

k -Limited Erasing Performed by Scattered Context Grammars

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Scattered Context Grammar

Scattered context grammar (SC grammar)

A SC grammar is a quadruple, $G = (V, T, P, S)$, where

V is a finite alphabet

T is a set of terminals, $T \subset V$

S is a starting symbol, $S \in (V - T)$

P is a finite set of productions of the form: $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)$;
 $A_1, \dots, A_n \in (V - T)$; $x_1, \dots, x_n \in V^*$

$$\blacksquare \text{len}((A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)) = n$$

$$\blacksquare \text{pos}(a_1 \dots a_i \dots a_n, i) = a_i$$

Propagating scattered context grammar (PSC grammar)

$$\blacksquare \text{every } (A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \text{ satisfies } x_1, \dots, x_n \in V^+$$

Generated Language

Derivation step

For $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$ and

$$u = u_1 A_1 \dots u_n A_n u_{n+1}$$

$$v = u_1 x_1 \dots u_n x_n u_{n+1}$$

we write $u \Rightarrow v [(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)]$

Generated language

$$L(G) = \{x \in T^* : S \Rightarrow^* x\}$$

Generative power

- $\mathcal{L}_{SC} = \mathcal{L}_{RE}$
- $\mathcal{L}_{CF} \subset \mathcal{L}_{PSC} \subseteq \mathcal{L}_{CS}$

Example

$G_1 = (\{A, B, C, S, a, b, c\}, \{a, b, c\}, P_1, S)$ with

$$P_1 = \{(S) \rightarrow (ABC), \\ (A, B, C) \rightarrow (aA, bB, cC), \\ (A, B, C) \rightarrow (\varepsilon, \varepsilon, \varepsilon)\}$$

$$S \Rightarrow ABC \Rightarrow aAbBcC \Rightarrow aaAbbBccC \Rightarrow aabbcc$$

$$L(G_1) = \{a^n b^n c^n : n \geq 0\}$$

Example

$G_2 = (\{A, B, C, S, a, b, c\}, \{a, b, c\}, P_2, S)$ with

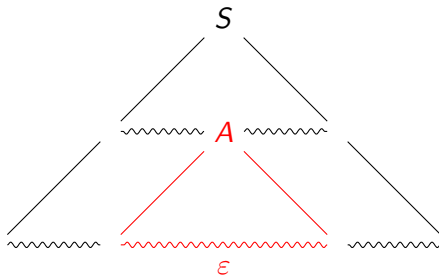
$$P_2 = \{(S) \rightarrow (\varepsilon), (S) \rightarrow (ABC), \\ (A, B, C) \rightarrow (aA, bB, cC), \\ (A, B, C) \rightarrow (a, b, c)\}$$

Symbols Erased During Derivation

Symbols erased during derivation

A symbol, A , is erased during a derivation if the frontier of the subtree rooted at A is ε .

- If the symbol A is erased, we write \check{A} ;
- otherwise the symbol is not erased and we write \hat{A} .

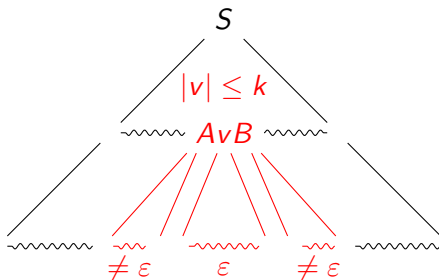


Nonterminals Erased in a k -Limited Way

Nonterminals erased in a k -limited way

For every $y \in L(G)$ there exists a derivation in which every sentential form x satisfies:

- 1 Every $x = uAvBw$, \hat{A} , \hat{B} , \check{v} , satisfies $|v| \leq k$.
- 2 Every $x = uAw$, \hat{A} , satisfies: if \check{u} or \check{w} , then $|u| \leq k$ or $|w| \leq k$, respectively.



Theorem

For every SC grammar, G , which erases its nonterminals in a k -limited way there exists a propagating SC grammar, \bar{G} , such that $L(G) = L(\bar{G})$.

Basic Idea—Demonstration

Example

$G_3 = (\{A, B, C, S, a, b, c\}, \{a, b, c\}, P_3, S)$ with

$$P_3 = \{(S) \rightarrow (ABC), (A, B, C) \rightarrow (\varepsilon, \varepsilon, \varepsilon), \\ (A) \rightarrow (aAA), (B) \rightarrow (bBB), (C) \rightarrow (cCC)\}$$

$$S \Rightarrow ABC \Rightarrow^3 aAbBcCC \Rightarrow aAbBcC \Rightarrow^3 aaAAbbBBccCC \Rightarrow^2 aabbcc$$

	$\langle S \rangle$		$(S) \rightarrow (ABC)$
\Rightarrow	$\langle A \rangle$	$\langle B \rangle$	$\langle C \rangle$
\Rightarrow^3	$\langle aAA \rangle$	$\langle bBB \rangle$	$\langle cCC \rangle$
\Rightarrow	$\langle aA \rangle$	$\langle bB \rangle$	$\langle cC \rangle$
\Rightarrow^3	$\langle a \rangle \langle aAA \rangle$	$\langle b \rangle \langle bBB \rangle$	$\langle c \rangle \langle cCC \rangle$
\Rightarrow	$\langle a \rangle \langle aA \rangle$	$\langle b \rangle \langle bB \rangle$	$\langle c \rangle \langle cC \rangle$
\Rightarrow	$\langle a \rangle \langle a \rangle$	$\langle b \rangle \langle b \rangle$	$\langle c \rangle \langle c \rangle$
\Rightarrow^6	aa	bb	cc

Basic Idea

Let $G = (V, T, P, S)$ be a grammar which erases its nonterminals in a k -limited way. Every application of $p = (A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)$ is simulated by a PSC grammar \bar{G} as follows:

- 1 Every CF component of a SC production is simulated independently.

$$\begin{aligned} & \langle z_{11} [p, 1] z_{12} \rangle \langle z_{21} B_1 A_2 z_{22} A_3 z_{23} \rangle \dots \langle z_{n1} A_n z_{n2} \rangle \\ \Rightarrow & \langle z_{11} x_1 [p, 1]' z_{12} \rangle \langle z_{21} B_1 A_2 z_{22} A_3 z_{23} \rangle \dots \langle z_{n1} A_n z_{n2} \rangle \end{aligned}$$

- 2 Transition to the next component of the SC production is performed.

$$\begin{aligned} & \langle z_{11} x_1 [p, 1]' z_{12} \rangle \langle z_{21} B_1 A_2 z_{22} A_3 z_{23} \rangle \dots \langle z_{n1} A_n z_{n2} \rangle \\ \Rightarrow & \langle z_{11} x_1 z_{12} \rangle \langle z_{21} B_1 [p, 2] z_{22} A_3 z_{23} \rangle \dots \langle z_{n1} A_n z_{n2} \rangle \end{aligned}$$

- 3 After the last component is simulated, transition to the following SC production, $q = (B_1, \dots, B_m) \rightarrow (y_1, \dots, y_m)$, is performed.

$$\begin{aligned} & \langle z_{11} x_1 z_{12} \rangle \langle z_{21} B_1 x_2 z_{22} x_3 z_{23} \rangle \dots \langle z_{n1} x_n [p, n]' z_{n2} \rangle \\ \Rightarrow & \langle z_{11} x_1 z_{12} \rangle \langle z_{21} [q, 1] x_2 z_{22} x_3 z_{23} \rangle \dots \langle z_{n1} x_n z_{n2} \rangle \end{aligned}$$

Finally, we replace symbols of the form $\langle a \rangle$ with a , where $a \in T$.

Construction of Symbols

- $\Psi = \{ \lfloor p, i \rfloor : p \in P, 1 \leq i \leq \text{len}(p) \}$
- $\Psi' = \{ \lfloor p, i \rfloor' : \lfloor p, i \rfloor \in \Psi \}$
- $\bar{N}_1 = \{ \langle x \rangle : x \in (V - T)^* \cup (V - T)^* T (V - T)^*, |x| \leq 2k + 1 \}$

For every $\langle x \rangle \in \bar{N}_1$ and $\lfloor p, i \rfloor \in \Psi$, define

$\text{lhs-replace}(\langle x \rangle, \lfloor p, i \rfloor) = \{ \langle x_1 \lfloor p, i \rfloor x_2 \rangle : x_1, x_2 \in V^*, x_1 \text{lhs}(\lfloor p, i \rfloor) x_2 = x \}$

- $\bar{N}_2 = \{ \langle x \rangle : \langle x \rangle = \text{lhs-replace}(\langle y \rangle, \lfloor p, i \rfloor), \langle y \rangle \in \bar{N}_1, \lfloor p, i \rfloor \in \Psi \}$

For every $\langle x \rangle \in \bar{N}_1$ and $\lfloor p, i \rfloor' \in \Psi'$, define

$\text{insert}(\langle x \rangle, \lfloor p, i \rfloor') = \{ \langle x_1 \lfloor p, i \rfloor' x_2 \rangle : x_1, x_2 \in V^*, x_1 x_2 = x \}$

- $\bar{N}_2' = \{ \langle x \rangle : \langle x \rangle = \text{insert}(\langle y \rangle, \lfloor p, i \rfloor'), \langle y \rangle \in \bar{N}_1, \lfloor p, i \rfloor' \in \Psi' \}$

Define the PSC grammar,

$$\bar{G} = (T \cup \bar{N}_1 \cup \bar{N}_2 \cup \bar{N}_2' \cup \{\bar{S}\}, T, \bar{P}, \bar{S})$$

Construction of Productions I

For every $x = \langle x_1 \rangle \langle x_2 \rangle \dots \langle x_n \rangle \in (\bar{N}_1 \cup \bar{N}_2 \cup \bar{N}'_2)^*$ for some $n \geq 1$, define

$$\text{join}(x) = x_1 x_2 \dots x_n$$

For every $x \in \bar{N}_1 \cup \bar{N}_2 \cup \bar{N}'_2$, define

$$\text{split}(x) = \{y : x = \text{join}(y)\}$$

1 Initialization

For every $p = (S) \rightarrow (x) \in P$, add
 $(\bar{S}) \rightarrow (\langle [p, 1] \rangle)$ to \bar{P}

2 Termination

For every $a \in T$, add
 $(\langle a \rangle) \rightarrow (a)$ to \bar{P}

Construction of Productions II

3 Simulation of one SC production's CF component

For every

- $\langle x_1[p, i]x_2 \rangle \in \text{lhs-replace}(\langle x \rangle, [p, i]), \langle x \rangle \in \bar{N}_1, [p, i] \in \Psi, x_1, x_2 \in V^*$
- $Y \in \text{split}(x_1 \text{ rhs}([p, i])[p, i]'x_2)$

add $(\langle x_1[p, i]x_2 \rangle) \rightarrow (Y)$ to \bar{P}

4 Transition to the next SC production's CF component

For every

- $\langle x \rangle \in \bar{N}_1$
- $X \in \text{insert}(\langle x \rangle, [p, i]')$, where $p \in P, i < \text{len}(p)$
- $\langle y \rangle \in \bar{N}_1$
- $Y \in \text{lhs-replace}(\langle y \rangle, [p, i + 1])$, where $q \in P$

add

1 $(X, \langle y \rangle) \rightarrow (\langle x \rangle, Y)$ to \bar{P}

2 If

- $\langle x \rangle = \langle y \rangle$
- $\text{pos}(X, l) = [p, i]'$, $\text{pos}(Y, m) = [p, i + 1]'$, $l < m$

add $(X) \rightarrow (Y)$ to \bar{P}

Construction of Productions III

5 Transition to the next SC production

For every

- $\langle x \rangle \in \bar{N}_1$
- $X \in \text{insert}(\langle x \rangle, \lfloor p, n \rfloor')$, where $p \in P$, $\text{len}(p) = n$
- $\langle y \rangle \in \bar{N}_1$
- $Y \in \text{lhs-replace}(\langle y \rangle, \lfloor q, 1 \rfloor)$, where $q \in P$

add

- 1 $(X, \langle y \rangle) \rightarrow (\langle x \rangle, Y)$ to \bar{P}
- 2 $(\langle y \rangle, X) \rightarrow (Y, \langle x \rangle)$ to \bar{P}
- 3 If $\langle x \rangle = \langle y \rangle$, add
 $(X) \rightarrow (Y)$ to \bar{P}
- 4 Finishing the simulation
 $(X) \rightarrow (\langle x \rangle)$ to \bar{P}

Summary and Future Investigation

Summary

- In general, in SC grammars ε -productions cannot be removed
- This removal is, however, possible under some conditions

Future investigation

- There are modifications of SC grammars which contain ε -productions and characterize all CS languages
- Is it possible to convert them to equivalent grammars which delete their nonterminals in a k -limited way?