Rewriting Over Word Monoids

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Abstract

This work discusses sequential rewriting over word monoids and also parallel version of rewriting. The first part focuses on sequential problematics, while the second part discusses parallel rewriting. In conclusion, both approaches will be compared from the generative power side.

For rewriting sentential forms of grammars is a standardly defined relation of a direct derivation which works with an alphabet—that is finitely many symbols. But this work modifies definition in terms of algebra and moreover in such a sense that works with finitely many strings. More precisely, the relation of a direct derivation is introduced as a relation over the free monoid generated by a finite set of strings.

In the context of sequentially rewriting over word monoids, a derivation step is performed only if rewritten sentential form occurs in the free monoids generated in this modified way. Consequently, even if the free monoids are generated by strings consisting of no more than two symbols, the resulting context-free grammars are as powerful as phrase-structure grammars.

Parallel variants deal with parallel rewriting over free monoids generated by finitely many strings. Parallelism is represented by E0L grammars, leading to a vigorous increase of the generative power of grammars, even if the generating strings of free monoids consist of no more than two symbols. E0L grammars are ET0L with a single set of rules, whereas ET0L grammars can be seen as generalized parallel versions of context-free grammars, except for three properties. First, there are finite many sets of rules. Second, the left-hand side of rules may contain terminals. Third, all symbols of string are simultaneously rewritten during a single derivation step. In conclusion, the relations between different families of languages are introduced.