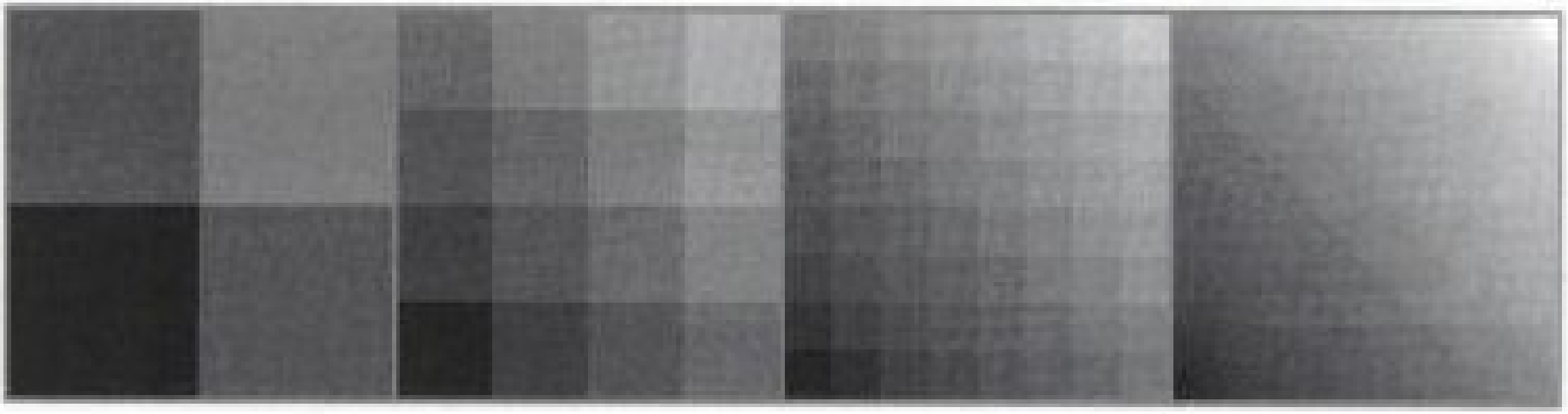
- *Example 8: automata A,* $\Sigma=\{0,1,2,3\}, I=(1,0), F=(\frac{1}{2},1)$

$$W_0=\begin{pmatrix}\frac{1}{2} & 0\\ 0 & 1\end{pmatrix}, W_1=\begin{pmatrix}\frac{1}{2} & \frac{1}{4}\\ 0 & 1\end{pmatrix}, W_2=\begin{pmatrix}\frac{1}{2} & \frac{1}{4}\\ 0 & 1\end{pmatrix}, W_3=\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ 0 & 1\end{pmatrix}$$



Grayscale images and WFA

- *Example 8: image corresponding to automata A*
  - *resolutions* 2x2, 4x4, 16x16, 256x256



Grayscale images and WFA

- note, that **deterministic ap-WFA** is **weaker** than **nondet. ap-WFA**
- image operations- matrices transformations
- **zooming**

  for an arbitrary multires. image f over Σ and word $u \in \Sigma^*$, let f_u denote the multiresolution image

  - $f_u(w) = f(uw), for\ every\ w \in \Sigma^*$

$$I_u = I * W_{a_1} * W_{a_2} * \ldots * W_{a_k}, where\ u = a_1 \ldots a_k$$

Conclusion

- all images of regular character and fractals can be with infinite precision described by regular expressions(finite automata)
- all grey-scale images can be described by nondet. ap-WFA
- advantage in image compression, when described by WFA
- basic image operations like zoom

References

[1] G. Rozenberg, A. Salomaa eds. Handbook of Formal Languages. Springer-Verlag. 3 vol., chap 10.

- all images in this presentation are taken from [1]