

# A Note on Scattered Context Grammars with Non-Context-Free Components

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# Scattered Context Grammar

## Scattered Context Grammar (SC Grammar)

$G = (V, T, P, S)$ , where

$V$  is a finite alphabet

$T$  is a set of terminals,  $T \subset V$

$S$  is the start symbol,  $S \in V - T$

$P$  is a finite set of productions of the form

$$(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n),$$

where  $A_1, \dots, A_n \in V - T$ ,  $x_1, \dots, x_n \in V^*$

## Propagating Scattered Context Grammar (PSC Grammar)

- each  $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)$  satisfies  $x_1, \dots, x_n \in V^+$

# SC Grammar—Derivation Step

## Derivation Step

For  $(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n) \in P$  and

$$u = u_1 A_1 \dots u_n A_n u_{n+1}$$

$$v = u_1 x_1 \dots u_n x_n u_{n+1}$$

we write  $u \Rightarrow v [(A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)]$

## Generated Language

$$L(G) = \{x \in T^* : S \Rightarrow^* x\}$$

## Generative Power

- $\mathcal{L}(SC) = \mathcal{L}(RE)$
- $\mathcal{L}(CF) \subset \mathcal{L}(PSC) \subseteq \mathcal{L}(CS)$

# SC Grammar—Example

## Production Length

$$\blacksquare \text{ len}((A_1, \dots, A_n) \rightarrow (x_1, \dots, x_n)) = |A_1 \dots A_n| = n$$

## Example

SC grammar  $G = (\{A, B, C, S, a, b, c\}, \{a, b, c\}, P, S)$  with

$$P = \{(S) \rightarrow (ABC), \\ (A, B, C) \rightarrow (aA, bB, cC), \\ (A, B, C) \rightarrow (\varepsilon, \varepsilon, \varepsilon)\}$$

$$S \Rightarrow ABC \Rightarrow aAbBcC \Rightarrow aaAbbBccC \Rightarrow aabbcc$$

$$L(G) = \{a^n b^n c^n : n \geq 0\}$$

# Linear Scattered Context Grammars

## Linear Scattered Context Grammar

- scattered context grammar  $G = (V, T, P, S)$
- $P$  is a finite set of productions of the following two forms:
  - 1  $(S) \rightarrow (x_1 A_1 \dots x_k A_k x_{k+1})$ , where  $A_i \in (V - T) - \{S\}$ ,  $x_i \in T^*$  for all  $1 \leq i \leq k$ , for some  $k \geq 1$ ,
  - 2  $(A_1, \dots, A_k) \rightarrow (z_1, \dots, z_k)$ , where  $A_i \in (V - T) - \{S\}$ , and either
    - $z_i = x_i B_i y_i$ , where  $x_i, y_i \in T^*$ ,  $B_i \in (V - T) - \{S\}$ , or
    - $z_i \in T^*$for all  $1 \leq i \leq k$ , for some  $k \geq 1$

## Degree $n$ of Linear Scattered Context Grammar

- $(S) \rightarrow (x_1 A_1 \dots x_n A_n x_{n+1}) \in P$  satisfies  $n \geq m$  for all  $(S) \rightarrow (y_1 A_1 \dots y_m A_m y_{m+1}) \in P$
- for each  $p \in P$ ,  $\text{len}(p)$  is constant for every grammar ( $\text{len}(p)$  does not depend on the degree)

# Right-Linear Scattered Context Grammars

## Right-Linear Scattered Context Grammar

- linear scattered context grammar  $G = (V, T, P, S)$
- $P$  is a finite set of productions of the following two forms:
  - 1  $(S) \rightarrow (x_1 A_1 \dots x_k A_k)$ , where  $A_i \in (V - T) - \{S\}$ ,  $x_i \in T^*$  for all  $1 \leq i \leq k$ , for some  $k \geq 1$ ,
  - 2  $(A_1, \dots, A_k) \rightarrow (z_1, \dots, z_k)$ , where  $A_i \in (V - T) - \{S\}$ , and either
    - $z_i = x_i B_i$ , where  $x_i \in T^*$ ,  $B_i \in (V - T) - \{S\}$ , or
    - $z_i \in T^*$for all  $1 \leq i \leq k$ , for some  $k \geq 1$

## Language Families

- $\mathcal{L}(SC, LIN, n)$  – linear scattered context grammars of degree  $n$
- $\mathcal{L}(SC, RLIN, n)$  – right-linear scattered context grammars of degree  $n$

# Main Results I

## Theorem

For each  $n \geq 1$ ,

$$\begin{aligned}\mathcal{L}(\text{SC}, \text{LIN}, n) &\subset \mathcal{L}(\text{SC}, \text{LIN}, n+1), \\ \mathcal{L}(\text{SC}, \text{RLIN}, n) &\subset \mathcal{L}(\text{SC}, \text{RLIN}, n+1), \\ \mathcal{L}(\text{SC}, \text{RLIN}, n) &\subset \mathcal{L}(\text{SC}, \text{LIN}, n).\end{aligned}$$

- $\mathcal{L}(\text{SC}, \text{LIN}) = \bigcup_{n=1}^{\infty} \mathcal{L}(\text{SC}, \text{LIN}, n)$
- $\mathcal{L}(\text{SC}, \text{RLIN}) = \bigcup_{n=1}^{\infty} \mathcal{L}(\text{SC}, \text{RLIN}, n)$

## Theorem

$$\begin{aligned}\mathcal{L}(\text{SC}, \text{LIN}) &\subset \mathcal{L}(\text{PSC}), \mathcal{L}(\text{CF}) - \mathcal{L}(\text{SC}, \text{LIN}) \neq \emptyset, \\ \mathcal{L}(\text{SC}, \text{RLIN}) &\subset \mathcal{L}(\text{PSC}), \mathcal{L}(\text{CF}) - \mathcal{L}(\text{SC}, \text{RLIN}) \neq \emptyset, \\ \mathcal{L}(\text{SC}, \text{RLIN}) &\subset \mathcal{L}(\text{SC}, \text{LIN}).\end{aligned}$$

# Main Results II

## Theorem (Positive Closure Properties)

Each family  $\mathcal{L}(SC, LIN, n)$  and  $\mathcal{L}(SC, RLIN, n)$ , where  $n \geq 1$ , is closed under union, reversal, homomorphism, inverse homomorphism, substitution with regular languages, concatenation with regular languages, intersection with regular languages, left and right quotient by regular languages.  $\mathcal{L}(SC, LIN)$  and  $\mathcal{L}(SC, RLIN)$  are closed under concatenation.

## Theorem (Negative Closure Properties)

Each family  $\mathcal{L}(SC, LIN, n)$ , where  $n \geq 1$ , is not closed under concatenation with linear languages. Each family  $\mathcal{L}(SC, RLIN, n)$ , where  $n \geq 1$ , is not closed under concatenation with  $\mathcal{L}(SC, RLIN, 2)$ .  $\mathcal{L}(SC, LIN)$  and  $\mathcal{L}(SC, RLIN)$  are not closed under intersection, complement and Kleene star.  $\mathcal{L}(SC, LIN)$  is not closed under substitution with linear languages.  $\mathcal{L}(SC, RLIN)$  is not closed under substitution with  $\mathcal{L}(SC, RLIN, 2)$ .



# Conclusion and Open Problems

- The proof of the previous theorems is based on the proof of the equivalence of (right) linear scattered context grammars and (right) linear simple matrix grammars
- We may want to know what is the power of scattered context grammars with context-sensitive and unrestricted components; clearly:
  - $\mathcal{L}(SC, CS) = \mathcal{L}(CS)$
  - $\mathcal{L}(SC, RE) = \mathcal{L}(RE)$
- Concerning the power of scattered context grammars, there remains the original open problem:

$$\mathcal{L}(CS) - \mathcal{L}(PSC) \stackrel{?}{=} \emptyset$$