

Parses and Their Generators in Scattered Context Grammars

Stanislav Židek

Department of Information Systems
Faculty of Information Technology
Brno University of Technology

Modern Theoretical Computer Science, 2009

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4 Summary

Introduction

Motivation, Previous Research

Motivation

- **parsing** is important part of formal language theory
- **parse**
 - sequence of rules that generated a sentence
 - useful piece of information
- obtaining **parses** by using scattered context grammars

Previous Research

- characterization of RE languages L by propagating scattered context grammars using “stuffing terminals” [2]
- sentences followed by their parses in matrix grammars (**Extended Szilard languages**) [3]

Scattered Context Grammar (SCG)

Definition

SCG is a 4-tuple $G = (V, T, P, S)$, where

- V is a finite set of symbols
- T is a finite set of terminals, $T \subseteq V$
- S is a starting symbol, $S \in (V - T)$
- P is a finite set of productions of the form

$$r : (A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n),$$

where

- $A_1, \dots, A_n \in (V - T)$ and $x_1, \dots, x_n \in V^*$
- r is a unique production label.

Propagating Scattered Context Grammar (PSCG)

Definition

PSCG is a 4-tuple $G = (V, T, P, S)$, where

- V, T, S is the same as in SCG
- P is a finite set of productions of the form

$$r : (A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n),$$

where

- $A_1, \dots, A_n \in (V - T)$ and $x_1, \dots, x_n \in V^+$
- r is a **unique** production label.

Note

$\text{lab}(G)$ – set of all production labels of grammar G

Relation of Direct Derivation (\Rightarrow)

Definition

Let $G = (V, T, P, S)$ be a SCG. Suppose that

- $r : (A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$
- $u = w_0 A_1 w_1 \dots w_{n-1} A_n w_n \in V^*$
- $v = w_0 x_1 w_1 \dots w_{n-1} x_n w_n \in V^*$

G directly derives v from u according to production r

$$u \Rightarrow v [r] \quad (\text{or simply } u \Rightarrow v)$$

Reflexive and Transitive Closure of \Rightarrow

\Rightarrow^* – reflexive and transitive closure of \Rightarrow

Suppose that $v \Rightarrow w_1 [p_1] \Rightarrow \dots \Rightarrow w_{n-1} [p_{n-1}] \Rightarrow u [p_n]$; we write

$$u \Rightarrow^* v [p_1 p_2 \dots p_n]$$

Generated Language

Definition

Let $G = (V, T, P, S)$ be a SCG.

$$L(G) = \{w : w \in T^* \wedge S \Rightarrow^* w [p_1 p_2 \dots p_n]\}$$

$p_1 p_2 \dots p_n$ – parse (derivation word, Szilard word, control word)

Generative Power

$$\mathcal{L}(CF) \subset \mathcal{L}(PSCG) \subseteq \mathcal{L}(CS) \subset \mathcal{L}(SCG) = \mathcal{L}(RE)$$

SCG Generating Sentences with Their Parses

Definition

Let $G = (V, T, P, S)$ be a SCG. G is a proper generator of its sentences with their parses (**SCG-P**) iff

$$L(G) = \{w\rho : w\rho \in T^* \wedge S \Rightarrow^* w\rho [\rho]\}$$

First Example

$$G_1 = (\{S, A, B, C, a, b, c\}, T = \{a, b, c\}, P_1, S)$$

$$\begin{aligned}P_1 = \{1 &: (S) \rightarrow (ABC), \\2 &: (A, B, C) \rightarrow (Aa, Bb, Cc), \\3 &: (A, B, C) \rightarrow (\varepsilon, \varepsilon, \varepsilon)\}\end{aligned}$$

Is G_1 SCG-P?

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Is G_1 SCG-P?

NO! ($\text{lab}(G_1) \not\subseteq T$)

SCG-P Examples II

Second Example

$$G_2 = (\{S, A, B, C, \$, a, b, c, 1, 2, 3\}, \{a, b, c, 1, 2, 3\}, P_2, S)$$

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Is G_2 SCG-P?

YES!

Example G_2 derivation

$$\begin{aligned}S &\Rightarrow ABC1\$ [1] \Rightarrow AaBbCc12\$ [2] \\&\Rightarrow AaaBbbCc122\$ [2] \Rightarrow aabbcc1223 [3]\end{aligned}$$

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Theorem

For every recursively enumerable language L there exists a propagating SCG-P G such that

$$L(G) = \bigcup_{w \in L} w\rho : w\rho \in T^* \wedge S \Rightarrow^* w\rho [\rho]$$

Basic Concept

- 1 for every RE language L there is SCG G such that $L(G)=L$
 - $\mathcal{L}(RE) = \mathcal{L}(SCG)$ (see references [1])
- 2 for every SCG G there is a propagating SCG-P G' generating sentences of G followed by their parses
 - construction will be shown

SCG-P Construction Idea I

Phase I

Phase I – Sentence Generation

Simultaneously generate sequence of productions by $\$_1$ and sequential forms with marked non-terminals (ε on the right hand side replaced by Y)

$$S' \Rightarrow X1\$_1ZS$$

$$\Rightarrow X12_1\$_1ZABC$$

[1 : $(S) \rightarrow (ABC)$]

$$\Rightarrow X12_12_2\$_1ZAa'Bb'Cc'$$

[2 : $(A, B, C) \rightarrow (Aa, Bb, Cc)$]

$$\Rightarrow X12_12_22_3\$_1ZYa'Yb'Yc'$$

[3 : $(A, B, C) \rightarrow (\varepsilon, \varepsilon, \varepsilon)$]

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SCG-P Construction Idea II

Phase II

Phase II – Generating Terminals

Move marked non-terminals before the parse (replace them by Y)

$$\begin{aligned} & X12_12_22_3\$_1ZYa' Yb' Yc' \\ \Rightarrow & X12_12_22_3\textcolor{red}{2\$}_2ZY\textcolor{blue}{a}' Yb' Yc' \\ \Rightarrow & \textcolor{red}{a}X12_12_22_323_a\$_2YY\textcolor{red}{Z}Y\textcolor{blue}{b}' Yc' \\ \Rightarrow & abX12_12_22_323_a3_b\$_2YYYY\textcolor{red}{Z}Y\textcolor{blue}{c}' \\ \Rightarrow & abc12_12_22_323_a3_b3_c\$_3YYYYYY\textcolor{red}{Y} \end{aligned}$$

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SCG-P Construction Idea III

Phase III

Phase III

Removing Y s Replace Y s by labels

$$\begin{aligned} & abc12_12_22_32\$_3 YYYYYYY \\ \Rightarrow & abc12_12_22_323_a3_b3_c3\$_3 YYYYYYY \\ \Rightarrow & abc12_12_22_323_a3_b3_c33\$_3 YYYYYY \\ \Rightarrow & \dots \\ \Rightarrow & abc12_12_22_323_a3_b3_c3333333\$_3 \\ \Rightarrow & \underbrace{abc}_{\text{sentence}} \underbrace{12_12_22_323_a3_b3_c33333334}_{\text{parse}} \end{aligned}$$

SCG-P Construction Idea III

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$$\begin{aligned} & abc12_12_22_32\$_3 \textcolor{red}{Y} \textcolor{blue}{YYYYYY} \\ \Rightarrow & abc12_12_22_323_a3_b3_c3\$_3 \textcolor{red}{Y} \textcolor{blue}{YYYYYY} \\ \Rightarrow & abc12_12_22_323_a3_b3_c33\$_3 \textcolor{red}{Y} \textcolor{blue}{YYYY} \\ \Rightarrow & \dots \\ \Rightarrow & abc12_12_22_323_a3_b3_c3333333\$_3 \\ \Rightarrow & \underbrace{abc}_{\text{sentence}} \underbrace{12_12_22_323_a3_b3_c33333334}_{\text{parse}} \end{aligned}$$

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SCG-P Formal Algorithm

Input and Output

Input: SCG $G = (V, T, P, S)$

Output: PSCG-P $G' = (V', T', P', S')$ such that

$$L(G') = \bigcup_{w \in L} w\rho : S \Rightarrow^* w\rho [\rho]$$

Preliminaries

Let $\Phi = \{a' : a \in T\}$. Define γ' and γ as follows:

$$\gamma'(a) = \begin{cases} a' & \text{if } a \in T \\ a & \text{if } a \in (V - T) \end{cases} \quad \gamma(x) = \begin{cases} Y & \text{if } x = \varepsilon \\ \gamma'(x) & \text{otherwise} \end{cases}$$

SCG-P Formal Algorithm II

Symbols

Terminals

$$T' = T \cup \text{lab}(G')$$

All Symbols

$$V' = T' \cup \Phi \cup \{S', X, Y, Z, \$_1, \$_2, \$_3\}$$

Note

Assume that $\text{lab}(G') \cap \text{alph}(L) = \emptyset$.

SCG-P Formal Algorithm III

Productions

Productions

Construct P' as follows:

- 1 Add $1 : (S) \rightarrow (X1\$_1ZS)$ and $1_\varepsilon : (S) \rightarrow (1\$_1S)$ to P' .
- 2 For every $r : (A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$ add
 $1_r : (\$, A_1, \dots, A_n) \rightarrow (1_r\$, \gamma(x_1), \dots, \gamma(x_n))$ to P' .
- 3 Add
 - 1 $2 : (\$_1) \rightarrow (2\$_2)$ and
 - 2 $2_\varepsilon : (\$_1) \rightarrow (2_\varepsilon\$_3)$ to P' .
- 4 For each $a \in T$ add
 - 1 $2_a : (X, \$_2, Z, a') \rightarrow (aX, 2_a\$, Y, Z)$ and
 - 2 $3_a : (X, \$_2, Z, a') \rightarrow (a, 3_a\$, Y, Y)$ to P' .
- 5 Add $3 : (\$, Y) \rightarrow (3, \$_3)$ to P' .
- 6 Add $4 : (\$, Y) \rightarrow (4)$.

Result

Every RE language can be characterized by **propagating** SCG-P.

Future Research

Propagating SCG (or other grammars) generating/accepting parse trees.

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References

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