Parallel Rewriting Over Word Monoids

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Overview



Motivation

Regulation of the grammatical parallelism

Introduction Classification

EOL Systems EPOL Systems WMEOL grammar Generative Power



- regulation of the grammatical parallelism
- grammar alphabet always strictly taken as finite set of letters
- parallel rewriting over word monoids easily increase generative power

word = context



- parallelism is represented with **EOL** grammar systems
- WME(P)OL grammars EOL grammars over word monoids
- SE(P)OL as WME(P)OL(2) WME(P)OL grammars with words of length 2 - symbiotic EOL grammars



EOL System

$$G = (V, T, P, S)$$

- V is the total alphabet,
- T is a finite set of terminals, $T \subseteq V$,
- *P* is a finite set of productions in the form

 $a \longrightarrow w$

with $a \in V$, and $w \in V^*$,

• *S* is the axiom, $S \in V$.

EPOL Systems



EPOL System

$$G = (V, T, P, S)$$

- V is the total alphabet,
- T is a finite set of terminals, $T \subseteq V$,
- *P* is a finite set of productions in the form

 $a \longrightarrow w$

with $a \in V$, and $w \in V^+$,

• *S* is the axiom, $S \in V$.

EPOL Systems



Derivation Step

Let $u = a_1 a_2 \dots a_n$, and $v = w_1 w_2 \dots w_n$. If there exists a production rule $a_i \longrightarrow w_i \in P$ for all $1 \le i \le n$, then

 $U \Longrightarrow V.$

Generated Language

$$L(G) = \{ w \in T^* \mid S \Longrightarrow^* w \}$$

Generative Power

 $\mathcal{L}(REG) \subset \mathcal{L}(CF) \subset \mathcal{L}(EOL) = \mathcal{L}(EPOL) \subset \mathcal{L}(CS) \subset \mathcal{L}(RE)$



EOL grammar on word monoid - WMEOL grammar

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WMEOL(i) grammar is a pair
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(G, W)

where

$$G = (V, T, P, S)$$

- G is an EOL grammar,
- W is the set of generators, finite language over V
- *i* is a grammar degree, if every $y \in W$ satisfies $|y| \le i$.

Derivation Step

Let $x, y \in W^*$:

- $x = a_1 a_2 \cdots a_n$, $y = y_1 y_2 \cdots y_n$
- *a_i* ∈ *V*, *y_i* ∈ *V**
- $1 \le i \le n$, and $n \ge 0$

If $a_i y_i \in P$ for all i = 1 ... n, then x directly derives y according to rules $a_1 \rightarrow y_1, a_2 \rightarrow y_2, ..., a_n \rightarrow y_n$, symbolically written as

$$x \Rightarrow_{(G,W)} y[a_1 \rightarrow y_1, \dots, a_n \rightarrow y_n]$$

Generated Language

$$L(G, W) = \left\{ w \in T^* | S \Rightarrow^*_{(G, W)} w \right\}$$

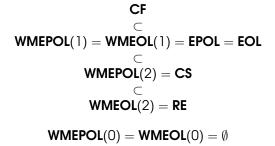
Generative Power

 $\mathcal{L}(REG) \subset \mathcal{L}(CF) \subset \mathcal{L}(EOL) = \mathcal{L}(EPOL) \subset \mathcal{L}(WMEOL) =$ $\mathcal{L}(WMEPOL) \subset \mathcal{L}(CS) = \mathcal{L}(WMEPOL(2)) \subset \mathcal{L}(RE) = \mathcal{L}(WMEOL(2))$



Generative Power





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Thank You For Your Attention !