

Scattered Context Grammars

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Scattered Context Grammar

Scattered Context Grammar

$$G = (V, T, P, S)$$

V is a finite alphabet

T is a set of terminals, $T \subset V$

S is the start symbol, $S \in V - T$

P is a finite set of productions of the form

$$(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n),$$

where $A_1, \dots, A_n \in V - T$, $x_1, \dots, x_n \in V^*$

Propagating Scattered Context Grammar

- each $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)$ satisfies $x_1, \dots, x_n \in V^+$

Derivation Step

Derivation Step

For $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$ and

$$u = u_1 A_1 \dots u_n A_n u_{n+1}$$

$$v = u_1 x_1 \dots u_n x_n u_{n+1}$$

we write $u \Rightarrow v [(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)]$

Generated Language

$$L(G) = \{x \in T^* : S \Rightarrow^* x\}$$

Generative Power

- $\mathcal{L}(SC) = \mathcal{L}(RE)$
- $\mathcal{L}(CF) \subset \mathcal{L}(PSC) \subseteq \mathcal{L}(CS)$

Example I

Example

Propagating scattered context grammar

$$G = (\{A, B, C, S, a, b, c\}, \{a, b, c\}, P, S)$$

with

$$P = \{(S) \rightarrow (ABC), \\ (A, B, C) \rightarrow (aA, bB, cC), \\ (A, B, C) \rightarrow (a, b, c)\}$$

Example of derivation

$$S \Rightarrow ABC \Rightarrow aAbBcC \Rightarrow aaAbbBccC \Rightarrow aaabbbccc$$

Generated language

$$L(G) = \{a^n b^n c^n : n \geq 1\}$$

Example II

Example

Propagating scattered context grammar

$$G = (\{S, W, X, Y, Z, A, a\}, \{a\}, P, S),$$

where

$$P = \{ \begin{array}{l} 1 : (S) \rightarrow (a), \\ 2 : (S) \rightarrow (aa), \\ 3 : (S) \rightarrow (WAXY), \\ 4 : (W, A, X, Y) \rightarrow (a, W, X, AAY), \\ 5 : (W, X, Y) \rightarrow (a, W, AXY), \\ 6 : (W, X, Y) \rightarrow (Z, Z, a), \\ 7 : (Z, A, Z) \rightarrow (Z, a, Z), \\ 8 : (Z, Z) \rightarrow (a, a) \end{array} \}$$

$$L = \{a^{2^n} : n \geq 0\}$$

$$\begin{array}{lll} S & \Rightarrow & WAXY \quad [3] \\ & \Rightarrow & aWX A^2 Y \quad [4] \\ & \Rightarrow & a^2 WAA^2 XY \quad [5] \\ & \Rightarrow & a^3 WAA X A^2 Y \quad [4] \\ & \Rightarrow & a^4 WAX A^4 Y \quad [4] \\ & \Rightarrow & a^5 WXA^6 Y \quad [4] \\ & \Rightarrow & a^6 WA^7 XY \quad [5] \\ & \Rightarrow & a^6 ZA^7 Z a \quad [6] \\ & \Rightarrow^7 & a^{13} ZZ a \quad [7^7] \\ & \Rightarrow & a^{16} \quad [8] \end{array}$$

Reduction – Definitions

Production length

$$\blacksquare \text{len}((A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)) = |A_1 \dots A_n| = n$$

Definitions

nonterminal complexity is the number of nonterminals in G

degree of context-sensitivity $\text{dcs}(G)$ is the number of context-sensitive productions in G

maximum context sensitivity $\text{mcs}(G)$ is the greatest number in

$$\{|\text{len}(p_i) - 1| : 1 \leq i \leq |P|\}$$

overall context sensitivity $\text{ocs}(G)$ is the sum of all members in

$$\{|\text{len}(p_i) - 1| : 1 \leq i \leq |P|\}$$

Reduction – Results I

Lemma

*There exists a scattered context grammar G such that G defines a **non-context-free** language and $\text{dcs}(G) = \text{mcs}(G) = \text{ocs}(G) = 1$.*

Proof

Consider a scattered context grammar

$$G = (\{S, A, B, C, D\}, \{a, b, c\}, P, S)$$

with

$$P = \{(S) \rightarrow (AC), \\ (A) \rightarrow (aAbB), \\ (A) \rightarrow (\varepsilon), \\ (C) \rightarrow (cCD), \\ (C) \rightarrow (\varepsilon), \\ (B, D) \rightarrow (\varepsilon, \varepsilon)\}$$

$$L(G) = \{a^n b^n c^n : n \geq 0\}$$

$$\text{dcs}(G) = \text{mcs}(G) = \text{ocs}(G) = 1$$

Reduction – Results II

Theorem

There are *context-sensitive* languages which *cannot* be described by a scattered context grammar $G = (V, T, P, S)$ satisfying $|V - T| = 1$.

Theorem

Every recursively enumerable language is generated by a scattered context grammar $G = (V, T, P, S)$ satisfying

$$|V - T| = 3, \text{ dcs}(G) = \infty, \text{ mcs}(G) = \infty, \text{ ocs}(G) = \infty.$$

Theorem

Every recursively enumerable language is generated by a scattered context grammar $G = (V, T, P, S)$ satisfying

$$|V - T| = 5, \text{ dcs}(G) = 2, \text{ mcs}(G) = 3, \text{ ocs}(G) = 6.$$

Theorem

Every recursively enumerable language is generated by a scattered context grammar $G = (V, T, P, S)$ satisfying

$$|V - T| = 8, \text{ dcs}(G) = 6, \text{ mcs}(G) = 1, \text{ ocs}(G) = 6.$$

Theorem

Every recursively enumerable language is generated by a scattered context grammar $G = (V, T, P, S)$ satisfying

$$|V - T| = 4, \text{ dcs}(G) = 4, \text{ mcs}(G) = 5, \text{ ocs}(G) = 20.$$

Context-Free and Context-Sensitive Productions

For a scattered context production p , if $\text{len}(p)$

$= 1$ then the production is **context-free**

≥ 2 then the production is **context-sensitive**

Theorem

Let $H = (M, T, R, S)$ be a phrase-structure grammar in Kuroda normal form. Then, there exists a scattered context grammar, $G = (V, T, P, E)$, that satisfies

- 1 $L(G) = L(H)$,
- 2 $|M| = |V| + 5$,
- 3 P contains 4 new context productions,
- 4 P contains 1 new context-free production.

Leftmost Derivations

Leftmost Derivation Step

For $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$ and

$$u = u_1 A_1 \dots u_n A_n u_{n+1}$$

$$v = u_1 x_1 \dots u_n x_n u_{n+1},$$

where $A_i \notin \text{alph}(u_i)$ for all $1 \leq i \leq n$, we write

$$u \xrightarrow{\text{lm}} v [(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)]$$

Theorem

Every context-sensitive language can be generated by a propagating scattered context grammar which uses only leftmost derivations.

Extended Propagating Scattered Context Grammar

An **extended propagating scattered context grammar** is a scattered context grammar

$$G = (V, T, P, S)$$

in which every

$$(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$$

satisfies $|x_1 \dots x_n| \geq n$

Theorem

Every context-sensitive language can be generated by an extended propagating scattered context grammar.

Unordered Scattered Context Grammar

Unordered Scattered Context Grammar

- scattered context grammar in which the order of context-free productions in a scattered context production is unimportant
- for $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$, a permutation $\pi : \{1, \dots, n\} \rightarrow \{1, \dots, n\}$, and

$$u = u_1 A_{\pi(1)} \dots u_n A_{\pi(n)} u_{n+1}$$

$$v = u_1 x_{\pi(1)} \dots u_n x_{\pi(n)} u_{n+1}$$

we write $u \Rightarrow v [(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)]$

Generative Power

- $\mathcal{L}(USC) = \mathcal{L}(P, \varepsilon)$
- $\mathcal{L}(PUSC) = \mathcal{L}(P) \subset \mathcal{L}(PSC)$

Open Problem

Are propagating scattered context grammars powerful enough to characterize all context-sensitive languages?

Open Problem

Can every recursively enumerable language be described by a scattered context grammar containing only two nonterminals?

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