

# *Dantzig's shortest Path Algorithm*

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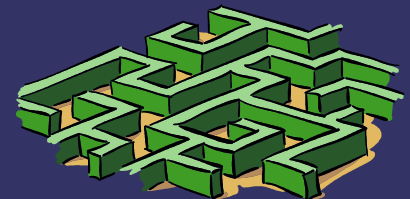
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Language Theory with Applications

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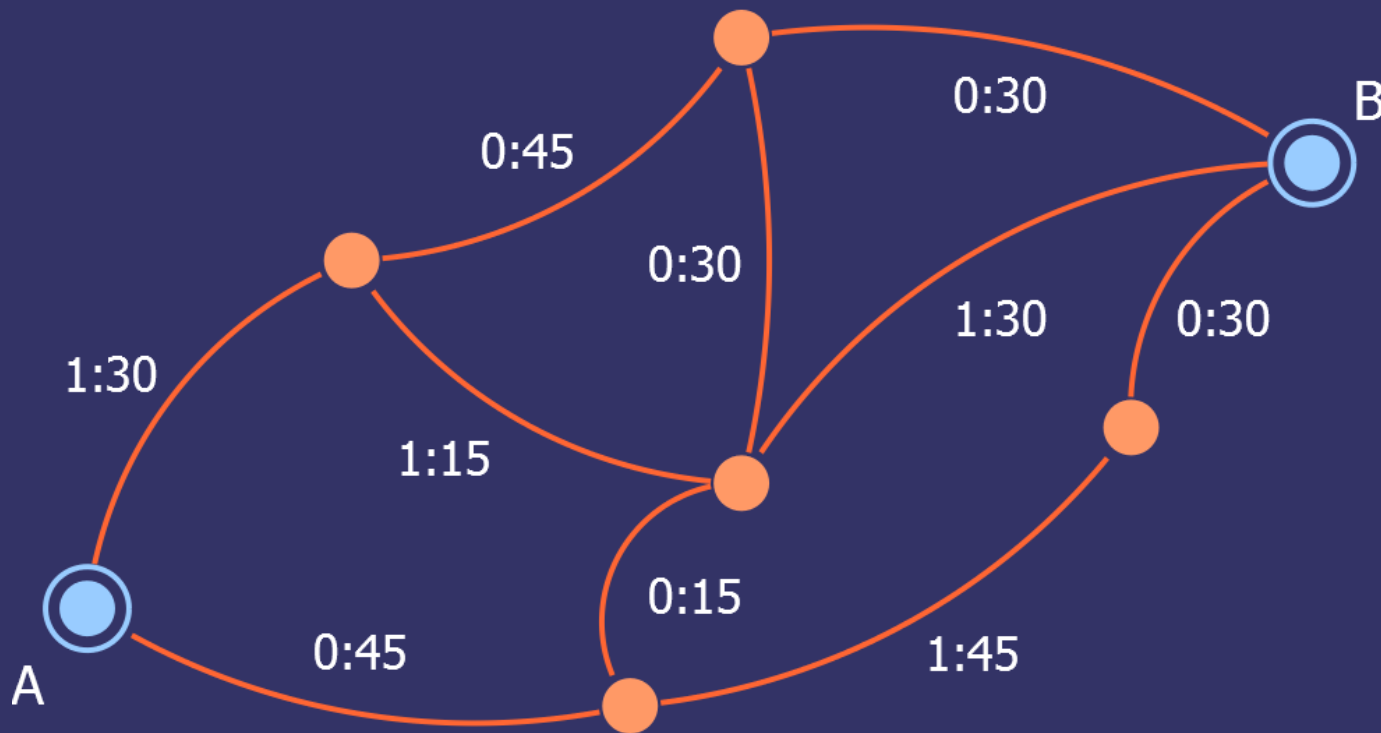
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Brno University of Technology



# What is it about?

- ⇒ You want to travel from point **A** to point **B**
- ⇒ And you need to be there *quickly*



# *What's the big deal?*

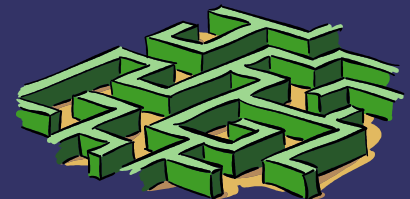
- ⇒ Just two questions:
  - Where do you want to go?
  - Which path do you want to take?
- ⇒ Pretty simple (?)





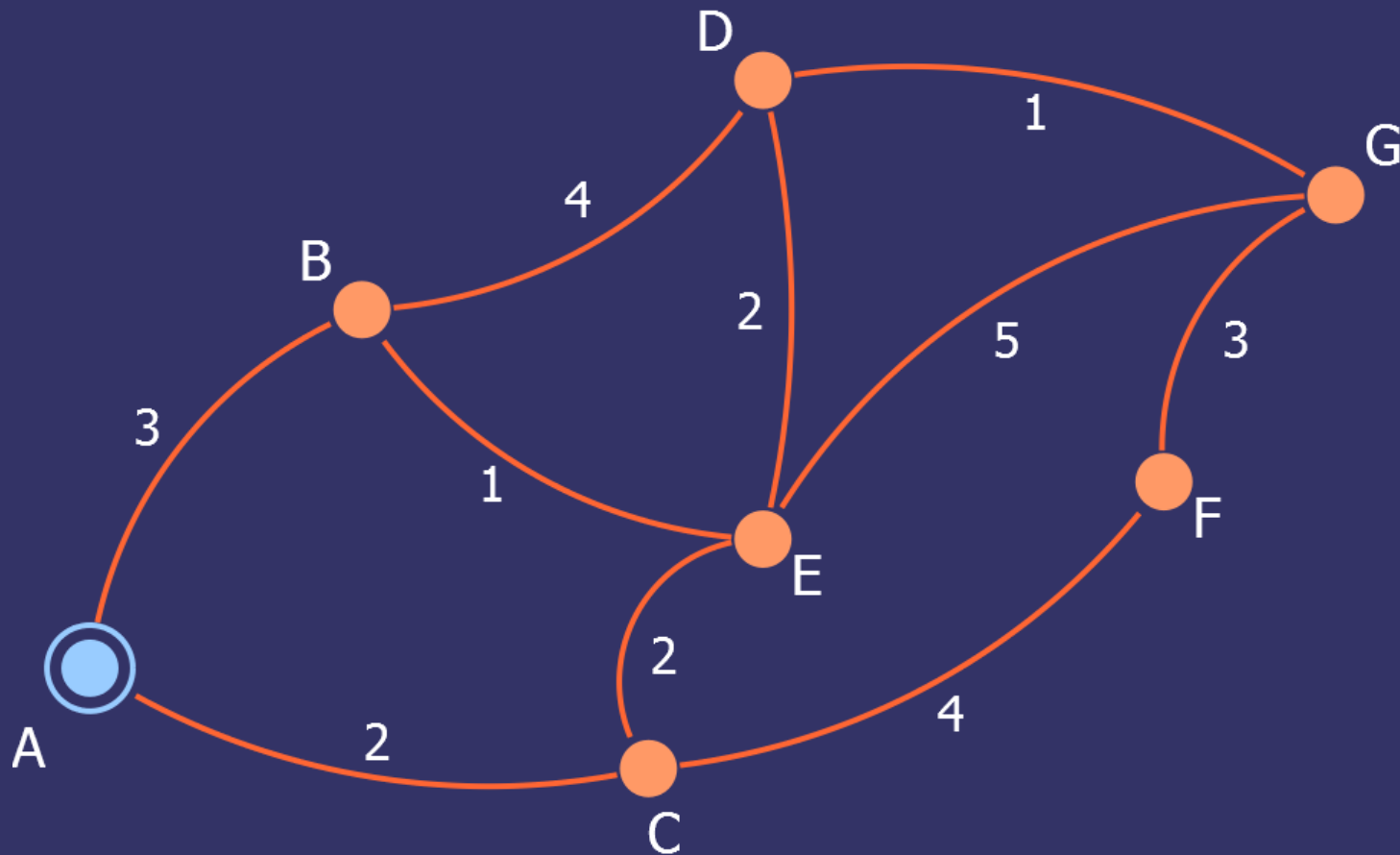
# *Dantzig's Algorithm*

- ➔ Presented in 1959
- ➔ Shortest paths from origin to all other nodes
- ➔ Only networks with nonnegative edges
- ➔ No backtracking to find shortest path itself
- ➔ Time complexity  $O(n^2)$  when certain conditions are met



# Example

- ➔ 7-node network with origin at A

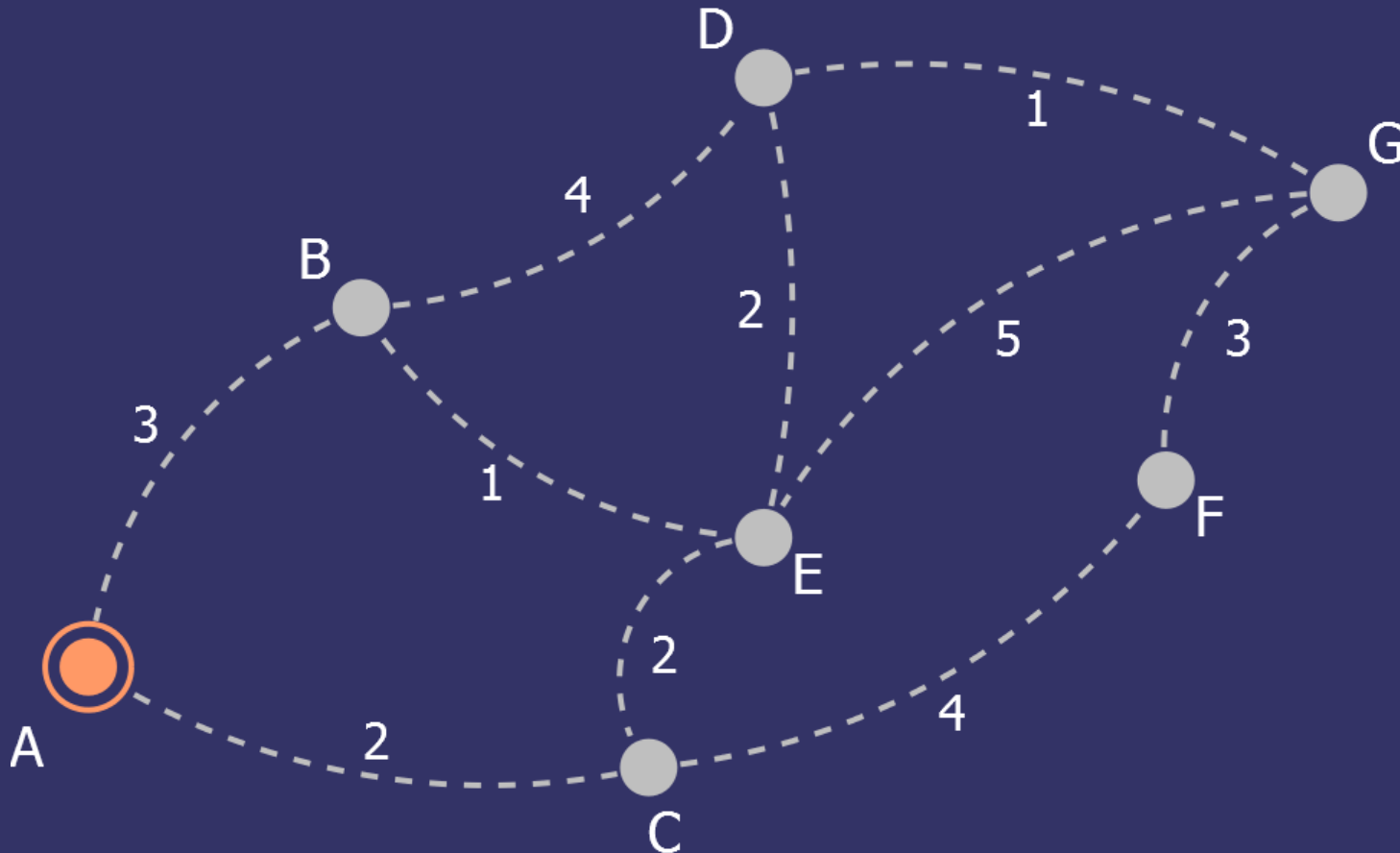


# Example: Step 0

Node	A	B	C	D	E	F	G
Distance	0						
Edges	AC-2 AB-3	BE-1 <del>BA-3</del> BD-4	<del>CA-2</del> CE-2 CF-4	DG-1 DE-2 DB-4	EB-1 EC-2 ED-2 EG-5	FG-3 FC-4	GD-1 GF-3 GE-5

⇒ Only origin A estimated

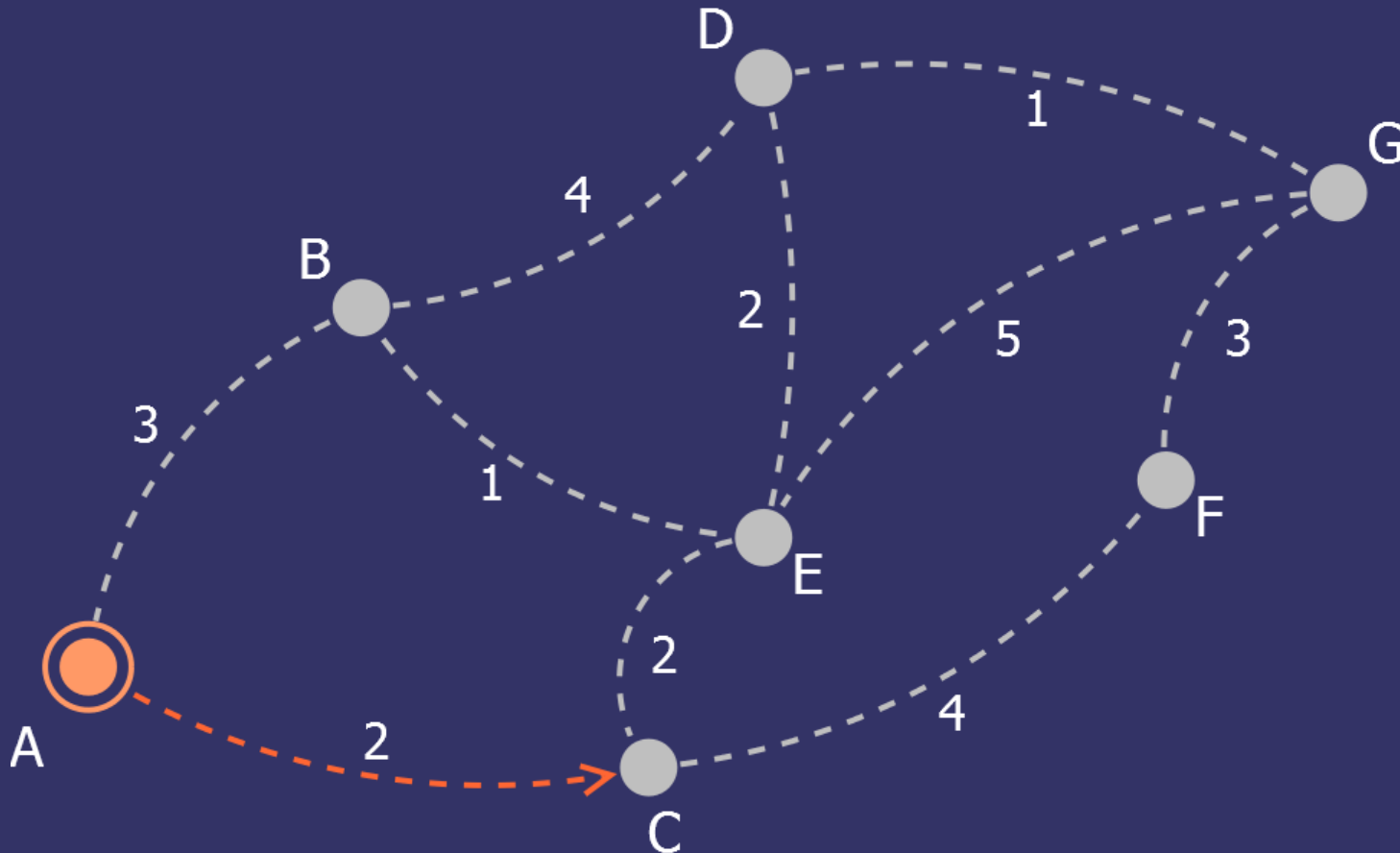
⇒ Edges sorted



# Example: Step 1

Node	A	B	C	D	E	F	G
Distance	0						
Edges	AC-2 AB-3	BE-1 <del>BA-3</del> BD-4	<del>CA-2</del> CE-2 CF-4	DG-1 DE-2 DB-4	EB-1 EC-2 ED-2 EG-5	FG-3 FC-4	GD-1 GF-3 GE-5

⇒ AC-2 (+0)

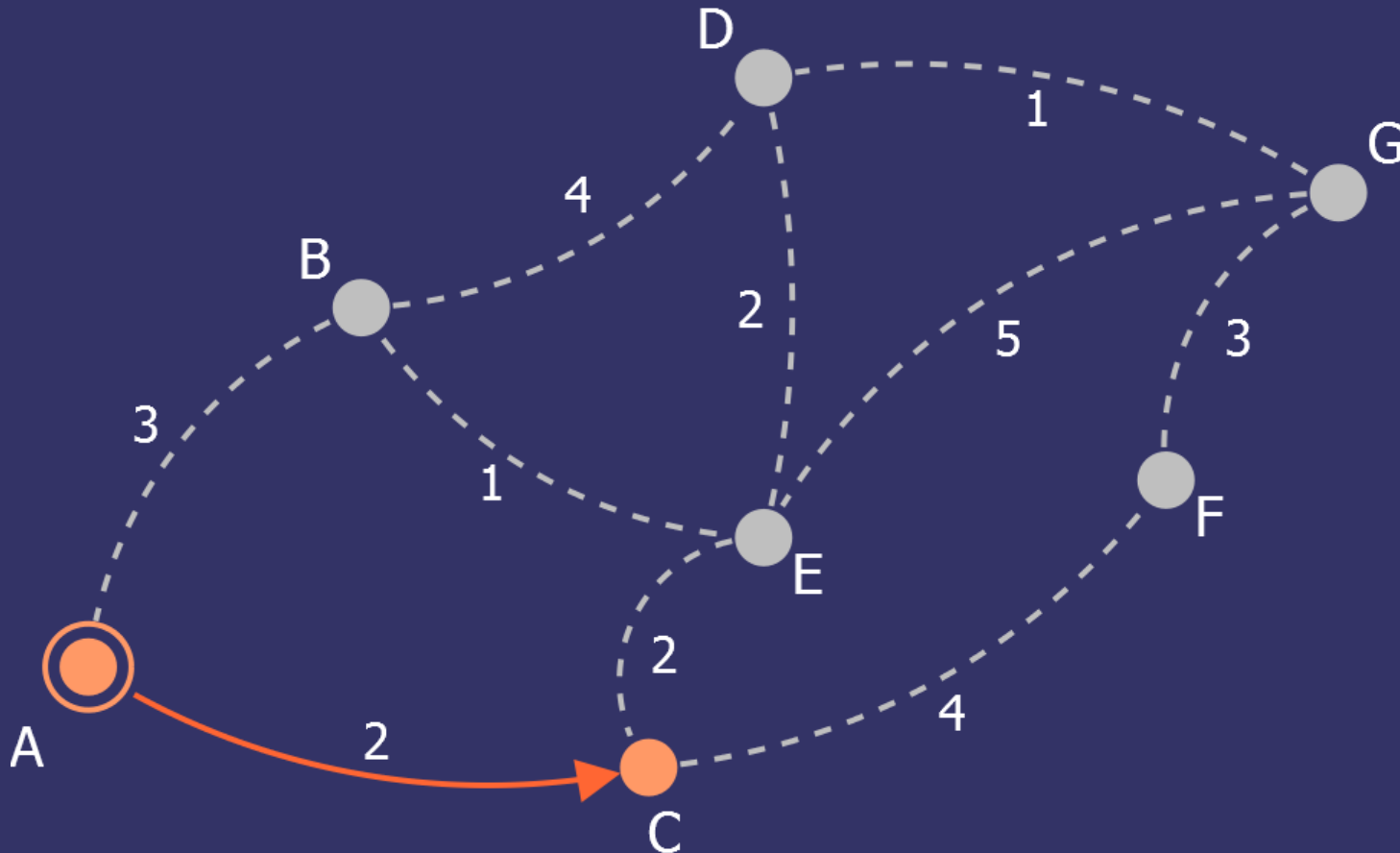




# Example: Step 1

Node	A	B	C	D	E	F	G
Distance	0		2				
Edges	<del>AC-2</del> AB-3	BE-1 <del>BA-3</del> BD-4	<del>CA-2</del> CE-2 CF-4	DG-1 DE-2 DB-4	EB-1 <del>EC-2</del> ED-2 EG-5	FG-3 <del>FC-4</del>	GD-1 GF-3 GE-5

⇒ AC-2 (+0)

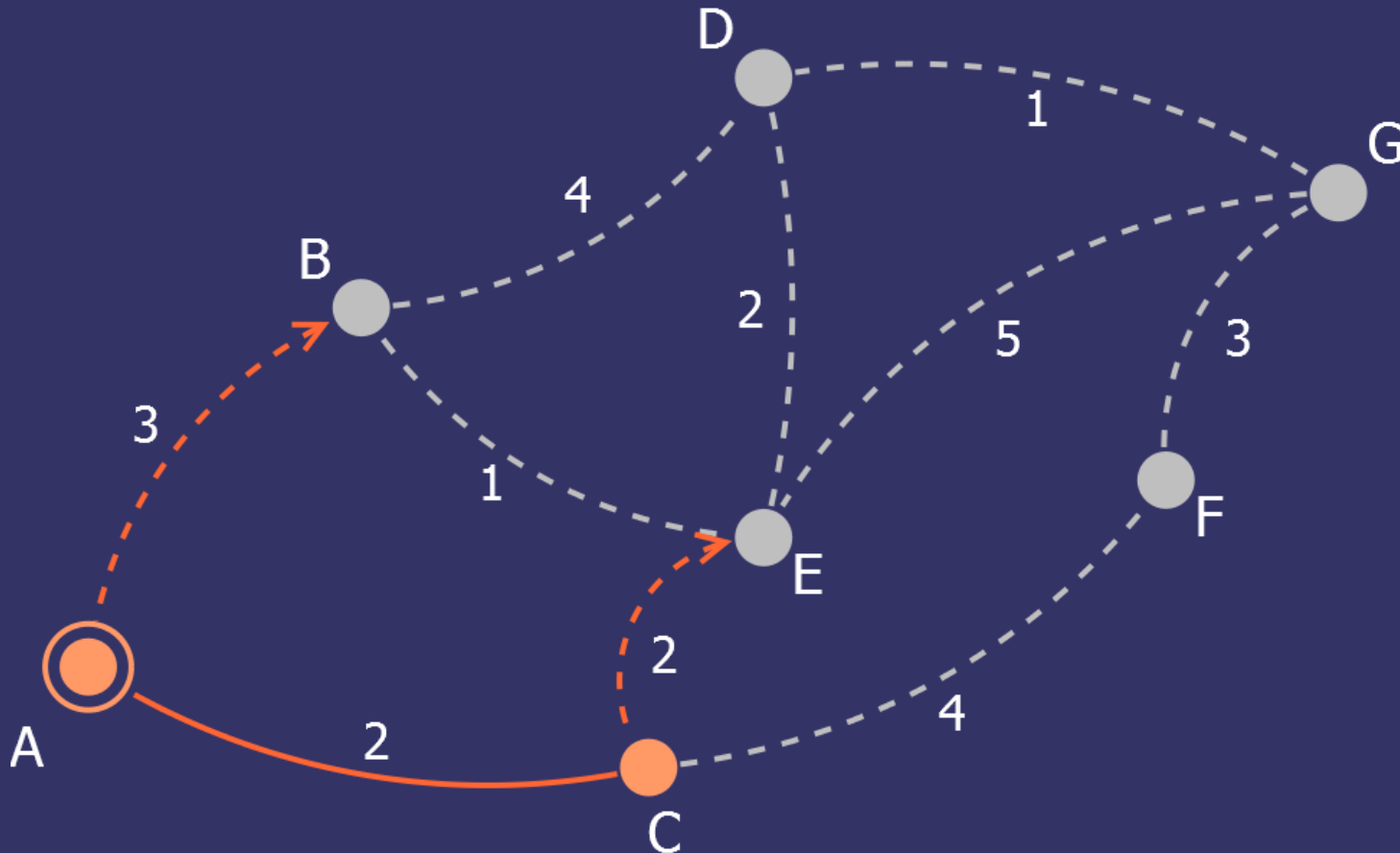


# Example: Step 2

Node	A	B	C	D	E	F	G
Distance	0		2				
Edges	<del>AC-2</del> AB-3	BE-1 <del>BA-3</del> BD-4	<del>CA-2</del> CE-2 CF-4	DG-1 DE-2 DB-4	EB-1 <del>EC-2</del> ED-2 EG-5	FG-3 <del>FC-4</del>	GD-1 GF-3 GE-5

⇒ AB-3 (+0)

⇒ CE-2 (+2)

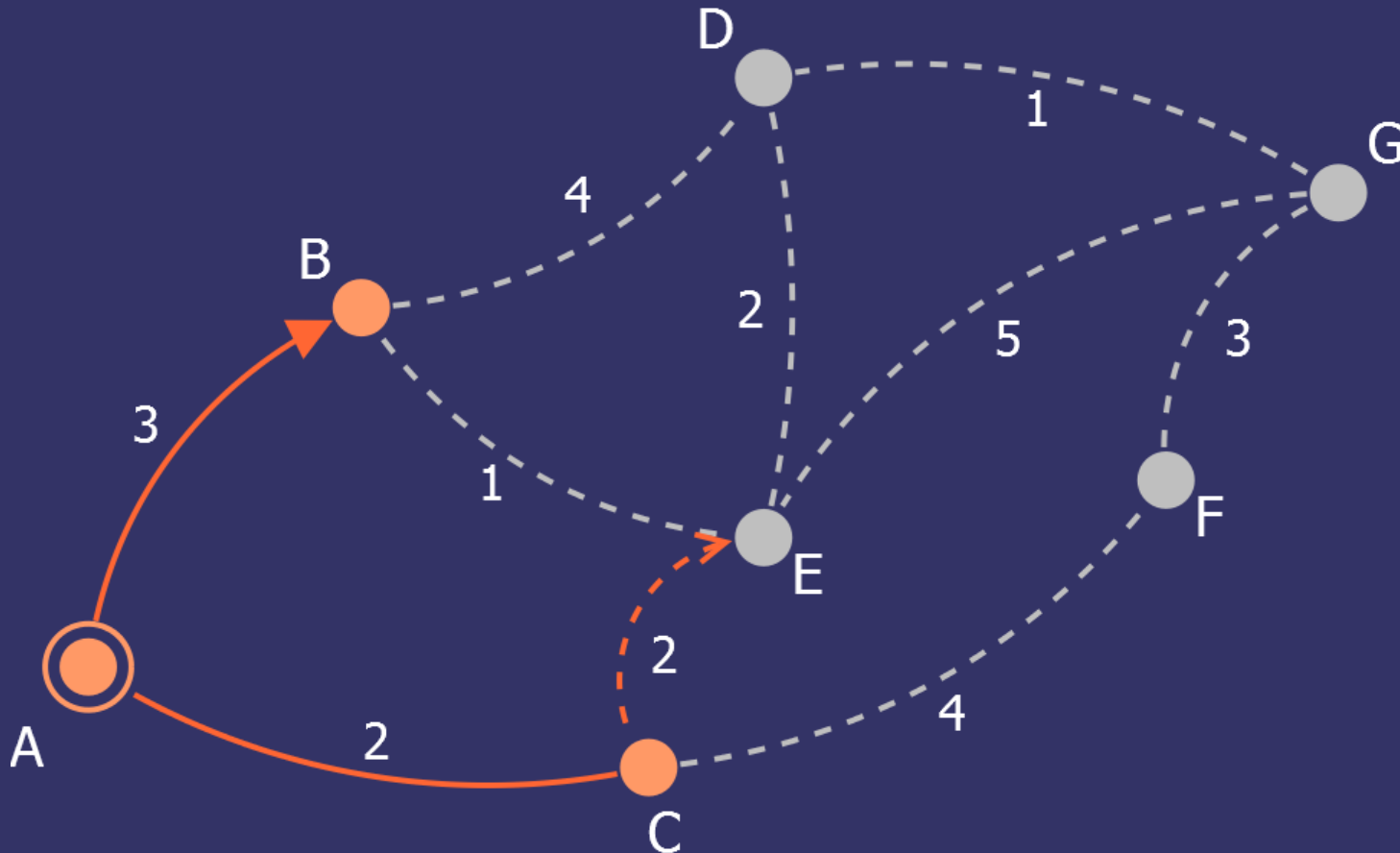


# Example: Step 2

Node	A	B	C	D	E	F	G
Distance	0	3	2				
Edges	<del>AC-2</del> <b>AB-3</b>	BE-1 <del>BA-3</del>	<del>CA-2</del> <b>CE-2</b> CF-4	DG-1 <del>DE-2</del> <del>DB-4</del>	<del>EB-1</del> <del>EC-2</del> ED-2 EG-5	FG-3 <del>FC-4</del>	GD-1 GF-3 GE-5

⇒ AB-3 (+0)

⇒ CE-2 (+2)

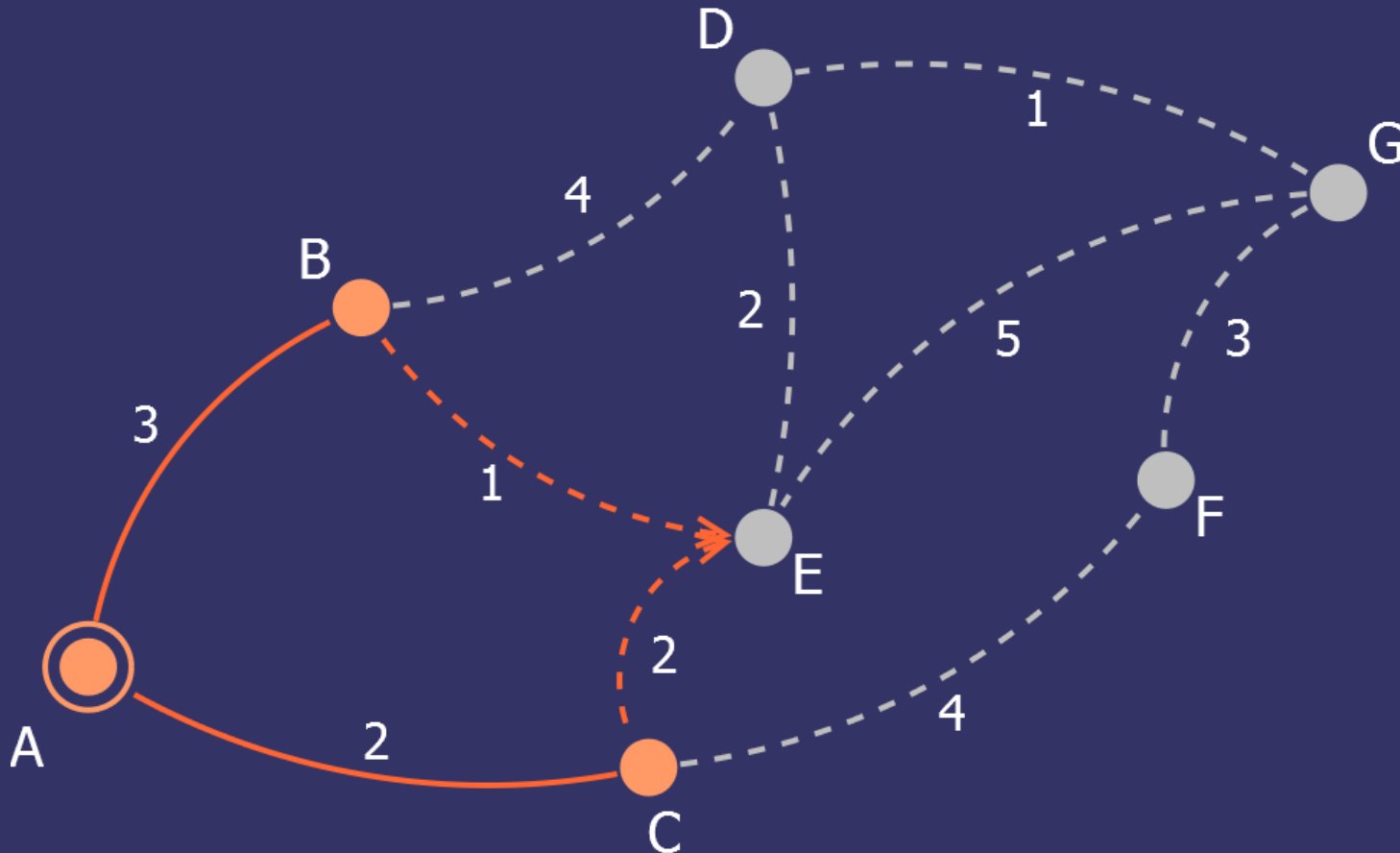


# Example: Step 3

Node	A	B	C	D	E	F	G
Distance	0	3	2				
Edges	<del>AC-2</del> <del>AB-3</del>	<b>BE-1</b> <del>BA-3</del> BD-4	<del>CA-2</del> <b>CE-2</b> CF-4	DG-1 DE-2 <del>DB-4</del>	<del>EB-1</del> <del>EC-2</del> ED-2 EG-5	FG-3 <del>FC-4</del>	GD-1 GF-3 GE-5

⇒ BE-1 (+3)

⇒ CE-2 (+2)

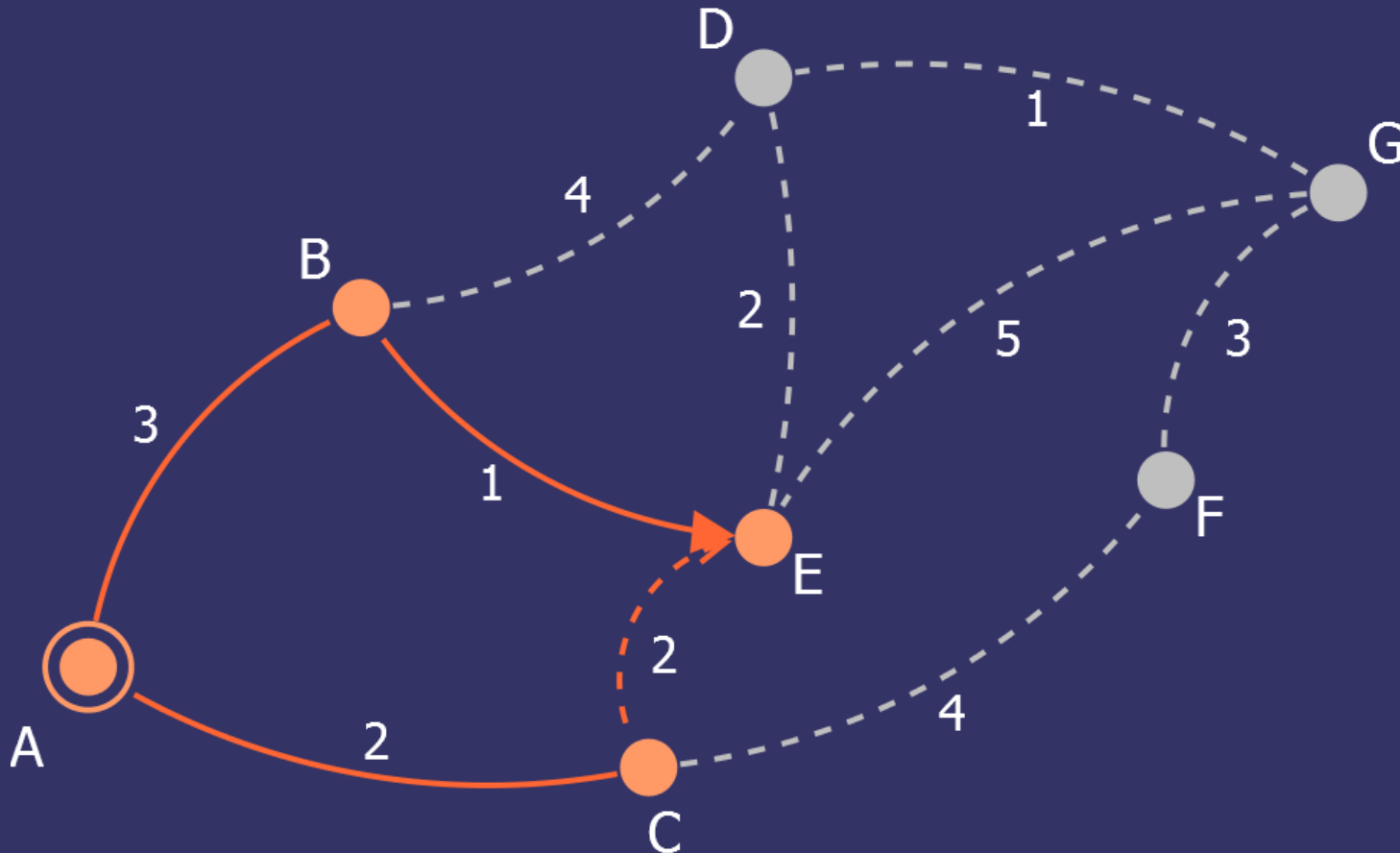


# Example: Step 3

Node	A	B	C	D	E	F	G
Distance	0	3	2		4		
Edges	<del>AC-2</del> <del>AB-3</del>	<del>BE-1</del> <del>BA-3</del> BD-4	<del>CA-2</del> <del>CE-2</del> CF-4	DG-1 <del>DE-2</del> <del>DB-4</del>	<del>EB-1</del> <del>EC-2</del> ED-2 EG-5	FG-3 FC-4	GD-1 GF-3 GE-5

⇒ BE-1 (+3)

⇒ CE-2 (+2)



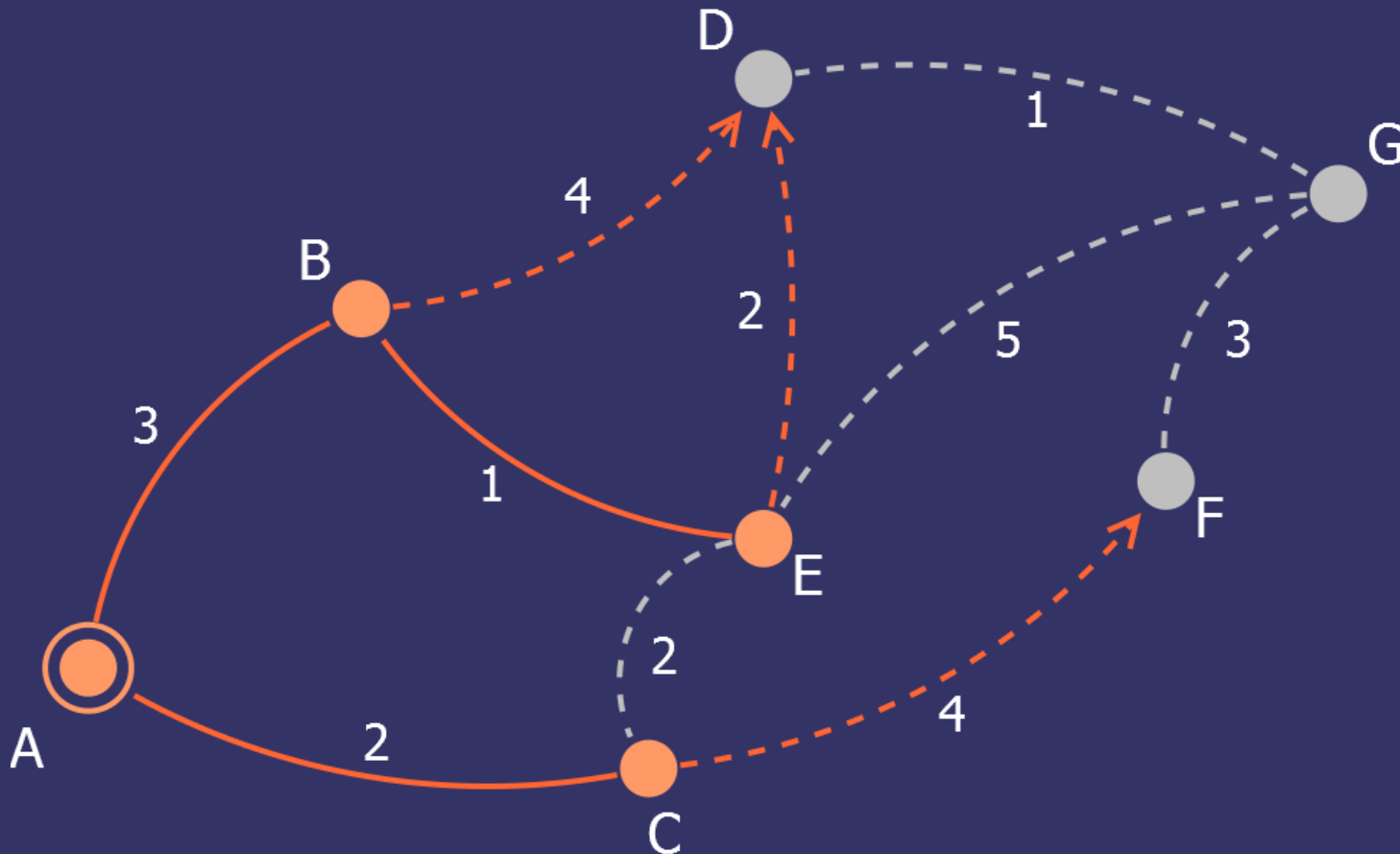
# Example: Step 4

Node	A	B	C	D	E	F	G
Distance	0	3	2		4		
Edges	<del>AC-2</del> <del>AB-3</del>	<del>BE-1</del> <del>BA-3</del> BD-4	<del>CA-2</del> <del>CE-2</del> CF-4	DG-1 <del>DE-2</del> <del>DB-4</del>	<del>EB-1</del> <del>EC-2</del> ED-2 EG-5	FG-3 FC-4	GD-1 GF-3 <del>GE-5</del>

⇒ BD-4 (+3)

⇒ CF-4 (+2)

⇒ ED-2 (+4)



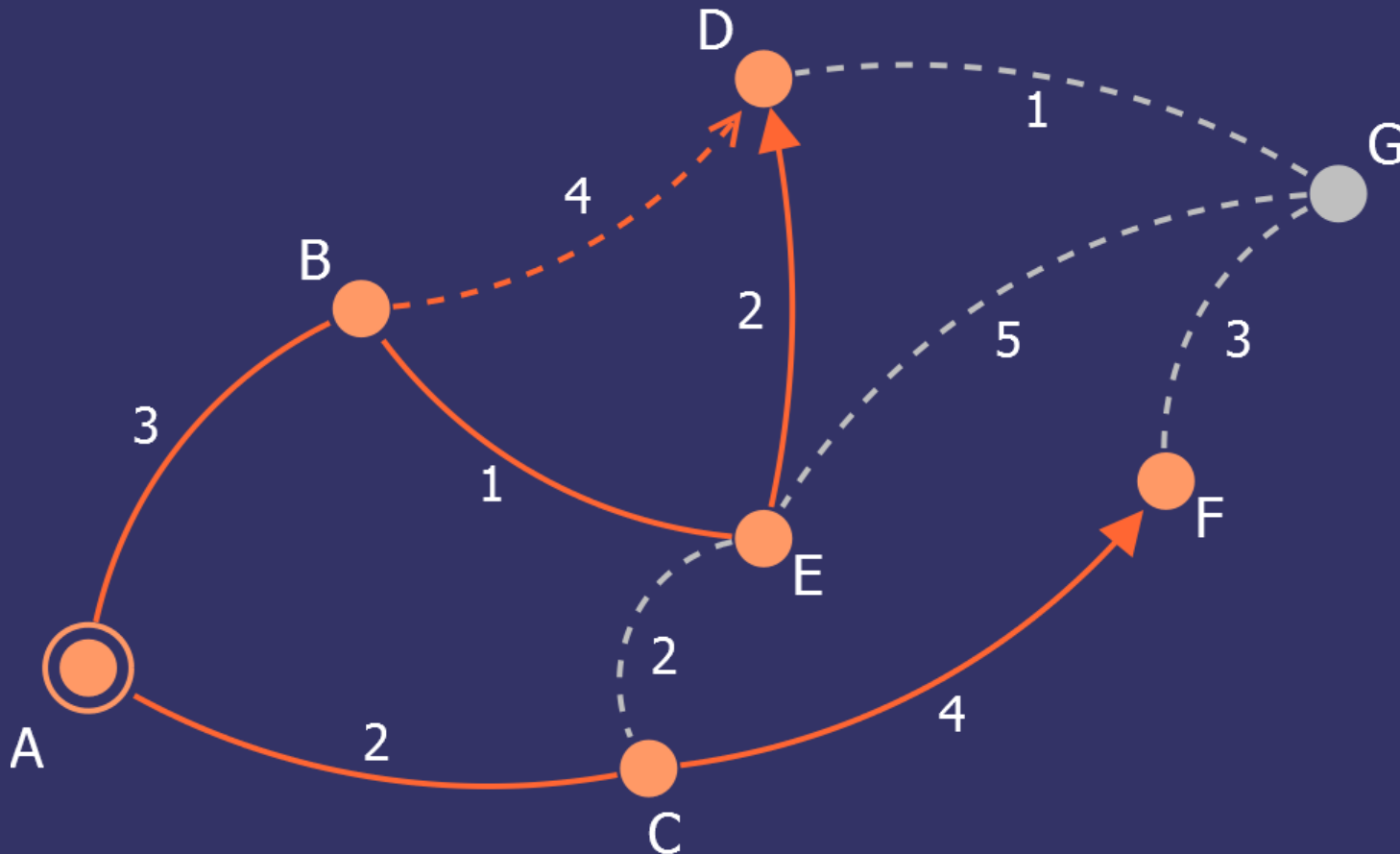
# Example: Step 4

Node	A	B	C	D	E	F	G
Distance	0	3	2	6	4	6	
Edges	<del>AC-2</del> <del>AB-3</del>	<del>BE-1</del> <del>BA-3</del> <del>BD-4</del>	<del>CA-2</del> <del>CE-2</del> <del>CF-4</del>	<del>DG-1</del> <del>DE-2</del> <del>DB-4</del>	<del>EB-1</del> <del>EC-2</del> <del>ED-2</del> EG-5	<del>FG-3</del> <del>FC-4</del>	<del>GD-1</del> <del>GF-3</del> <del>GE-5</del>

⇒ BD-4 (+3)

⇒ CF-4 (+2)

⇒ ED-2 (+4)



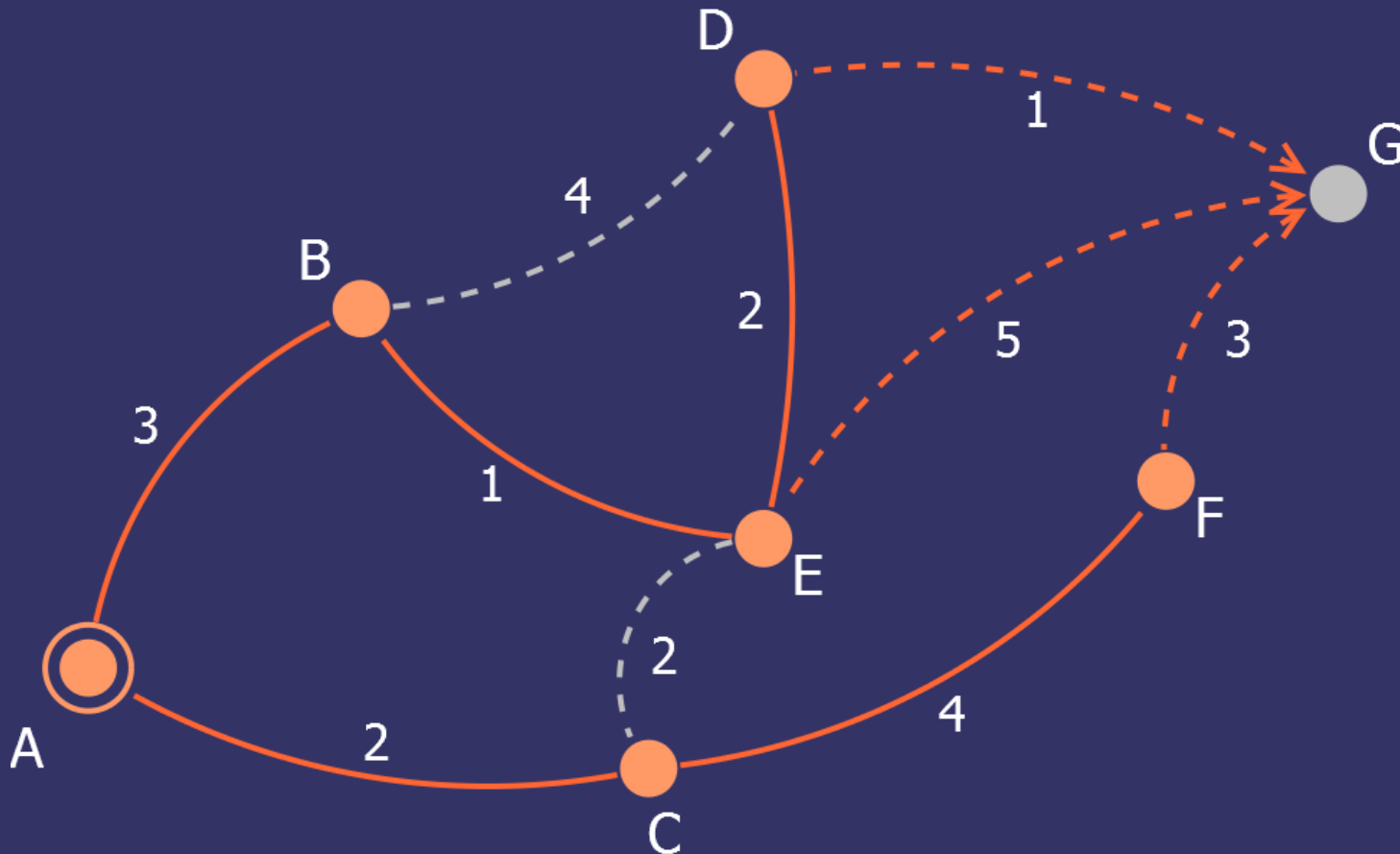
# Example: Step 5

Node	A	B	C	D	E	F	G
Distance	0	3	2	6	4	6	
Edges	<del>AC-2</del> <del>AB-3</del>	<del>BE-1</del> <del>BA-3</del> <del>BD-4</del>	<del>CA-2</del> <del>CE-2</del> <del>CF-4</del>	<b>DG-1</b> <del>DE-2</del> <del>DB-4</del>	<del>EB-1</del> <del>EC-2</del> <del>ED-2</del> <b>EG-5</b>	<b>FG-3</b> <del>FC-4</del>	<del>GD-1</del> <del>GF-3</del> <del>GE-5</del>

⇒ DG-1 (+6)

⇒ EG-5 (+4)

⇒ FG-3 (+6)





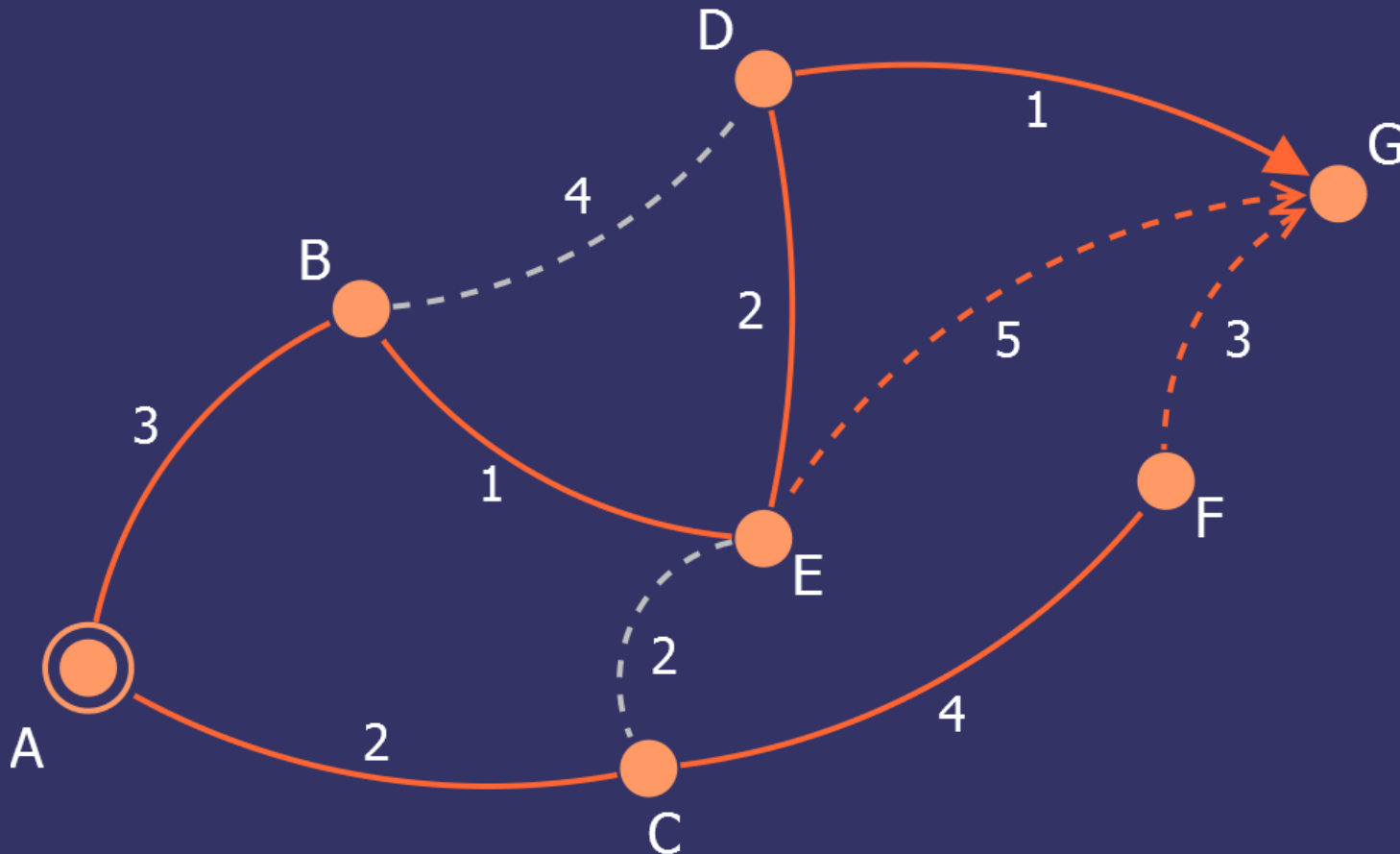
# Example: Step 5

Node	A	B	C	D	E	F	G
Distance	0	3	2	6	4	6	7
Edges	<del>AC-2</del> <del>AB-3</del>	<del>BE-1</del> <del>BA-3</del> <del>BD-4</del>	<del>CA-2</del> <del>CE-2</del> <del>CF-4</del>	<del>DG-1</del> <del>DE-2</del> <del>DB-4</del>	<del>EB-1</del> <del>EC-2</del> <del>ED-2</del> <del>EG-5</del>	<del>FG-3</del> <del>FC-4</del>	<del>GD-1</del> <del>GF-3</del> <del>GE-5</del>

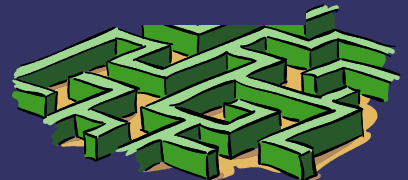
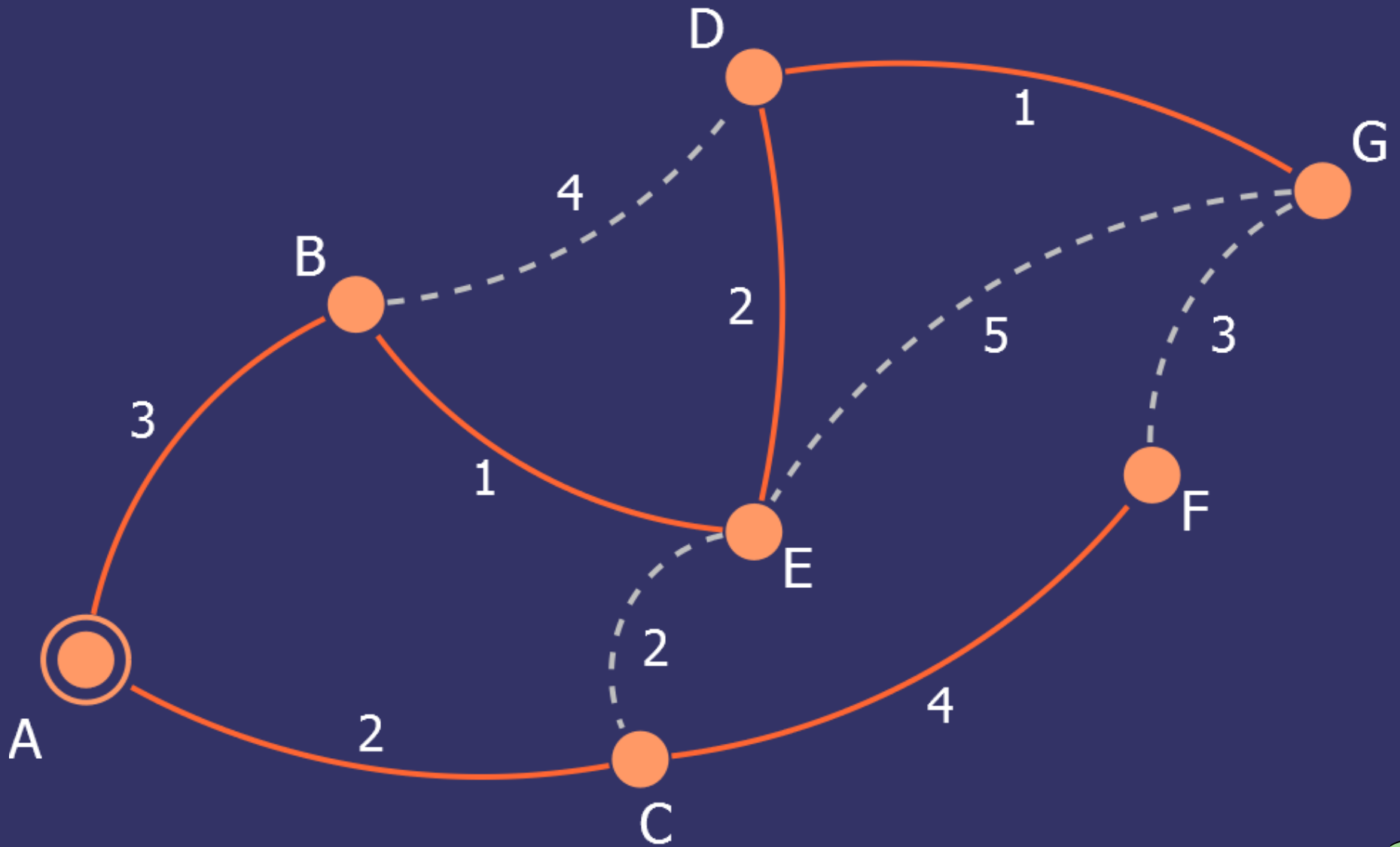
⇒ DG-1 (+6)

⇒ EG-5 (+4)

⇒ FG-3 (+6)



# Example: Step 6



# Questions?

- ➔ Dantzig, G.B.: *On the shortest path route through a network*. Management science, 6, 1960, s. 187-190.

