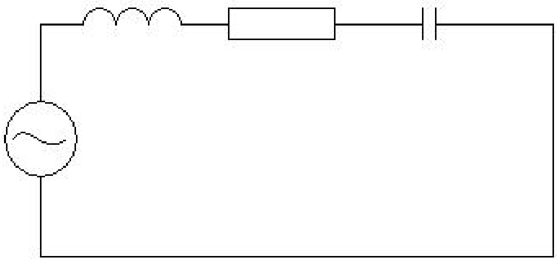

$$i' = \frac{1}{L}(u - R \cdot i - u_c) \tag{1}$$

$$u_c' = \frac{1}{C} i \tag{2}$$

We derive $u_c'$ from equation [2] and we obtain

$$u_c'' = \frac{1}{C} i' \tag{3}$$

then we replace variable $i'$ from [1] into [3] and we get

$$u_c'' = \frac{1}{C} \cdot \frac{1}{L}(u - R \cdot i - u_c) \tag{4}$$

Repeatedly, we substitute variable $i$ from [2] into equation [4]

$$u_c'' = \frac{1}{C} \cdot \frac{1}{L}(u - R \cdot C \cdot u_c' - u_c) \tag{5}$$

and we modify it into form

$$L \cdot C \cdot u_c'' + R \cdot C \cdot u_c' + u_c = u \tag{6}$$

We set concrete numerical values
$L = 7,5 \cdot 10^{-3} H$, $C = 10^{-6} F$, $R = 200\Omega$, $u = \sin(2000)t$

$$L \cdot C = 7,5 \cdot 10^{-9}$$

$$R \cdot C = 2 \cdot 10^{-4}$$

Firstly, we replace variables in equation [6]

$$7,5 \cdot 10^{-9} u_c'' + 2 \cdot 10^{-4} u_c' + u_c = \sin(2000)t \tag{7}$$

which leads to characteristic equation

$$7,5 \cdot 10^{-9}\lambda^2 + 2 \cdot 10^{-4}\lambda + 1 = 0$$

$$\lambda_{1,2} = \frac{-2 \cdot 10^{-4} \pm \sqrt{4 \cdot 10^{-8} - 4 \cdot 7,5 \cdot 10^{-9}}}{2 \cdot 7,5 \cdot 10^{-9}}$$

$$\lambda_{1,2} = \frac{-2 \cdot 10^{-4} \pm 10^{-4}}{1,5 \cdot 10^{-8}}$$

$$\lambda_1 = -\frac{2}{3}10^4, \ \lambda_2 = -2 \cdot 10^4. \tag{8}$$

Homogeneous solution for **real roots** is given by

$$u_{ch} = C_1.e^{-\frac{2}{3}10^4 t} + C_2.e^{-2 \cdot 10^4 t} \tag{9}$$

The right-hand side of complex roots solution will be expressed by

$$u_{cp} = A\sin(2000)t + B\cos(2000)t \tag{10}$$

and before replacement we derive variable $u_{cp}$ to $u'_{cp}$ a $u''_{cp}$ using equation [10]

$$u'_{cp} = A \cdot 2000 \cdot \cos(2000)t - B \cdot 2000 \cdot \sin(2000)t \tag{11}$$

$$u''_{cp} = -A \cdot 4 \cdot 10^6 . \sin(2000)t - B \cdot 4 \cdot 10^6 \cos(2000)t \tag{12}$$

Now we substitute equations [10], [11] and [12] into [7] and we obtain

$$7,5 \cdot 10^{-9}[-4A \cdot 10^6 \cdot \sin(2000)t - 4B \cdot 10^6 \cdot \cos(2000)t] + 2 \cdot 10^{-4}[2000A \cdot \cos(2000)t -$$
$$-2000B \cdot \sin(2000)t] + A \cdot \sin(2000)t + B \cdot \cos(2000)t = \sin(2000)t \tag{13}$$

We compare coefficients on left-hand and right-hand side of equation [13] and we get system of two linear equations

$$7,5 \cdot 10^{-9} \cdot (-4)A \cdot 10^6 + 2 \cdot 10^{-4}(-2000)B + A = 1$$

$$7,5 \cdot 10^{-9} \cdot (-4)B \cdot 10^6 + 2 \cdot 10^{-4} \cdot 2000A + B = 0$$

and we calculate

$$0,97 \cdot A - 0,4 \cdot B = 1$$

$$0,4 \cdot A + 0,97 \cdot B = 0$$

$$A = -2,425 \cdot B$$
$$0,97 \cdot (-2,425)B - 0,4 \cdot B = 1$$
$$-2,75225 \cdot B = 1$$

$$B \doteq -0,36333909$$
$$A \doteq 0,88109728$$

General solution is given by

$$u_c = u_{ch} + u_{cp}$$
$$u_c = C_1 \cdot e^{-\frac{2}{3}10^4 t} + C_2 \cdot e^{-2 \cdot 10^4 t} +$$
$$+0,88109728 \sin(2000t) - 0,36333909 \cos(2000t) \tag{14}$$

Initial values are set to zero.

$$u_c(0) = 0 \qquad i(0) = 0 \quad \Longrightarrow \quad u_c^{'}(0) = 0$$

By substituting condition $u_c(0) = 0$ into [14] we calculate

$$0 = C_1 + C_2 - 0,36333909 \tag{15}$$

and before using substitution $u_c^{'}(0) = 0$ we derive equation [14]

$$u_c^{'} = (-\frac{2}{3}10^4)C_1 e^{-\frac{2}{3}10^4 t} - 2 \cdot 10^4 C_2 e^{-2 \cdot 10^4 t} +$$
$$+0,88109728 \cdot 2000 \cdot cos(2000t) + 0,36333909 \cdot 2000 \cdot sin(2000t) \tag{16}$$

Now we substitute $u_c^{'}(0) = 0$ into [16]

$$0 = (-\frac{2}{3}10^4)C_1 - 2 \cdot 10^4 C_2 + 1762,19456 \tag{17}$$

and we get system of two linear equations from [15] and [17]

$$C_1 + C_2 = 0,36333909$$
$$\frac{2}{3}10^4 \cdot C_1 + 2 \cdot 10^4 C_2 = 1762,19456$$

i. e. we calculate

$$C_1 = \frac{\begin{vmatrix} 0,36333909 & 1 \\ 1762,19456 & 2 \cdot 10^4 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ \frac{2}{3}10^4 & 2 \cdot 10^4 \end{vmatrix}} = \frac{5504,58724}{2 \cdot 10^4 - \frac{2}{3}10^4} = 0,412844043$$

$$C_2 = \frac{\begin{vmatrix} 1 & 0,36333909 \\ \frac{2}{3}10^4 & 1762,19456 \end{vmatrix}}{2 \cdot 10^4 - \frac{2}{3}10^4} = -0,049504953$$

Finally, we substitute $C_1$ and $C_2$ into [14] and the result is given by

$$\mathbf{u_c} = \mathbf{0,412844043} \cdot \mathbf{e}^{-\frac{2}{3}\mathbf{10^4 t}} - \mathbf{0,049504953} \cdot \mathbf{e}^{-\mathbf{2 \cdot 10^4 t}} + \\ + \mathbf{0,88109728} \sin(\mathbf{2000t}) - \mathbf{0,36333909} \cos(\mathbf{2000t}) \tag{18}$$

For calculating variable $i$ we modify equation [2] into

$$i = C \cdot u_c^{'} \tag{19}$$

and the first we derive $u_c$ from [18]

$$u_c^{'} = 0,412844043(-\frac{2}{3}10^4)e^{-\frac{2}{3}10^4 t} - 0,049504953(-2 \cdot 10^4)e^{-2 \cdot 10^4 t} + \\ + 0,88109728 \cdot 2000 \cos(2000t) + 0,36333909 \cdot 2000 \sin(2000t)$$

$$u_c^{'} = -2752,29362 \cdot e^{-\frac{2}{3}10^4 t} + 990,09906 \cdot e^{-2 \cdot 10^4 t} + \\ + 1762,19456 \cdot \cos(2000t) + 726,67818 \cdot \sin(2000t) \tag{20}$$

Then we substitute $u_c^{'}$ from [20] into [19] and the result is

$$i = 10^{-6}[-2752,29362 \cdot e^{-\frac{2}{3}10^4 t} + 990,09906 \cdot e^{-2\cdot 10^4 t} +$$

$$+726,67818 \cdot \sin(2000t) + 1762,19456 \cdot \cos(2000t)]$$

$$\mathbf{i = -2,75229362 \cdot 10^{-3} e^{-\frac{2}{3}10^4 t} + 0,99009906 \cdot 10^{-3} e^{-2t} +}$$

$$\mathbf{+0,72667818 \cdot 10^{-3} \sin(2000t) + 1,76219456 \cdot 10^{-3} \cos(2000t)} \quad (21)$$