Uniform Regulated Rewriting in Parallel

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Overview

- **Motivation**
- **Semi-Parallel Uniform Rewriting**
  - Scattered Context Grammars
  - Regulated Scattered Context Grammar
  - RE = SCAT(.2/4) = SCAT(2/4.)
- **Parallel Uniform Rewriting**
  - EIL Systems
  - Regulated EIL Grammar
  - RE = EIL(.2) = EIL(2.)

Motivation

- parallel grammars - big variety of producing strings
- negatively affecting theoretical and practical informatics
- **permutation-based form** strings
- semi-parallel language generation by **scattered context grammars**
- totally parallel generation of languages generated by **EIL grammars**

Scattered Context Grammar

$$G = (V, T, P, S)$$

- V is the total alphabet,
- T is a finite set of terminals, $T \subset V$,
- P is a finite set of productions in the form
$$(A_1, \dots, A_n) \longrightarrow (x_1, \dots, x_n)$$
where $n \geq 1, A_i \in V \setminus T, x_i \in V^*$ for all $1 \leq i \leq n$,
- S is the start symbol, $S \in V \setminus T$.

Note

If $x_i \in V^+$ for all $p \in P$, it is a propagating scattered context grammar.

Generative Power

Derivation Step

For every $u = u_1 A_1 u_2 A_2 \dots u_n A_n u_{n+1}$, $v = u_1 x_1 u_2 x_2 \dots u_n x_n u_{n+1}$ and $p \colon (A_1, A_2, \dots, A_n) \longrightarrow (x_1, x_2, \dots, x_n) \in P$,

$$u \Longrightarrow v \; [p] \text{ in } G.$$

Generated Language

$$L(G) = \{x \in T^* : S \Longrightarrow^* x\}$$

Generative Power

$$\mathcal{L}(REG) \subset \mathcal{L}(CF) \subset \mathcal{L}(PSC) \subseteq \mathcal{L}(CS) \subset \mathcal{L}(SC) = \mathcal{L}(RE)$$

Note

Context-free grammars are a special case of scattered context grammars where $n = 1$.

Example

1. $(S) \longrightarrow (aAbAcA)$
2. $(A, A, A) \longrightarrow (aA, bA, cA)$
3. $(A, A, A) \longrightarrow (\varepsilon, \varepsilon, \varepsilon)$

1. $(S) \longrightarrow (ABC)$
2. $(A, B, C) \longrightarrow (aA, bB, cC)$
3. $(A, B, C) \longrightarrow (a, b, c)$

$\underline{S} \Longrightarrow a\underline{A}b\underline{B}c\underline{C}$ [1] $\Longrightarrow aa\underline{A}bb\underline{B}cc\underline{C}$ [2] $\Longrightarrow aabbcc$ [3]

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Generated Language

$$L(G) = \{a^n b^n c^n \mid n > 0\}$$

Definition

Let $G = (N, T, P, S)$ be a **RE** grammar, and let $V = N \cup T$. Set

$$F(G) = \left\{x \in V^* | S \Rightarrow_G^+ x\right\}$$

and

$$\Delta(G) = \left\{x \in F(G)^* | x \Rightarrow_G^* y,\ y \in T^*\right\} \qquad \square$$

Regulated Scattered Context Grammar

$\textbf{SCAT}[.\textbf{i}/\textbf{j}] = \{\textbf{L} = L(G),$ where $G = (V, T, P, S)$ is a SC grammar such that $\Delta(G) \subseteq T^*(K)^*,$ where K is a finite language consisting of equally long strings with $card(K) = i$ and $card((K)) = j\}$

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Queue Grammar

Queue Grammar (Type 0 Generative Power)

- $R \subseteq (V \times (W - F)) \times (V^* \times W)$ is a finite relation
- for every $a \in V$
- $(a, b, x, c) \in R$
- $u, v \in V^*W$,
- $u = arb$
- $v = rxc$
- $a \in V; r, x \in V^*; b, c \in W$

$$u \Rightarrow v[(a, b, x, c)] \text{ in } G$$

$$u \Rightarrow v$$

Example

- $G = (V, T, W, F, R, g)$
- $V = \{S, A, a, b\}$
- $T = \{a, b\}$
- $W = \{Q, f\}$
- $F = \{f\}$
- $g = SQ$
- $R = \{p_1, p_2\}$
- $p_1 = (S, Q, Aaa, Q)$
- $p_2 = (A, Q, bb, f)$

$$g = SQ \Longrightarrow AaaQ\ [p_1] \Longrightarrow aabbf\ [p_2]$$

Lemma

Let $L \in \mathbf{RE}$. Then, there exists a queue grammar $Q = (V, T, W, F, R, g)$ satisfying these two properties

(i) $L = L(G)$;

(ii) *Q derives every $w \in L(Q)$ in this way*

$$\begin{aligned} g &\Rightarrow_Q^i a_1 u_1 b_1 \\ &\Rightarrow_Q u_1 x_1 y_1 c_1 \quad [(a_1, b_1, x_1 y_1, c_1)] \\ &\Rightarrow_Q^j y_1 z_1 d \end{aligned}$$

where $i, j \geq 1$, $w = y_1 z_1$, $x_1, u_1 \in V^*$, $y_1, z_1 \in T^*$, $b_1, c_1 \in W$ and $d \in F$.

RE = SCAT(.2/4) = SCAT(2/4.)

Lemma

Let $L \in \textbf{RE}$. *Then, there exists a scattered context grammar* $G = (\{A, B, C, D, S\} \cup T, T, P, S)$ *so that* $L(G) = (L)$ *and*

$$\Delta(G) \subseteq (\{A^t B^{n-t} C, A^t B^{n-t} D\})^* T^*$$

for some $t, n \geq 1$.

- *simulate Queue grammar by SCAT(.2/4) and SCAT(2/4.)*

Conclusion

- RE = SCAT(.2/4) = SCAT(2/4.)

EIL - extended $\langle k, l \rangle$L System

$$G = (V, T, P, S)$$

- V is the total alphabet,
- T is a finite set of terminals, $T \subseteq V$,
- P is a finite set of productions in the form

$$(e_1, a, e_2) \longrightarrow w$$

  with $a \in V$, $w, e_1, e_2 \in V^*$ such that $|e_1| \leq k$, $|e_2| \leq l$,
- S is the axiom, $S \in V^+$.

Note

Constants k and l represent the maximum size of the environment on the left and right side of the cell respectively.

Regulated EIL Grammar

$$\textbf{EIL[.j]} = \{\mathbf{L} = L(G), \text{ where } G = (V, T, P, S) \text{ is an EIL grammar such that } card((F(G)) - T) = j \text{ and } F(G) \subseteq T^*(w)^*, \text{ where } w \in (V - T)^*\}$$

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Proof of RE = EIL(.2) = EIL(2.)

Lemma

- *any EIL grammar G can by transformed to an equivalent grammar* $G' = (\{S, 0, 1\} \cup T, T, P, S)$ *so that for every* $x \in F(G')$,
$$x \in T^*(w)^*$$
or
$$x \in (w)^* T^*$$
where $w \in \{0, 1\}^*$.
- *RE = EIL(.2) = EIL(2.)*

Open questions

- Can we do this for propagating grammars?

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Thank You For Your Attention !