Jumping Pure Grammars

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Outline of this talk

- Based on

  Křivka, Z., Kučera, J., and Meduna, A.: Jumping Pure Grammars.
  In: The Computer Journal, 2018.

Contents of this talk:

Introduction

Motivation

- Classical grammars and automata work strictly continuously

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- Adaptation of classical models to work on strings discontinuously

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Let $z = uxv$. By using $x \to y$, G rewrites uxv to uyv.

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Let $z = uxv$. By using $x \rightarrow y$, G rewrites uxv to uyv.

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1. selects an occurrence of x in z;
2. erase x from z;
3. G inserts y anywhere in uv.

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2. derivations start from a starting string (axiom) σ;
3. every string they derive from σ belongs to the generated language.

Example: Classical Grammar

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Classical Grammar

1. Starting nonterminal S. Rules:

$$S \rightarrow a, S \rightarrow aa$$

Trivially, generated language is $\{a, aa\}$.

Example

Pure Grammars

1. Starting string S. Rules:

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   Generated language is $\{a\}^{+}$.

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   Then, generated language is $\{S, a, aa\}$.

2. Starting string a. Rules:

$$a \rightarrow aa$$

   Generated language is $\{a\}^+$.

3. Starting string aa. Rules:

$$a \rightarrow \varepsilon$$

   Generated language is $\{\varepsilon, a, aa\}$.

Example: Jumping Grammar

Example

Jumping Grammar

1. Starting nonterminal S. Rules:

$$S \rightarrow aS, S \rightarrow b$$

   Trivially, generated language is $\{a\}^*\{b\}\{a\}^*$.

Example

Jumping Pure Grammar

1. Starting string ab. Rules:

$$a \rightarrow a$$

Generated language is $\{ab, ba\}$.

Preliminaries & Definitions

- For an alphabet of symbols Σ, Σ^* denotes the set of all strings over Σ.
- Algebraically, Σ^* represents the free monoid generated by Σ under concatenation.
- The unit of Σ^* is denoted by ε (the empty string).
- $\Sigma^+ = \Sigma^* - \{\varepsilon\}$.
- Any $L \subseteq \Sigma^*$ is a language over Σ.
- Let $a \in \Sigma$ and $w \in L$,
  - $|w|$ denotes the length of w and
  - $|w|_a$ denotes the number of occurrences of a in w.
- **REG** $\subset$ **CF** $\subset$ **CS**
  - **REG**, **CF**, and **CS** denote the families of regular, context-free, and context-sensitive languages, respectively.

Definition (Pure Grammars)

A pure grammar (PG for short) is a triplet, $G = (\Sigma, P, \sigma)$, where

- Σ is an alphabet;
- P is a finite relation from Σ^+ to Σ^*;
- $\sigma \in \Sigma^+$ is the start string.

Any member $(x, y) \in P$ is called a rule and written as $x \rightarrow y$

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Definition (Propagating Pure Grammars)

If for every $x \rightarrow y \in P$, $y \neq \varepsilon$, G is propagating.

Definition (Context-Free Pure Grammars)

If for every $x \rightarrow y \in P$, $|x| = 1$, G is context-free (CFPG for short).

Definition (Derivation Modes)

Let $u, v \in \Sigma^*$.
Derivation step according to a mode:

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   (a) right mode: $uxwv \,{}_{j}\!\!\Rightarrow uwyv$ in G iff $x \rightarrow y \in P$ or
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3. Parallel mode: $u \ {}_p\!\Rightarrow v$ in G iff there exist $x_1 \rightarrow y_1, x_2 \rightarrow y_2, \ldots, x_n \rightarrow y_n \in P$ such that $u = x_1 x_2 \cdots x_n$ and $v = y_1 y_2 \cdots y_n$, where $n \geq 0$;

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Definition (Generated Language)

For $h \in \{s, j, p, jp\}$, $L(G, {}_{h}\!\Rightarrow) = \{x \mid \sigma \;{}_{h}\!\Rightarrow^* x\}$.

Example (1)

Consider CFPG $G = (\{a, b, c, d\}, P, a)$ with

$$P = \{a \to abcd, a \to a, b \to b, c \to c, d \to d\}$$

Modes ${}_s\Rightarrow$ and ${}_p\Rightarrow$:

$$a \;{}_s\Rightarrow abcd \;{}_s\Rightarrow abcd \;{}_s\Rightarrow abcdbcd \;{}_s\Rightarrow abcdbcdbcd$$
$$a \;{}_p\Rightarrow abcd \;{}_p\Rightarrow abcd \;{}_p\Rightarrow abcdbcd \;{}_p\Rightarrow abcdbcdbcd$$

$L(G, {}_s\Rightarrow) = L(G, {}_p\Rightarrow) = \{a\}\{bcd\}^* \in$ **REG**

Modes ${}_j\Rightarrow$ and ${}_{jp}\Rightarrow$:

$$a \;{}_j\Rightarrow abcd \;{}_j\Rightarrow bacd \;{}_j\Rightarrow badc \;{}_j\Rightarrow bdabcdc$$
$$a \;{}_{jp}\Rightarrow abcd \;{}_{jp}\Rightarrow badc \;{}_{jp}\Rightarrow bdabcdc$$

$L(G, {}_j\Rightarrow) = L(G, {}_{jp}\Rightarrow) = \{w \mid |w|_a = 1, |w|_b = |w|_c = |w|_d\} \in$ **CS** $-$ **CF**

Example (2)

Our goal: Language $L = \{a\}^+ \cup \{b\}^+$ with $\Sigma = \{a, b\}$:

1. In classical CF grammar, G_1 with ${}_s\Rightarrow$:

Examples

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2. In CFPG, $G_2 = (\{a, b\}, P, \sigma)$ with ${}_s\Rightarrow$; $\sigma = ?$. P :

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   $a \rightarrow b$ or $c \rightarrow a$, $c \rightarrow b$

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3. In classical CF grammar with jumping, simply use G_1 with ${}_j\Rightarrow$. For instance,

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   $L(G_1, {}_j\Rightarrow) = \{a\}^+ \cup \{b\}^+$.

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Our goal: Language $L = \{a\}^+ \cup \{b\}^+$ with $\Sigma = \{a, b\}$:

3. In CFPG, $G_3 = (\{a, b\}, P, a)$ with ${}_p\Rightarrow$ and P:

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4. In CFPG grammar with ${}_{jp}\Rightarrow$, simply use G_3 with ${}_{jp}\Rightarrow$.

   For instance, $a\ {}_{jp}\Rightarrow b\ {}_{jp}\Rightarrow aa\ {}_{jp}\Rightarrow bbb\ {}_{jp}\Rightarrow aaa\ {}_{jp}\Rightarrow \cdots$
   $L(G_3, {}_{jp}\Rightarrow) = \{a\}^+ \cup \{b\}^+$.

J $=$ Jumping, **S** $=$ Sequential, **P** $=$ Parallel, **CF** $=$ Context-Free.
For a language family **X**, its propagating variant is $\mathbf{X}^{-\varepsilon}$.

1. **S** $= \{L(G, {}_s\!\Rightarrow) \mid G$ is a PG$\}$;
2. **JS** $= \{L(G, {}_j\!\Rightarrow) \mid G$ is a PG$\}$;
3. **P** $= \{L(G, {}_p\!\Rightarrow) \mid G$ is a PG$\}$;
4. **JP** $= \{L(G, {}_{jp}\!\Rightarrow) \mid G$ is a PG$\}$;
5. **SCF** $= \{L(G, {}_s\!\Rightarrow) \mid G$ is a CFPG$\}$;
6. **JSCF** $= \{L(G, {}_j\!\Rightarrow) \mid G$ is a CFPG$\}$;
7. **PCF** $= \{L(G, {}_p\!\Rightarrow) \mid G$ is a CFPG$\}$;
8. **JPCF** $= \{L(G, {}_{jp}\!\Rightarrow) \mid G$ is a CFPG$\}$;

Results

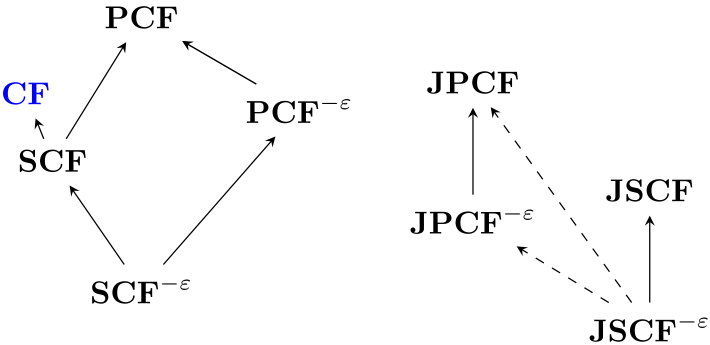
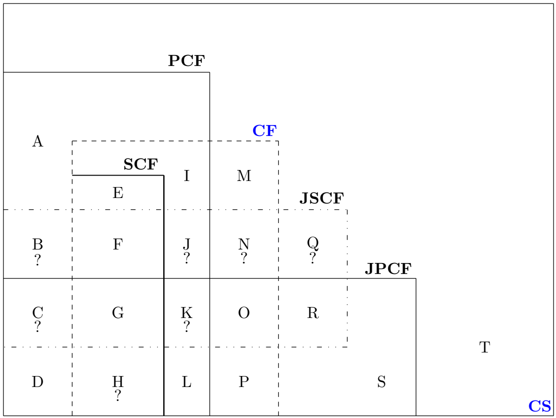


Figure: Dashed arrow = open problem. No connection = incomparability.

Note: Two language families X and Y are incomparable iff $X \not\subseteq Y$ and $Y \not\subseteq X$.


PCF
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D
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F
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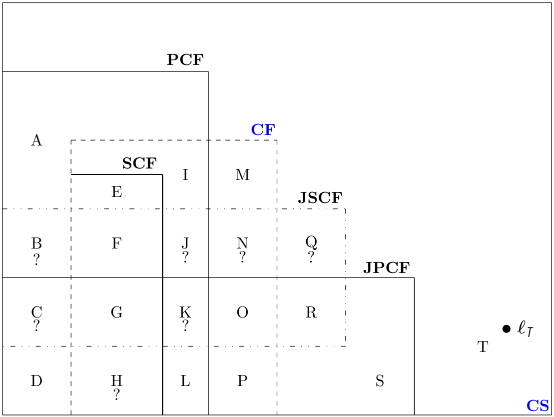
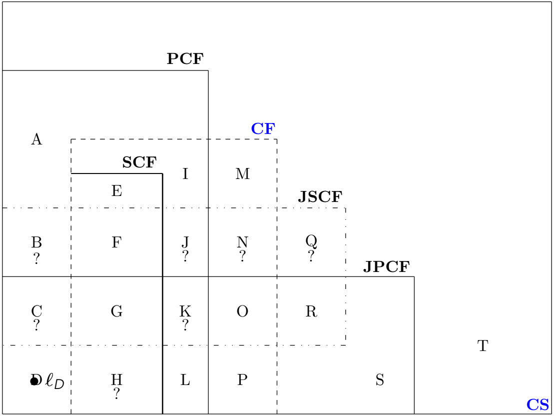
O
P
Q ?
R
S
T



$\ell_T = \{a^p \mid p \text{ is a prime}\} \in$ **CS** $-$ (**PCF** $\cup$ **CF** $\cup$ **JSCF** $\cup$ **JPCF**)

Idea $\ell_T \notin$ **JPCF**: $a \to \varepsilon \notin P$, so $\sigma = a^2$. We need $a^2 \ {}_{jp}\!\Rightarrow a^3$ by $a \to aa$ and $a \to a$, but we get a^4 as well.

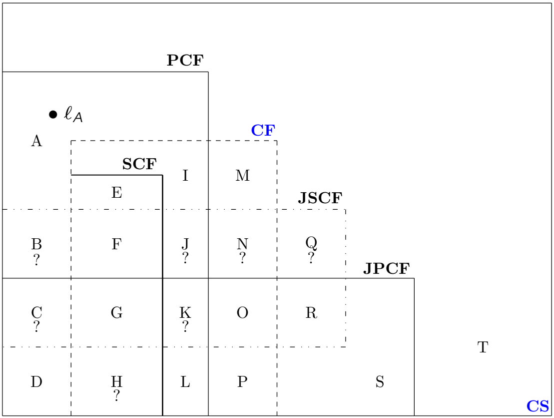
Then, $\mathbf{JPCF}_u = \mathbf{PCF}_u \supset \mathbf{JSCF}_u = \mathbf{SCF}_u$.



$\ell_D = \{a^{2^n} \mid n \geq 0\} = (\mathbf{PCF} \cap \mathbf{JPCF}) - (\mathbf{CF} \cup \mathbf{JSCF})$

Idea $\ell_D \in \mathbf{PCF} \cap \mathbf{JPCF}$: Take rule $a \to aa$ with $\sigma = a$.

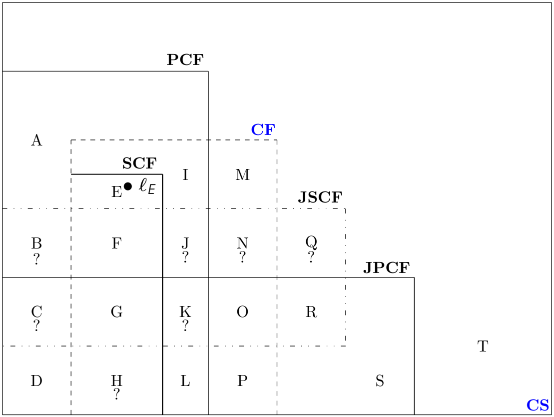
Idea $\ell_D \notin \mathbf{CF} \cup \mathbf{JSCF}$: ℓ_D is not semilinear.



$\ell_A = \{a^{2^n} b^{2^n} \mid n \geq 0\} = \mathbf{PCF} - (\mathbf{CF} \cup \mathbf{JSCF} \cup \mathbf{JPCF})$

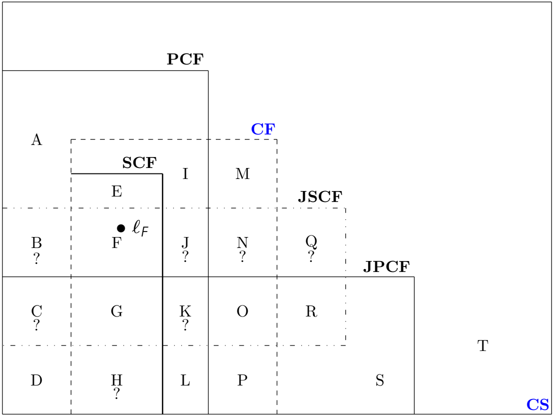
Idea $\ell_A \in$ **PCF**: Take rules $a \rightarrow aa$ and $b \rightarrow bb$ with $\sigma = ab$.

Idea $\ell_A \notin$ **JPCF**: Show the proof by contradiction.



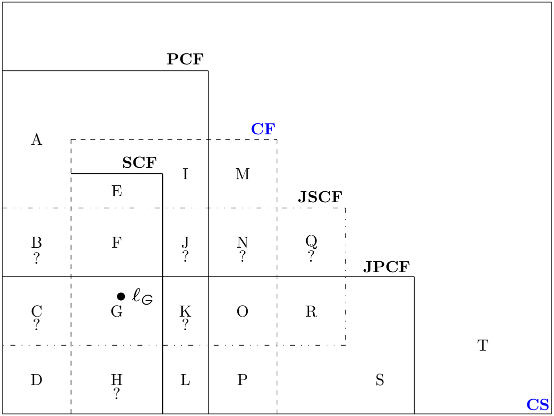
$\ell_E = \{a^n c b^n \mid n \geq 0\} \in$ **SCF** $-$ (**JSCF** $\cup$ **JPCF**)

Idea $\ell_E \in$ **SCF**: Take rule $c \rightarrow acb$ with $\sigma = c$.

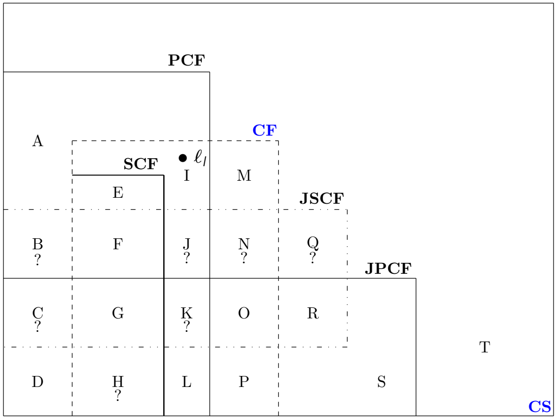


$\ell_F = \{aa, aab, aac, aabc\} \in (\mathbf{SCF} \cap \mathbf{JSCF}) - \mathbf{JPCF}$

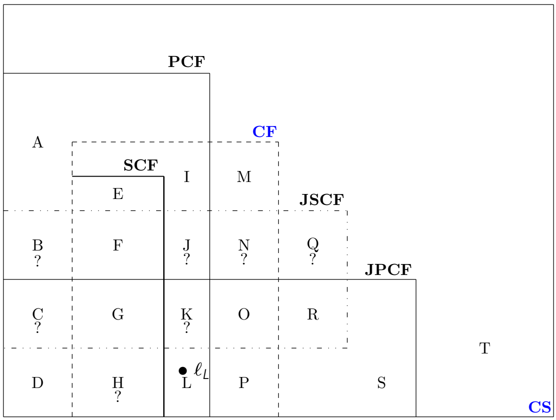
Idea for $\ell_F \in \mathbf{SCF} \cap \mathbf{JSCF}$: Take $\sigma = aabc$ and rules $b \to \varepsilon$ and $c \to \varepsilon$.



$$\ell_G = \{a\}^+ \in \mathbf{SCF} \cap \mathbf{JSCF} \cap \mathbf{JPCF}$$



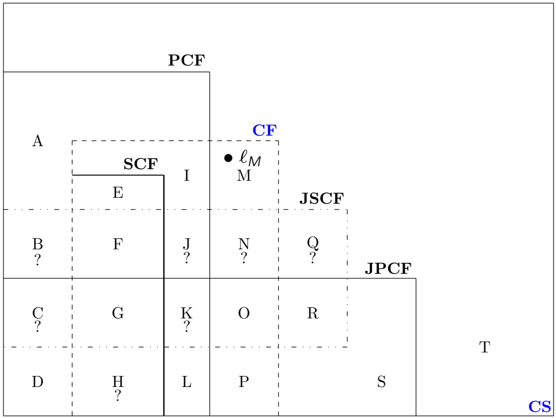
$$\ell_I = \{aabb, ccdd\} \in (\mathbf{PCF} \cap \mathbf{CF}) - (\mathbf{SCF} \cup \mathbf{JSCF} \cup \mathbf{JPCF})$$



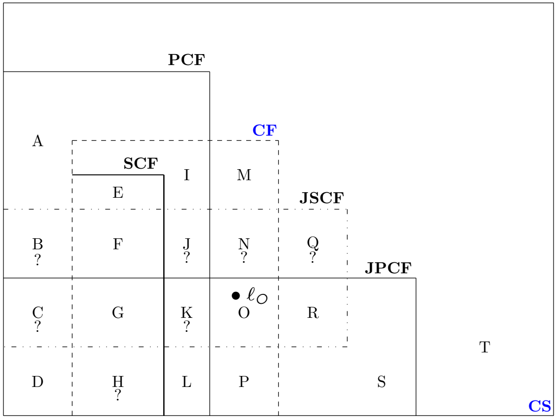
$\ell_L = \{ab, cd, dc\} \in$ (**PCF** $\cap$ **CF** $\cap$ **JPCF**) $-$ (**SCF** $\cup$ **JSCF**)

We need to rewrite two symbols in parallel such as with $a \to c, b \to d, c \to d, d \to c$ with $\sigma = ab$.

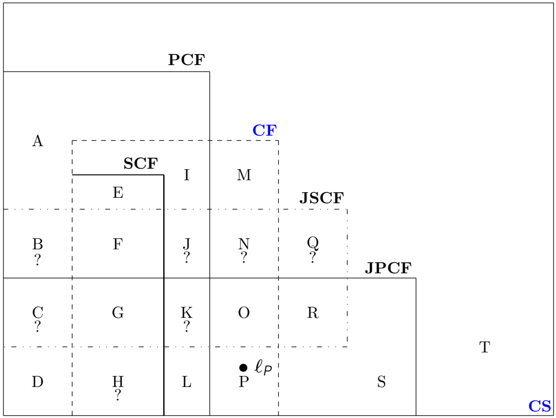
For instance, $ab \ {}_{p}\!\Rightarrow cd \ {}_{p}\!\Rightarrow dc$ or $ab \ {}_{jp}\!\Rightarrow dc$.



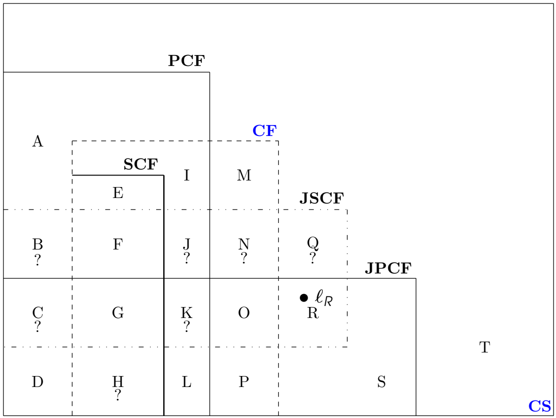
$$\ell_M = \{a^n b^n \mid n \geq 1\} \in \mathbf{CF} - (\mathbf{PCF} \cup \mathbf{JSCF} \cup \mathbf{JPCF})$$



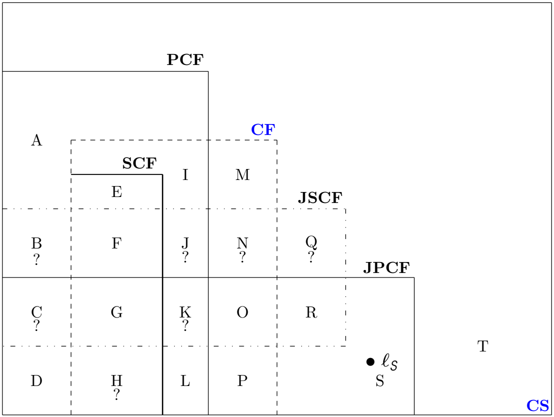
$\ell_O = \{aabb, abab, abba, baab, baba, bbaa\} \in$ (**CF** $\cap$ **JSCF** $\cap$ **JPCF**) $-$ **PCF**



$\ell_P = \{aabb, ccdd, cdcd, cddc, dccd, dcdc, ddcc\} \in$ $(\mathbf{CF} \cap \mathbf{JPCF}) - (\mathbf{PCF} \cup \mathbf{JSCF})$



$$\ell_R = \left\{ w \;\middle|\; \begin{array}{l} |w|_a - 1 = |w|_b = |w|_c, \\ w \in \{a,b,c\}^+ \end{array} \right\} \in (\mathbf{JSCF} \cap \mathbf{JPCF}) - (\mathbf{CF} \cup \mathbf{PCF})$$



$$\ell_S = \{\hat{a}\hat{b}\hat{c}\} \cup \left\{ w \;\middle|\; \begin{array}{l} |w|_a - 1 = |w|_b = |w|_c, \\ w \in \{a, b, c\}^+ \end{array} \right\} \in$$

JPCF $-$ (**CF** $\cup$ **PCF** $\cup$ **JSCF**)

$$
\begin{tikzcd}
\mathbf{PCF}_u^{-\varepsilon} \arrow[r, dash] \arrow[d] & \mathbf{JPCF}_u^{-\varepsilon} \arrow[d] \\
\mathbf{PCF}_u \arrow[r, dash] & \mathbf{JPCF}_u \\
\mathbf{SCF}_u \arrow[u] \arrow[r, dash] & \mathbf{JSCF}_u \arrow[u] \\
\mathbf{SCF}_u^{-\varepsilon} \arrow[u] \arrow[uuu, bend right] \arrow[r, dash] & \mathbf{JSCF}_u^{-\varepsilon} \arrow[u] \arrow[uuu, bend right]
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathbf{PCF}_u^{-\varepsilon} \arrow[r, dash] \arrow[d] & \mathbf{JPCF}_u^{-\varepsilon} \arrow[d] \\
\mathbf{PCF}_u \arrow[r, dash] & \mathbf{JPCF}_u \\
\mathbf{SCF}_u \arrow[r, dash] \arrow[u] & \mathbf{JSCF}_u \arrow[u] \\
\mathbf{SCF}_u^{-\varepsilon} \arrow[r, dash] \arrow[u] \arrow[uuu, bend right] & \mathbf{JSCF}_u^{-\varepsilon} \arrow[u] \arrow[uuu, bend right]
\end{tikzcd}
$$

Note: $\mathbf{SCF}_u$ and $\mathbf{PCF}_u^{-\varepsilon}$ are incomparable.

Conclusion

Conclusion, Future Work

- Open Problems
- Closure Properties
- Decidability (Emptiness, Universality, ...)
- Left-jumps and Right-jumps in Pure Grammars

Thank You For Your Attention!