MAT05

Největší spol. dělitel polynomů

$f = x^5 + x + 1$

$g = 2x^4 + x^3 + 2x^2 + 1$

$$(x^5+x+1) = (2x^4+x^3+2x^2+1)\cdot\left(\tfrac{1}{2}x - \tfrac{1}{4}\right) + \left(-\tfrac{3}{4}x^3 + \tfrac{1}{2}x^2 + \tfrac{1}{2}x + \tfrac{5}{4}\right)$$

$-\left(x^5 + \tfrac{1}{2}x^4 + x^3 + \tfrac{1}{2}x\right)$

$-\tfrac{1}{2}x^4 - x^3 - \tfrac{1}{2}x + 1$

$-\left(-\tfrac{1}{2}x^4 - \tfrac{1}{4}x^3 - \tfrac{1}{2}x^2 - \tfrac{1}{4}\right)$

$-\tfrac{3}{4}x^3 + \tfrac{1}{2}x^2 - \tfrac{1}{2}x + \tfrac{5}{4}$

$$(2x^4+x^3+2x^2+1) = \left(-\tfrac{3}{4}x^3 + \tfrac{1}{2}x^2 + \tfrac{1}{2}x + \tfrac{5}{4}\right)\cdot\left(-\tfrac{8}{3}x - \tfrac{28}{9}\right) + (x^2+x+1)\cdot\tfrac{44}{9}$$

$-\left(2x^4 - \tfrac{4}{3}x^3 - \tfrac{4}{3}x^2 - \tfrac{10}{3}x\right)$

$\tfrac{7}{3}x^3 + \tfrac{10}{3}x^2 + \tfrac{10}{3}x + 1$

$-\left(\tfrac{7}{3}x^3 - \tfrac{14}{9}x^2 - \tfrac{14}{9}x - \tfrac{35}{9}\right)$

$\tfrac{44}{9}x^2 + \tfrac{44}{9}x + \tfrac{44}{9}$

$$\left(-\tfrac{3}{4}x^3 + \tfrac{1}{2}x^2 + \tfrac{1}{2}x + \tfrac{5}{4}\right) = (x^2+x+1)\cdot\left(-\tfrac{3}{4}x + \tfrac{5}{4}\right)$$

$-\left(-\tfrac{3}{4}x^3 - \tfrac{3}{4}x^2 - \tfrac{3}{4}x\right)$

$\tfrac{5}{4}x^2 + \tfrac{5}{4}x + \tfrac{5}{4}$

$-\left(\tfrac{5}{4}x^2 + \tfrac{5}{4}x + \tfrac{5}{4}\right)$

$\underline{\underline{0}}$

$$\underline{\underline{NSD(f, g) = (x^2 + x + 1)}}$$

Pokračování

MAT05

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$$f = g \cdot \left(\frac{1}{2}x - \frac{1}{4}\right) + h$$
$$g = h \cdot \left(-\frac{8}{3}x - \frac{28}{9}\right) + d$$

$$d = g + h \cdot \left(\frac{8}{3}x + \frac{28}{9}\right) =$$
$$= g + \left(f - g\left(\frac{1}{2}x - \frac{1}{4}\right)\right)\left(\frac{8}{3}x + \frac{28}{9}\right) =$$
$$= g + f\left(\frac{8}{3}x + \frac{28}{9}\right) - g\left(\frac{1}{2}x - \frac{1}{4}\right)\left(\frac{8}{3}x + \frac{28}{9}\right) =$$
$$= f\left(\frac{8}{3}x + \frac{28}{9}\right) + g\left(1 - \left(\frac{1}{2}x - \frac{1}{4}\right)\cdot\left(\frac{8}{3}x + \frac{28}{9}\right)\right) =$$
$$= f\left(\frac{8}{3}x + \frac{28}{9}\right) + g\left(-\frac{4}{3}x^2 - \frac{8}{9}x + \frac{16}{9}\right) =$$
$$= \frac{44}{9}\left(x^2 + x + 1\right) = d$$

$$\left(x^2 + x + 1\right) = f \cdot \left(\frac{6}{11}x + \frac{7}{11}\right) + g \cdot \left(-\frac{3}{11}x^2 - \frac{3}{11}x + \frac{4}{11}\right)$$

Hornerovo schéma

najděte celočíselné kořeny polynomu

$$f = x^6 - 5x^4 + 3x^3 + 6x^2 - 7x + 2$$

p kořen $\Rightarrow p \mid 2$

$p \in \{\pm 1, \pm 2\}$

	1	0	-5	3	6	-7	2		
-1	1	-1	-4	7	-1	-6	8	X	$\Rightarrow$ -1 není kořen
1	1	1	-4	-1	5	-2	0	✓	$\Rightarrow$ 1 kořen
1	1	2	-2	-3	2	0			$\Rightarrow$ 1 2-násobný
1	1	3	1	-2	0				$\Rightarrow$ 1 3-násobný
1	1	4	5	3	X				
2	1	5	11	20	X				$\Rightarrow$ 2 není kořen
-2	1	1	-1	0	✓				$\Rightarrow$ -2 kořen
-2	1	-1	1	X					$\Rightarrow x^2 + x - 1$

$$\boxed{f = (x-1)^3 \cdot (x+2)\,(x^2+x-1)}$$