

# Cooperating Distributed Grammar Systems

Jiří Techet    Tomáš Masopust    (Alexander Meduna)

Department of Information Systems  
Faculty of Information Technology  
Brno University of Technology  
Božetěchova 2, Brno 61266, Czech Republic

Modern Formal Language Theory, 2007

# CD Grammar Systems

## CD Grammar System

A cooperating distributed (CD) grammar system of degree  $n$ ,  $n \geq 1$ , is a construct

$$\Gamma = (N, T, S, P_1, \dots, P_n)$$

where

$N, T, S$  are defined as usual

$P_i$  is a finite set of context-free productions, called component of  $\Gamma$ ,  
for each  $i \in \{1, \dots, n\}$

## Notation

- the  $i$ th grammar  $G_i = (N, T, P_i, S)$
- $y \not\Rightarrow z$  –  $y$  does not directly derive  $z$

# CD Grammar Systems – Derivation Modes I

## Terminating Mode

For each  $i = 1, \dots, n$ , terminating derivation by the  $i$ th component

$$x \Rightarrow_i^t y$$

iff

- 1**  $x \Rightarrow^* y$  in  $G_i = (N, T, P_i, S)$  and
- 2**  $y \not\Rightarrow z$  for all  $z \in (N \cup T)^*$

## Note

$y \in T^*$  is not required

# CD Grammar Systems – Derivation Modes II

## **$k$ -Step Derivation in $G_i$**

$$x \Rightarrow_i^k y$$

iff  $x \Rightarrow^k y$  in  $G_i$

## **At Most $k$ -Step Derivation in $G_i$**

$$x \Rightarrow_i^{\leq k} y$$

iff  $x \Rightarrow^j y$  in  $G_i$  for some  $j \leq k$

## **At Least $k$ -Step Derivation in $G_i$**

$$x \Rightarrow_i^{\geq k} y$$

iff  $x \Rightarrow^j y$  in  $G_i$  for some  $j \geq k$

# CD Grammar Systems – Generated Language

## Set of Derivation Modes

$$D = \{*, t\} \cup \{\leq k, = k, \geq k : k = 1, 2, \dots\}$$

## Set of Possible Derivations

$$F(G_j, u, f) = \{v : u \xRightarrow{f} v\}, \text{ where } j \in \{1, \dots, n\}, f \in D, u \in V^*$$

## Generated Language

$$L_f(\Gamma) = \{ w \in T^* : \text{there are } v_0, v_1, \dots, v_m \text{ such that} \\ v_i \in F(G_{j_i}, v_{i-1}, f), i = 1, \dots, m, j_i \in \{1, \dots, n\}, \\ v_0 = S, v_m = w, \text{ for some } m \geq 1 \}$$

# CD Grammar Systems – Example I

## Example

$$\Gamma = (\{S, A, A', B, B'\}, \{a, b, c\}, S, P_1, P_2)$$

where

$$P_1 = \{S \rightarrow S, S \rightarrow AB, A' \rightarrow A, B' \rightarrow B\}$$

$$P_2 = \{A \rightarrow aA'b, B \rightarrow cB', A \rightarrow ab, B \rightarrow c\}$$

$$L_f(\Gamma) = \{a^n b^n c^m : m, n \geq 1\}, f \in \{= 1, \geq 1, *, t\} \cup \{\leq k : k \geq 1\}$$

$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) = \{a^n b^n c^n : n \geq 1\}$$

$$L_{=k}(\Gamma) = L_{\geq k}(\Gamma) = \emptyset, k \geq 3$$

# CD Grammar Systems – Example II

## Example

$$\Gamma = (\{S, A\}, \{a\}, S, P_1, P_2, P_3)$$

where

$$P_1 = \{S \rightarrow AA\}$$

$$P_2 = \{A \rightarrow S\}$$

$$P_3 = \{A \rightarrow a\}$$

$$L_t(\Gamma) = \{a^{2^n} : n \geq 1\}$$

# Denotation of CD Language Families

## Denotation of CD Language Families

$$CD_x^y(f)$$

where

$f$  derivation mode,  $f \in D$

$y$  ■ nothing – no  $\varepsilon$ -productions

■  $\varepsilon$  –  $\varepsilon$ -productions allowed

$x$  ■  $n$  – degree at most  $n$ ,  $n \geq 1$

■  $\infty$  – the number of components is not limited

### Note

$CD_\infty(=)$  the union of all families  $CD_\infty(=k)$  for  $k = 1, 2, \dots$

$CD_\infty(\geq)$  the union of all families  $CD_\infty(\geq k)$  for  $k = 1, 2, \dots$



# CD Grammar Systems – Generative Power

## Theorem

- $CD_{\infty}^y(f) = \mathcal{L}(CF)$ , for all  $f \in \{= 1, \geq 1, *\} \cup \{\leq k : k \geq 1\}$
- $\mathcal{L}(CF) = CD_1^y(f) \subset CD_2^y(f) \subseteq CD_r^y(f) \subseteq CD_{\infty}^y(f) \subseteq \mathcal{L}(M)$ , for all  $f \in \{= k, \geq k : k \geq 2\}, r \geq 3$
- $CD_r^y(\geq k) \subseteq CD_r^y(\geq k + 1)$
- $CD_{\infty}^y(\geq) \subseteq CD_{\infty}^y(=)$
- $\mathcal{L}(CF) = CD_1^y(t) = CD_2^y(t) \subset CD_3^y(t) = CD_{\infty}^y(t) = \mathcal{L}(ET0L)$

# Hybrid CD Grammar Systems

- combination of different modes is possible

## Hybrid CD Grammar System

$$\Gamma = (N, T, S, (P_1, f_1), \dots, (P_n, f_n))$$

where

$N, T, S, P_i$  are defined as in the case of CD grammar systems

$f_i$  is the mode of the  $i$ th component,  $f_i \in D$  for all  $i \in \{1, \dots, n\}$

## Generated Language

$$L(\Gamma) = \{w \in T^* : \text{there are } v_0, v_1, \dots, v_m \text{ such that} \\ v_i \in F(G_{j_i}, v_{i-1}, f_{j_i}), i = 1, \dots, m, j_i \in \{1, \dots, n\}, \\ v_0 = S, v_m = w, \text{ for some } m \geq 1\}$$

# Denotation of Hybrid CD Language Families

## Denotation of Hybrid CD Language Families

$$XCD_{x,v}^y(f)$$

where

$x, y, f$  are defined as in the case of CD grammar systems

- $v$     ■  $m$  – each  $P_i$  contains at most  $m$  productions,  $m \geq 1$ 
  - $\infty$ , nothing – any number of productions
- $X$     ■ nothing – nondeterministic
  - $D$  – deterministic (for each  $P_i$ ,  $A \rightarrow u, A \rightarrow w \in P_i$  satisfy  $u = w$ )
  - $H$  – hybrid (then,  $(f)$  is not written)

### Example

$HCD_n^\varepsilon$  is a hybrid CD grammar system with at most  $n$  components

# Hybrid CD Grammar Systems – Generative Power


## Theorem


- $CD_{\infty,\infty}(f) = CD_{\infty,1}(f) = \mathcal{L}(CF)$ , for all  $f \in \{=, 1, \geq 1, *\} \cup \{\leq k : k \geq 1\}$
- $\mathcal{L}(CF) \subset CD_{\infty,1}^{\varepsilon}(t) \subset CD_{\infty,2}^{\varepsilon}(t) \subseteq CD_{\infty,3}^{\varepsilon}(t) \subseteq CD_{\infty,4}^{\varepsilon}(t) \subseteq CD_{\infty,5}^{\varepsilon}(t) = CD_{\infty,\infty}^{\varepsilon}(t) = \mathcal{L}(ET0L)$
- $CD_{n,m}(f) \subset CD_{n+1,m}(f)$ ,  $f \in \{*, t\}$
- $CD_{n,m}(f) \subset CD_{n,m+1}(f)$ ,  $f \in \{*, t\}$


## Theorem

- $\mathcal{L}(CF) \subset HCD_1 \subset HCD_2 \subseteq HCD_3 \subseteq HCD_4 = HCD_{\infty} = \mathcal{L}(M, ac)$
- $\mathcal{L}(ET0L) \subset HCD_4$
- $CD_{\infty}(=) \subset HCD_3$

# Bibliography

 Grammar systems.  
<http://www.sztaki.hu/mms/bib.html>.

 E. Csuhaj-Varjú and J. Dassow.  
On cooperating distributed grammar systems.  
*Elektronische Informationsverarbeitung und Kybernetik*, 26:49–63,  
1990.

 R. Meersman and G. Rozenberg.  
Cooperating grammar systems.  
In *MFCS 78*, pages 364–373, 1978.