ONE CCD CAMERA MEASUREMENT OF VEHICLE VELOCITY

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Abstract: The paper deals with the problem of determining the instantaneous velocity of vehicles from a series of one-camera images. The model of vehicle trajectory is described, and the relationship between the image and real coordinates is expressed. Relationships are derived to obtain the necessary a priori information on the height above the road. Accuracy analysis for the designed processing method is performed.

Keywords: computer vision, digital photogrammetry, CCD camera

1. INTRODUCTION

For measuring a vehicle velocity, we can use a series (time sequence) of CCD camera. In order to calculate the velocity, we need to know a distinct point on the vehicle images (usually derived from the car plate), to identify it in all images of the series, and subsequently transform its position from image coordinates (area $\mathbf{N}^2$) to real coordinates (area $\mathbf{R}^3$). As this transformation is not generally straightforward a certain a priori information is required. We can use (Lisztwan 2001):

- the constant value of the coordinate z. For example, in passenger vehicles the height of the car plate above the surface of the road can be regarded as constant. A certain error must be taken into account – the height can vary in dependence on vehicle loading. Moreover, in cross-country vehicles, lorries or buses the positions of the car plate considerably differ.
- a known difference between the height of two points (Δz) – *vertical information.* Also in this case the car plate is a good example as its dimensions are standardized.
- a known distance between two points lying in a plane parallel to the plane of the road ($m = |[x_1, y_1, z], [x_2, y_2, z]|$) – *horizontal information.* The known width of the car plate can be used similarly as in the preceding case. But a higher relative accuracy will be attained (the width of the car plate is greater that its height).

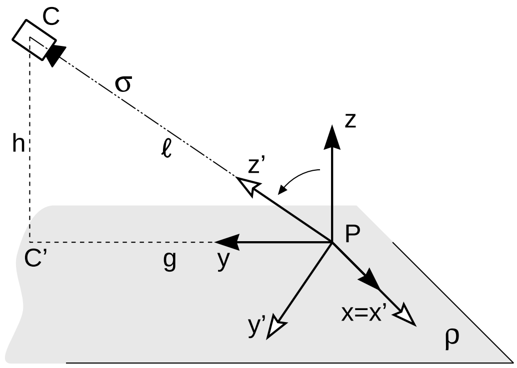

Fig. 1. Location and orientation of coordinate systems

- Combination of the horizontal and vertical information. If both the horizontal and the vertical distance between two arbitrary points within a visible space are known, their height above the road can be determined (coordinate z).

2. IMAGING MODEL

For the purpose of measuring the position or speed of a vehicle, a modified model of general transformation of coordinates is applied. The number of imaging parameters (and therefore the number of degrees of freedom) is limited to those really necessary for model application – that is its ability to represent reality.

2.1 *Space-camera tranformation*

The anti-clockwise orthonormal coordinate system in the real space is applied for easier association of coordinates x, y in the real space to coordinates u, w in the image. The origin P of the coordinate system is situated at the intersection point of the camera optical axis σ and the plane ρ of the road. Axes x and y also lie at the plane of the road. Axis y lies on the projection of the camera optical axis into the plane of the road. Transformation of the real-space coordinates to image coordinates takes place in three phases: (1) rotation round axis x, (2) central projection, and (3) rotation round the camera optical axis σ.

2.1.1. *Rotation of coordinate axes round axis x* To enable application to the model of the central projection physically realized by the camera lens, the coordinate system must rotate round axis x in such a way that the direction of axis z be identical with the camera optical axis σ, as it is shown in Fig. 1. It can be derived for the coordinates:

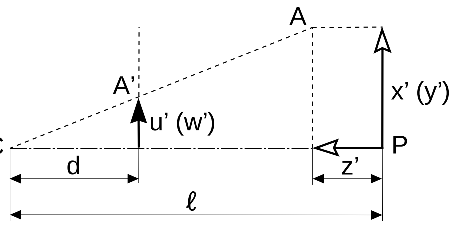

Fig. 2. Mathematical model of central projection

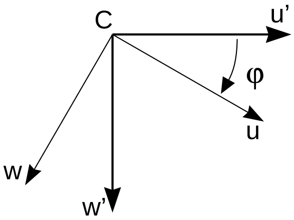

Fig. 3. Rotation of coordinate axes round axis σ

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{h}{\ell} & -\frac{g}{\ell} \\ 0 & \frac{g}{\ell} & \frac{h}{\ell} \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \qquad (1)$$

where

$$\ell = \sqrt{h^2 + g^2}. \qquad (2)$$

2.1.2. *Central projection* The camera lens projects the objects in the area of interest onto the active area of the CCD chip in the image. This can be modeled by central projection, on the assumption that various errors in the optical system are neglected. The following equation applies to the model in Fig. 2:

$$\begin{bmatrix} u' \\ w' \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} \times \left[\frac{d}{\ell - z'} \right] \qquad (3)$$

By substituting from (1) we get:

$$u' = d \frac{x\ell}{\ell^2 - yg - zh} \qquad (4)$$

$$w' = d \frac{yh - zg}{\ell^2 - yg - zh} \qquad (5)$$

2.1.3. *Rotation round the camera optical axis* Coordinates u', w' could be used as the final values of image coordinates u, v on the assumption that u axis is parallel to the road plane ρ. However, this is not possible in reality. On the contrary, the camera is aimed in such a way that the horizontal edge of the car plate is scanned parallel with axis u. That is why rotation of coordinate axes round the camera optical axis σ is introduced in the model (Fig. 3):

$$\begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \times \begin{bmatrix} u' \\ w' \end{bmatrix} \qquad (6)$$

Application of (6) to equations (4) and (5) results in the final imaging model $(x, y, z) \rightarrow (u, w)$:

$$u = \frac{d(x\ell \cos\varphi + (yh - zg)\sin\varphi)}{\ell^2 - yg - zh} \quad (7)$$

$$w = \frac{d(x\ell \sin\varphi - (yh - zg)\cos\varphi)}{\ell^2 - yg - zh} \quad (8)$$

2.2 *Camera-space transformation*

In velocity measurement, the object must be located (coordinates x, y, z) using the known coordinates u, v. We get two equations with three unknowns – a certain a priori information on the real scene is required. The coordinate z appears to be most useful. Then the remaining coordinates x, y can be determined through reverse transformation:

$$x = u'\ell \frac{h - z}{hd + w'g} \quad (9)$$

$$y = \frac{w'\ell^2 + z(gd - w'h)}{hd + w'g} \quad (10)$$

The values u', w' can be obtained through tranformation reverse to transformation (6):

$$\begin{bmatrix} u' \\ w' \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \times \begin{bmatrix} u \\ w \end{bmatrix} \quad (11)$$

The final equations for x and y can be adapted as follows:

$$x = \frac{\ell(h - z)(u\cos\varphi - w\sin\varphi)}{hd + (u\sin\varphi + w\cos\varphi)g} \quad (12)$$

$$y = \frac{(\ell^2 - hz)(u\sin\varphi + w\cos\varphi) + zgd}{hd + (u\sin\varphi + w\cos\varphi)g} \quad (13)$$

3. UNKNOWN HEIGHT ELIMINATION

3.1 *Vertical information*

The difference Δz of the height of two points lying on one straight line vertical to the plane of the road (in other words: x-th and y-th coordinates of the points are identical). Let the distance between these points be $2n$. Then it holds:

$$z_1 = z - n \quad (14)$$

$$z_2 = z + n \quad (15)$$

Two equations are available for calculation:

(1) $x_1 = x_2$. Substituting after (9), (14) and (15) and after adaptation we get:

$$z = h - n\frac{hd(u'_2 + u'_1) + g(u'_2 w'_1 + u'_1 w'_2)}{hd(u'_2 - u'_1) + g(u'_2 w'_1 - u'_1 w'_2)} \quad (16)$$

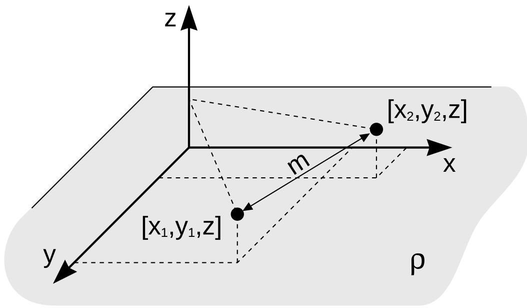

Fig. 4. Horizontal information

(2) $y_1 = y_2$. Substituting after (10), (14) and (15) and after adaptation we get:

$$z = h - n\Bigg(1 + +2\frac{h^2 d w'_1 - g^2 d w'_2 - hgd^2 + w'_1 w'_2 hg}{d\ell^2(w'_2 - w'_1)}\Bigg) \quad (17)$$

It should be noted that values u'_i, w'_i transformed from values must be substituted, according to (11).

3.2 *Horizontal information*

If the distance m between two points lying in one plane parallel to the plane of the road (Fig. 4) is known, their height above the road can be determined from the image in which they are contained. The basic equation is the relationship expressing the distance between the two points:

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = m^2 \quad (18)$$

By substituting

$$a_1 = u'_1 \frac{\ell h}{hd + w'_1 g} - u'_2 \frac{\ell h}{hd + w'_2 g}, \quad (19)$$

$$a_2 = w'_1 \frac{\ell^2}{hd + w'_1 g} - w'_2 \frac{\ell^2}{hd + w'_2 g} \quad (20)$$

equation (18) then has the form:

$$\left(a_1 - \frac{a_1}{h} z\right)^2 + \left(a_2 - \frac{a_2}{h} z\right)^2 = m^2 \quad (21)$$

The result then has the form:

$$z = h\left(1 - \frac{m}{\sqrt{a_1^2 + a_2^2}}\right) \quad (22)$$

Calculating substitutions a_1, a_2, values u'_i, w'_i transformed from values u_i, w_i according to (11) must be substituted.

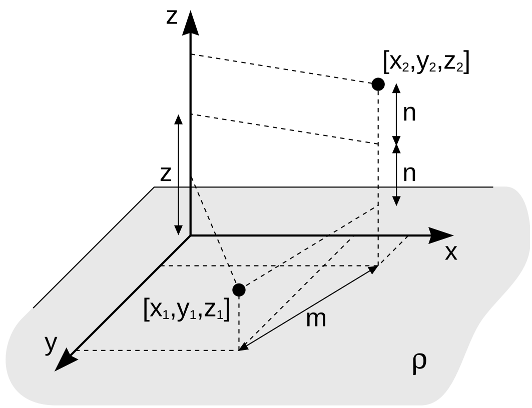

Fig. 5. Horizontal and vertical information

3.3 *Horizontal and vertical information*

If the horizontal and vertical distances (i.e. m and $2n$) between two arbitrary points in the visible space are known, their height above the road can be determined from the image in which they can be identified. The derivation procedure is similar to those in the previous cases. Let us use equation (18) for the distance between two points. After substitution:

$$a_1 = -u_1' \frac{\ell}{hd + w_1' g} \tag{23}$$

$$a_2 = -u_2' \frac{\ell}{hd + w_2' g} \tag{24}$$

$$b_1 = -w_1' \frac{\ell}{h} \frac{\ell}{hd + w_1' g} \tag{25}$$

$$b_2 = -w_2' \frac{\ell}{h} \frac{\ell}{hd + w_2' g} \tag{26}$$

using equation (9) and relationships (14) and (15), we get

$$x_1 - x_2 = -h(a_1 - a_2) + z(a_1 - a_2) - -n(a_1 + a_2) \tag{27}$$

Similarly, using equation (10) and relationships (14) and (15), we get

$$y_1 - y_2 = -h(b_1 - b_2) + z(b_1 - b_2) - -n(b_1 + b_2) \tag{28}$$

The result is in the form:

$$z = h + \frac{n(a_2^2 - a_1^2 + b_2^2 - b_1^2)}{c} - -\frac{\sqrt{m^2 c - 4n^2 (a_1 b_2 + a_2 b_1)^2}}{c} \tag{29}$$

where $c = (a_2 - a_1)^2 + (b_2 - b_1)^2$. The relationship holds for $m > 0, n \geq 0$. Attentive reader will note that for $n = 0$ we get a form similar to equation (22) – it is not identical as substitutions a_1, a_2 are defined in a slightly different way. As in the previous cases, the transformed values u_i', w_i' must be used for substitution.

4. VELOCITY CALCULATION

Instantaneous velocity is defined as a derivation of the covered trajectory according to time:

$$v = \frac{\partial s}{\partial t} \tag{30}$$

In reality, instantaneous velocity can be determined as:

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \tag{31}$$

for sufficiently small Δt.

The process of instantaneous velocity measurement consists in measuring the distance covered by the vehicle in the interval between two images. The absolute position of the vehicle in space must be located in both images. For this reason one particular point on the vehicle must be reliably identified in both images with the greatest possible accuracy. The most convenient seem to be the corner points of the car plate. The car plate is an object that can be identified with relative accuracy, and whose dimensions and approximate height above the road are standardized.

The velocity can be calculated according to the formula

$$v = \frac{\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}}{\Delta t} \tag{32}$$

The coordinate z is taken into account. Even if the height of the car plate should be constant, it can be calculated for each image separately using equations in chapter 3. Thus the deviations due to the unevenness of the surface or to other effects can be considered.

5. MEASUREMENT ACCURACY

To increase the measurement accuracy, passive and active means can be used. Active means are used to prevent errors or an increase in errors, e.g. by using correctly focused and set cameras, suitable illumination, and accurate calibration. Passive means eliminate or reduce the impact of the already existing errors.

5.1 *Active means of accuracy increase*

The most important and obvious aspect of correct velocity measurement is calibration. Acquisition of the most accurate parameters of the camera system (i.e. h, g, d and φ parameters) decisively affects the measurement as it introduces a systematic error.

Another aspect is a suitable aiming of the camera as the measurement error varies in different parts of the image. A rectangular image represents a trapezoid area on the road. Obviously, the same number of pixels is contained in a smaller real area within the lower part than that in the upper part. The measurement of the position is therefore more accurate in the lower part. For velocity measurement, however, we need to know the difference between the positions. For greater distances, the same absolute position error results in a smaller relative distance error. The last image of the processed sequence can contain the distinct point nearest to the lower edge of the image. It should be asked which is the highest position in the image where a distinct point can be identified so that the relative error of distance determination would not exceed the required limit.

Let δS be the relative error of distance determination and Δu or Δw the absolute error of the determination of the coordinate u or w. The relative error relates to the significant point $[x_0, y_0]$ in the second image. Then it holds:

$$\delta S = \frac{\sqrt{D_u + D_w}}{\sqrt{D_{xy}}} \tag{33}$$

where

$$D_u = \left(\Delta u \frac{dx}{du}(u,w)\right)^2 + \left(\Delta u \frac{dy}{du}(u,w)\right)^2 \tag{34}$$

$$D_w = \left(\Delta w \frac{dx}{du}(u,w)\right)^2 + \left(\Delta w \frac{dy}{du}(u,w)\right)^2 \tag{35}$$

$$D_{xy} = (x(u,w) - x_0)^2 + (y(u,w) - y_0)^2 \tag{36}$$

Error estimation was performed for parameters obtained at the experimental crossroads in Brno on Božetěchova: $h = 4.15$ m, $g = 22.5$ m, $d = 1660$ px, $\varphi = -0.02$ rad. Point $[u, w] = [0, 143]$ was taken as a distinct point of the second image which corresponds to values $x_0 = 0$ m, $y_0 = 7.5$ m.

It is evident from the graph in Fig. 6 that the optimum location for a distinct point identification in the first image is in the concave area around coordinate $w = -80$ where the error falls below 0.9 %. In Fig. 7 a slight decrease towards the edges of the image (note the scale of the error) can be seen. It can be said that the relative error curve approximates a hyperbolic paraboloid with the centre round the point $u = 0, w = -80$. Nevertheless, the increase in error following the shift of the distinct point in the first image to a quarter of the height of the image is not too steep.

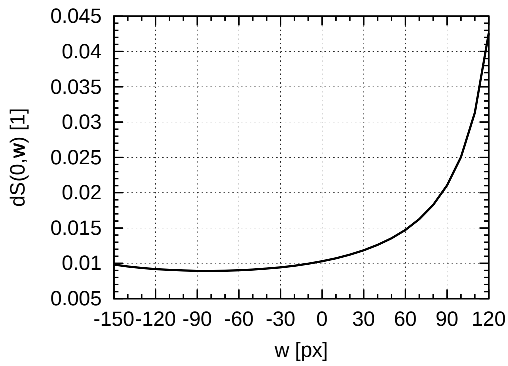

Fig. 6. Relative error curve for $u = 0$

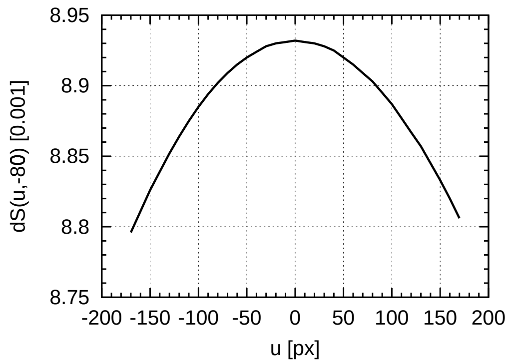

Fig. 7. Relative error curve for $w = -80$

At a shorter distance between the images the error is substantially greater.

Based on the analysis of the relative error behaviour, convenient locations can be found in the image where the vehicle distinct points can be identified.

5.2 *Passive means of accuracy increase*

The main passive means of accuracy increase is the averaging of a number of measured values.

In a velocity measurement, more than two images of a vehicle can be obtained. Velocity is calculated for each pair of successive images, and then the obtained values are averaged. A simple arithmetic mean or a weighted average can be used. The weight can be determined as the reciprocal value of the sum of location errors at the beginning and end of the respective interval. To enhance the calculation, the weights can be calculated in advance.

For vehicle location, more than one distinct point can be used. The covered distance is determined for each point separately, and then their average is calculated. If the car plate is used for identification of distinct points, the four corner points are chosen.

6. CONCLUSION

A model of the movement of vehicles was described in this paper. The relationship between the image and real coordinates was expressed, enabling reconstruction of the position of a distinct point and measurement of the velocity of an object using one camera. The height of the point must be a priori known. The height can be calculated from the known dimensions of the object. For velocity measurement, the most convenient proved to be the horizontal width of the car plate – the so called horizontal information.

The relationship was derived for the measurement error. It was shown by experiment that if the calibration parameters are correctly set, the relative measurement error of about 1 % can be attained. However, it holds only if the entire image is used as a measurement area. If only one part of the image is used, the error increases.

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