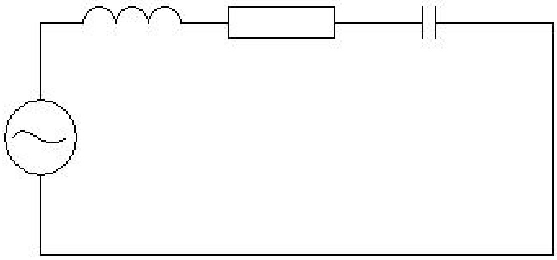

$$i' = \frac{1}{L}(u - Ri - u_c) \tag{1}$$

$$u_c' = \frac{1}{C}i \tag{2}$$

We derive u_c' from equation [2] and we obtain

$$u_c'' = \frac{1}{C}i' \tag{3}$$

then we replace variable i' from [1] into [3] and we get

$$u_c'' = \frac{1}{C}\frac{1}{L}(u - Ri - u_c). \tag{4}$$

Repeatedly, we substitute variable i from [2] into equation [4]

$$u_c'' = \frac{1}{C}\frac{1}{L}(u - RCu_c' - u_c) \tag{5}$$

and we modify it into form

$$LCu_c'' + RCu_c' + u_c = u. \tag{6}$$

We set concrete numerical values
$L = 10^{-2}H$, $C = 10^{-6}F$, $R = 200\Omega$, $u = \sin(1000)t$

$$L \cdot C = 10^{-8}$$

$$R \cdot C = 210^{-4}$$

Firstly, we replace variables in equation [6]

$$10^{-8}u_c'' + 2 \cdot 10^{-4}u_c' + u_c = \sin(1000)t \tag{7}$$

which give us characteristic equation

$$10^{-8}\lambda^2 + 2\cdot 10^{-4}\lambda + 1 = 0$$

$$\lambda_{1,2} = \frac{-2\cdot 10^{-4} \pm \sqrt{4\cdot 10^{-8} - 4\cdot 10^{-8}}}{2\cdot 10^{-8}}$$

$$\lambda_{1,2} = \frac{-2\cdot 10^{-4} \pm 0}{2\cdot 10^{-8}}$$

with the double root

$$\lambda_{1,2} = -10^4. \tag{8}$$

Homogeneous solution for **double root** is given by

$$u_{C_H} = e^{-10^4 t}(C_1 t + C_2). \tag{9}$$

The right-hand side of double root solution will be expressed by

$$u_{cp} = A\sin(1000)t + B\cos(1000)t \tag{10}$$

and before replacement we derive variable u_{cp} to u'_{cp} a u''_{cp} using equation [10]

$$u'_{cp} = A1000\cos(1000)t - B1000\sin(1000)t \tag{11}$$

$$u''_{cp} = -A10^6\sin(1000)t - B\cdot 10^6\cos(1000)t \tag{12}$$

Now we substitute equations [10], [11] and [12] into [7] and we obtain

$$10^{-8}[-A\cdot 10^6\cdot\sin(1000)t - B\cdot 10^6\cdot\cos(1000)t] + 2\cdot 10^{-4}[1000A\cdot\cos(1000)t-$$
$$-1000B\cdot\sin(1000)t] + A\cdot\sin(1000)t + B\cdot\cos(1000)t = \sin(1000)t \tag{13}$$

We compare coefficients on left-hand and right-hand side of equation [13] and we get system of two linear equations

$$10^{-8}(-A)\cdot 10^6 - 2\cdot 10^{-4}\cdot 1000B + A = 1$$

$$10^{-8}(-B)\cdot 10^6 + 2\cdot 10^{-4}\cdot 1000A + B = 0$$

and we calculate

$$0,99A - 0,2B = 1$$
$$0,2A + 0,99B = 0$$

$$A = -4,95B$$
$$0,99(-4,95)B - 0,2B = 1$$
$$-5,1005B = 1$$

$$B \doteq -0,19605921$$
$$A \doteq 0,97049309$$

General solution is given by

$$u_c = u_{ch} + u_{cp}$$

which in our case is

$$u_c = e^{-10^4 t}(C_1 \cdot t + C_2) + 0,97049309 \sin(1000)t - 0,196059209 \cos(1000)t \quad (14)$$

Initial values are set to zero.

$$u_c(0) = 0 \qquad i(0) = 0 \quad \Longrightarrow \quad u_c^{'}(0) = 0$$

By substituting condition $u_c(0) = 0$ into [14] we calculate

$$C_2 = 0,196059209 \quad (15)$$

and before using substitution $u_c^{'}(0) = 0$ we derive equation [14]

$$\begin{aligned} u_c^{'} &= (-10^4) \cdot e^{-10^4 t}(C_1 \cdot t + C_2) + e^{-10^4 t} \cdot C_1 + \\ &\quad +0,97049309 \cdot 1000 \cos(1000)t + 0,196059209 \cdot 1000 \sin(1000)t \end{aligned}$$

$$\begin{aligned} u_c^{'} &= (-10^4)\cdot e^{-10^4 t}(C_1\cdot t + C_2) + e^{-10^4 t}\cdot C_1 + \\ &+970,49309\cos(1000)t + 196,059209\sin(1000)t \end{aligned} \tag{16}$$

Now we substitute $u_c^{'}(0) = 0$ into [16]

$$0 = -10^4 C_2 + C_1 + 970,49309$$

and we replace C_2 from equation [15]

$$0 = -10^4\cdot 0,196059209 + C_1 + 970,49309$$

$$C_1 = 990,099002 \tag{17}$$

Finally, we substitute [15] and [17] into [14] and the result is given by

$$\begin{aligned} \mathbf{u_c} &= \mathbf{e^{-10^4 t}(990,099002t + 0,196059209)} + \\ &+\mathbf{0,97049309}\sin(\mathbf{1000})\mathbf{t} - \mathbf{0,196059209}\cos(\mathbf{1000})\mathbf{t} \end{aligned} \tag{18}$$

For calculating variable i we modify equation [2] into

$$i = C\cdot u_c^{'} \tag{19}$$

and the first we derive u_c from [18]

$$\begin{aligned} u_c^{'} &= -10^4 e^{-10^4 t}(990,099002t + 0,196059209) + e^{-10^4 t}990,099002 + \\ &+0,97049309\cdot 1000\cos(1000t) + 0,196059209\cdot 1000\sin(1000t) \end{aligned}$$

$$u_c^{'} = e^{-10^4 t}(9900990,02\cdot t - 2950,691029) + 196,059209\sin(1000t) + 970,49309\cos(1000t) \tag{20}$$

Then we substitute $u_c^{'}$ from [20] into [19] and the result is

$$i = 10^{-6}[e^{-10^4 t}(9900990,02\cdot t - 2950,691029) +$$

$$+196,059209\sin(1000t) + 970,49309\cos(1000t)]$$

$$\begin{aligned} \mathbf{i} &= \mathbf{e^{-10^4 t}(9,90099002t - 29,50691029\cdot 10^{-4})} + \\ &+\mathbf{1,96059209\cdot 10^{-4}}\sin(\mathbf{1000t}) + \mathbf{9,7049309\cdot 10^{-4}}\cos(\mathbf{1000t}) \end{aligned} \tag{21}$$