Turing Machines

A Model of Computation

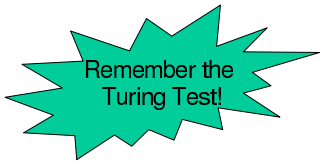
Alan Turing

- Born in 1912
- 1922: Troubles in School
- 1936: Turing Machine
- WWII: Bletchley Park
- 1954: Suicide?!

Original Motivation

Modeling "Human Computers"

What are "Human Computers"?



Original Motivation

Modeling "Human Computers"

What are "Human Computers"?

(might be a biased picture)

Today's Perspective

Modeling "Human Computers"

What are "Human Computers"?

Today's Perspective

Modeling "Computers"

What are "Computers"?

Today's Perspective

Modeling "Computers"

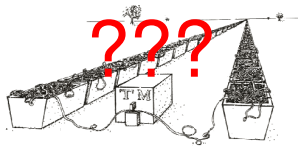
What are "Computers"?

- Storage Device
  - read/write access
  - finite size (conceptually arbitrarily large)
- Control Unit
  - defines which step to do next
  - aka CPUs/Programs

Today's Perspective

Modeling "Computers"

What are "Computers"?



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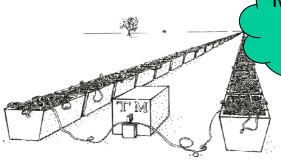

Modeling… But what for???

Today's Perspective

Modeling "Computers"

What are "Computers"?

Modeling… But what for???

Reasoning about
- algorithms
- computational problems

Treating them as Mathematical Objects!!!

Algorithms as Mathematical Objects

- TMs are meant for formulating & proving general statements about algorithms:
  - What is co…
  - How m… …ed to so…
- TMs ar…
  - program…g
  - real computations
  - browsing the web

So what?! Turing Machines and real computers?!

Relevance of TMs: What is Computable by TMs?

LCMs can do anything that could be described as "rule of thumb" or "purely mechanical".

This is sufficiently well established that it is now agreed amongst logicians that "calculable by means of an LCM" is the correct accurate rendering of such phrases.

(Turing in 1948 on his Logical Computing Machine)

Relevance of TMs: Church-Turing Thesis

LCMs can do anything that could be described as "rule of thumb" or "purely mechanical".

- Historically: A lot of models are equivalent to TMs (i.e., they describe the same set of algorithms)
  - Lambda Calculus
  - Partially Recursive Functions
  - …
- Practically: All known computer systems are equivalent to TMs.

Relevance of TMs: How efficient are TMs?

All reasonable models of computation are polynomially related to the TM wrt. their time performance.

This is established by simulation arguments…

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A Pentium 4 can be simulated by a TM with runtime n^k for some fixed k.

Relevance of TMs: Extended Church-Turing Thesis

All reasonable models of computation are polynomially related to the TM wrt. their time performance.

This is established by simulation arguments…
…. it's a thesis – not a theorem.

DNA-Computing

Quantum-Computing

Relevance of TMs: Extended Church-Turing Thesis

TMs can simulate real computers efficiently

TMs have a mathematically simple structure

TMs are the ideal vehicle to build a Theory on Efficient Computability

This Lecture

- Definition of TMs
- Execution of TMs
- Multi-Tape TMs
- Non-Deterministic TMs