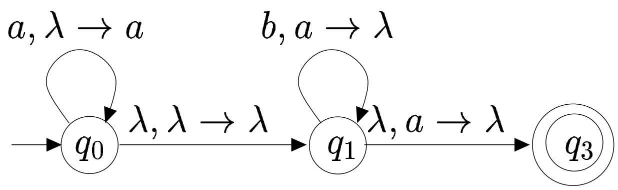
CSCI 2400 – Models of Computation

Solutions to the Practice Midterm Exam 2

Problem 1.



The initial stack symbol is \$. State q_0 reads the a's and pushes them into the stack. State q_1 reads the b's and pops an a from the stack for each input b. Finally, state q_3 is the accept state which the automaton enters only if there is an a in the stack, which means that the numbers of a's was more than the number of b's.

Problem 2.

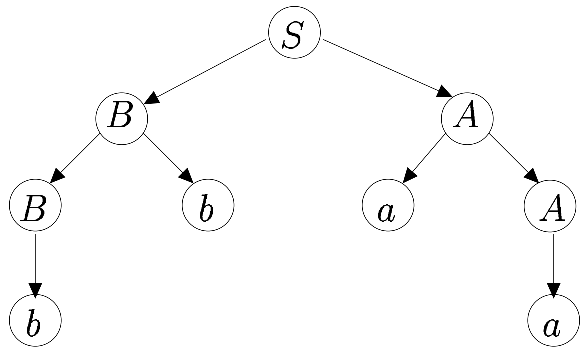
(a)

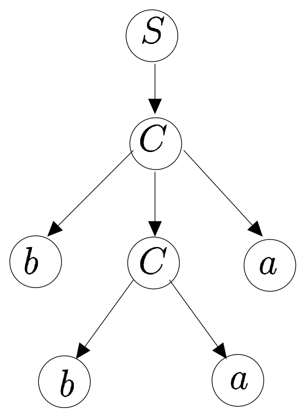
$$S \to aSa|bSb|A$$
$$A \to aAb|\lambda$$

(b)

$$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abAba \Rightarrow abaAbba \Rightarrow abaaAbbba \Rightarrow abaabbba$$

Problem 3. Yes, the grammar is ambiguous. The reason is that there is string generated by the grammar that has two different derivation trees. This string is bbaa. The two derivation trees are:





Problem 4.

$$\begin{aligned}
&S \to AV_1\\
&V_1 \to T_bV_2\\
&V_2 \to BT_a\\
&A \to AV_3\\
&V_3 \to BT_a\\
&A \to a\\
&B \to BV_4\\
&V_4 \to T_aA\\
&B \to b\\
&T_a \to a\\
&T_b \to b
\end{aligned}$$

Problem 5. Let m be the parameter of the pumping Lemma. We choose to pump the string $a^m b^m c^m$.

The string vxy has length at most m. Notice that v cannot be simultaneously in a^m and b^m, since if we repeat v (we take v^2xy^2) then a's are mixed with b's and the resulting string is not in the language L. Similarly, v cannot be simultaneously in b^m and c^m. A similar observation holds for y.

Now let's consider the case where vxy is completely within a^m. If we pump up the string vxy and we take v^2xy^2, then the resulting string has the form $a^{m+k}b^m c^m$, for some $k > 0$ (since $|vy| \geq 1$), which is not in the language L. For similar reasons vxy cannot be completely within b^m. If the vxy is completely within c^m then if we take uv^0xy^0z, then the resulting string has the form $a^m b^m c^{m-k}$, which clearly is not in the language.

Now we consider the case where vxy is such that v is completely within a^m, and y is completely within b^m. If pump up the string vxy once and we obtain the string v^2xy^2, then the resulting string has the form $a^{m+k_1}b^{m+k_2}c^m$, with $k_1 + k_2 \geq 1$ (since $|vy| \geq 1$), which is not in the language L. In the case where vxy is such that v is completely within b^m, and y is completely within c^m, we take the the string uv^0xy^0z, which gives $a^m b^{m-k_1} c^{m-k_2}$, which clearly is not in the language.

Therefore, in all cases the resulting string is not in the language L, and therefore the language L is not context-free.

Problem 6.

We know that the language $L_1 = \{ww^R : w \in \{a,b\}^*\}$ is context-free, since it is generated by the context-free grammar:

$$S \to aSa|bSb|\lambda$$

The language $L_2 = \{abbaabba\}$ is regular, since it is described by the regular expression $abbaabba$. Therefore, the language $\overline{L_2}$ is regular too (the complement of a regular language is regular). We have:

$$L = L_1 \cap \overline{L}_2.$$

From Theorem 8.5 of the book, we have that the intersection of a context-free language with a regular language is context-free. Therefore, the language L is context-free.