Regulated Rewriting

Introduction

Gist: Only a selected subset of productions can be applied during a derivation step

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Example:

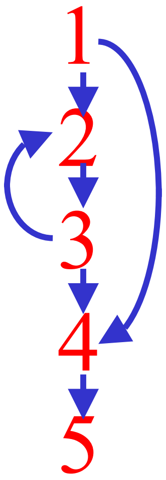
1: $S \rightarrow AB$

2: $A \rightarrow aA$

3: $B \rightarrow bBc$

4: $A \rightarrow a$

5: $B \rightarrow bc$



$$\begin{aligned} S &\Rightarrow_1 AB \\ &\Rightarrow_2 aAB \\ &\Rightarrow_3 aAbBc \\ &\Rightarrow_2 aaAbBc \\ &\Rightarrow_3 aaAbbBcc \\ &\Rightarrow_4 aaabbBcc \\ &\Rightarrow_5 aaabbbccc \end{aligned}$$

$$L(G) = \{a^n b^n c^n : n \geq 1\}$$

Matrix Grammar

Definition: A *matrix grammar* is a pair $H = (G, M)$, where:

- $G = (N, T, P, S)$ is a CFG
- M is a finite language over P ($M \subseteq P^*$)

Example:

$H_1 = (G, M)$

$M = \{1, 23, 45\}$

$G = (N, T, P, S)$

$N = \{S, A, B\}$

$T = \{a, b, c\}$

$$P = \{\ 1: S \rightarrow AB$$
$$2: A \rightarrow aA$$
$$3: B \rightarrow bBc$$
$$4: A \rightarrow a$$
$$5: B \rightarrow bc\ \}$$

Derivation Step in MG

Definition: For $x, y \in (N \cup T)^*$, $m \in M$,

$$x \Rightarrow y\ [m] \text{ in } H$$

if there are $x_0, x_1, \ldots, x_n$ such that $x = x_0$, $x_n = y$, and

1. $x_0 \Rightarrow x_1\ [p_1] \Rightarrow x_2\ [p_2] \Rightarrow \ldots \Rightarrow x_n\ [p_n]$ in G, and
2. $m = p_1 p_2 \ldots p_n$

Example:

$S \Rightarrow AB\ [1] \Rightarrow aAbBc\ [23] \Rightarrow aabbcc\ [45]$ in H_1

$L(H_1) = \{a^n b^n c^n \mid n \geq 1\}$

Random Context Grammar

Definition: A *random context grammar* is a pair $H = (G, R)$, where:

- $G = (N, T, P, S)$ is a CFG
- R is a finite relation from P to N

Derivation step: For $x, y \in V^*, p \in P$,

$$x \Rightarrow y\ [p] \text{ in } H, \text{ if}$$

1. $x \Rightarrow y\ [p]$ in G and

2. $R(p) \subseteq alph(x)$

Note: $alph(x)$ denotes the set of all symbols appearing in x

Example of RCG

Note: If $p: A \rightarrow x \in P$, $R(p) = Q$, we write

$$(p: A \rightarrow x, Q)$$

$(S \rightarrow ABC, \emptyset)$

$(A \rightarrow aA', \{B\})$

$(B \rightarrow bB', \{C\})$

$(C \rightarrow cC', \{A'\})$

$(A' \rightarrow A, \{B'\})$

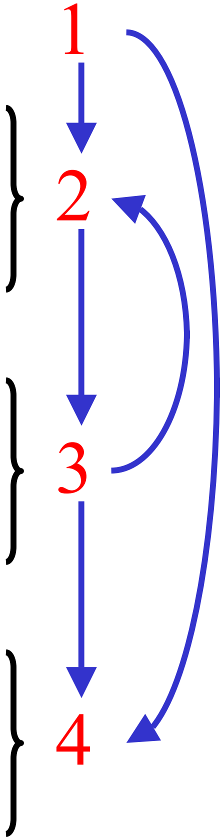
$(B' \rightarrow B, \{C'\})$

$(C' \rightarrow C, \{A\})$

$(A \rightarrow a, \{B\})$

$(B \rightarrow b, \{C\})$

$(C \rightarrow c, \emptyset)$



$S \Rightarrow ABC$	1
$\Rightarrow^3 aA'bB'cC'$	2
$\Rightarrow^3 aAbBcC$	3
$\Rightarrow^3 aaA'bbB'ccC'$	2
$\Rightarrow^3 aaAbbBccC$	3
…	…
$\Rightarrow^3 a^nAb^nBc^nC$	3
$\Rightarrow^3 a^{n+1}b^{n+1}c^{n+1}$	4

$$L(H) = \{a^k b^k c^k : k \geq 1\}$$

Programmed Grammar

Definition: A *programmed grammar* is a pair $H = (G, R)$, where:

- $G = (N, T, P, S)$ is a CFG
- R is a finite relation on P

Derivation step: For $(x, p), (y, q) \in V^{*} \times P$,

$$(x, p) \Rightarrow (y, q) \text{ in } H\text{, if}$$

1. $x \Rightarrow y\ [p]$ in G

2. $q \in R(p)$

Generated language: $L(H) = \{x : x \in T^{*}$, $(S, p) \Rightarrow^{*} (x, p')$ for some $p, p' \in P\}$

Example of PG

Note: If $p: A \rightarrow x \in P$, $R(p) = Q$, we write

$$(p: A \rightarrow x, Q)$$

$(1: S \rightarrow ABC, \{2, 5\})$

$(2: A \rightarrow aA, \{3\})$

$(3: B \rightarrow bB, \{4\})$

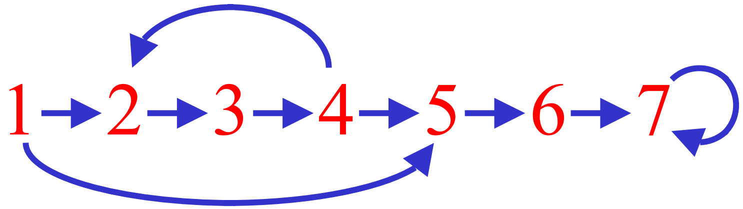
$(4: C \rightarrow cC, \{2, 5\})$

$(5: A \rightarrow a, \{6\})$

$(6: B \rightarrow b, \{7\})$

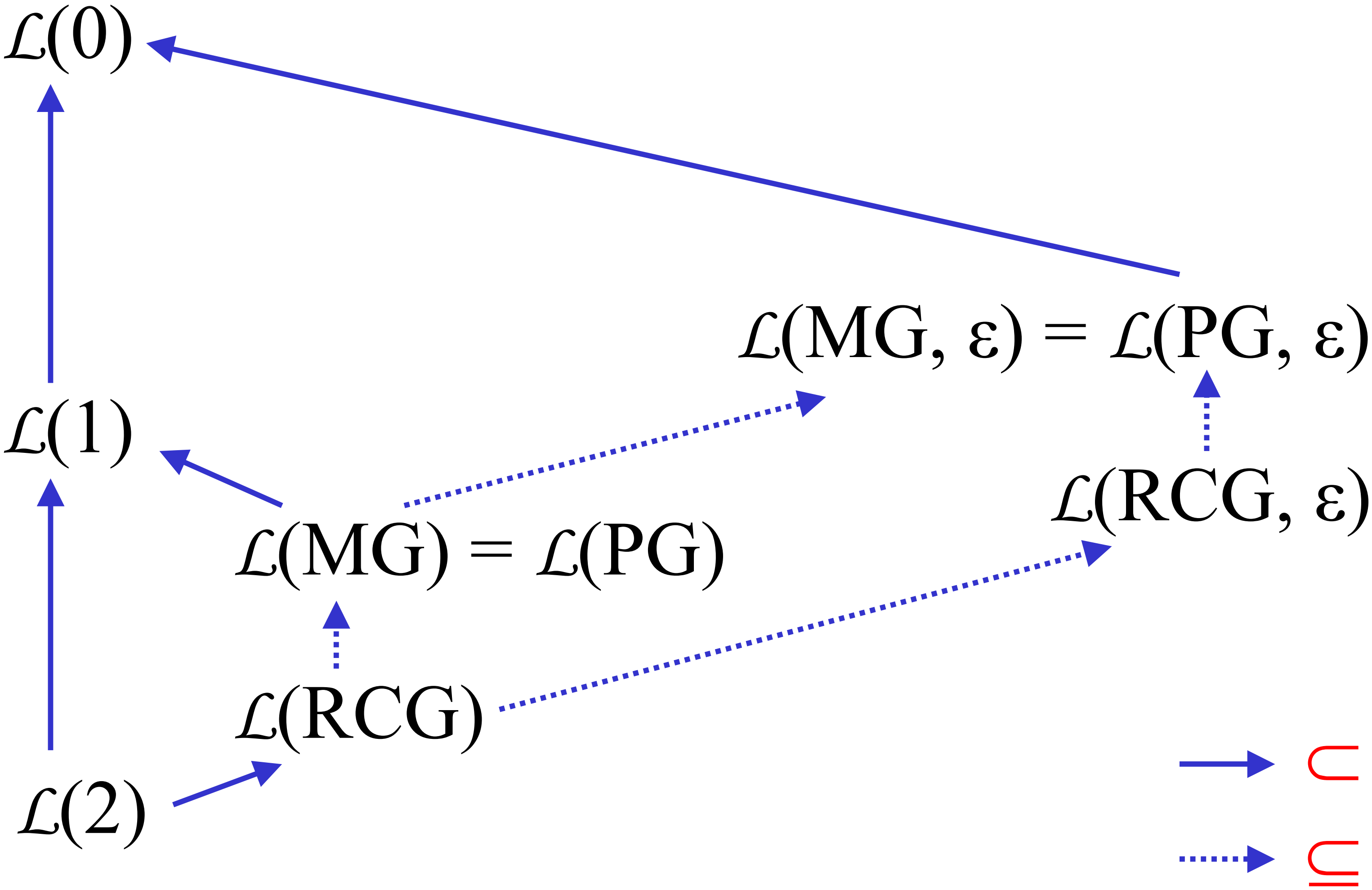
$(7: C \rightarrow c, \{7\})$

$$\begin{aligned}(S, 1) &\Rightarrow (ABC, 2)\\ &\Rightarrow (aABC, 3)\\ &\Rightarrow (aAbBC, 4)\\ &\Rightarrow (aAbBcC, 5)\\ &\Rightarrow (aabBcC, 6)\\ &\Rightarrow (aabbcC, 7)\\ &\Rightarrow (aabbcc, 7)\end{aligned}$$



$L(H) = \{a^n b^n c^n : n \geq 1\}$

Generative Power



$\mathcal{L}(0)$

$\mathcal{L}(\mathrm{MG}, \varepsilon) = \mathcal{L}(\mathrm{PG}, \varepsilon)$

$\mathcal{L}(1)$

$\mathcal{L}(\mathrm{RCG}, \varepsilon)$

$\mathcal{L}(\mathrm{MG}) = \mathcal{L}(\mathrm{PG})$

$\mathcal{L}(\mathrm{RCG})$

$\mathcal{L}(2)$

→ $\subset$

⋯→ $\subseteq$