RC circuit

1 Problem

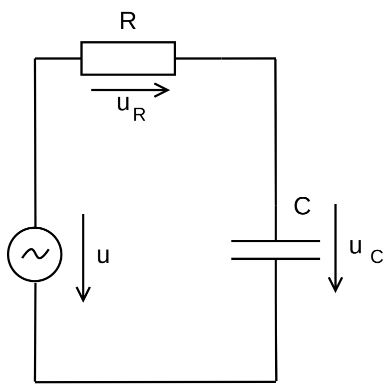

Figure 1:

$$\begin{aligned} u &= \sin(\omega t) \\ u'_C &= \tfrac{i}{C} \quad \Rightarrow \quad i = u'_C C \end{aligned}$$

$$u = u_C + Ri = u_C + u'_C RC$$

$$u'_C = \frac{1}{RC}u - \frac{1}{RC}u_C, \quad a = \frac{1}{RC}$$

$$u'_C + au_C = au, \quad u_C(0) = 0 \tag{1}$$

1.1 Analytic solution

Finding solution of homogenous differential equation

$$u'_C + au_C = 0$$

that leads to **characteristic equation**

$$\begin{aligned} \lambda + a &= 0 \\ \lambda &= -a \end{aligned}$$

General solution of homogenous differential equation

$$u_C = K(t)e^{\lambda t} + u_{Cp}$$

solving particular integral (right-hand side of Eq. 1 - function $\sin(\omega t)$)

$$\begin{aligned} u_{Cp} &= A\sin(\omega t) + B\cos(\omega t) \\ u'_{Cp} &= A\omega\cos(\omega t) - B\omega\sin(\omega t) \end{aligned}$$

We substitute u_{Cp} a u'_{Cp} into Eq. (1)

$$A\omega\cos(\omega t) - B\omega\sin(\omega t) + Aa\sin(\omega t) + Ba\cos(\omega t) = a\sin(\omega t)$$

$$\begin{aligned} A\omega + Ba &= 0 \\ Aa - B\omega &= a \end{aligned}$$

$$\begin{aligned} A &= \tfrac{a^2}{a^2+\omega^2} \\ B &= -\tfrac{a\omega}{a^2+\omega^2} \end{aligned}$$

General solution of non-homogenous differential Eq. (1)

$$u_C = K(t)e^{-at} - \frac{a\omega}{a^2+\omega^2}\cos(\omega t) + \frac{a^2}{a^2+\omega^2}\sin(\omega t)$$

Substituting initial value $u_C(0) = 0$ we have **particular solution**

$$u_C = \frac{a\omega}{a^2+\omega^2}e^{-at} - \frac{a\omega}{a^2+\omega^2}\cos(\omega t) + \frac{a^2}{a^2+\omega^2}\sin(\omega t) \tag{2}$$

1.2 Homework

Find analytic solution of u_C for RC circuit with $u = \sin(\omega t)$, where $\omega =$ 1 rad/s. Compare analytic solution with numerical computation of differential equation u'_C in TKSL