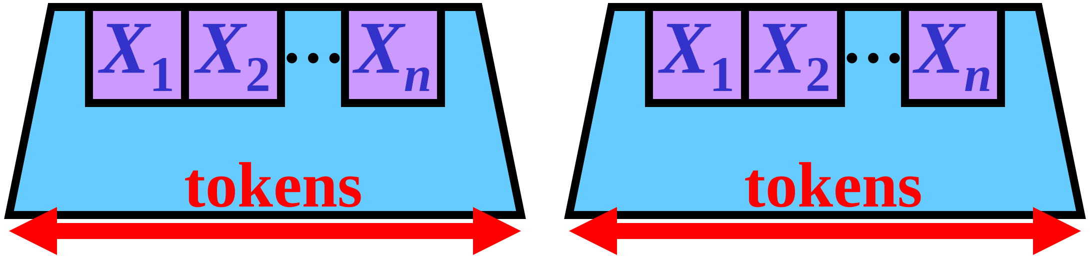
Bottom-Up Parsing

Bottom-Up Parsing: Problems

1) Two or more rules have the same *handle*

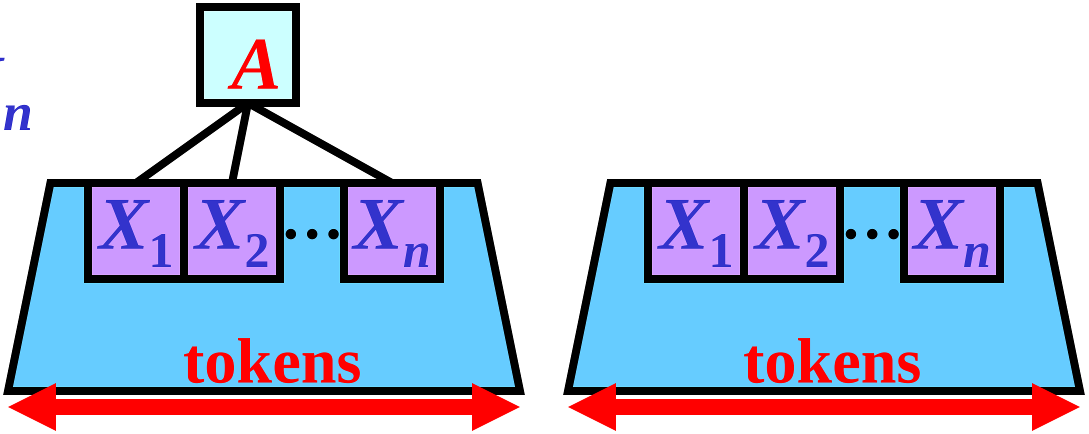


Note: A *handle* is the right-hand side of a rule.

Bottom-Up Parsing: Problems

1) Two or more rules have the same *handle*

r_1: $A \rightarrow X_1 X_2 \ldots X_n$



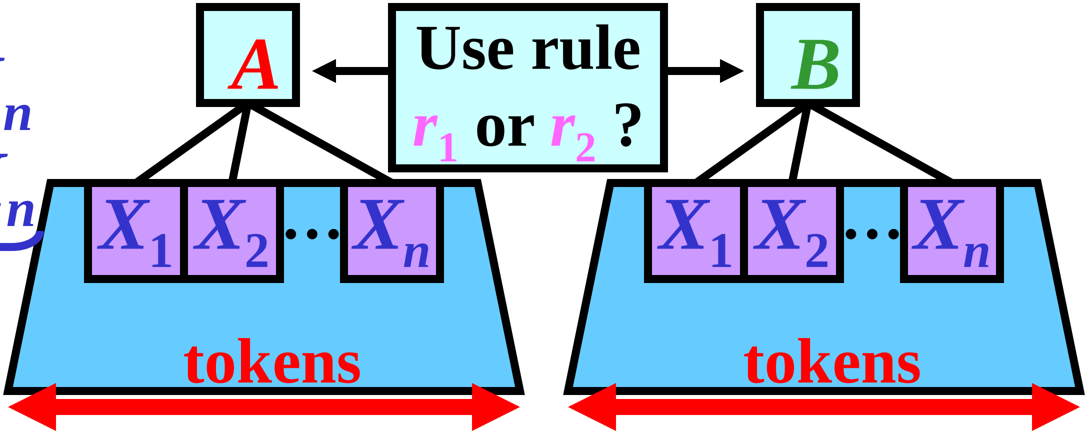
Note: A *handle* is the right-hand side of a rule.

Bottom-Up Parsing: Problems

1) Two or more rules have the same *handle*

r_1: $A \rightarrow X_1X_2\ldots X_n$

r_2: $B \rightarrow \underbrace{X_1X_2\ldots X_n}_{\text{handle}}$



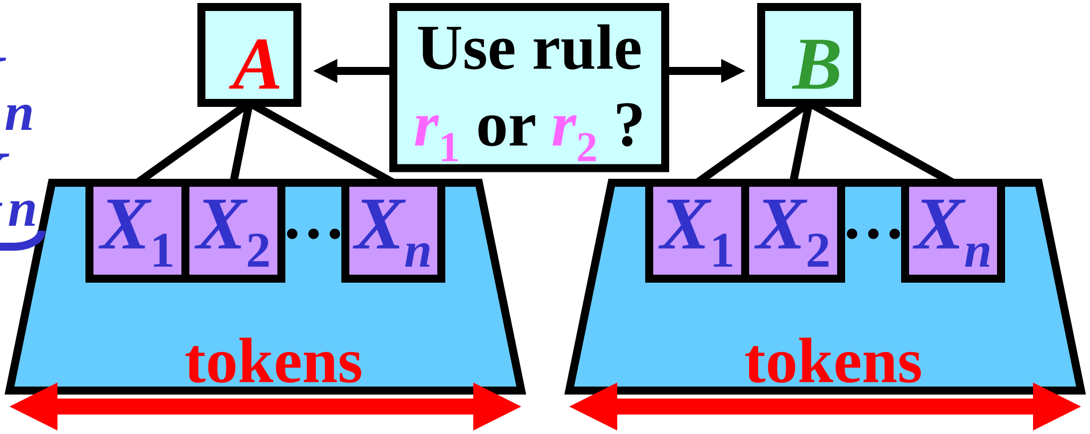
Note: A *handle* is the right-hand side of a rule.

Bottom-Up Parsing: Problems

1) Two or more rules have the same *handle*

r_1: $A \rightarrow X_1X_2\ldots X_n$

r_2: $B \rightarrow \underbrace{X_1X_2\ldots X_n}_{\text{handle}}$



Use rule r_1 or r_2 ?

Note: A *handle* is the right-hand side of a rule.

---

2) Ambiguous grammars

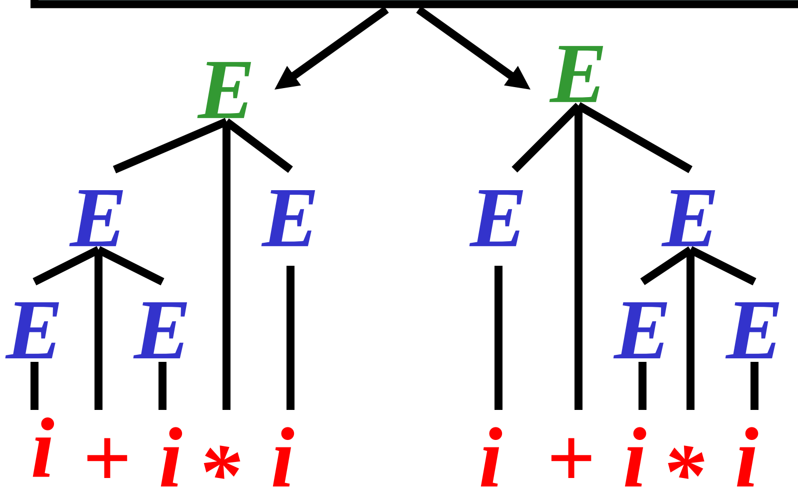
$G_{expr2} = (N, T, P, E)$, where

$N = \{E\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 1: $E \rightarrow E+E$, 2: $E \rightarrow E*E$,

3: $E \rightarrow (E)$, 4: $E \rightarrow i$ $\}$

Which of these tree to create?



Bottom-Up Parsers

1) Operator-precedence parser

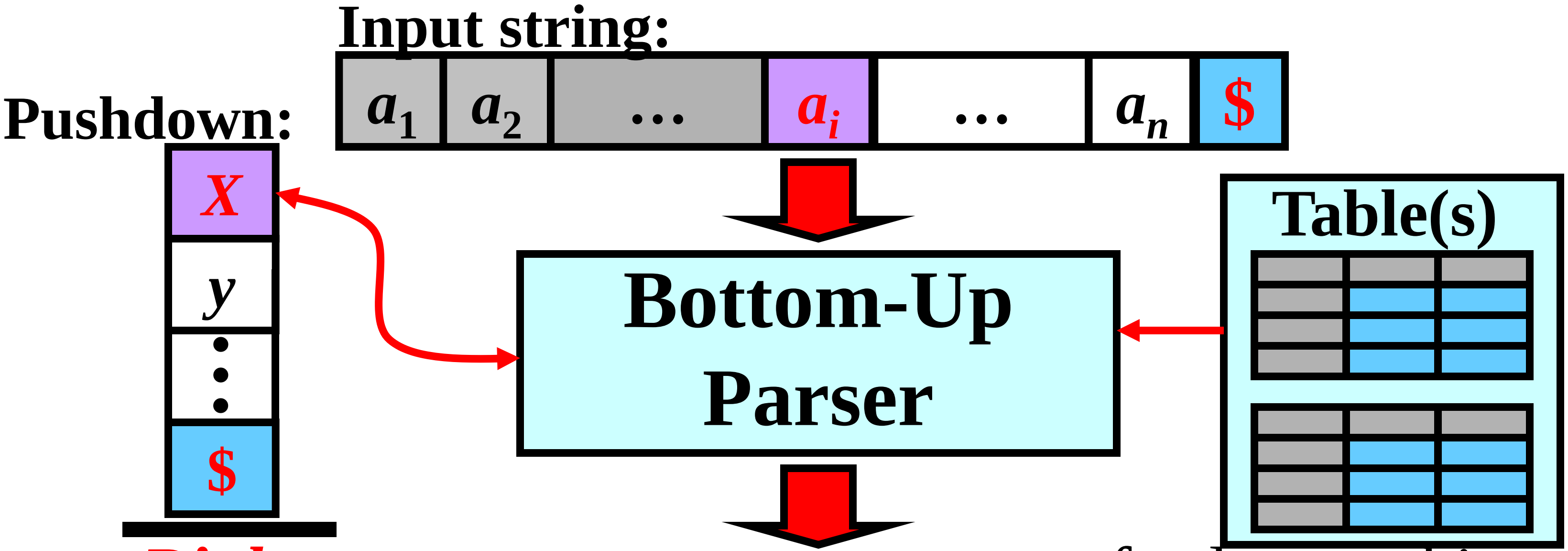
- the least powerful, but simple & easy-to-make

---

2) LR parser

- the most powerful

- **Model of Bottom-Up parser:**



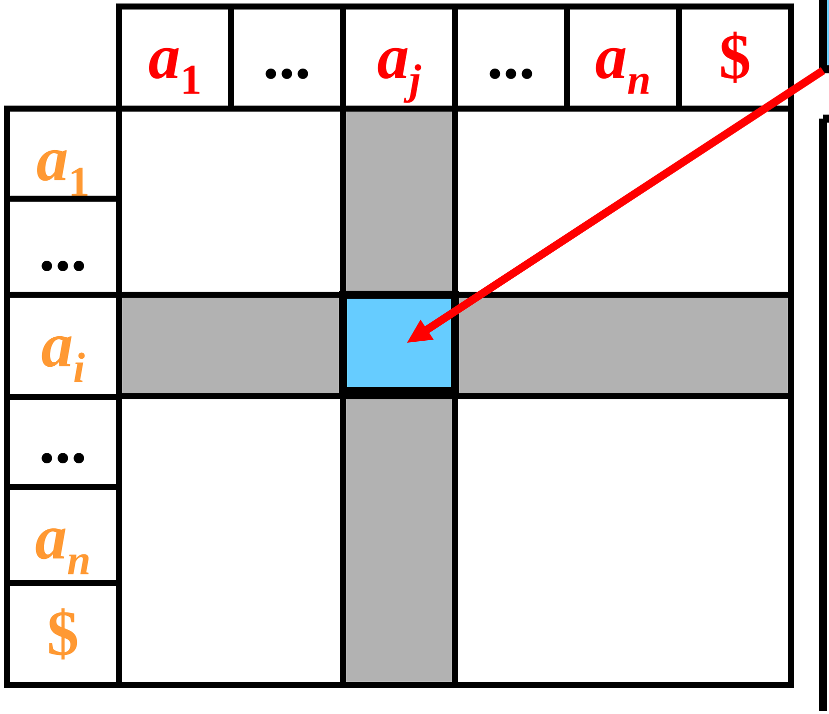
Right parse = **reverse** sequence of rules used in the **rightmost derivation** of the tokenized source program

Operator-Precedence Parser

- No two distinct nonterminals have the same handle
- No ε-rules.

- Let $G = (N, T, P, S)$ be CFG, where $T = \{a_1, a_2, \dots , a_n\}$

Precedence-table:

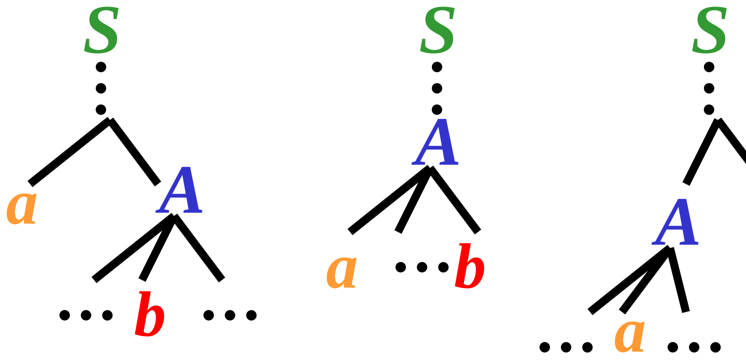


	a_1	…	a_j	…	a_n	\$
a_1						
…						
a_i						
…						
a_n						
\$						

Table[a_i , a_j] ∈ {<, =, >, *blank*}

Illustration of meaning of <, =, >:

$a < b$ $a = b$ $a > b$



Operator-Precedence Parser: Algorithm

- **Input:** Precedence-table for $G = (N, T, P, S)$; $x \in T^*$
- **Output:** Right parse of x if $x \in L(G)$; otherwise, **error**

- **Method:**

```
• Push $ onto the pushdown;
• repeat
    • let a = the topmost terminal on the pushdown and
          b = the current token
    • case Table[a, b] of:
        • = : push(b) & read next b from input string
        • < : replace a with a< on the pushdown &
              push(b) & read next b from input string
        • > : if <y is the pushdown top string and r: A → y ∈ P
              then replace <y with A & write r to output
              else error
        • blank : if a = $ and b = $ then success
                                          else error
  until success or error
```

Operator-Precedence Parser: Example

$G_{expr2} = (N, T, P, E)$, where $N = \{E\}$, $T = \{i, +, *, (,)\}$,
$P = \{\ 1: E \to E+E,\ 2: E \to E*E,\ 3: E \to (E),\ 4: E \to i\}$

Precedence-table for G_{expr2}:

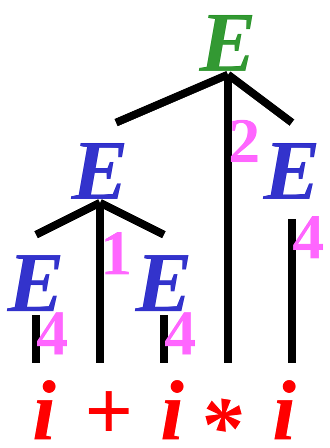
Input token

Pushdown topmost terminal

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Note: Operator associativity and precedence rules underlie the precedence table:

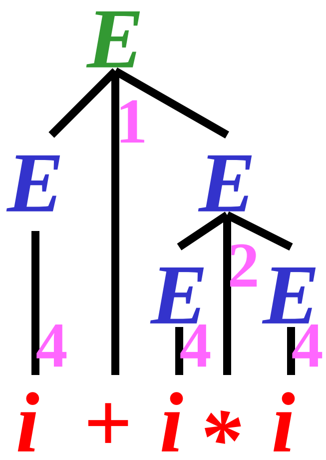
☹ **Wrong tree:**



Right parse:

44142

☺ **Right tree:**



Right parse:

44421

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Input string: *i* + *i* * *i* $

Pushdown	Op	Input	Rule

Rules:

1: $E \rightarrow E+E$
2: $E \rightarrow E*E$
3: $E \rightarrow (E)$
4: $E \rightarrow i$

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Input string: *i* + *i* * *i* $

Pushdown	Op	Input	Rule
$	<	*i*+*i***i*$	

Rules:

1: $E \rightarrow E+E$

2: $E \rightarrow E*E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Input string: *i* + *i* * *i* $

Pushdown	Op	Input	Rule
$	<	*i*+*i***i*$	
$<*i*	>	+*i***i*$	4: *E* → *i*

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Input string: *i* + *i* * *i* $

Pushdown	Op	Input	Rule
$	<	*i*+*i**i*$	
$<*i*	>	+*i**i*$	4: *E* → *i*
$*E*	<	+*i**i*$	

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Input string: *i* + *i* * *i* $

Pushdown	Op	Input	Rule
$	<	*i*+*i**i*$	
$<*i*	>	+*i**i*$	4: $E \rightarrow i$
$*E*	<	+*i**i*$	
$<*E*+	<	*i**i*$	

Rules:

1: $E \rightarrow E+E$

2: $E \rightarrow E*E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Rules:

1: *E* → *E*+*E*
2: *E* → *E***E*
3: *E* → (*E*)
4: *E* → *i*

Input string: *i* + *i* * *i* $

Pushdown	Op	Input	Rule
$	<	*i*+*i***i*$	
$<*i*	>	+*i***i*$	4: *E* → *i*
$*E*	<	+*i***i*$	
$<*E*+	<	*i***i*$	
$<*E*+<*i*	>	**i*$	4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	(	)	i	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Input string: i + i * i $

Pushdown	Op	Input	Rule
$	<	i+i*i$	
$<i	>	+i*i$	4: E → i
$E	<	+i*i$	
$<E+	<	i*i$	
$<E+<i	>	*i$	4: E → i
$<E+E	<	*i$	

Rules:

1: E → E+E
2: E → E*E
3: E → (E)
4: E → i

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Input string: *i* + *i* * *i* $

Pushdown	Op	Input	Rule
$	<	*i*+*i**i*$	
$<*i*	>	+*i**i*$	4: *E* → *i*
$*E*	<	+*i**i*$	
$<*E*+	<	*i**i*$	
$<*E*+<*i*	>	**i*$	4: *E* → *i*
$<*E*+*E*	<	**i*$	
$<*E*+<*E**	<	*i*$	

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Input string: ***i* + *i* * *i* $**

Pushdown	Op	Input	Rule
$	<	*i*+*i***i*$	
$<*i*	>	+*i***i*$	4: *E* → *i*
$*E*	<	+*i***i*$	
$<*E*+	<	*i***i*$	
$<*E*+<*i*	>	**i*$	4: *E* → *i*
$<*E*+*E*	<	**i*$	
$<*E*+<*E**	<	*i*$	
$<*E*+<*E**<*i*	>	$	4: *E* → *i*

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Rules:

1: $E \rightarrow E+E$

2: $E \rightarrow E*E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Input string: *i* + *i* * *i* $

Pushdown	Op	Input	Rule
$	<	*i*+*i**i*$	
$<*i*	>	+*i**i*$	4: $E \rightarrow i$
$*E*	<	+*i**i*$	
$<*E*+	<	*i**i*$	
$<*E*+<*i*	>	**i*$	4: $E \rightarrow i$
$<*E*+*E*	<	**i*$	
$<*E*+<*E**	<	*i*$	
$<*E*+<*E**<*i*	>	$	4: $E \rightarrow i$
$<*E*+<*E***E*	>	$	2: $E \rightarrow E*E$

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Rules:

1: $E \to E+E$

2: $E \to E*E$

3: $E \to (E)$

4: $E \to i$

Input string: $i + i * i \$$

Pushdown	Op	Input	Rule
$\$$	$<$	$i+i*i\$$	
$\$<i$	$>$	$+i*i\$$	4: $E \to i$
$\$E$	$<$	$+i*i\$$	
$\$<E+$	$<$	$i*i\$$	
$\$<E+<i$	$>$	$*i\$$	4: $E \to i$
$\$<E+E$	$<$	$*i\$$	
$\$<E+<E*$	$<$	$i\$$	
$\$<E+<E*<i$	$>$	$\$$	4: $E \to i$
$\$<E+<E*E$	$>$	$\$$	2: $E \to E*E$
$\$<E+E$	$>$	$\$$	1: $E \to E+E$

Operator-Precedence Parsing: Example

	+	*	(	)	i	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
\$	<	<	<		<	

Input string: $i + i * i \$$

Pushdown	Op	Input	Rule
$\$$	<	$i+i*i\$$	
$\$<i$	>	$+i*i\$$	4: $E \rightarrow i$
$\$E$	<	$+i*i\$$	
$\$<E+$	<	$i*i\$$	
$\$<E+<i$	>	$*i\$$	4: $E \rightarrow i$
$\$<E+E$	<	$*i\$$	
$\$<E+<E*$	<	$i\$$	
$\$<E+<E*<i$	>	$\$$	4: $E \rightarrow i$
$\$<E+<E*E$	>	$\$$	2: $E \rightarrow E*E$
$\$<E+E$	>	$\$$	1: $E \rightarrow E+E$
$\$E$		$\$$	

Rules:

1: $E \rightarrow E+E$
2: $E \rightarrow E*E$
3: $E \rightarrow (E)$
4: $E \rightarrow i$

Operator-Precedence Parsing: Example

	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Input string: *i* + *i* * *i* $

Pushdown	Op	Input	Rule
$	<	*i*+*i***i*$	
$<*i*	>	+*i***i*$	4: *E* → *i*
$*E*	<	+*i***i*$	
$<*E*+	<	*i***i*$	
$<*E*+<*i*	>	**i*$	4: *E* → *i*
$<*E*+*E*	<	**i*$	
$<*E*+<*E**	<	*i*$	
$<*E*+<*E**<*i*	>	$	4: *E* → *i*
$<*E*+<*E***E*	>	$	2: *E* → *E***E*
$<*E*+*E*	>	$	1: *E* → *E*+*E*
$*E*		$	

Rules:

1: *E* → *E*+*E*

2: *E* → *E***E*

3: *E* → (*E*)

4: *E* → *i*

Success

Right parse: 44421

Construction of Precedence Table 1/5

- Let $G_{expr} = (N, T, P, E)$, where $N = \{E\}$,

$T = \{(,), \mathbf{id}_1, \mathbf{id}_2, \dots, \mathbf{id}_m, \mathbf{op}_1, \mathbf{op}_2, \dots \mathbf{op}_n\}$,

$P = \{\ E \to (E), E \to \mathbf{id}_1, E \to \mathbf{id}_2, \dots, E \to \mathbf{id}_m,$

$E \to E\ \mathbf{op}_1\ E, E \to E\ \mathbf{op}_2\ E, \dots, E \to E\ \mathbf{op}_n\ E\ \}$

Note: $\mathbf{id}_1, \mathbf{id}_2, \dots, \mathbf{id}_m$ are identifiers,

$\mathbf{op}_1, \mathbf{op}_2, \dots \mathbf{op}_n$ are different operators

---

1) Precedence of operators:

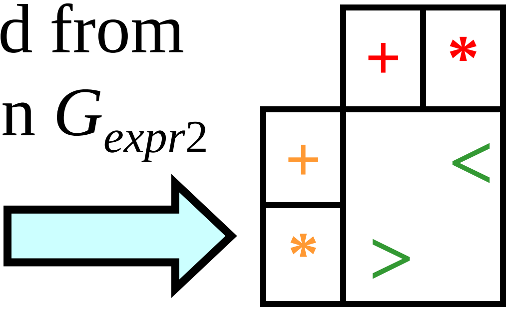
- If $\mathbf{op}_i$ has **<u>higher precedence</u>** than $\mathbf{op}_j$ then

$$\boxed{\mathbf{op}_i > \mathbf{op}_j \text{ and } \mathbf{op}_j < \mathbf{op}_i}$$

Example: Precedence-table part derived from the precedence of operators in G_{expr2}

$* > +$

$+ < *$



	+	*
+		<
*	>	

Construction of Precedence Table 2/5

2) Associativity:

Note:

- $\mathbf{op}_i$ is left-associative ⇔ $a\ \mathbf{op}_i\ b\ \mathbf{op}_i\ c = (a\ \mathbf{op}_i\ b)\ \mathbf{op}_i\ c$
- $\mathbf{op}_i$ is right-associative ⇔ $a\ \mathbf{op}_i\ b\ \mathbf{op}_i\ c = a\ \mathbf{op}_i\ (b\ \mathbf{op}_i\ c)$

- Let $\mathbf{op}_i$ and $\mathbf{op}_j$ have **equal precedence**
  - If $\mathbf{op}_i$ and $\mathbf{op}_j$ are **left associative** then

    $\mathbf{op}_i > \mathbf{op}_j$ and $\mathbf{op}_j > \mathbf{op}_i$

  - If $\mathbf{op}_i$ and $\mathbf{op}_j$ are **right associative** then

    $\mathbf{op}_i < \mathbf{op}_j$ and $\mathbf{op}_j < \mathbf{op}_i$

Example: Precedence-table part derived from the associativity of operators in G_{expr2}

+ is left-associative
*** is left-associative**

	+	*
+	>	
*		>

Construction of Precedence Table 3/5

3) Identifiers:

- If $a \in T$ may **precede** id_i, then $a < \mathrm{id}_i$
- If $a \in T$ may **follow** id_i, then $\mathrm{id}_i > a$

Example: Precedence-table part for identifiers

$i * (i + i) * i

$, (, +, * may precede *i*

i * (*i* + *i*)* *i* $

*, +,), $ may follow *i*

	+	*	(	)	*i*	$
+					<	
*					<	
(					<	
)						
i	>	>		>		>
$					<	

Construction of Precedence Table 4/5

4) Parentheses:

- A pair of parentheses: $(=)$
- Let $a \in T - \{), \$\}$. Then, $(< a$
- Let $a \in T - \{(, \$\}$. Then, $a >)$
- Let $a \in T$ and a may **precede** (. Then, $a < ($
- Let $a \in T$ and a may **follow**). Then, $) > a$

Example: Precedence-table part for parentheses.

$\$(i + ((i * (i + (i + i)))))$

$\$$, (, $*$, + may precede (

$(((((i + i)+ i)* i)) + i)\$$

+, $*$,), $\$$ may follow)

	+	*	(	)	i	$
+			<	>		
*			<	>		
(	<	<	<	=	<	
)	>	>		>		>
i				>		
$			<			

Construction of Precedence Table 5/5

5) End Marker $\$$

- Let $\mathbf{op}_i$ be any operator. Then:

$\$ < \mathbf{op}_i$ and $\mathbf{op}_i > \$$

Example: Precedence-table part for end-markers.

$\$ < +$

$\$ < *$

$+ > \$$

$* > \$$

	+	*	$\$$
+			>
*			>
$\$$	<	<	

Construction of Precedence Table 5/5

5) End Marker \$

- Let $\mathbf{op}_i$ be any operator. Then:

$$\$ < \mathbf{op}_i \text{ and } \mathbf{op}_i > \$$$

Example: Precedence-table part for end-markers.

$\$ < +$
$\$ < *$
$+ > \$$
$* > \$$

	+	*	\$
+			>
*			>
\$	<	<	

Summary:

	+	*	(	)	*i*	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
\$	<	<	<		<	

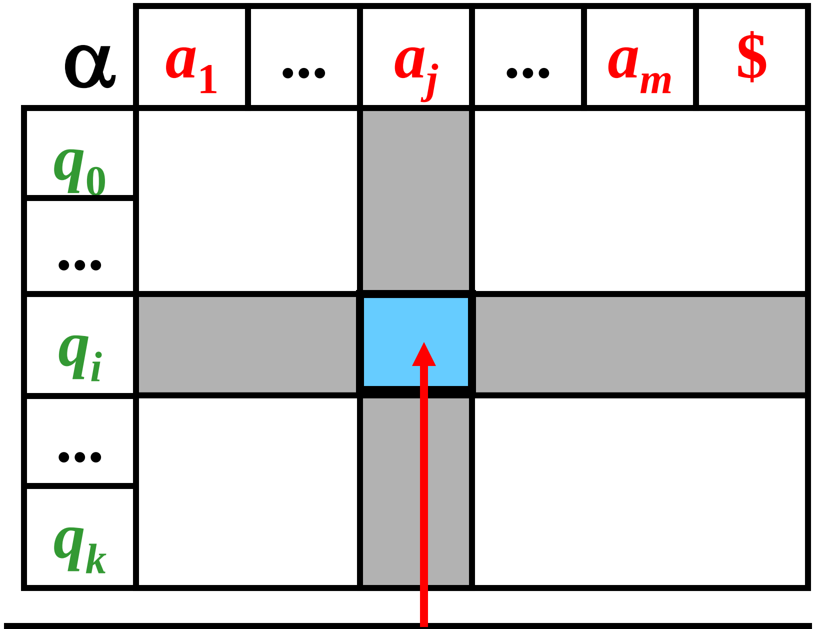
LR-Parser

- Let $G = (N, T, P, S)$ be a CFG, where $N = \{\mathbf{A_1}, \mathbf{A_2}, \dots , \mathbf{A_n}\}$, $T = \{\mathbf{a_1}, \mathbf{a_2}, \dots , \mathbf{a_m}\}$

- LR-parser is a EPDA, M, with states $Q = \{\mathbf{q_0}, \mathbf{q_1}, \dots, \mathbf{q_k}\}$, where $\mathbf{q_0}$ is the start state.
- M is based on LR table that has these two parts
  - **1) Action part**
  - **2) Go-to part**

Action Part & Go-to Part

Action Part:



α	a_1	…	a_j	…	a_m	$
q_0						
…						
q_i						
…						
q_k						

$\alpha[q_i , a_j] =$ **1** or **2** or **3** or **4**

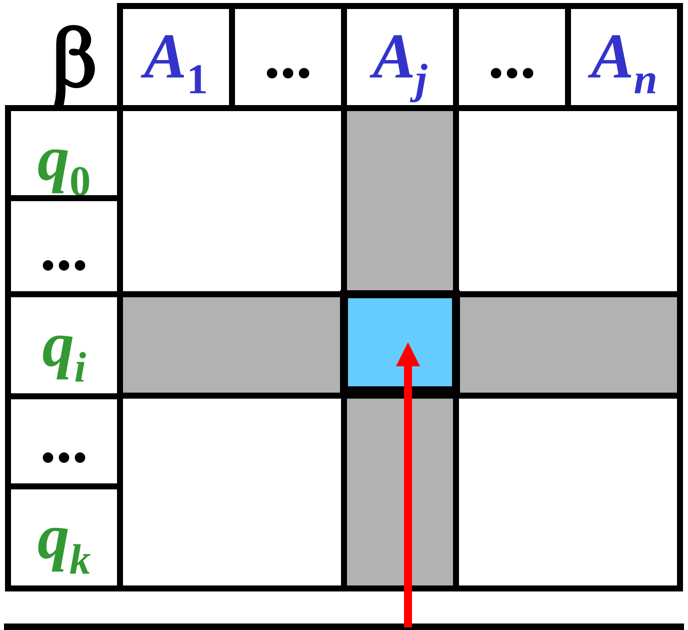
1) **sq**: **s** = **s**hift, $q \in Q$

2) **rp**: **r** = **r**educe, $p \in P$

3) ☺ **: success**

4) **blank: error**

Go-to Part:



β	A_1	…	A_j	…	A_n
q_0					
…					
q_i					
…					
q_k					

$\beta[q_i , A_j] =$ **1** or **2**

1) **q**: $q \in Q$

2) **blank**

LR-Parser: Algorithm

- **Input:** LR-table for $G = (N, T, P, S)$; $x \in T^*$
- **Output:** Right parse of x if $x \in L(G)$; otherwise, **error**

- **Method:**

```
• push($\langle \$, q_0 \rangle$) onto pushdown; $state := q_0$;
• repeat
    • let $a$ = the current token
    case $\alpha[state, a]$ of:
        • $sq$: push($\langle a, q \rangle$) & read next $a$ from input string &
              $state := q$;
        • $rp$: if $p: A \rightarrow X_1X_2\ldots X_n \in P$ and
              $\langle ?, q \rangle\langle X_1, ? \rangle\langle X_2, ? \rangle\ldots\langle X_n, ? \rangle$ is pushdown top
              then $state := \beta[q, A]$ &
              replace $\langle X_1, ? \rangle\langle X_2, ? \rangle\ldots\langle X_n, ? \rangle$ with $\langle A, state \rangle$
              on the pushdown & write $r$ to output
              else error
        • ☺: success
        • blank: error
until success or error
```

LR-Parser: Example 1/2

$K = (N, T, P, S)$, where $N = \{S, A\}$, $T = \{i, o, (,)\}$,

$P = \{1\text{: } S \to SoA,\ 2\text{: } S \to A,\ 3\text{: } A \to i,\ 4\text{: } A \to (S)\}$

LR-table for *K*:

α	*i*	*o*	(	)	$
0	s3		s4		
1		s6			☺
2		r2		r2	r2
3		r3		r3	r3
4	s3		s4		
5		s6		s8	
6	s3		s4		
7		r1		r1	r1
8		r4		r4	r4

Action part for *K*

Go-to part for *K*

β	*S*	*A*
0	1	2
1		
2		
3		
4	5	2
5		
6		7
7		
8		

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: ***i o i $***

Pushdown	St.	Input	Enter	Rule

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: ***i o i $***

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	*ioi*$	$\alpha[0, i]$ = s3	

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i\ o\ i\ \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0\rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0\rangle\langle i, 3\rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i\ o\ i\ \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0\rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0\rangle\langle i, 3\rangle$	3	$oi\$$	$\alpha[3, o] = r3$ $\beta[0, A] = 2$	3: $A \rightarrow i$

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i\ o\ i\ \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0\rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0\rangle\langle i, 3\rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0\rangle\langle A, 2\rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: *i o i $*

Pushdown	St.	Input	Enter	Rule
$\langle\$, 0\rangle$	0	*ioi*$	$\alpha[0, i] = s3$	
$\langle\$, 0\rangle\langle i, 3\rangle$	3	*oi*$	$\alpha[3, o] = r3$ $\beta[0, A] = 2$	3: $A \rightarrow i$
$\langle\$, 0\rangle\langle A, 2\rangle$	2	*oi*$	$\alpha[2, o] = r2$ $\beta[0, S] = 1$	2: $S \rightarrow A$

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i\ o\ i\ \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0\rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0\rangle\langle i, 3\rangle$	3	$oi\$$	$\alpha[3, o] = r3$ $\beta[0, A] = 2$	3: $A \rightarrow i$
$\langle \$, 0\rangle\langle A, 2\rangle$	2	$oi\$$	$\alpha[2, o] = r2$ $\beta[0, S] = 1$	2: $S \rightarrow A$
$\langle \$, 0\rangle\langle S, 1\rangle$	1	$oi\$$	$\alpha[1, o] = s6$	

LR-Parser: Example 2/2

Rules: 1: *S* → *SoA*, 2: *S* → *A*, 3: *A* → *i*, 4: *A* → (*S*)

Input string: ***i o i $***

Pushdown	St.	Input	Enter	Rule
⟨$, 0⟩	0	*ioi*$	α[0, *i*] = s3	
⟨$, 0⟩⟨*i*, 3⟩	3	*oi*$	α[3, *o*] = r3	3: *A* → *i*
			β[0, *A*] = 2	
⟨$, 0⟩⟨*A*, 2⟩	2	*oi*$	α[2, *o*] = r2	2: *S* → *A*
			β[0, *S*] = 1	
⟨$, 0⟩⟨*S*, 1⟩	1	*oi*$	α[1, *o*] = s6	
⟨$, 0⟩⟨*S*, 1⟩⟨*o*, 6⟩	6	*i*$	α[6, *i*] = s3	

LR-Parser: Example 2/2

Rules: 1: *S* → *SoA*, 2: *S* → *A*, 3: *A* → *i*, 4: *A* → (*S*)

Input string: *i o i* $

Pushdown	St.	Input	Enter	Rule
⟨$, 0⟩	0	*ioi*$	α[0, *i*] = s3	
⟨$, 0⟩⟨*i*, 3⟩	3	*oi*$	α[3, *o*] = r3	3: *A* → *i*
			β[0, *A*] = 2	
⟨$, 0⟩⟨*A*, 2⟩	2	*oi*$	α[2, *o*] = r2	2: *S* → *A*
			β[0, *S*] = 1	
⟨$, 0⟩⟨*S*, 1⟩	1	*oi*$	α[1, *o*] = s6	
⟨$, 0⟩⟨*S*, 1⟩⟨*o*, 6⟩	6	*i*$	α[6, *i*] = s3	
⟨$, 0⟩⟨*S*, 1⟩⟨*o*, 6⟩⟨*i*, 3⟩	3	$	α[3, $] = r3	3: *A* → *i*

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i\ o\ i\ \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0\rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0\rangle\langle i, 3\rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0\rangle\langle A, 2\rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$
			$\beta[0, S] = 1$	
$\langle \$, 0\rangle\langle S, 1\rangle$	1	$oi\$$	$\alpha[1, o] = s6$	
$\langle \$, 0\rangle\langle S, 1\rangle\langle o, 6\rangle$	6	$i\$$	$\alpha[6, i] = s3$	
$\langle \$, 0\rangle\langle S, 1\rangle\langle o, 6\rangle\langle i, 3\rangle$	3	$\$$	$\alpha[3, \$] = r3$	3: $A \rightarrow i$
			$\beta[6, A] = 7$	

LR-Parser: Example 2/2

Rules: 1: *S* → *SoA*, 2: *S* → *A*, 3: *A* → *i*, 4: *A* → (*S*)

Input string: *i o i* $

Pushdown	St.	Input	Enter	Rule
⟨$, 0⟩	0	*ioi*$	α[0, *i*] = s3	
⟨$, 0⟩⟨*i*, 3⟩	3	*oi*$	α[3, *o*] = r3	3: *A* → *i*
			β[0, *A*] = 2	
⟨$, 0⟩⟨*A*, 2⟩	2	*oi*$	α[2, *o*] = r2	2: *S* → *A*
			β[0, *S*] = 1	
⟨$, 0⟩⟨*S*, 1⟩	1	*oi*$	α[1, *o*] = s6	
⟨$, 0⟩⟨*S*, 1⟩⟨*o*, 6⟩	6	*i*$	α[6, *i*] = s3	
⟨$, 0⟩⟨*S*, 1⟩⟨*o*, 6⟩⟨*i*, 3⟩	3	$	α[3, $] = r3	3: *A* → *i*
			β[6, *A*] = 7	
⟨$, 0⟩⟨*S*, 1⟩⟨*o*, 6⟩⟨*A*, 7⟩	7	$	α[7, $] = r1	1: *S* → *SoA*

LR-Parser: Example 2/2

Rules: 1: $S \to SoA$, 2: $S \to A$, 3: $A \to i$, 4: $A \to (S)$

Input string: $i\ o\ i\ \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0\rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0\rangle\langle i, 3\rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \to i$
			$\beta[0, A] = 2$	
$\langle \$, 0\rangle\langle A, 2\rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \to A$
			$\beta[0, S] = 1$	
$\langle \$, 0\rangle\langle S, 1\rangle$	1	$oi\$$	$\alpha[1, o] = s6$	
$\langle \$, 0\rangle\langle S, 1\rangle\langle o, 6\rangle$	6	$i\$$	$\alpha[6, i] = s3$	
$\langle \$, 0\rangle\langle S, 1\rangle\langle o, 6\rangle\langle i, 3\rangle$	3	$\$$	$\alpha[3, \$] = r3$	3: $A \to i$
			$\beta[6, A] = 7$	
$\langle \$, 0\rangle\langle S, 1\rangle\langle o, 6\rangle\langle A, 7\rangle$	7	$\$$	$\alpha[7, \$] = r1$	1: $S \to SoA$
			$\beta[0, S] = 1$	

LR-Parser: Example 2/2

Rules: 1: $S \rightarrow SoA$, 2: $S \rightarrow A$, 3: $A \rightarrow i$, 4: $A \rightarrow (S)$

Input string: $i\ o\ i\ \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0\rangle$	0	$ioi\$$	$\alpha[0, i] = s3$	
$\langle \$, 0\rangle\langle i, 3\rangle$	3	$oi\$$	$\alpha[3, o] = r3$	3: $A \rightarrow i$
			$\beta[0, A] = 2$	
$\langle \$, 0\rangle\langle A, 2\rangle$	2	$oi\$$	$\alpha[2, o] = r2$	2: $S \rightarrow A$
			$\beta[0, S] = 1$	
$\langle \$, 0\rangle\langle S, 1\rangle$	1	$oi\$$	$\alpha[1, o] = s6$	
$\langle \$, 0\rangle\langle S, 1\rangle\langle o, 6\rangle$	6	$i\$$	$\alpha[6, i] = s3$	
$\langle \$, 0\rangle\langle S, 1\rangle\langle o, 6\rangle\langle i, 3\rangle$	3	$\$$	$\alpha[3, \$] = r3$	3: $A \rightarrow i$
			$\beta[6, A] = 7$	
$\langle \$, 0\rangle\langle S, 1\rangle\langle o, 6\rangle\langle A, 7\rangle$	7	$\$$	$\alpha[7, \$] = r1$	1: $S \rightarrow SoA$
			$\beta[0, S] = 1$	
$\langle \$, 0\rangle\langle S, 1\rangle$	1	$\$$	$\alpha[1, \$] =$ ☺	

LR-Parser: Example 2/2

Rules: 1: *S* → *SoA*, 2: *S* → *A*, 3: *A* → *i*, 4: *A* → (*S*)

Input string: *i o i* $

Pushdown	St.	Input	Enter	Rule
⟨$, 0⟩	0	*ioi*$	α[0, *i*] = s3	
⟨$, 0⟩⟨*i*, 3⟩	3	*oi*$	α[3, *o*] = r3	3: *A* → *i*
			β[0, *A*] = 2	
⟨$, 0⟩⟨*A*, 2⟩	2	*oi*$	α[2, *o*] = r2	2: *S* → *A*
			β[0, *S*] = 1	
⟨$, 0⟩⟨*S*, 1⟩	1	*oi*$	α[1, *o*] = s6	
⟨$, 0⟩⟨*S*, 1⟩⟨*o*, 6⟩	6	*i*$	α[6, *i*] = s3	
⟨$, 0⟩⟨*S*, 1⟩⟨*o*, 6⟩⟨*i*, 3⟩	3	$	α[3, $] = r3	3: *A* → *i*
			β[6, *A*] = 7	
⟨$, 0⟩⟨*S*, 1⟩⟨*o*, 6⟩⟨*A*, 7⟩	7	$	α[7, $] = r1	1: *S* → *SoA*
			β[0, *S*] = 1	
⟨$, 0⟩⟨*S*, 1⟩	1	$	α[1, $] = ☺	

Success

Right parse: 3231

Construction of LR Table: Introduction

- **One parsing algorithm but many algorithms for the construction of LR table.**

Basic algorithms for the construction of LR table:

1) **Simple LR (SLR)**: the least powerful, but simple and few states

2) **Canonical LR**: more powerful, but many states

3) **Lookahead LR (LALR)**: the best because the most powerful and the same number of states as SLR

Extended Grammar with a "Dummy Rule"

Gist: Grammar with special "starting rule"

Definition: Let $G = (N, T, P, S)$ be a CFG, $S' \notin N$. *Extended grammar* for G is grammar $G' = (N \cup \{S'\}, T, P \cup \{S' \rightarrow S\}, S')$.

Why a dummy rule? When $S' \rightarrow S$ is used and the input token is endmarker, then **syntax analysis is successfully completed**.

Example:

$K = (N, T, P, S)$, where $N = \{S, A\}$, $T = \{i, o, (,)\}$,
$P = \{1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$

Extended grammar for K:
$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$

Construction of LR Table: Items

Gist: Item is a rule of CFG with • in the right-hand side of rule.

Definition: Let $G = (N, T, P, S)$ be a CFG, $A \rightarrow x \in P$, $x = yz$. Then, $A \rightarrow y \bullet z$ is an *item*.

Example: Consider $S \rightarrow SoA$

All items for $S \rightarrow SoA$ are:

$S \rightarrow \bullet SoA$, $S \rightarrow S \bullet oA$, $S \rightarrow So \bullet A$, $S \rightarrow SoA \bullet$

Meaning: $A \rightarrow y \bullet z$ means that if y appears on the pushdown top and a prefix of the input is eventually reduced to z, then yz (= x) as a handle can be reduced to A according to $A \rightarrow x$.

Closure of Item: Algorithm

Note: Closure(*I*) is the set of items defined by the following algorithm:

- **Input:** $G = (N, T, P, S)$; item I
- **Output:** *Closure*(I)

- **Method:**
- *Closure*(I) := $\{I\}$;
- **Apply the following rule until** *Closure*(I) **cannot be changed:**
    - **if** $A \rightarrow y \bullet Bz \in$ *Closure*(I) **and** $B \rightarrow x \in P$ **then** add $B \rightarrow \bullet x$ to *Closure*(I)

Closure of Item: Example 1/2

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0: S' \rightarrow S,\ 1: S \rightarrow SoA,\ 2: S \rightarrow A,\ 3: A \rightarrow i,\ 4: A \rightarrow (S)\}$

Task: $Closure(I)$ for $I = S' \rightarrow \bullet S$

$Closure(I) := \{S' \rightarrow \bullet S\}$

---

1) $S' \rightarrow \bullet S \in Closure(I)$ **&** $S \rightarrow SoA \in P$**:**
add $S \rightarrow \bullet SoA$ **to** $Closure(I)$

$Closure(I)$ **=** $\{S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA\}$

---

2) $S' \rightarrow \bullet S \in Closure(I)$ **&** $S \rightarrow A \in P$**:**
add $S \rightarrow \bullet A$ **to** $Closure(I)$

$Closure(I)$ **=** $\{S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A\}$

Closure of Item: Example 2/2

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{$**i**, ***o***, **(**, **)**$\}$,
$P = \{0\text{: } S' \rightarrow S,\ 1\text{: } S \rightarrow SoA,\ 2\text{: } S \rightarrow A,\ 3\text{: } A \rightarrow i,\ 4\text{: } A \rightarrow (S)\}$

---

3) $S \rightarrow \bullet A \in \mathit{Closure}(I)$ **&** $A \rightarrow i \in P$**:**
add $A \rightarrow \bullet i$ **to** $\mathit{Closure}(I)$

$\mathit{Closure}(I) = \{S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i\}$

---

4) $S \rightarrow \bullet A \in \mathit{Closure}(I)$ **&** $A \rightarrow (S) \in P$**:**
add $A \rightarrow \bullet(S)$ **to** $\mathit{Closure}(I)$

Summary:

$\mathit{Closure}(I) = \{S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\}$

Set $\Theta_U(\boldsymbol{I})$ for grammar G

Gist: For a symbol *U* and set of items *I*, $\Theta_U(I)$ denotes union of all closures of the form *Closure*($A \rightarrow yU{\bullet}z$), where $A \rightarrow y{\bullet}Uz$ is in *I*.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\boldsymbol{I}$ be a set of items, and $\boldsymbol{U} \in T \cup N$. Then,

$\Theta_U(\boldsymbol{I}) = \{j: j \in Closure(\boldsymbol{A} \rightarrow \boldsymbol{yU{\bullet}z}), \boldsymbol{A} \rightarrow \boldsymbol{y{\bullet}Uz} \in \boldsymbol{I}\}$

Set $\Theta_U(\boldsymbol{I})$ for grammar G

Gist: For a symbol U and set of items I, $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\boldsymbol{I}$ be a set of items, and $\boldsymbol{U} \in T \cup N$. Then,

$\Theta_U(\boldsymbol{I}) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in \boldsymbol{I}\}$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,

$P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,

$\boldsymbol{I} = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(\boldsymbol{I})$

Set $\Theta_U(\boldsymbol{I})$ for grammar G

Gist: For a symbol U and set of items I, $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\boldsymbol{I}$ be a set of items, and $\boldsymbol{U} \in T \cup N$. Then,

$\Theta_U(\boldsymbol{I}) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in \boldsymbol{I}\}$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,

$P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,

$\boldsymbol{I} = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(\boldsymbol{I})$

Set $\Theta_U(\boldsymbol{I})$ for grammar G

Gist: For a symbol U and set of items I, $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \to yU\bullet z)$, where $A \to y\bullet Uz$ is in I.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\boldsymbol{I}$ be a set of items, and $\boldsymbol{U} \in T \cup N$. Then,
$\Theta_U(\boldsymbol{I}) = \{j: j \in Closure(\boldsymbol{A} \to \boldsymbol{yU\bullet z}), \boldsymbol{A} \to \boldsymbol{y\bullet Uz} \in \boldsymbol{I}\}$

Example:

$H = (N, T, P, \boldsymbol{S'})$, where $N = \{\boldsymbol{S'}, \boldsymbol{S}, \boldsymbol{A}\}$, $T = \{\boldsymbol{i}, \boldsymbol{o}, \boldsymbol{(}, \boldsymbol{)}\}$,
$P = \{0\text{: } \boldsymbol{S'} \to \boldsymbol{S}, 1\text{: } \boldsymbol{S} \to \boldsymbol{SoA}, 2\text{: } \boldsymbol{S} \to \boldsymbol{A}, 3\text{: } \boldsymbol{A} \to \boldsymbol{i}, 4\text{: } \boldsymbol{A} \to \boldsymbol{(S)}\}$,
$\boldsymbol{I} = \{\boldsymbol{S} \to \boldsymbol{So}\bullet\boldsymbol{A}, \boldsymbol{S} \to \bullet\boldsymbol{A}, \boldsymbol{A} \to \bullet\boldsymbol{(S)}\}$

Task: $\Theta_{\boldsymbol{A}}(\boldsymbol{I})$

$Closure(\boldsymbol{S} \to \boldsymbol{SoA}\bullet)$

Set $\Theta_U(I)$ for grammar G

Gist: For a symbol U and set of items I, $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \to yU\bullet z)$, where $A \to y\bullet Uz$ is in I.

Definition: Let $G = (N, T, P, S)$ be a CFG, I be a set of items, and $U \in T \cup N$. Then,
$\Theta_U(I) = \{j: j \in Closure(A \to yU\bullet z), A \to y\bullet Uz \in I\}$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0: S' \to S, 1: S \to SoA, 2: S \to A, 3: A \to i, 4: A \to (S)\}$,
$I = \{S \to So\bullet A, S \to \bullet A, A \to \bullet(S)\}$

Task: $\Theta_A(I)$

$Closure(S \to SoA\bullet)$

Set $\Theta_U(\boldsymbol{I})$ for grammar G

Gist: For a symbol U and set of items I, $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\boldsymbol{I}$ be a set of items, and $\boldsymbol{U} \in T \cup N$. Then,
$\Theta_U(\boldsymbol{I}) = \{j : j \in Closure(\boldsymbol{A} \rightarrow \boldsymbol{yU\bullet z}), \boldsymbol{A} \rightarrow \boldsymbol{y\bullet Uz} \in \boldsymbol{I}\}$

Example:

$H = (N, T, P, \boldsymbol{S'})$, where $N = \{\boldsymbol{S'}, \boldsymbol{S}, \boldsymbol{A}\}$, $T = \{\boldsymbol{i}, \boldsymbol{o}, \boldsymbol{(}, \boldsymbol{)}\}$,
$P = \{0: \boldsymbol{S'} \rightarrow \boldsymbol{S}, 1: \boldsymbol{S} \rightarrow \boldsymbol{SoA}, 2: \boldsymbol{S} \rightarrow \boldsymbol{A}, 3: \boldsymbol{A} \rightarrow \boldsymbol{i}, 4: \boldsymbol{A} \rightarrow \boldsymbol{(S)}\}$,
$\boldsymbol{I} = \{\boldsymbol{S} \rightarrow \boldsymbol{So}\bullet\boldsymbol{A}, \boldsymbol{S} \rightarrow \bullet\boldsymbol{A}, \boldsymbol{A} \rightarrow \bullet\boldsymbol{(S)}\}$

Task: $\Theta_A(\boldsymbol{I})$

$Closure(\boldsymbol{S} \rightarrow \boldsymbol{SoA}\bullet) \cup Closure(\boldsymbol{S} \rightarrow \boldsymbol{A}\bullet)$

Set $\Theta_U(\boldsymbol{I})$ for grammar G

Gist: For a symbol U and set of items I, $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\boldsymbol{I}$ be a set of items, and $\boldsymbol{U} \in T \cup N$. Then,

$$\Theta_U(\boldsymbol{I}) = \{j: j \in Closure(\boldsymbol{A} \rightarrow \boldsymbol{yU\bullet z}), \boldsymbol{A} \rightarrow \boldsymbol{y\bullet Uz} \in \boldsymbol{I}\}$$

Example:

$H = (N, T, P, \boldsymbol{S'})$, where $N = \{\boldsymbol{S'}, \boldsymbol{S}, \boldsymbol{A}\}$, $T = \{\boldsymbol{i}, \boldsymbol{o}, \boldsymbol{(}, \boldsymbol{)}\}$,

$P = \{0: \boldsymbol{S'} \rightarrow \boldsymbol{S}, 1: \boldsymbol{S} \rightarrow \boldsymbol{SoA}, 2: \boldsymbol{S} \rightarrow \boldsymbol{A}, 3: \boldsymbol{A} \rightarrow \boldsymbol{i}, 4: \boldsymbol{A} \rightarrow \boldsymbol{(S)}\}$,

$\boldsymbol{I} = \{\boldsymbol{S} \rightarrow \boldsymbol{So}\bullet\boldsymbol{A}, \boldsymbol{S} \rightarrow \bullet\boldsymbol{A}, \boldsymbol{A} \rightarrow \bullet\boldsymbol{(S)}\}$

Task: $\Theta_{A}(\boldsymbol{I})$

$Closure(\boldsymbol{S} \rightarrow \boldsymbol{SoA}\bullet) \cup Closure(\boldsymbol{S} \rightarrow \boldsymbol{A}\bullet) = \{\boldsymbol{S} \rightarrow \boldsymbol{SoA}\bullet, \boldsymbol{S} \rightarrow \boldsymbol{A}\bullet\}$

Set $\Theta_U(\boldsymbol{I})$ for grammar G

Gist: For a symbol U and set of items I, $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\boldsymbol{I}$ be a set of items, and $\boldsymbol{U} \in T \cup N$. Then,

$$\Theta_U(\boldsymbol{I}) = \{j: j \in Closure(\boldsymbol{A} \rightarrow \boldsymbol{yU\bullet z}), \boldsymbol{A} \rightarrow \boldsymbol{y\bullet Uz} \in \boldsymbol{I}\}$$

Example:

$H = (N, T, P, \boldsymbol{S'})$, where $N = \{\boldsymbol{S'}, \boldsymbol{S}, \boldsymbol{A}\}$, $T = \{\boldsymbol{i}, \boldsymbol{o}, \boldsymbol{(}, \boldsymbol{)}\}$,

$P = \{0\text{: } \boldsymbol{S'} \rightarrow \boldsymbol{S}$, $1\text{: } \boldsymbol{S} \rightarrow \boldsymbol{SoA}$, $2\text{: } \boldsymbol{S} \rightarrow \boldsymbol{A}$, $3\text{: } \boldsymbol{A} \rightarrow \boldsymbol{i}$, $4\text{: } \boldsymbol{A} \rightarrow \boldsymbol{(S)}\}$,

$\boldsymbol{I} = \{\boldsymbol{S} \rightarrow \boldsymbol{So\bullet A}, \boldsymbol{S} \rightarrow \bullet\boldsymbol{A}, \boldsymbol{A} \rightarrow \bullet\boldsymbol{(S)}\}$

Task: $\Theta_{\boldsymbol{A}}(\boldsymbol{I})$

$Closure(\boldsymbol{S} \rightarrow \boldsymbol{SoA}\bullet) \cup Closure(\boldsymbol{S} \rightarrow \boldsymbol{A}\bullet) = \{\boldsymbol{S} \rightarrow \boldsymbol{SoA}\bullet, \boldsymbol{S} \rightarrow \boldsymbol{A}\bullet\}$

Task: $\Theta_{\boldsymbol{(}}(\boldsymbol{I})$

Set $\Theta_U(\boldsymbol{I})$ for grammar G

Gist: For a symbol *U* and set of items *I*, $\Theta_U(I)$ denotes union of all closures of the form *Closure*($A \rightarrow yU\bullet z$), where $A \rightarrow y\bullet Uz$ is in *I*.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\boldsymbol{I}$ be a set of items, and $\boldsymbol{U} \in T \cup N$. Then,

$\Theta_U(\boldsymbol{I}) = \{j: j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in \boldsymbol{I}\}$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,

$P = \{0: S' \rightarrow S, 1: S \rightarrow SoA, 2: S \rightarrow A, 3: A \rightarrow i, 4: A \rightarrow (S)\}$,

$\boldsymbol{I} = \{S \rightarrow So\bullet A, S \rightarrow \bullet A, A \rightarrow \bullet(S)\}$

Task: $\Theta_A(\boldsymbol{I})$

$Closure(S \rightarrow SoA\bullet) \cup Closure(S \rightarrow A\bullet) = \{S \rightarrow SoA\bullet, S \rightarrow A\bullet\}$

Task: $\Theta_((\boldsymbol{I})$

Set $\Theta_U(\boldsymbol{I})$ for grammar G

Gist: For a symbol U and set of items I, $\Theta_U(I)$ denotes union of all closures of the form $Closure(A \rightarrow yU\bullet z)$, where $A \rightarrow y\bullet Uz$ is in I.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\boldsymbol{I}$ be a set of items, and $\boldsymbol{U} \in T \cup N$. Then,

$$\Theta_U(\boldsymbol{I}) = \{j : j \in Closure(A \rightarrow yU\bullet z), A \rightarrow y\bullet Uz \in \boldsymbol{I}\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,

$P = \{0: S' \rightarrow S,\ 1: S \rightarrow SoA,\ 2: S \rightarrow A,\ 3: A \rightarrow i,\ 4: A \rightarrow (S)\}$,

$\boldsymbol{I} = \{S \rightarrow So\bullet A,\ S \rightarrow \bullet A,\ A \rightarrow \bullet(S)\}$

Task: $\Theta_A(\boldsymbol{I})$

$Closure(S \rightarrow SoA\bullet) \cup Closure(S \rightarrow A\bullet) = \{S \rightarrow SoA\bullet,\ S \rightarrow A\bullet\}$

Task: $\Theta_{(}(\boldsymbol{I})$

$Closure(A \rightarrow (\bullet S))$

Set $\Theta_U(\boldsymbol{I})$ for grammar G

Gist: For a symbol U and set of items I, $\Theta_U(I)$ denotes union of all closures of the form $\mathit{Closure}(A \to yU\bullet z)$, where $A \to y\bullet Uz$ is in I.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\boldsymbol{I}$ be a set of items, and $\boldsymbol{U} \in T \cup N$. Then,

$$\Theta_U(\boldsymbol{I}) = \{j: j \in \mathit{Closure}(A \to yU\bullet z), A \to y\bullet Uz \in \boldsymbol{I}\}$$

Example:

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,

$P = \{0: S' \to S,\ 1: S \to SoA,\ 2: S \to A,\ 3: A \to i,\ 4: A \to (S)\}$,

$\boldsymbol{I} = \{S \to So\bullet A,\ S \to \bullet A,\ A \to \bullet(S)\}$

Task: $\Theta_A(\boldsymbol{I})$

$\mathit{Closure}(S \to SoA\bullet) \cup \mathit{Closure}(S \to A\bullet) = \{S \to SoA\bullet,\ S \to A\bullet\}$

Task: $\Theta_((\boldsymbol{I})$

$\mathit{Closure}(A \to (\bullet S)) = \{A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\}$

Set Θ_G for Grammar *G*

Note: Set Θ_G for grammar *G* is the set of sets of items defined by the following algorithm:

- **Input:** Extended $G = (N, T, P, S')$
- **Output:** Θ_G for grammar *G*

- **Method:**
- $\Theta_G := \{Closure(S' \rightarrow \bullet S)\};$
- **for each** $I \in \Theta_G$ **and** $U \in N \cup T$

  if $\Theta_U(I) \neq \emptyset$ **then** include set $\Theta_U(I)$ into Θ_G

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{\mathbf{i}, \mathbf{o}, \mathbf{(}, \mathbf{)}\}$,
$P = \{0: S' \to S, 1: S \to S\mathbf{o}A, 2: S \to A, 3: A \to \mathbf{i}, 4: A \to \mathbf{(}S\mathbf{)}\}$

Initialization:

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0: S' \to S,\ 1: S \to SoA,\ 2: S \to A,\ 3: A \to i,\ 4: A \to (S)\}$

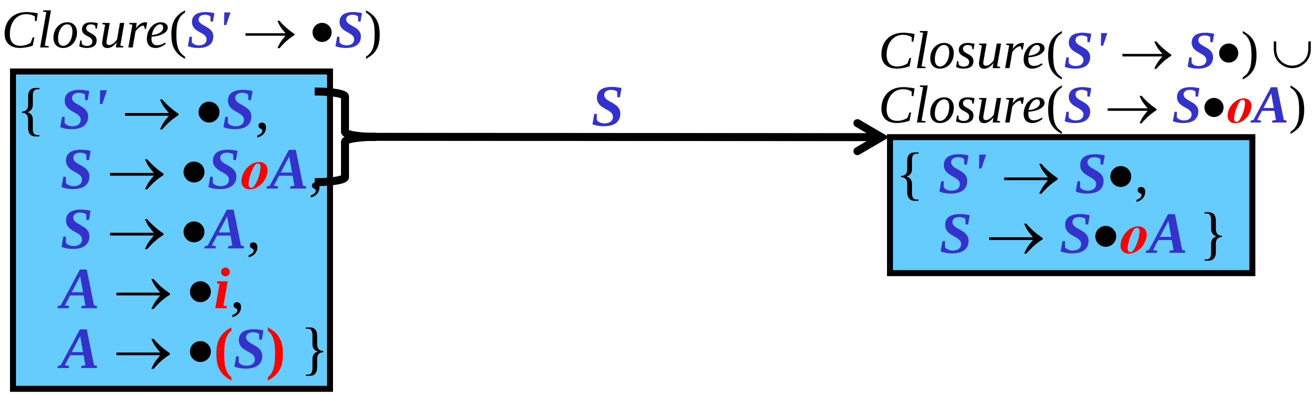
Initialization: $Closure(S' \to \bullet S)$

$$\boxed{\begin{aligned}\{\ & S' \to \bullet S,\\ & S \to \bullet SoA,\\ & S \to \bullet A,\\ & A \to \bullet i,\\ & A \to \bullet (S)\ \}\end{aligned}}$$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0\text{: } S' \to S,\ 1\text{: } S \to SoA,\ 2\text{: } S \to A,\ 3\text{: } A \to i,\ 4\text{: } A \to (S)\}$

Initialization:



$Closure(S' \to \bullet S)$

$\{ S' \to \bullet S,$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet (S) \}$

S

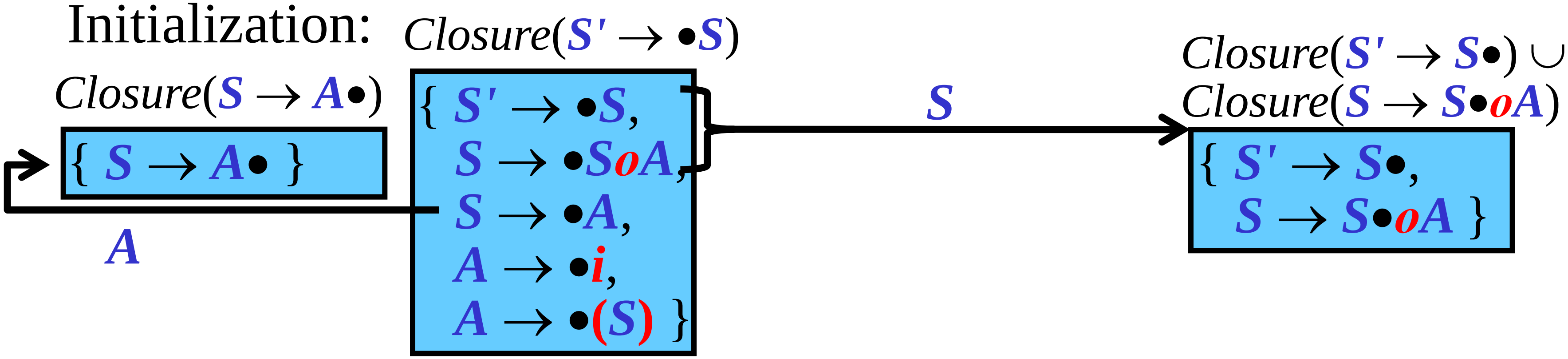
$Closure(S' \to S\bullet) \cup$
$Closure(S \to S\bullet oA)$

$\{ S' \to S\bullet,$
$S \to S\bullet oA \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0\text{: } S' \to S,\ 1\text{: } S \to SoA,\ 2\text{: } S \to A,\ 3\text{: } A \to i,\ 4\text{: } A \to (S)\}$

Initialization:



Closure$(S' \to \bullet S)$

$\{ S' \to \bullet S,$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet (S) \}$

Closure$(S \to A\bullet)$

$\{ S \to A\bullet \}$

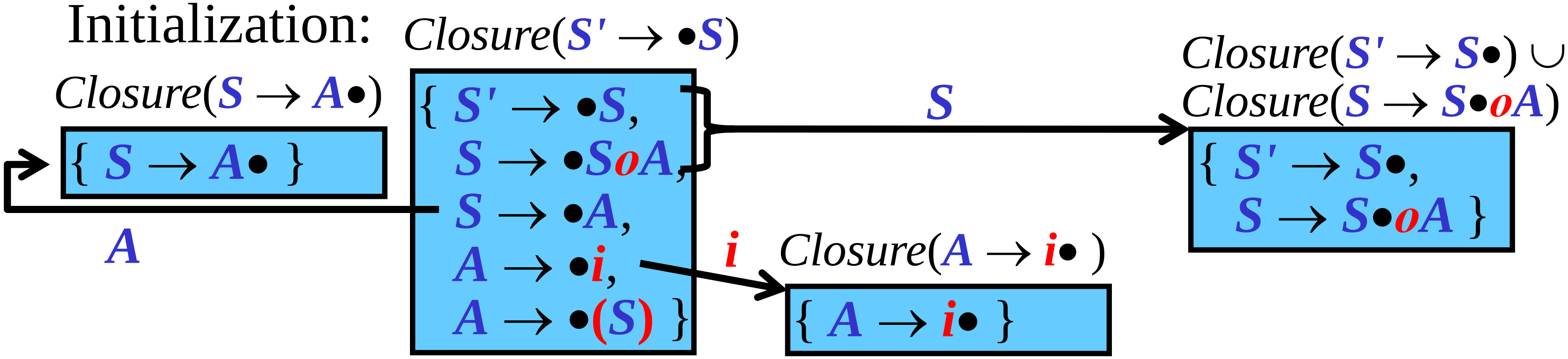
Closure$(S' \to S\bullet) \cup$
Closure$(S \to S\bullet oA)$

$\{ S' \to S\bullet,$
$S \to S\bullet oA \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0: S' \to S,\ 1: S \to SoA,\ 2: S \to A,\ 3: A \to i,\ 4: A \to (S)\}$

Initialization:



$Closure(S' \to \bullet S)$

$\{ S' \to \bullet S,$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet (S) \}$

$Closure(S' \to S\bullet) \cup$
$Closure(S \to S\bullet oA)$

$\{ S' \to S\bullet,$
$S \to S\bullet oA \}$

$Closure(S \to A\bullet)$

$\{ S \to A\bullet \}$

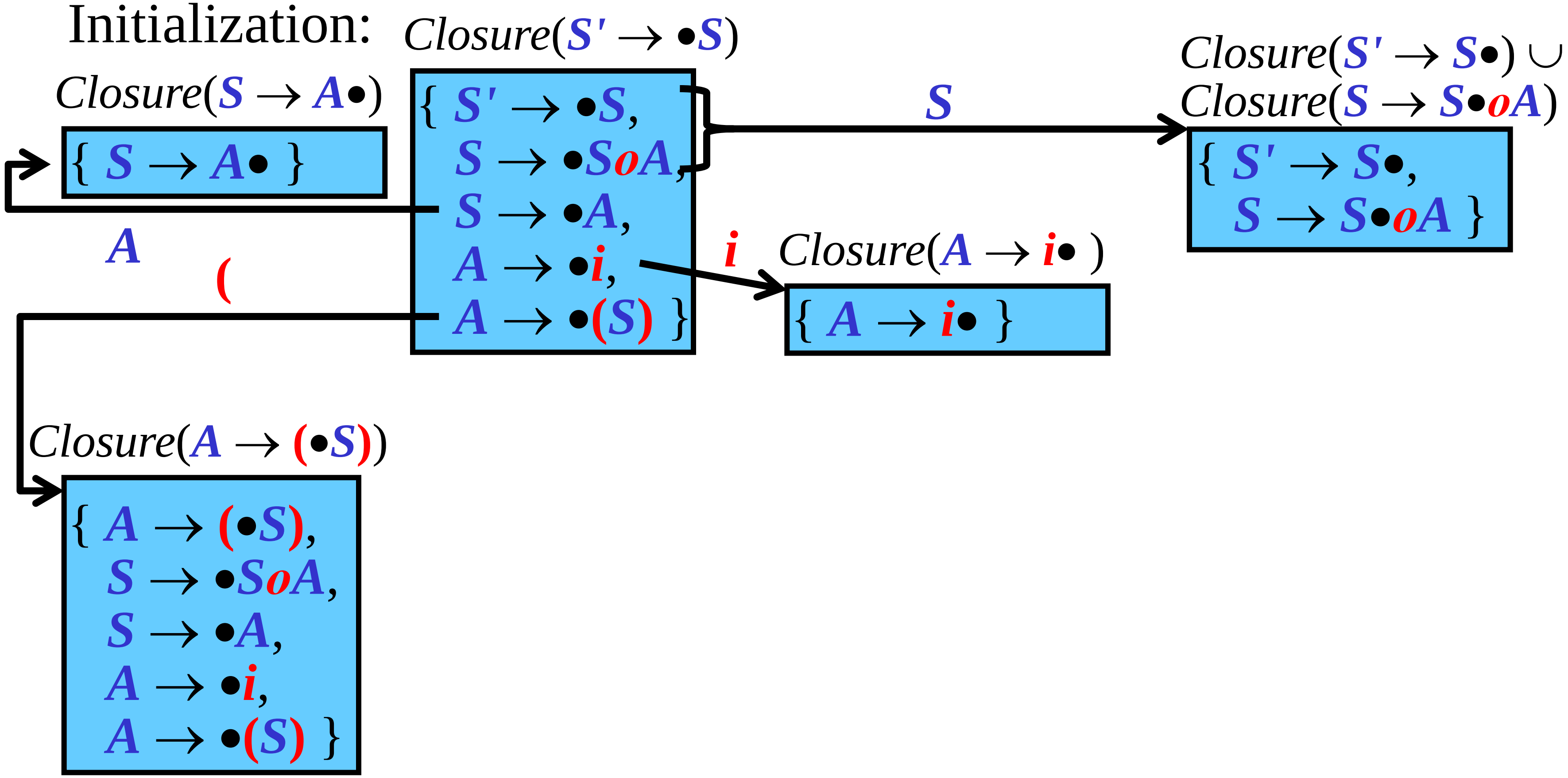
$Closure(A \to i\bullet)$

$\{ A \to i\bullet \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0: S' \to S,\ 1: S \to SoA,\ 2: S \to A,\ 3: A \to i,\ 4: A \to (S)\}$

Initialization:



Closure$(S' \to \bullet S)$

$\{ S' \to \bullet S,$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet(S) \}$

Closure$(S \to A\bullet)$

$\{ S \to A\bullet \}$

Closure$(S' \to S\bullet) \cup$
Closure$(S \to S\bullet oA)$

$\{ S' \to S\bullet,$
$S \to S\bullet oA \}$

Closure$(A \to i\bullet)$

$\{ A \to i\bullet \}$

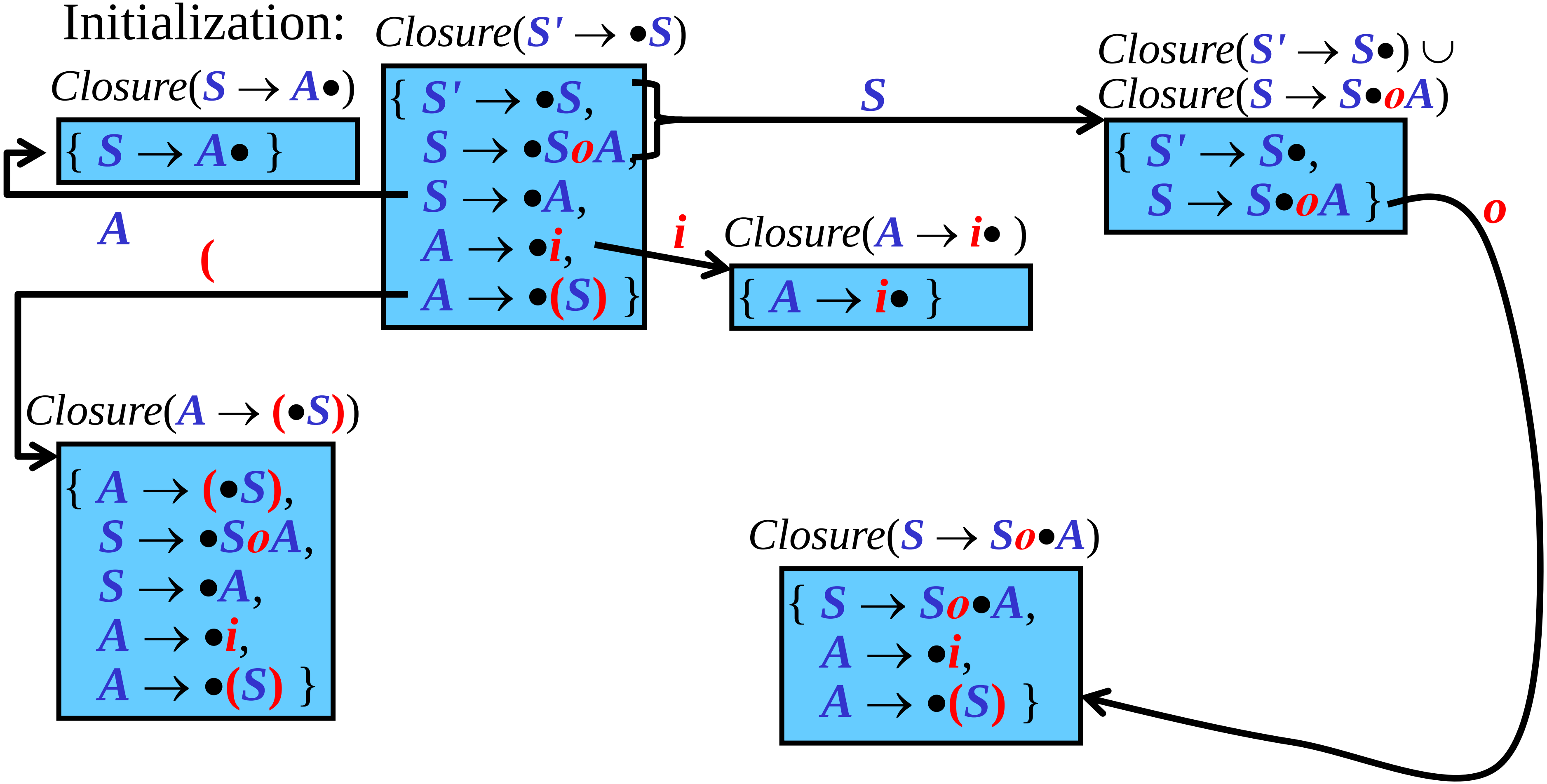
Closure$(A \to (\bullet S))$

$\{ A \to (\bullet S),$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet(S) \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0: S' \to S,\ 1: S \to SoA,\ 2: S \to A,\ 3: A \to i,\ 4: A \to (S)\}$

Initialization:



$Closure(S' \to \bullet S)$

$\{ S' \to \bullet S,$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet(S) \}$

$Closure(S \to A\bullet)$

$\{ S \to A\bullet \}$

$Closure(S' \to S\bullet) \cup Closure(S \to S\bullet oA)$

$\{ S' \to S\bullet,$
$S \to S\bullet oA \}$

$Closure(A \to i\bullet)$

$\{ A \to i\bullet \}$

$Closure(A \to (\bullet S))$

$\{ A \to (\bullet S),$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet(S) \}$

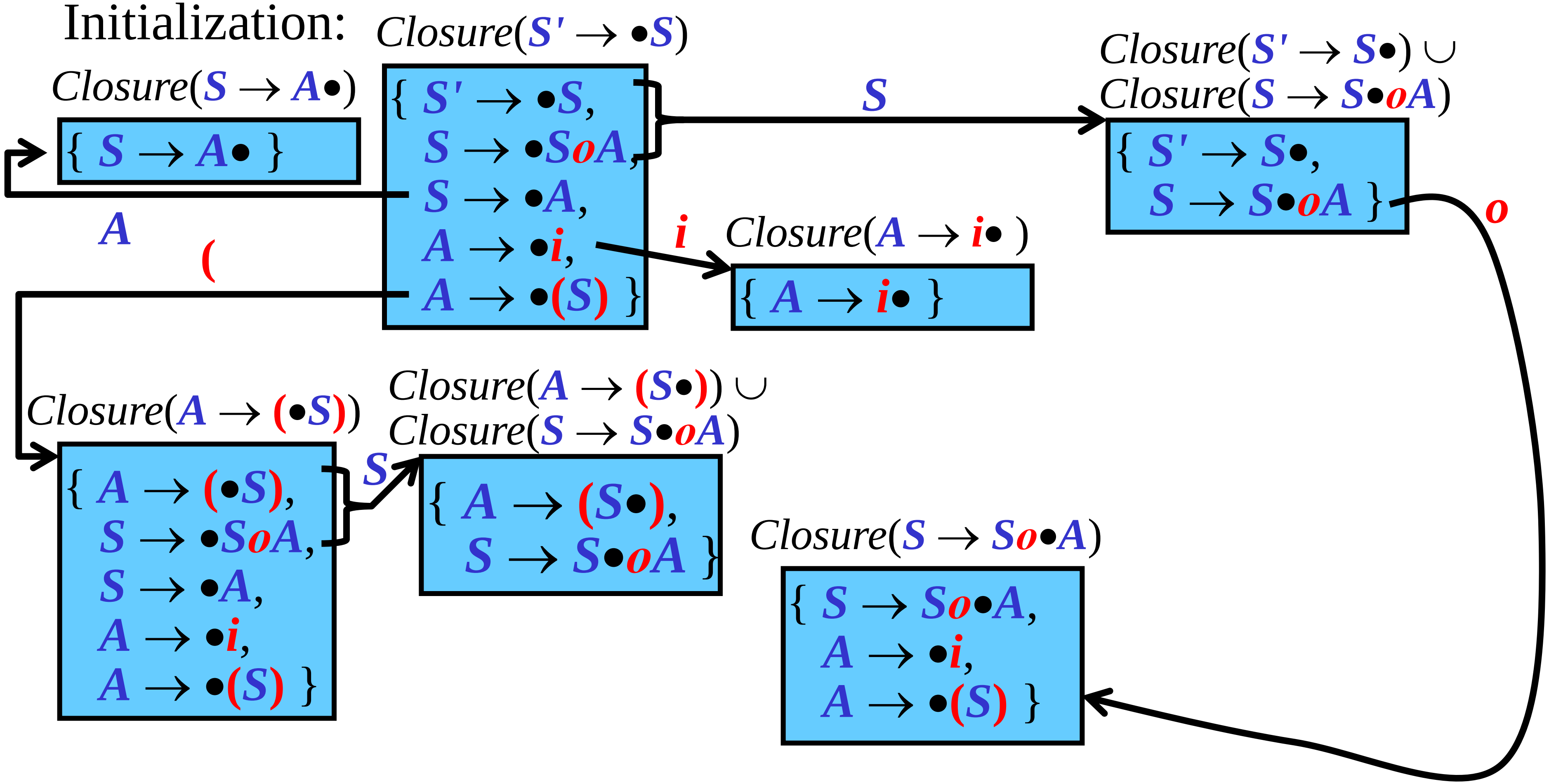
$Closure(S \to So\bullet A)$

$\{ S \to So\bullet A,$
$A \to \bullet i,$
$A \to \bullet(S) \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0: S' \to S,\ 1: S \to SoA,\ 2: S \to A,\ 3: A \to i,\ 4: A \to (S)\}$

Initialization:



Closure($S' \to \bullet S$)

$\{ S' \to \bullet S,$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet (S) \}$

Closure($S \to A\bullet$)

$\{ S \to A\bullet \}$

Closure($S' \to S\bullet$) ∪
Closure($S \to S\bullet oA$)

$\{ S' \to S\bullet,$
$S \to S\bullet oA \}$

Closure($A \to i\bullet$)

$\{ A \to i\bullet \}$

Closure($A \to (\bullet S)$)

$\{ A \to (\bullet S),$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet (S) \}$

Closure($A \to (S\bullet)$) ∪
Closure($S \to S\bullet oA$)

$\{ A \to (S\bullet),$
$S \to S\bullet oA \}$

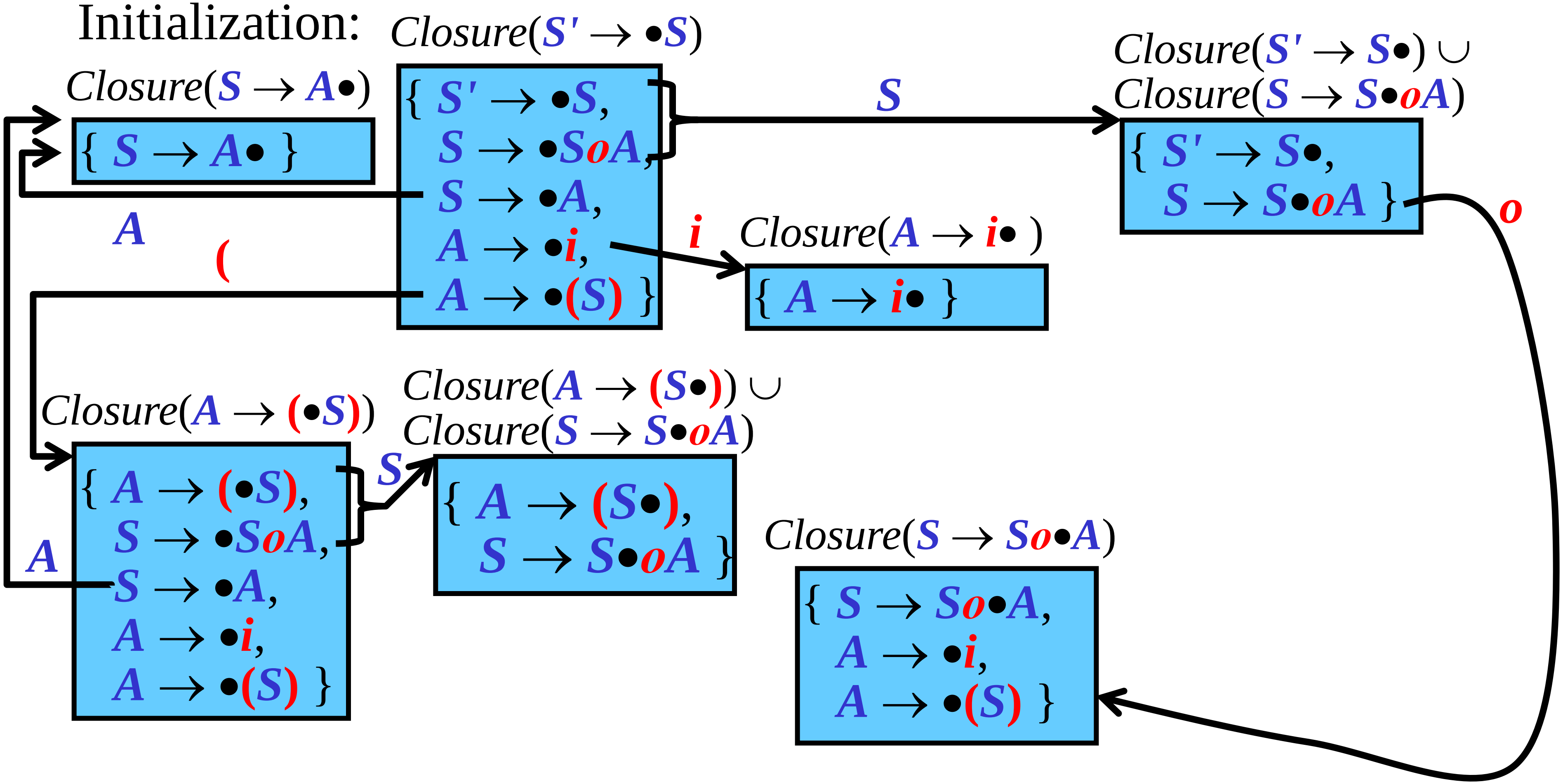
Closure($S \to So\bullet A$)

$\{ S \to So\bullet A,$
$A \to \bullet i,$
$A \to \bullet (S) \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0: S' \to S,\ 1: S \to SoA,\ 2: S \to A,\ 3: A \to i,\ 4: A \to (S)\}$

Initialization:



$Closure(S' \to \bullet S)$

$\{ S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet (S) \}$

$Closure(S \to A\bullet)$

$\{ S \to A\bullet \}$

$Closure(S' \to S\bullet) \cup Closure(S \to S\bullet oA)$

$\{ S' \to S\bullet,\ S \to S\bullet oA \}$

$Closure(A \to i\bullet)$

$\{ A \to i\bullet \}$

$Closure(A \to (\bullet S))$

$\{ A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet (S) \}$

$Closure(A \to (S\bullet)) \cup Closure(S \to S\bullet oA)$

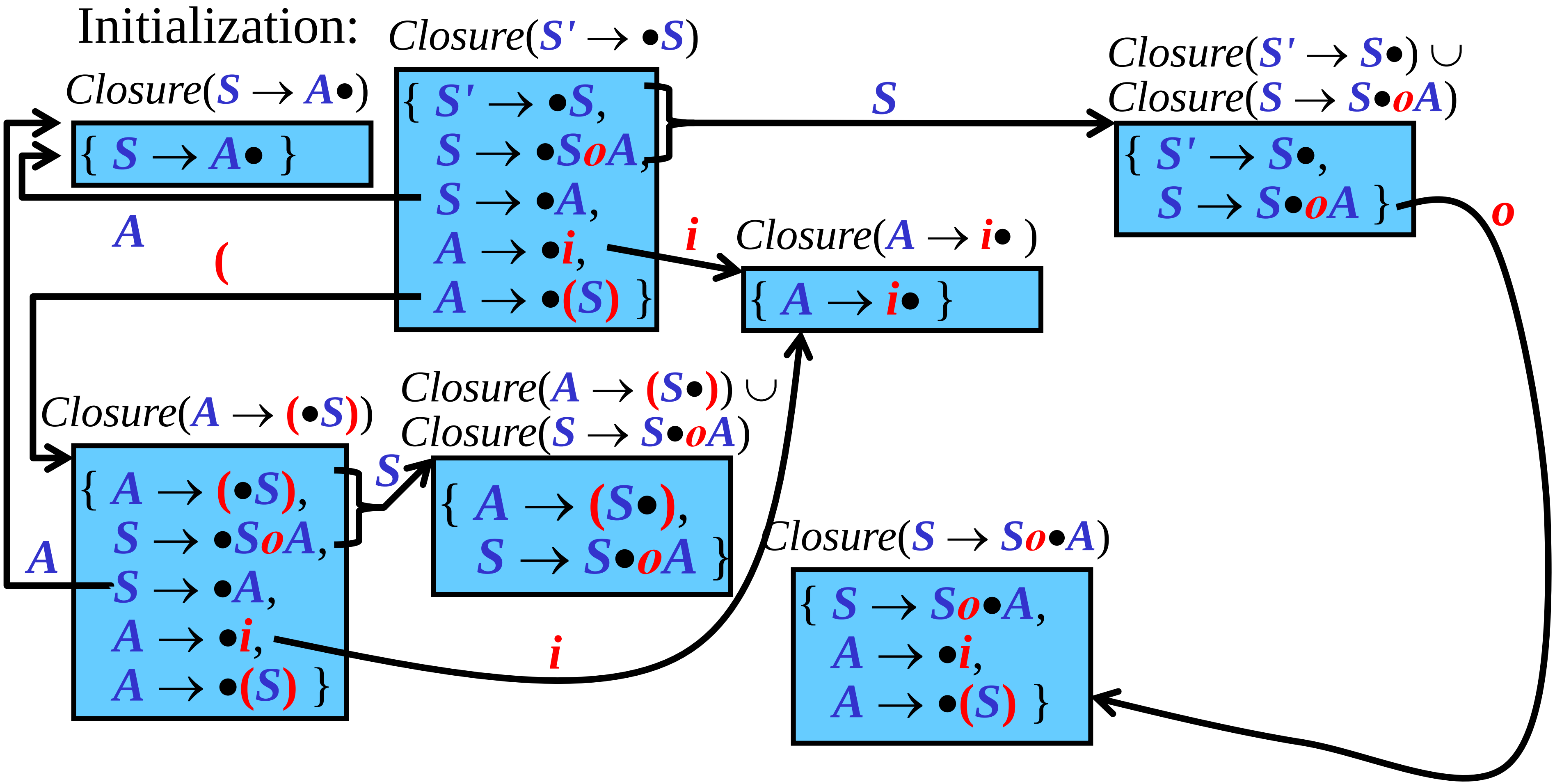
$\{ A \to (S\bullet),\ S \to S\bullet oA \}$

$Closure(S \to So\bullet A)$

$\{ S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet (S) \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0\text{: } S' \rightarrow S,\ 1\text{: } S \rightarrow SoA,\ 2\text{: } S \rightarrow A,\ 3\text{: } A \rightarrow i,\ 4\text{: } A \rightarrow (S)\}$



Initialization: $Closure(S' \rightarrow \bullet S)$

$\{\ S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\ \}$

$Closure(S \rightarrow A\bullet)$

$\{\ S \rightarrow A\bullet\ \}$

$Closure(S' \rightarrow S\bullet) \cup Closure(S \rightarrow S\bullet oA)$

$\{\ S' \rightarrow S\bullet,\ S \rightarrow S\bullet oA\ \}$

$Closure(A \rightarrow i\bullet\ )$

$\{\ A \rightarrow i\bullet\ \}$

$Closure(A \rightarrow (\bullet S))$

$\{\ A \rightarrow (\bullet S),\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\ \}$

$Closure(A \rightarrow (S\bullet)) \cup Closure(S \rightarrow S\bullet oA)$

$\{\ A \rightarrow (S\bullet),\ S \rightarrow S\bullet oA\ \}$

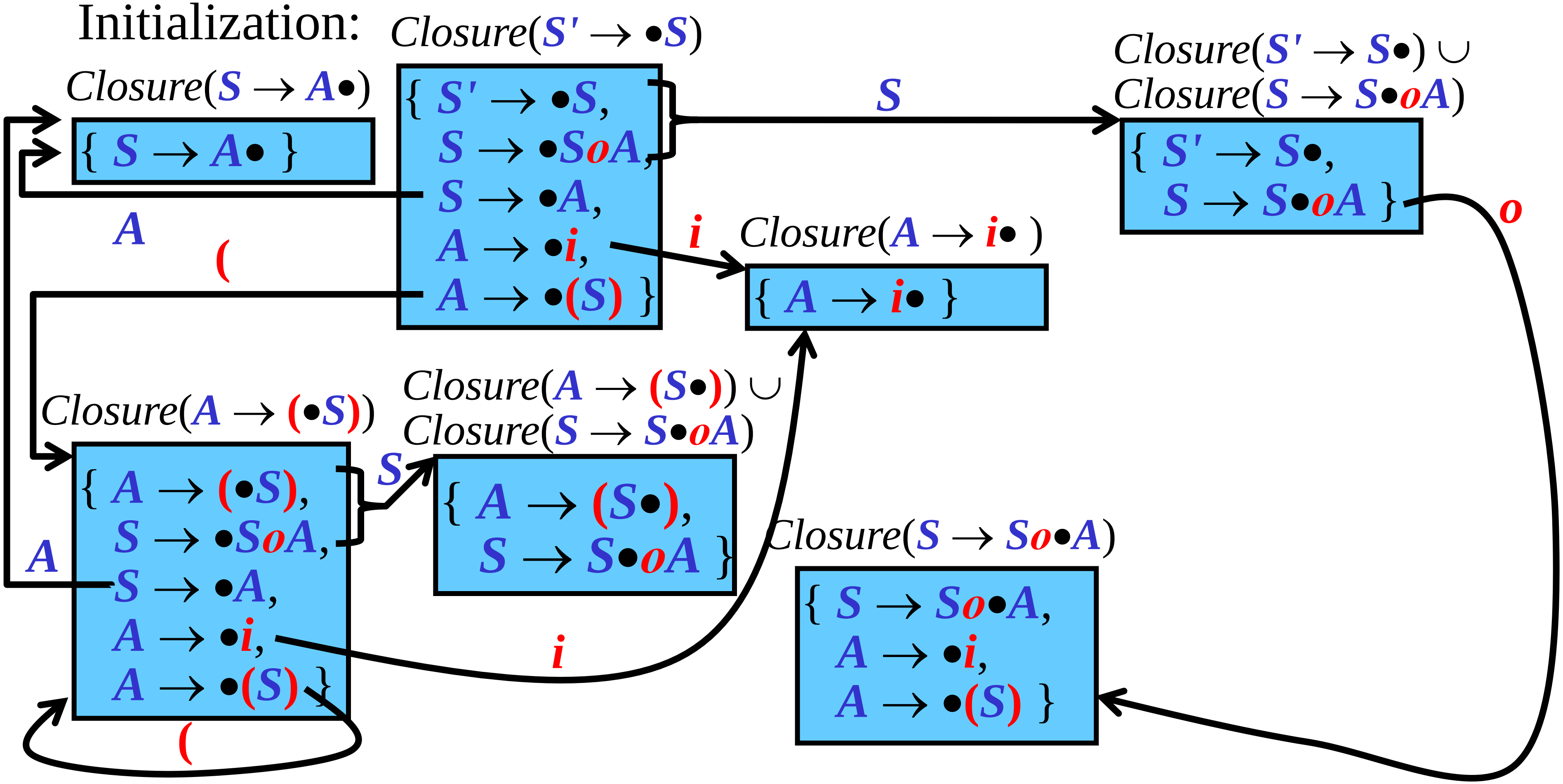
$Closure(S \rightarrow So\bullet A)$

$\{\ S \rightarrow So\bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\ \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0{:}\ S' \to S,\ 1{:}\ S \to SoA,\ 2{:}\ S \to A,\ 3{:}\ A \to i,\ 4{:}\ A \to (S)\}$

Initialization:



Closure$(S' \to \bullet S)$
$\{ S' \to \bullet S,$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet (S) \}$

Closure$(S \to A\bullet)$
$\{ S \to A\bullet \}$

Closure$(S' \to S\bullet) \cup$
Closure$(S \to S\bullet oA)$
$\{ S' \to S\bullet,$
$S \to S\bullet oA \}$

Closure$(A \to i\bullet\ )$
$\{ A \to i\bullet \}$

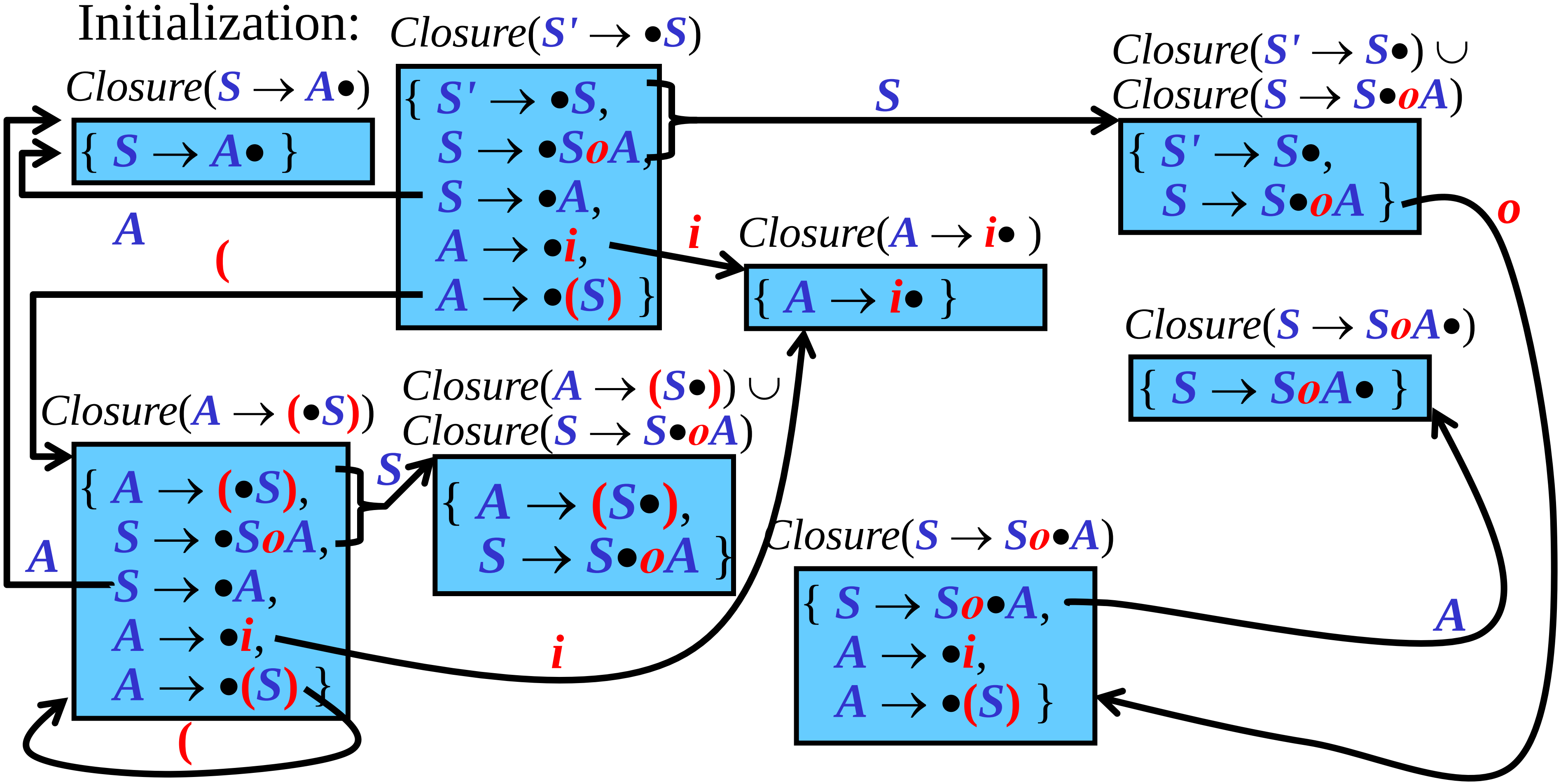
Closure$(A \to (\bullet S))$
$\{ A \to (\bullet S),$
$S \to \bullet SoA,$
$S \to \bullet A,$
$A \to \bullet i,$
$A \to \bullet (S) \}$

Closure$(A \to (S\bullet)) \cup$
Closure$(S \to S\bullet oA)$
$\{ A \to (S\bullet),$
$S \to S\bullet oA \}$

Closure$(S \to So\bullet A)$
$\{ S \to So\bullet A,$
$A \to \bullet i,$
$A \to \bullet (S) \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0\text{: } S' \rightarrow S,\ 1\text{: } S \rightarrow SoA,\ 2\text{: } S \rightarrow A,\ 3\text{: } A \rightarrow i,\ 4\text{: } A \rightarrow (S)\}$



Initialization: $Closure(S' \rightarrow \bullet S)$

$\{ S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet (S) \}$

$Closure(S \rightarrow A\bullet)$

$\{ S \rightarrow A\bullet \}$

$Closure(S' \rightarrow S\bullet) \cup Closure(S \rightarrow S\bullet oA)$

$\{ S' \rightarrow S\bullet,\ S \rightarrow S\bullet oA \}$

$Closure(A \rightarrow i\bullet)$

$\{ A \rightarrow i\bullet \}$

$Closure(S \rightarrow SoA\bullet)$

$\{ S \rightarrow SoA\bullet \}$

$Closure(A \rightarrow (\bullet S))$

$\{ A \rightarrow (\bullet S),\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet (S) \}$

$Closure(A \rightarrow (S\bullet)) \cup Closure(S \rightarrow S\bullet oA)$

$\{ A \rightarrow (S\bullet),\ S \rightarrow S\bullet oA \}$

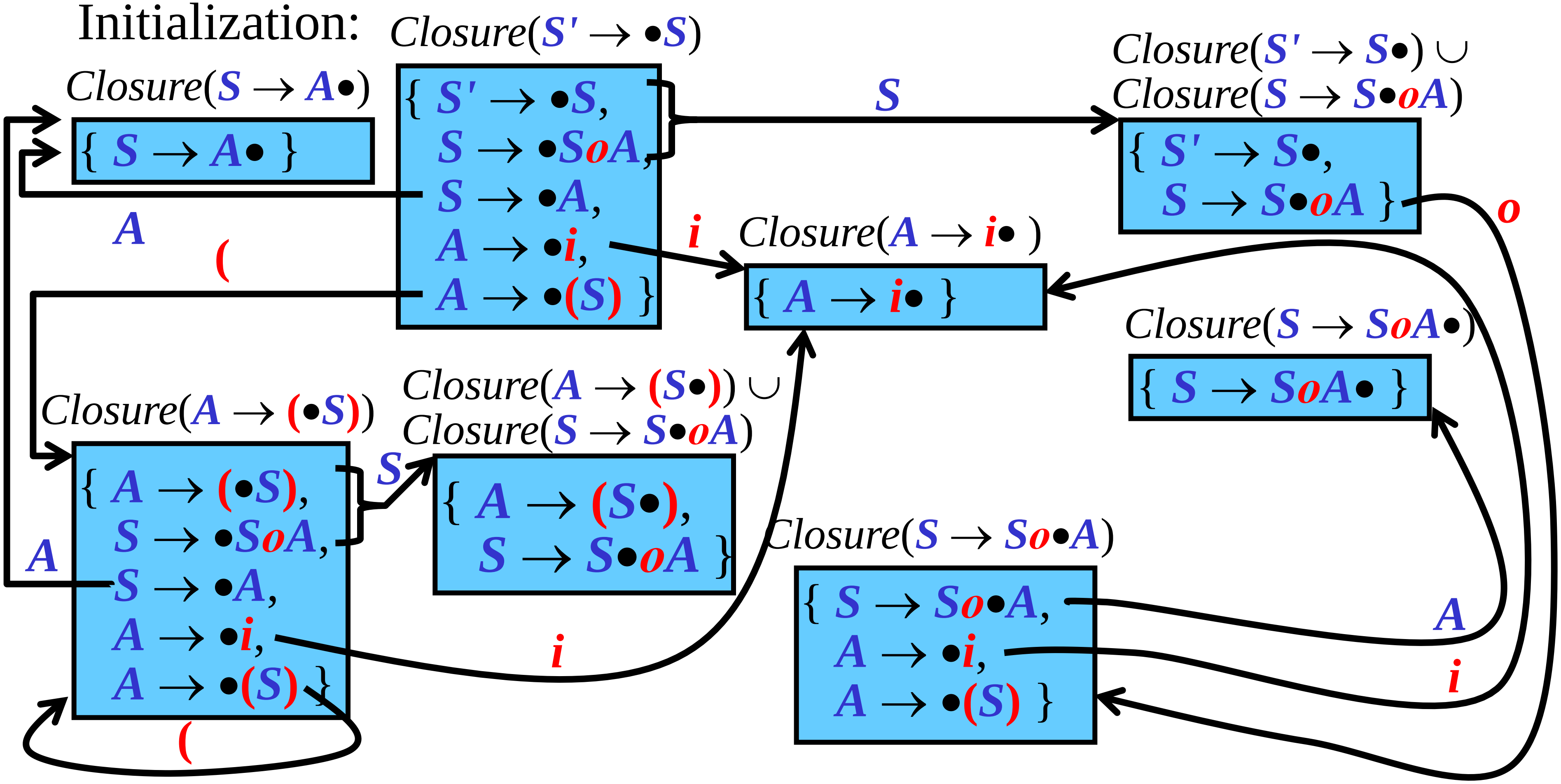
$Closure(S \rightarrow So\bullet A)$

$\{ S \rightarrow So\bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet (S) \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0: S' \to S,\ 1: S \to SoA,\ 2: S \to A,\ 3: A \to i,\ 4: A \to (S)\}$

Initialization:



$Closure(S' \to \bullet S)$
$\{ S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S) \}$

$Closure(S \to A\bullet)$
$\{ S \to A\bullet \}$

$Closure(S' \to S\bullet) \cup Closure(S \to S\bullet oA)$
$\{ S' \to S\bullet,\ S \to S\bullet oA \}$

$Closure(A \to i\bullet)$
$\{ A \to i\bullet \}$

$Closure(S \to SoA\bullet)$
$\{ S \to SoA\bullet \}$

$Closure(A \to (\bullet S))$
$\{ A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S) \}$

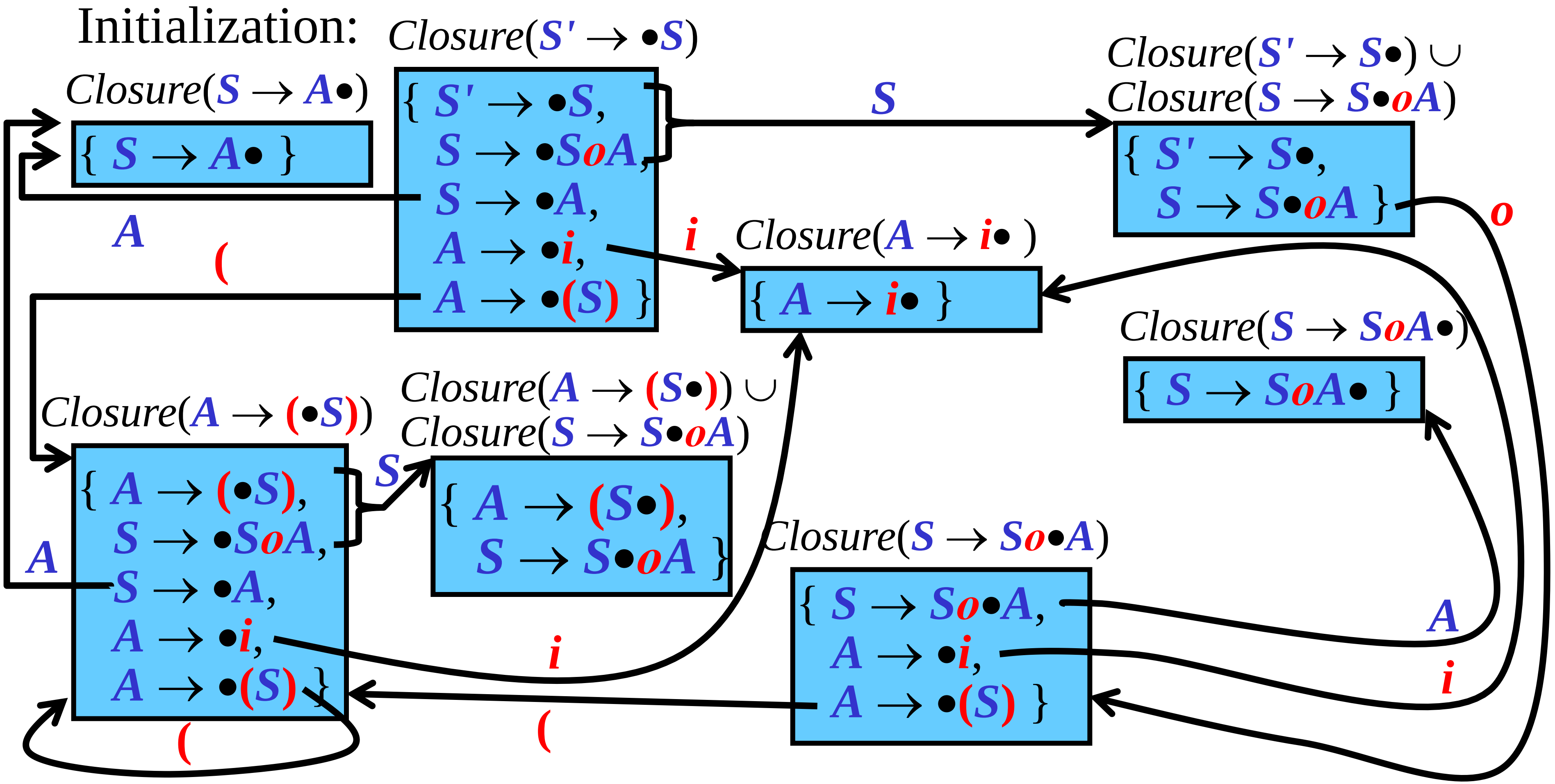
$Closure(A \to (S\bullet)) \cup Closure(S \to S\bullet oA)$
$\{ A \to (S\bullet),\ S \to S\bullet oA \}$

$Closure(S \to So\bullet A)$
$\{ S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet(S) \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,

$P = \{0\text{: } S' \to S,\ 1\text{: } S \to SoA,\ 2\text{: } S \to A,\ 3\text{: } A \to i,\ 4\text{: } A \to (S)\}$



Initialization: $Closure(S' \to \bullet S)$

$\{ S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S) \}$

$Closure(S \to A\bullet)$

$\{ S \to A\bullet \}$

$Closure(S' \to S\bullet) \cup Closure(S \to S\bullet oA)$

$\{ S' \to S\bullet,\ S \to S\bullet oA \}$

$Closure(A \to i\bullet)$

$\{ A \to i\bullet \}$

$Closure(S \to SoA\bullet)$

$\{ S \to SoA\bullet \}$

$Closure(A \to (\bullet S))$

$\{ A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S) \}$

$Closure(A \to (S\bullet)) \cup Closure(S \to S\bullet oA)$

$\{ A \to (S\bullet),\ S \to S\bullet oA \}$

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Set Θ_G: Example

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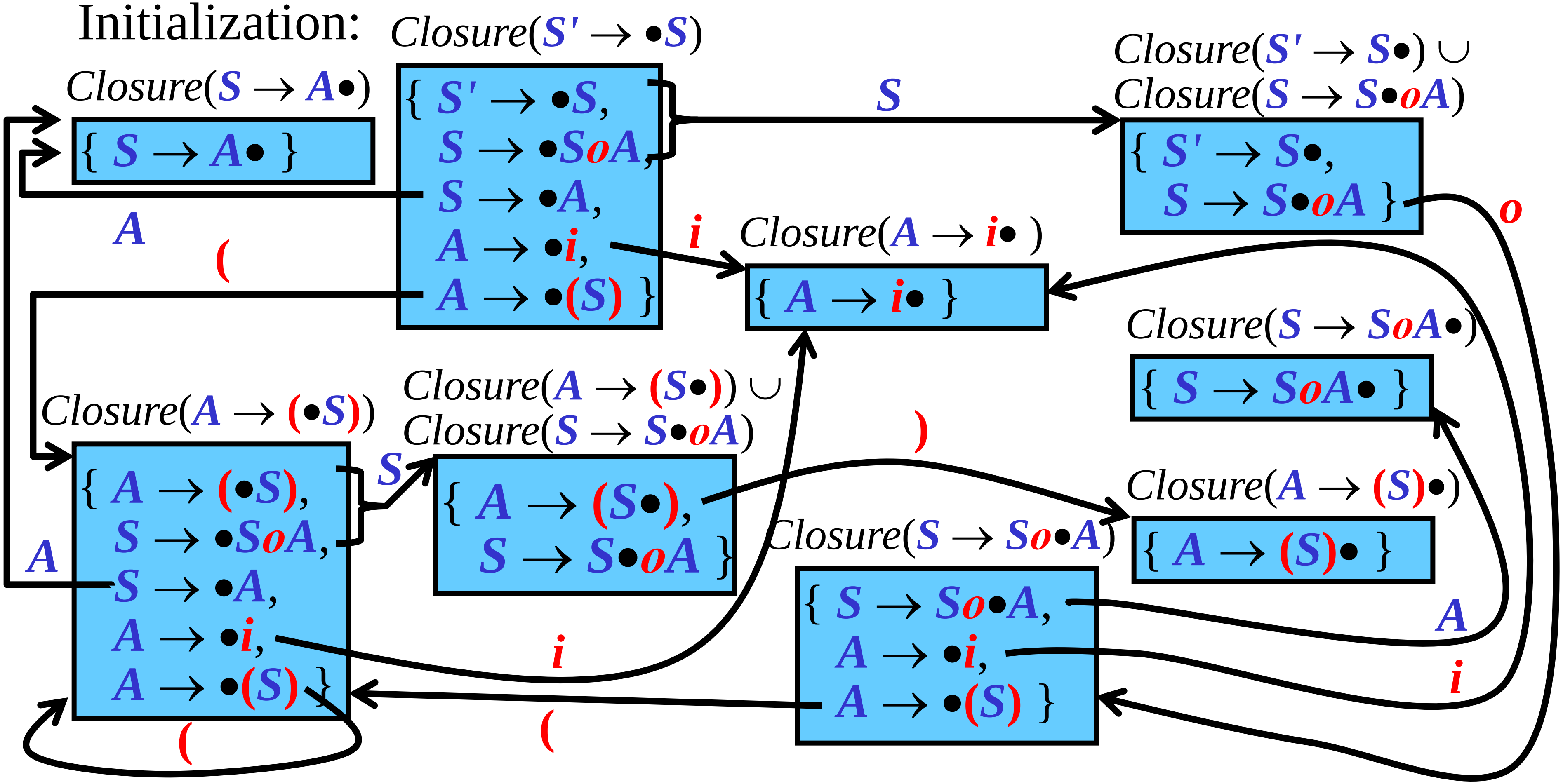
Initialization:



Closure$(S' \rightarrow \bullet S)$

$\{ S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S) \}$

Closure$(S \rightarrow A\bullet)$

$\{ S \rightarrow A\bullet \}$

Closure$(S' \rightarrow S\bullet) \cup$ *Closure*$(S \rightarrow S\bullet oA)$

$\{ S' \rightarrow S\bullet,\ S \rightarrow S\bullet oA \}$

Closure$(A \rightarrow i\bullet)$

$\{ A \rightarrow i\bullet \}$

Closure$(S \rightarrow SoA\bullet)$

$\{ S \rightarrow SoA\bullet \}$

Closure$(A \rightarrow (\bullet S))$

$\{ A \rightarrow (\bullet S),\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S) \}$

Closure$(A \rightarrow (S\bullet)) \cup$ *Closure*$(S \rightarrow S\bullet oA)$

$\{ A \rightarrow (S\bullet),\ S \rightarrow S\bullet oA \}$

Closure$(A \rightarrow (S)\bullet)$

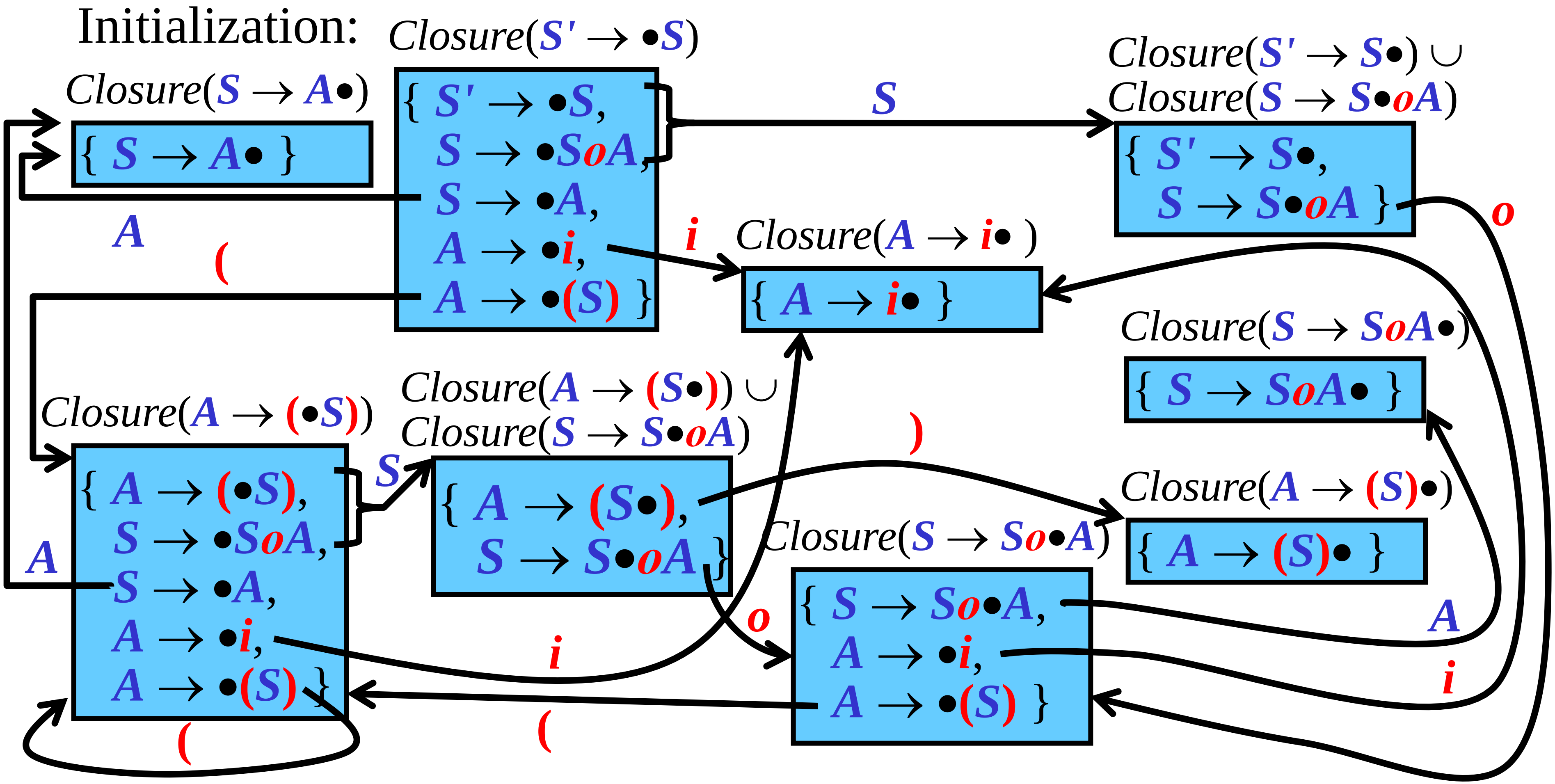
$\{ A \rightarrow (S)\bullet \}$

Closure$(S \rightarrow So\bullet A)$

$\{ S \rightarrow So\bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S) \}$

Set Θ_G: Example

$H = (N, T, P, S')$, where $N = \{S', S, A\}$, $T = \{i, o, (,)\}$,
$P = \{0\!: S' \to S,\ 1\!: S \to SoA,\ 2\!: S \to A,\ 3\!: A \to i,\ 4\!: A \to (S)\}$



Initialization: $Closure(S' \to \bullet S)$

$\{\ S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\ \}$

$Closure(S \to A\bullet)$

$\{\ S \to A\bullet\ \}$

$Closure(S' \to S\bullet) \cup Closure(S \to S\bullet oA)$

$\{\ S' \to S\bullet,\ S \to S\bullet oA\ \}$

$Closure(A \to i\bullet\ )$

$\{\ A \to i\bullet\ \}$

$Closure(S \to SoA\bullet)$

$\{\ S \to SoA\bullet\ \}$

$Closure(A \to (\bullet S))$

$\{\ A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\ \}$

$Closure(A \to (S\bullet)) \cup Closure(S \to S\bullet oA)$

$\{\ A \to (S\bullet),\ S \to S\bullet oA\ \}$

$Closure(A \to (S)\bullet)$

$\{\ A \to (S)\bullet\ \}$

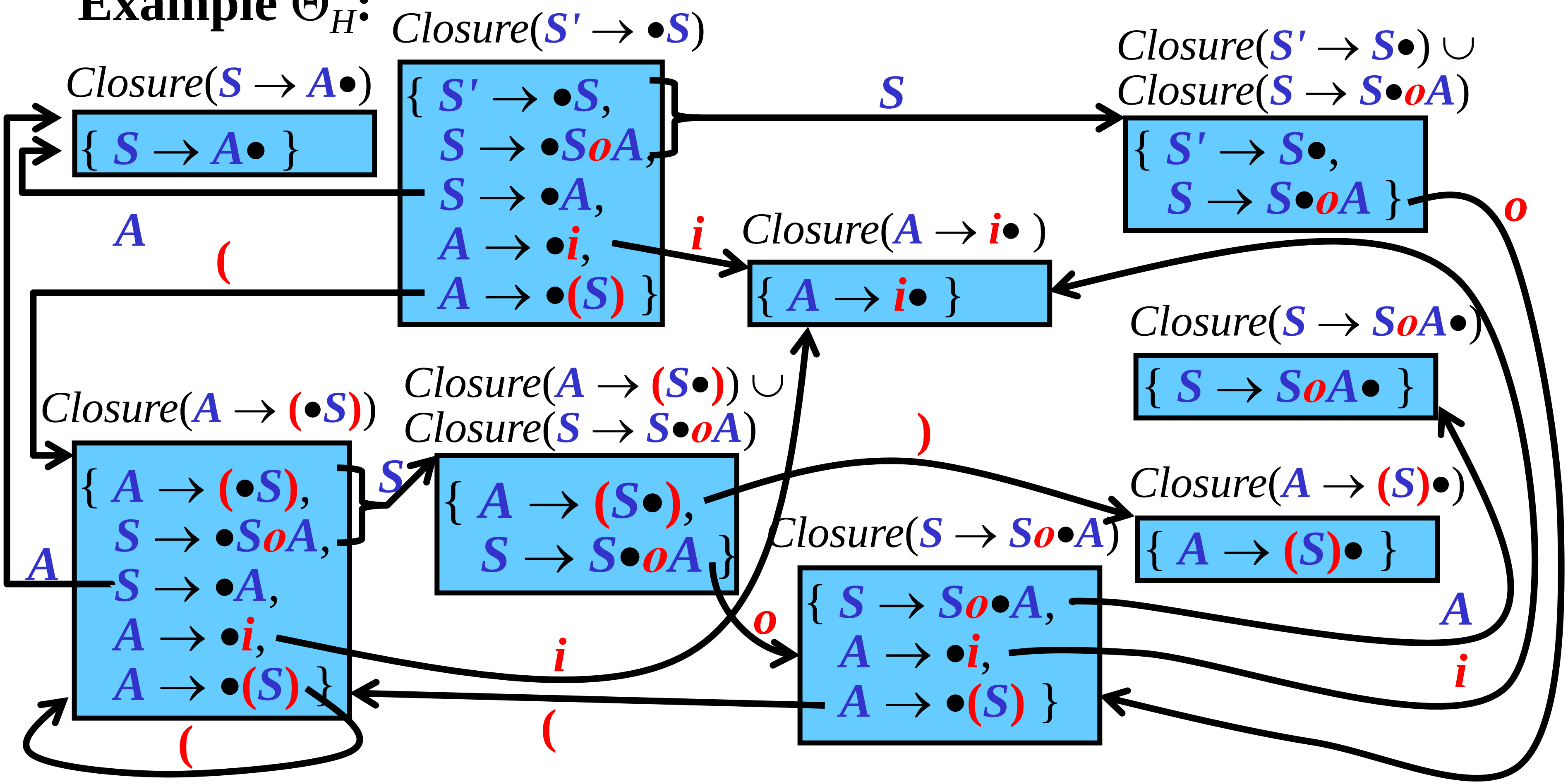
$Closure(S \to So\bullet A)$

$\{\ S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet(S)\ \}$

Naming of members in set Θ_G

Name the elements of Θ_G as I_0 to I_n, where n+1 is number of elements in Θ_G. The member with $S' \to \bullet S$ is I_0.

Example Θ_H:



$Closure(S' \to \bullet S)$

$\{ S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S) \}$

$Closure(S \to A\bullet)$

$\{ S \to A\bullet \}$

$Closure(S' \to S\bullet) \cup Closure(S \to S\bullet oA)$

$\{ S' \to S\bullet,\ S \to S\bullet oA \}$

$Closure(A \to i\bullet\ )$

$\{ A \to i\bullet \}$

$Closure(S \to SoA\bullet)$

$\{ S \to SoA\bullet \}$

$Closure(A \to (\bullet S))$

$\{ A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S) \}$

$Closure(A \to (S\bullet)) \cup Closure(S \to S\bullet oA)$

$\{ A \to (S\bullet),\ S \to S\bullet oA \}$

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$Closure(S \to So\bullet A)$

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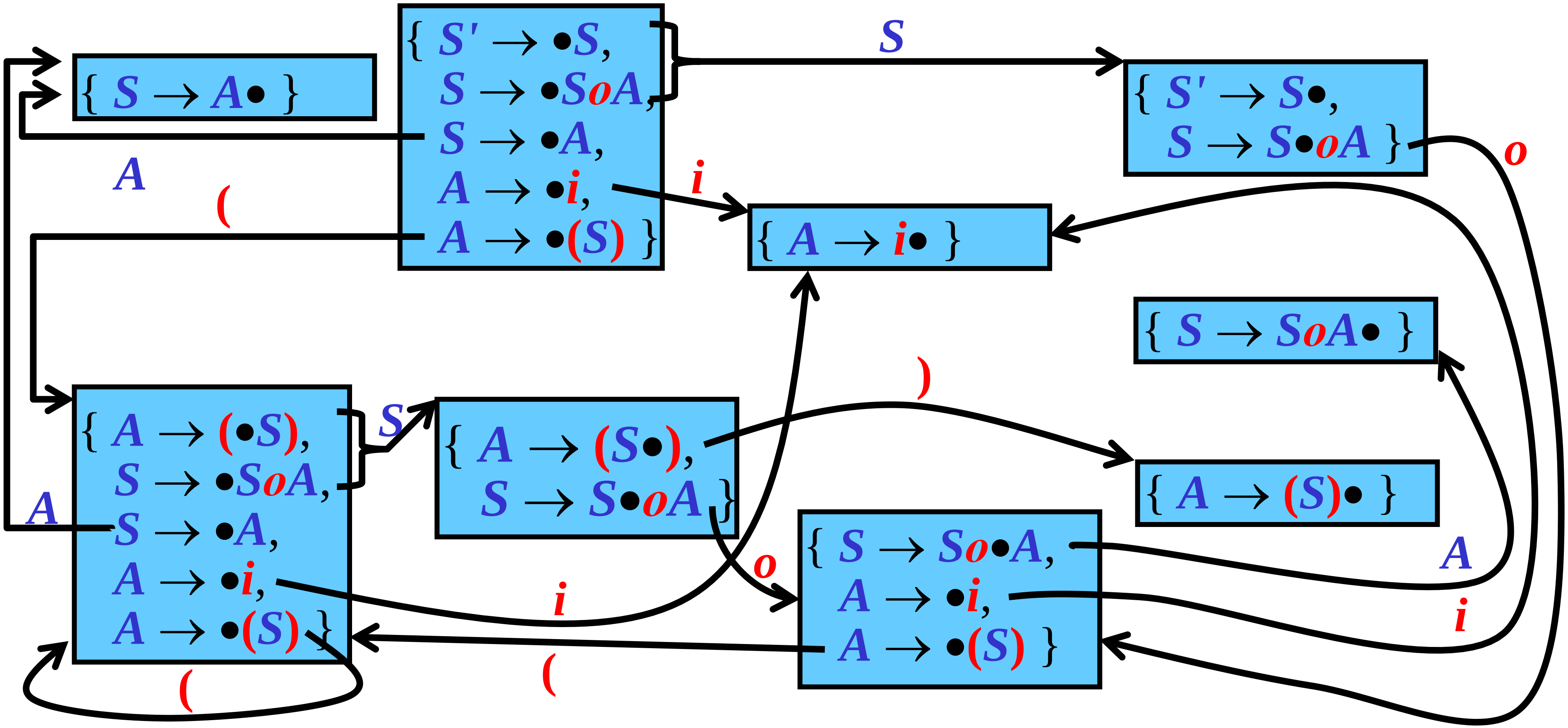


$\{ S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S) \}$

$\{ S \to A\bullet \}$

$\{ S' \to S\bullet,\ S \to S\bullet oA \}$

$\{ A \to i\bullet \}$

$\{ S \to SoA\bullet \}$

$\{ A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S) \}$

$\{ A \to (S\bullet),\ S \to S\bullet oA \}$

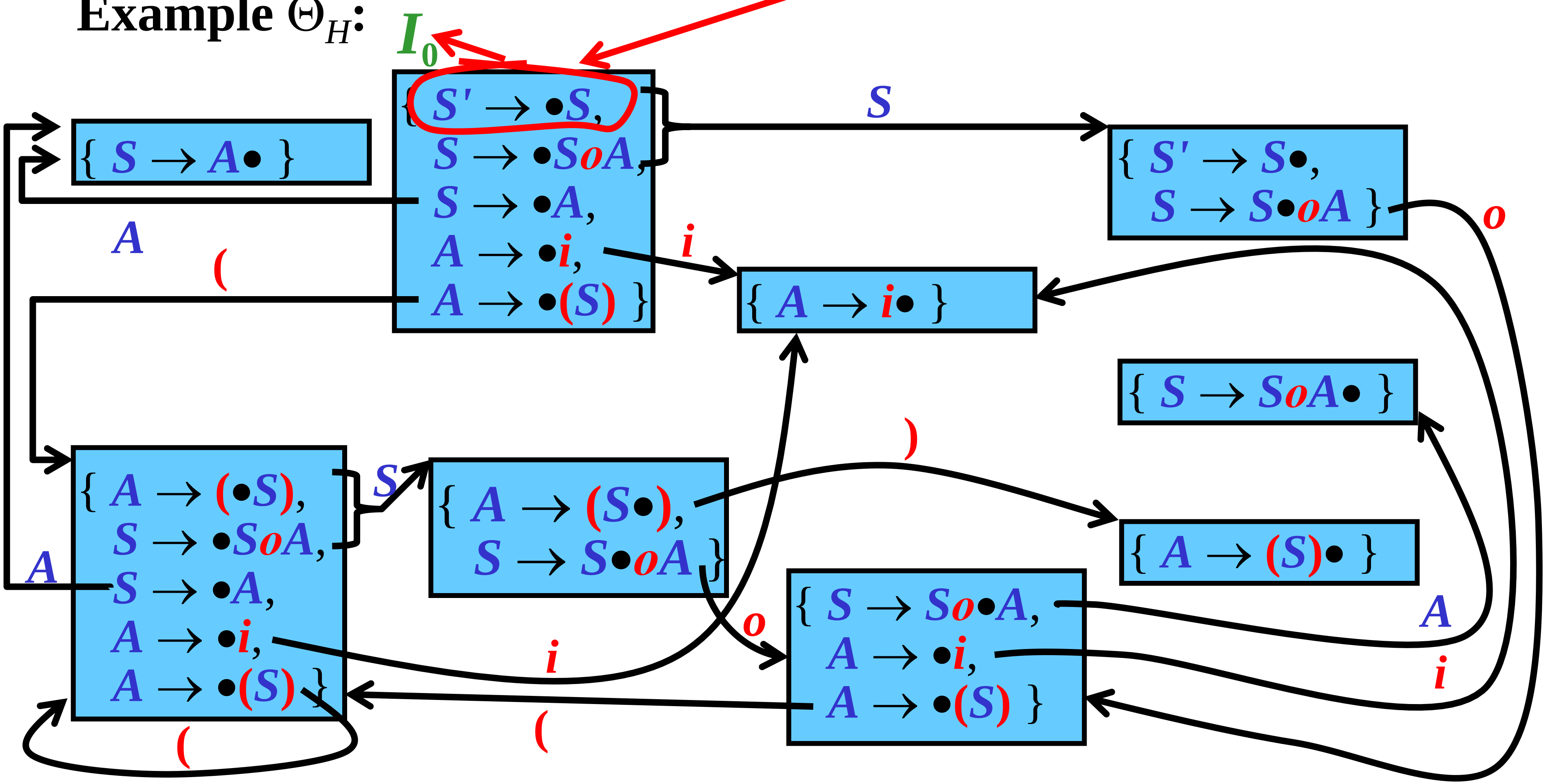
$\{ A \to (S)\bullet \}$

$\{ S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet(S) \}$

Naming of members in set Θ_G

Name the elements of Θ_G as I_0 to I_n, where $n+1$ is number of elements in Θ_G. The member with $S' \rightarrow \bullet S$ is I_0.

Example Θ_H:



I_0: $\{ S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet (S) \}$

$\{ S \rightarrow A\bullet \}$

$\{ S' \rightarrow S\bullet,\ S \rightarrow S\bullet oA \}$

$\{ A \rightarrow i\bullet \}$

$\{ S \rightarrow SoA\bullet \}$

$\{ A \rightarrow (\bullet S),\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet (S) \}$

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$\{ A \rightarrow (S)\bullet \}$

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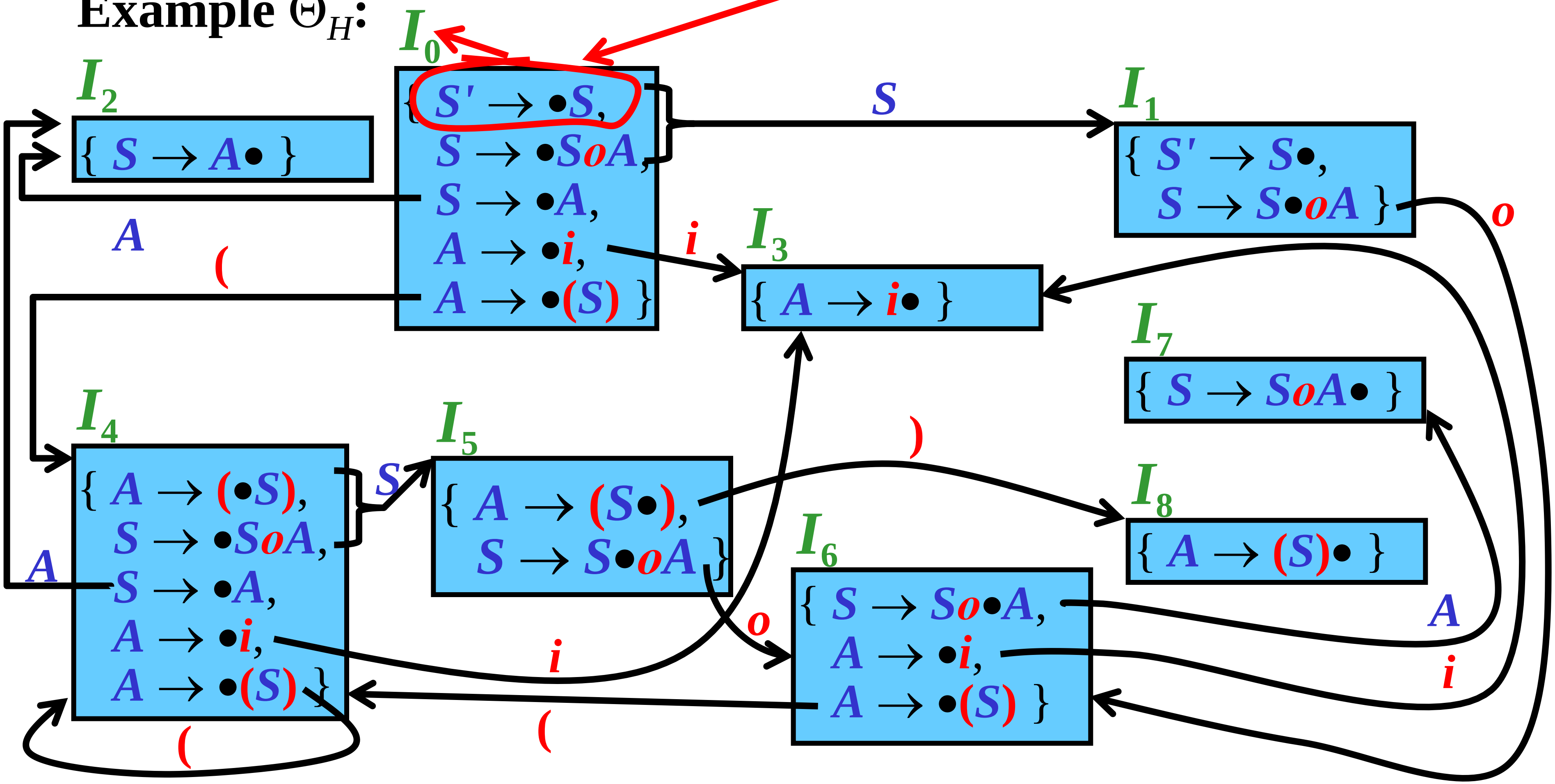
Naming of members in set Θ_G

Name the elements of Θ_G as I_0 to I_n, where n+1 is number of elements in Θ_G. The member with $S' \to \bullet S$ is I_0.

Example Θ_H:



Construction of LR-table: SLR Algorithm

- **Input:** Extended $G = (N, T, P, S')$; Θ_G;
  $Follow(A)$ for all $A \in N$
- **Output:** LR-table for G (α = Action part, β = Go-to part)

- **Method:**

```
• StatesOfTable := $\Theta_G$; StartState := Closure($S' \to \bullet S$);
• for each $x \in \Theta_G$ do
• for each $I \in x$ do
    • case $I$ of
        • $I = A \to y \bullet Xz$, where $X \in N$:
          $\beta[x, X] := \Theta_X(x)$
        • $I = A \to y \bullet Xz$, where $X \in T$:
          $\alpha[x, X] := s\Theta_X(x)$
        • $I = S' \to S \bullet$: $\alpha[x, \$] :=$ ☺
        • $I = A \to y \bullet$ ($A \neq S'$):
          for each $a \in Follow(A)$ do $\alpha[x, a] := rp$,
          where $p$ is a label of rule $A \to y$
```

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0: \{S' \to \bullet S, S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet (S)\},$
$I_1: \{S' \to S\bullet, S \to S\bullet oA\}, I_2: \{S \to A\bullet\}, I_3: \{A \to i\bullet\},$
$I_4: \{A \to (\bullet S), S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet (S)\},$
$I_5: \{A \to (S\bullet), S \to S\bullet oA\}, I_6: \{S \to So\bullet A, A \to \bullet i, A \to \bullet (S)\},$
$I_7: \{S \to SoA\bullet\}, I_8: \{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0							

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0:\{S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_1:\{S' \to S\bullet,\ S \to S\bullet oA\},\ I_2:\{S \to A\bullet\},\ I_3:\{A \to i\bullet\},$
$I_4:\{A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_5:\{A \to (S\bullet),\ S \to S\bullet oA\},\ I_6:\{S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_7:\{S \to SoA\bullet\},\ I_8:\{A \to (S)\bullet\}\}$

Task: LR-table for K

	α					β	
	i	o	(	)	$	S	A
I_0							

$S' \to \bullet S \in I_0,\ S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

Construction of LR-table: Example 1/5

$$\begin{aligned}
&\Theta_H = \{I_0:\{S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},\\
&I_1:\{S' \to S\bullet,\ S \to S\bullet oA\},\ I_2:\{S \to A\bullet\},\ I_3:\{A \to i\bullet\},\\
&I_4:\{A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},\\
&I_5:\{A \to (S\bullet),\ S \to S\bullet oA\},\ I_6:\{S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},\\
&I_7:\{S \to SoA\bullet\},\ I_8:\{A \to (S)\bullet\}\}
\end{aligned}$$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0						I_1	

$S' \to \bullet S \in I_0$, $S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0{:}\{S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_1{:}\{S' \to S\bullet,\ S \to S\bullet oA\},\ I_2{:}\{S \to A\bullet\},\ I_3{:}\{A \to i\bullet\},$
$I_4{:}\{A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_5{:}\{A \to (S\bullet),\ S \to S\bullet oA\},\ I_6{:}\{S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_7{:}\{S \to SoA\bullet\},\ I_8{:}\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	o	(	)	$	S	A
I_0						I_1	

$S' \to \bullet S \in I_0,\ S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \to \bullet SoA \in I_0,\ S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0{:}\{S' \to \bullet S,\ S \to \bullet S o A,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet (S)\},$
$I_1{:}\{S' \to S\bullet,\ S \to S\bullet o A\},\ I_2{:}\{S \to A\bullet\},\ I_3{:}\{A \to i\bullet\},$
$I_4{:}\{A \to (\bullet S),\ S \to \bullet S o A,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet (S)\},$
$I_5{:}\{A \to (S\bullet),\ S \to S\bullet o A\},\ I_6{:}\{S \to S o\bullet A,\ A \to \bullet i,\ A \to \bullet (S)\},$
$I_7{:}\{S \to S o A\bullet\},\ I_8{:}\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0						I_1	

$S' \to \bullet S \in I_0,\ S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \to \bullet S o A \in I_0,\ S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0{:}\{S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_1{:}\{S' \rightarrow S\bullet,\ S \rightarrow S\bullet oA\},\ I_2{:}\{S \rightarrow A\bullet\},\ I_3{:}\{A \rightarrow i\bullet\},$
$I_4{:}\{A \rightarrow (\bullet S),\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_5{:}\{A \rightarrow (S\bullet),\ S \rightarrow S\bullet oA\},\ I_6{:}\{S \rightarrow So\bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_7{:}\{S \rightarrow SoA\bullet\},\ I_8{:}\{A \rightarrow (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	***o***	**(**	**)**	**$**	***S***	***A***
I_0						I_1	

$S' \rightarrow \bullet S \in I_0,\ S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet SoA \in I_0,\ S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet A \in I_0,\ A \in N$: $\beta[I_0, A] := \Theta_A(I_0) = I_2$

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0:\{S' \to \bullet S, S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet (S)\},$
$I_1:\{S' \to S\bullet, S \to S\bullet oA\}, I_2:\{S \to A\bullet\}, I_3:\{A \to i\bullet\},$
$I_4:\{A \to (\bullet S), S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet (S)\},$
$I_5:\{A \to (S\bullet), S \to S\bullet oA\}, I_6:\{S \to So\bullet A, A \to \bullet i, A \to \bullet (S)\},$
$I_7:\{S \to SoA\bullet\}, I_8:\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0						I_1	I_2

$S' \to \bullet S \in I_0, S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \to \bullet SoA \in I_0, S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \to \bullet A \in I_0, A \in N$: $\beta[I_0, A] := \Theta_A(I_0) = I_2$

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0{:}\{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\},$
$I_1{:}\{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2{:}\{S \rightarrow A\bullet\}, I_3{:}\{A \rightarrow i\bullet\},$
$I_4{:}\{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\},$
$I_5{:}\{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6{:}\{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet (S)\},$
$I_7{:}\{S \rightarrow SoA\bullet\}, I_8{:}\{A \rightarrow (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0						I_1	I_2

$S' \rightarrow \bullet S \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet SoA \in I_0, S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet A \in I_0, A \in N: \beta[I_0, A] := \Theta_A(I_0) = I_2$

$A \rightarrow \bullet i \in I_0, i \in T: \alpha[I_0, i] := s\Theta_i(I_0) = sI_3$

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0{:}\{S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_1{:}\{S' \rightarrow S\bullet,\ S \rightarrow S\bullet oA\},\ I_2{:}\{S \rightarrow A\bullet\},\ I_3{:}\{A \rightarrow i\bullet\},$
$I_4{:}\{A \rightarrow (\bullet S),\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_5{:}\{A \rightarrow (S\bullet),\ S \rightarrow S\bullet oA\},\ I_6{:}\{S \rightarrow So\bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_7{:}\{S \rightarrow SoA\bullet\},\ I_8{:}\{A \rightarrow (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3					I_1	I_2

$S' \rightarrow \bullet S \in I_0,\ S \in N{:}\ \beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet SoA \in I_0,\ S \in N{:}\ \beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet A \in I_0,\ A \in N{:}\ \beta[I_0, A] := \Theta_A(I_0) = I_2$

$A \rightarrow \bullet i \in I_0,\ i \in T{:}\ \alpha[I_0, i] := s\Theta_i(I_0) = sI_3$

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0:\{S' \rightarrow \bullet S, S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet(S)\},$
$I_1:\{S' \rightarrow S\bullet, S \rightarrow S\bullet oA\}, I_2:\{S \rightarrow A\bullet\}, I_3:\{A \rightarrow i\bullet\},$
$I_4:\{A \rightarrow (\bullet S), S \rightarrow \bullet SoA, S \rightarrow \bullet A, A \rightarrow \bullet i, A \rightarrow \bullet(S)\},$
$I_5:\{A \rightarrow (S\bullet), S \rightarrow S\bullet oA\}, I_6:\{S \rightarrow So\bullet A, A \rightarrow \bullet i, A \rightarrow \bullet(S)\},$
$I_7:\{S \rightarrow SoA\bullet\}, I_8:\{A \rightarrow (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3					I_1	I_2

$S' \rightarrow \bullet S \in I_0,\ S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet SoA \in I_0,\ S \in N: \beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \rightarrow \bullet A \in I_0,\ A \in N: \beta[I_0, A] := \Theta_A(I_0) = I_2$

$A \rightarrow \bullet i \in I_0,\ i \in T: \alpha[I_0, i] := s\Theta_i(I_0) = sI_3$

$A \rightarrow \bullet(S) \in I_0,\ (\in T: \alpha[I_0, (] := s\Theta_((I_0) = sI_4$

Construction of LR-table: Example 1/5

$\Theta_H = \{I_0:\{S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_1:\{S' \to S\bullet,\ S \to S\bullet oA\},\ I_2:\{S \to A\bullet\},\ I_3:\{A \to i\bullet\},$
$I_4:\{A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_5:\{A \to (S\bullet),\ S \to S\bullet oA\},\ I_6:\{S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_7:\{S \to SoA\bullet\},\ I_8:\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2

$S' \to \bullet S \in I_0,\ S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \to \bullet SoA \in I_0,\ S \in N$: $\beta[I_0, S] := \Theta_S(I_0) = I_1$

$S \to \bullet A \in I_0,\ A \in N$: $\beta[I_0, A] := \Theta_A(I_0) = I_2$

$A \to \bullet i \in I_0,\ i \in T$: $\alpha[I_0, i] := s\Theta_i(I_0) = sI_3$

$A \to \bullet(S) \in I_0,\ (\in T$: $\alpha[I_0, (] := s\Theta_((I_0) = sI_4$

Construction of LR-table: Example 2/5

$\Theta_H = \{I_0{:}\{S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_1{:}\{S' \rightarrow S\bullet,\ S \rightarrow S\bullet oA\},\ I_2{:}\{S \rightarrow A\bullet\},\ I_3{:}\{A \rightarrow i\bullet\},$
$I_4{:}\{A \rightarrow (\bullet S),\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_5{:}\{A \rightarrow (S\bullet),\ S \rightarrow S\bullet oA\},\ I_6{:}\{S \rightarrow So\bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_7{:}\{S \rightarrow SoA\bullet\},\ I_8{:}\{A \rightarrow (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1							

Construction of LR-table: Example 2/5

$\Theta_H = \{I_0\text{:}\{S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_1\text{:}\{S' \to S\bullet,\ S \to S\bullet oA\},\ I_2\text{:}\{S \to A\bullet\},\ I_3\text{:}\{A \to i\bullet\},$
$I_4\text{:}\{A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_5\text{:}\{A \to (S\bullet),\ S \to S\bullet oA\},\ I_6\text{:}\{S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_7\text{:}\{S \to SoA\bullet\},\ I_8\text{:}\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1							

$S' \to S\bullet \in I_1$: $\alpha[I_1, \$]$:= ☺

Construction of LR-table: Example 2/5

$\Theta_H = \{I_0{:}\{S' \to \bullet S, S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_1{:}\{S' \to S\bullet, S \to S\bullet oA\}, I_2{:}\{S \to A\bullet\}, I_3{:}\{A \to i\bullet\},$
$I_4{:}\{A \to (\bullet S), S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_5{:}\{A \to (S\bullet), S \to S\bullet oA\}, I_6{:}\{S \to So\bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_7{:}\{S \to SoA\bullet\}, I_8{:}\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1					☺		

$S' \to S\bullet \in I_1$: $\alpha[I_1, \$] :=$ ☺

Construction of LR-table: Example 2/5

$\Theta_H = \{I_0{:}\{S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_1{:}\{S' \rightarrow S\bullet,\ S \rightarrow S\bullet oA\},\ I_2{:}\{S \rightarrow A\bullet\},\ I_3{:}\{A \rightarrow i\bullet\},$
$I_4{:}\{A \rightarrow (\bullet S),\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_5{:}\{A \rightarrow (S\bullet),\ S \rightarrow S\bullet oA\},\ I_6{:}\{S \rightarrow So\bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_7{:}\{S \rightarrow SoA\bullet\},\ I_8{:}\{A \rightarrow (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1					☺		

$S' \rightarrow S\bullet \in I_1$: $\alpha[I_1, \$] :=$ ☺

$S \rightarrow S\bullet oA \in I_1,\ o \in T$: $\alpha[I_1, o] := \text{s}\Theta_o(I_1) = \text{s}I_6$

Construction of LR-table: Example 2/5

$\Theta_H = \{I_0:\{S' \to \bullet S, S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_1:\{S' \to S\bullet, S \to S\bullet oA\}, I_2:\{S \to A\bullet\}, I_3:\{A \to i\bullet\},$
$I_4:\{A \to (\bullet S), S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_5:\{A \to (S\bullet), S \to S\bullet oA\}, I_6:\{S \to So\bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_7:\{S \to SoA\bullet\}, I_8:\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$\$$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		

$S' \to S\bullet \in I_1$: $\alpha[I_1, \$] :=$ ☺

$S \to S\bullet oA \in I_1$, $o \in T$: $\alpha[I_1, o] := s\Theta_o(I_1) = sI_6$

Construction of LR-table: Example 3/5

$\Theta_H = \{I_0{:}\{S' \to \bullet S, S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_1{:}\{S' \to S\bullet, S \to S\bullet oA\}, I_2{:}\{S \to A\bullet\}, I_3{:}\{A \to i\bullet\},$
$I_4{:}\{A \to (\bullet S), S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_5{:}\{A \to (S\bullet), S \to S\bullet oA\}, I_6{:}\{S \to So\bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_7{:}\{S \to SoA\bullet\}, I_8{:}\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2							

Construction of LR-table: Example 3/5

$\Theta_H = \{I_0{:}\{S' \rightarrow \bullet S,\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_1{:}\{S' \rightarrow S\bullet,\ S \rightarrow S\bullet oA\},\ I_2{:}\{S \rightarrow A\bullet\},\ I_3{:}\{A \rightarrow i\bullet\},$
$I_4{:}\{A \rightarrow (\bullet S),\ S \rightarrow \bullet SoA,\ S \rightarrow \bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_5{:}\{A \rightarrow (S\bullet),\ S \rightarrow S\bullet oA\},\ I_6{:}\{S \rightarrow So\bullet A,\ A \rightarrow \bullet i,\ A \rightarrow \bullet(S)\},$
$I_7{:}\{S \rightarrow SoA\bullet\},\ I_8{:}\{A \rightarrow (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2							

$S \rightarrow A\bullet \in I_2$, $Follow(S) = \{o,), \$\}$:

$\alpha[I_2, o] = \alpha[I_2,)] = \alpha[I_2, \$] := r2$

Construction of LR-table: Example 3/5

$\Theta_H = \{I_0:\{S' \to \bullet S, S \to \bullet S o A, S \to \bullet A, A \to \bullet i, A \to \bullet (S)\},$
$I_1:\{S' \to S\bullet, S \to S\bullet o A\}, I_2:\{S \to A\bullet\}, I_3:\{A \to i\bullet\},$
$I_4:\{A \to (\bullet S), S \to \bullet S o A, S \to \bullet A, A \to \bullet i, A \to \bullet (S)\},$
$I_5:\{A \to (S\bullet), S \to S\bullet o A\}, I_6:\{S \to S o\bullet A, A \to \bullet i, A \to \bullet (S)\},$
$I_7:\{S \to S o A\bullet\}, I_8:\{A \to (S)\bullet\}\}$

Task: LR-table for K

	α					β	
	i	o	(	)	\$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		r2		r2	r2		

$S \to A\bullet \in I_2$, $Follow(S) = \{o,), \$\}$:

$\alpha[I_2, o] = \alpha[I_2,)] = \alpha[I_2, \$] :=$ r2

Construction of LR-table: Example 4/5

$\Theta_H = \{I_0:\{S' \to \bullet S, S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_1:\{S' \to S\bullet, S \to S\bullet oA\}, I_2:\{S \to A\bullet\}, I_3:\{A \to i\bullet\},$
$I_4:\{A \to (\bullet S), S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_5:\{A \to (S\bullet), S \to S\bullet oA\}, I_6:\{S \to So\bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_7:\{S \to SoA\bullet\}, I_8:\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		r2		r2	r2		
I_3							

Construction of LR-table: Example 4/5

$\Theta_H = \{I_0{:}\{S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet (S)\},$
$I_1{:}\{S' \to S\bullet,\ S \to S\bullet oA\},\ I_2{:}\{S \to A\bullet\},\ I_3{:}\{A \to i\bullet\},$
$I_4{:}\{A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet (S)\},$
$I_5{:}\{A \to (S\bullet),\ S \to S\bullet oA\},\ I_6{:}\{S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet (S)\},$
$I_7{:}\{S \to SoA\bullet\},\ I_8{:}\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	o	(	)	\$	S	A
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		r2		r2	r2		
I_3							

$A \to i\bullet \in I_3$, $Follow(A) = \{o,), \$\}$:

$\alpha[I_3, o] = \alpha[I_3,)] = \alpha[I_3, \$] :=$ r3

Construction of LR-table: Example 4/5

$\Theta_H = \{I_0:\{S' \to \bullet S,\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_1:\{S' \to S\bullet,\ S \to S\bullet oA\},\ I_2:\{S \to A\bullet\},\ I_3:\{A \to i\bullet\},$
$I_4:\{A \to (\bullet S),\ S \to \bullet SoA,\ S \to \bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_5:\{A \to (S\bullet),\ S \to S\bullet oA\},\ I_6:\{S \to So\bullet A,\ A \to \bullet i,\ A \to \bullet(S)\},$
$I_7:\{S \to SoA\bullet\},\ I_8:\{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		r2		r2	r2		
I_3		r3		r3	r3		

$A \to i\bullet \in I_3$, $Follow(A) = \{o,), \$\}$:

$\alpha[I_3, o] = \alpha[I_3,)] = \alpha[I_3, \$] :=$ r3

Construction of LR-table: Example 4/5

$\Theta_H = \{I_0: \{S' \to \bullet S, S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_1: \{S' \to S\bullet, S \to S\bullet oA\}, I_2: \{S \to A\bullet\}, I_3: \{A \to i\bullet\},$
$I_4: \{A \to (\bullet S), S \to \bullet SoA, S \to \bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_5: \{A \to (S\bullet), S \to S\bullet oA\}, I_6: \{S \to So\bullet A, A \to \bullet i, A \to \bullet(S)\},$
$I_7: \{S \to SoA\bullet\}, I_8: \{A \to (S)\bullet\}\}$

Task: LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		r2		r2	r2		
I_3		r3		r3	r3		

Construct the rest analogically.

$A \to i\bullet \in I_3$, $Follow(A) = \{o,), \$\}$:

$\alpha[I_3, o] = \alpha[I_3,)] = \alpha[I_3, \$] := r3$

Construction of LR-table: Example 5/5

Final LR-table for *K*

	α					β	
	i	*o*	(	)	$	*S*	*A*
I_0	sI_3		sI_4			I_1	I_2
I_1		sI_6			☺		
I_2		r2		r2	r2		
I_3		r3		r3	r3		
I_4	sI_3		sI_4			I_5	I_2
I_5		sI_6		sI_8			
I_6	sI_3		sI_4				I_7
I_7		r1		r1	r1		
I_8		r4		r4	r4		

Renaming the states

Rename the states:

Old	**New**
I_0	0
I_1	1
I_2	2
I_3	3
I_4	4
I_5	5
I_6	6
I_7	7
I_8	8

LR-table for *K* with the renamed states:

α	***i***	***o***	**(**	**)**	**$**
0	s3		s4		
1		s6			☺
2		r2		r2	r2
3		r3		r3	r3
4	s3		s4		
5		s6		s8	
6	s3		s4		
7		r1		r1	r1
8		r4		r4	r4

β	***S***	***A***
0	1	2
1		
2		
3		
4	5	2
5		
6		7
7		
8		