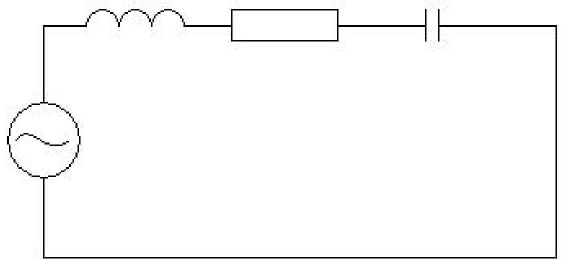

$$i' = \frac{1}{L}(u - R \cdot i - u_c) \tag{1}$$

$$u_c' = \frac{1}{C} i \tag{2}$$

We derive u_c' from equation [2] and we obtain

$$u_c'' = \frac{1}{C} i' \tag{3}$$

then we replace variable i' from [1] into [3] and we get

$$u_c'' = \frac{1}{C} \cdot \frac{1}{L}(u - R \cdot i - u_c) \tag{4}$$

Repeatedly, we substitute variable i from [2] into equation [4]

$$u_c'' = \frac{1}{C} \cdot \frac{1}{L}(u - R \cdot C \cdot u_c' - u_c) \tag{5}$$

and we modify it into form

$$L \cdot C \cdot u_c'' + R \cdot C \cdot u_c' + u_c = u \tag{6}$$

We set concrete numerical values
$L = 2,5 \cdot 10^{-2} H$, $C = 5 \cdot 10^{-5} F$, $R = 20 \Omega$, $u = \sin(1000)t$

$$L \cdot C = \frac{5}{4} \cdot 10^{-6}$$

$$R \cdot C = 10^{-3}$$

Firstly, we replace variables in equation [6]

$$\frac{5}{4} \cdot 10^{-6} u_c'' + 10^{-3} u_c' + u_c = \sin(1000)t \tag{7}$$

which leads to characteristic equation

$$\frac{5}{4}\cdot 10^{-6}\lambda^2 + 10^{-3}\lambda + 1 = 0$$

$$\lambda_{1,2} = \frac{10^{-3} \pm \sqrt{10^{-6} - 4\cdot\frac{5}{4}\cdot 10^{-6}}}{2\cdot\frac{5}{4}\cdot 10^{-6}}$$

$$\lambda_{1,2} = \frac{10^{-3}}{2,5\cdot 10^{-6}} \pm \frac{\sqrt{10^{-6} - 5\cdot 10^{-6}}}{2,5\cdot 10^{-6}}$$

$$\lambda_{1,2} = -0.4\cdot 10^{3} \pm \frac{2\cdot 10^{-3}}{2,5\cdot 10^{-6}}$$

with complex roots

$$\lambda_{1,2} = -400 \pm j\cdot 800. \tag{8}$$

Homogeneous solution for **complex roots** is given by

$$u_{ch} = e^{-400t}(C_1\cdot\cos(800)t + C_2\cdot\sin(800)t) \tag{9}$$

The right-hand side of complex roots solution will be expressed by

$$u_{cp} = A\sin(1000)t + B\cos(1000)t \tag{10}$$

and before replacement we derive variable u_{cp} to u'_{cp} a u''_{cp} using equation [10]

$$u'_{cp} = A\cdot 1000\cdot\cos(1000)t - B\cdot 1000\cdot\sin(1000)t \tag{11}$$

$$u''_{cp} = -A\cdot 10^{6}\cdot\sin(1000)t - B\cdot 10^{6}\cdot\cos(1000)t \tag{12}$$

Now we substitute equations [10], [11] and [12] into [7] and we obtain

$$\begin{gathered}\tfrac{5}{4}\cdot 10^{-6}[-A\cdot 10^{6}\cdot\sin(1000)t - B\cdot 10^{6}\cdot\cos(1000)t]+\\ +10^{-3}[A\cdot 1000\cdot\cos(1000)t - B\cdot 1000B\cdot\sin(1000)t]+\\ +A\cdot\sin(1000)t + B\cdot\cos(1000)t = \sin(1000)t\end{gathered} \tag{13}$$

We compare coefficients on left-hand and right-hand side of equation [13] and we get system of two linear equations

$$\frac{5}{4}\cdot 10^{-6}\cdot(-A\cdot 10^{6}) + 10^{-3}(-B\cdot 1000) + A = 1$$

$$\frac{5}{4} \cdot 10^{-6} \cdot (-B \cdot 10^{6}) + 10^{-3}(A \cdot 1000) + B = 0$$

and we calculate

$$-\frac{5}{4} \cdot A - B + A = 1$$
$$A - \frac{5}{4} \cdot B + B = 0$$

$$A = \frac{1}{4} \cdot B$$
$$-\frac{5}{16} \cdot B - B + \frac{1}{4} \cdot B = 1$$
$$-\frac{17}{16} \cdot B = 1$$

$$B = -\frac{16}{17}$$
$$A = -\frac{4}{17}$$

General solution is given by

$$u_c = u_{ch} + u_{cp}$$
$$\begin{aligned} u_c \;&=\; e^{-400t}[C_1 \cdot \cos(800)t + C_2 \cdot \sin(800)t] + \\ &\quad -\tfrac{4}{17}\sin(1000)t - \tfrac{16}{17}\cos(1000)t \end{aligned} \tag{14}$$

Initial values are set to zero.

$$u_c(0) = 0 \qquad i(0) = 0 \quad \Longrightarrow \quad u_c^{'}(0) = 0$$

By substituting condition $u_c(0) = 0$ into [14] we calculate

$$0 = C_1 - \frac{16}{17}$$
$$C_1 = \frac{16}{17} \tag{15}$$

and before using substitution $u_c^{'}(0) = 0$ we derive equation [14]

$$\begin{aligned} u_c^{'} &= -400e^{-400t}[C_1 \cdot \cos(800)t + C_2 \cdot \sin(800)t] + e^{-400t}[-C_1 \cdot 800 \sin(800)t + \\ &\quad +C_2 \cdot 800 \cos(800)t] - \tfrac{4}{17} \cdot 1000 \cdot \cos(1000t) + \tfrac{16}{17} \cdot 1000 \cdot \sin(1000t) \end{aligned} \tag{16}$$

Now we substitute $u_c^{'}(0) = 0$ into [16]

$$0 = (-400) \cdot C_1 + 800 \cdot C_2 - \frac{4}{17} \cdot 1000$$

and we replace C_1 from equation [15]

$$0 = -400 \cdot \frac{16}{17} + 800 \cdot C_2 + \frac{4000}{17}$$

$$C_2 = \frac{13}{17} \tag{17}$$

Finally, we substitute C_1 and C_2 into [14] and the result is given by

$$\mathbf{u_c} = \mathbf{e^{-400t}}[\frac{\mathbf{16}}{\mathbf{17}} \cdot \cos(\mathbf{800})\mathbf{t} + \frac{\mathbf{13}}{\mathbf{17}} \cdot \sin(\mathbf{800})\mathbf{t}] - \frac{\mathbf{4}}{\mathbf{17}} \sin(\mathbf{1000})\mathbf{t} - \frac{\mathbf{16}}{\mathbf{17}} \cos(\mathbf{1000})\mathbf{t} \tag{18}$$

For calculating variable i we modify equation [2] into

$$i = C \cdot u_c^{'} \tag{19}$$

and the first we derive u_c from [18]

$$\begin{aligned} u_c^{'} &= -400 \cdot e^{-400t}[\frac{16}{17} \cdot \cos(800)t + \frac{13}{17} \cdot \sin(800)t] + \\ &\quad e^{-400t}[-\frac{16}{17} \cdot 800 \sin(800)t - \frac{13}{17} \cdot 800 \cos(800)t] + \\ &\quad -\frac{4}{17} \cdot 1000 \cos(1000)t + \frac{16}{17} \cdot 1000 \sin(1000)t \end{aligned}$$

$$u_c^{'} = e^{-400t}[\frac{-400 \cdot 16}{17} \cdot \cos(800)t + \frac{-400 \cdot 13}{17} \cdot \sin(800)t] +$$

$$e^{-400t}[\frac{-800\cdot 16}{17}\sin(800)t-\frac{800\cdot 13}{17}\cos(800)t]+$$
$$-\frac{4}{17}\cos(1000)t+\frac{16}{17}\sin(1000)t$$

$$\begin{aligned} u_c^{'} &= e^{-400t}[400\cdot\cos(800)t\cdot\tfrac{9}{17}-400\cdot\sin(800)t\cdot\tfrac{43}{17}]+ \\ &\quad +1000\cdot\tfrac{-4}{17}\cos(1000)t+1000\cdot\tfrac{-16}{17}\sin(1000)t \end{aligned} \tag{20}$$

Then we substitute $u_c^{'}$ from [20] into [19] and the result is

$$\mathbf{i}=-\frac{\mathbf{9}}{\mathbf{170}}\cdot\sin(\mathbf{800})\mathbf{t}+\frac{\mathbf{1}}{\mathbf{85}}\mathbf{e}^{-\mathbf{400t}}\cos(\mathbf{800})\mathbf{t}+\frac{\mathbf{4}}{\mathbf{85}}\sin(\mathbf{1000})\mathbf{t}-\frac{\mathbf{1}}{\mathbf{85}}\cos(\mathbf{1000})\mathbf{t} \tag{21}$$