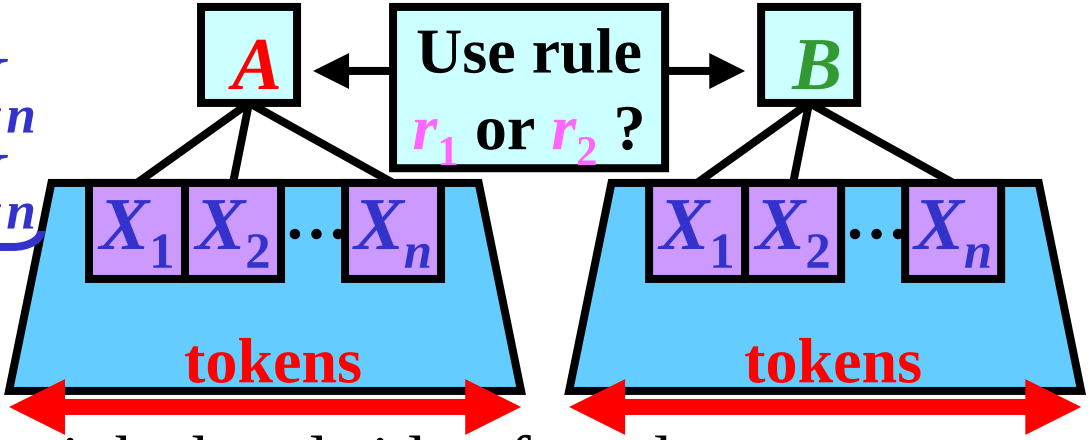
Part IX.
Bottom-Up Parsing

Bottom-Up Parsing: Problems

1) Two or more rules have the same *handle*

r_1: $A \to X_1X_2\ldots X_n$

r_2: $B \to \underbrace{X_1X_2\ldots X_n}_{\text{handle}}$



Use rule r_1 or r_2 ?

Note: A *handle* is the right-hand side of a rule.

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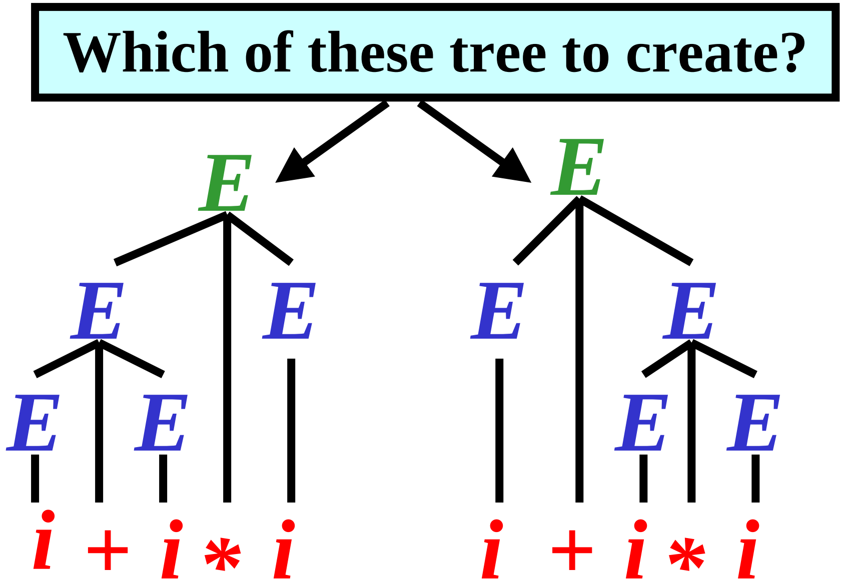
2) Ambiguous grammars

$G_{expr2} = (N, T, P, E)$, where

$N = \{E\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 1: $E \to E+E$, 2: $E \to E*E$,

3: $E \to (E)$, 4: $E \to i$ $\}$



Which of these tree to create?

Bottom-Up Parsers

1) Operator-precedence parser

- the least powerful, but simple & easy-to-make

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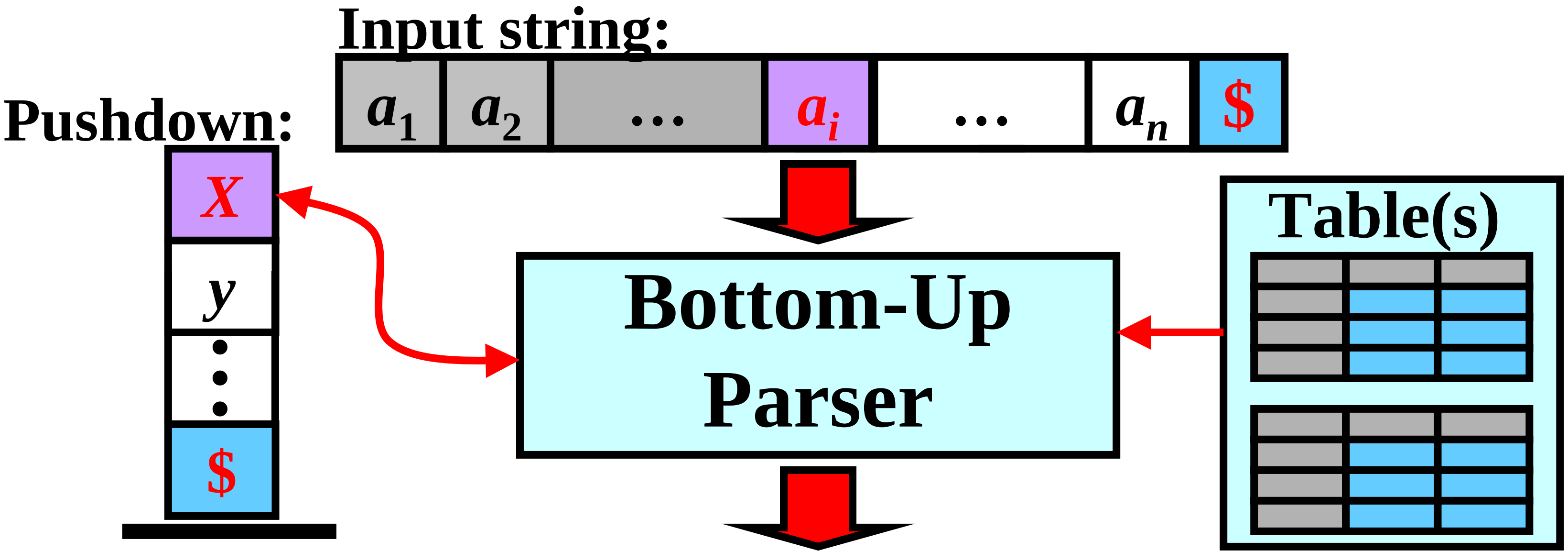
2) LR parser

- the most powerful

- **Model of Bottom-Up parser:**



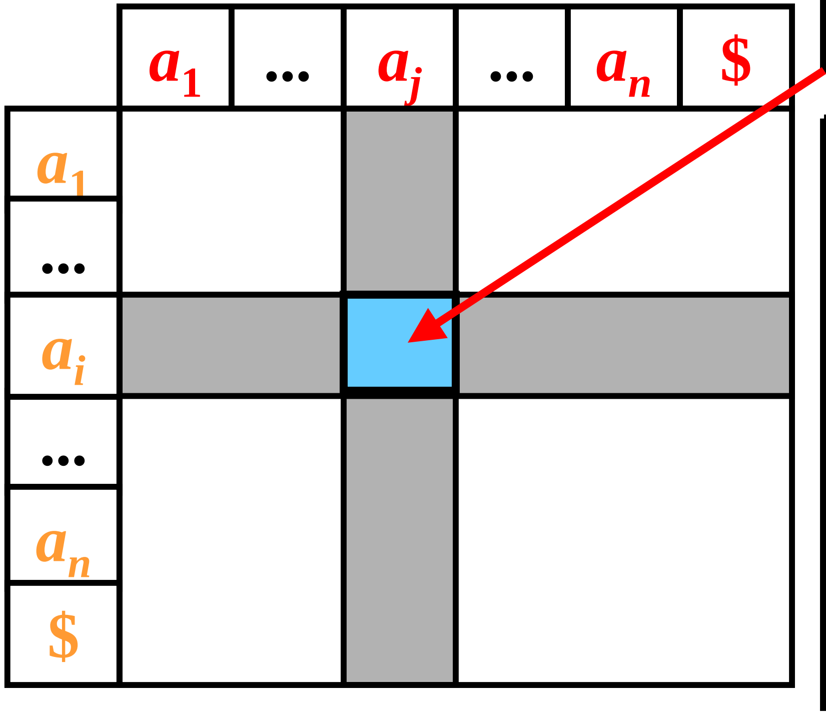
Right parse = **reverse** sequence of rules used in the **rightmost derivation** of the tokenized source program

Operator-Precedence Parser

- No two distinct nonterminals have the same handle
- No ε-rules.

- Let $G = (N, T, P, S)$ be CFG, where $T = \{a_1, a_2, \dots, a_n\}$

Precedence-table:

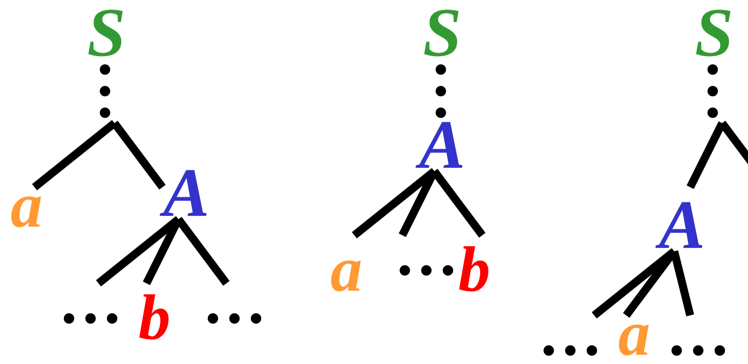


	a_1	...	a_j	...	a_n	$
a_1						
...						
a_i						
...						
a_n						
$						

Table$[a_i, a_j] \in \{<, =, >, \textit{blank}\}$

Illustration of meaning of <, =, >:

$a < b$ $a = b$ $a > b$



Operator-Precedence Parser: Algorithm

- **Input:** Precedence-table for $G = (N, T, P, S)$; $x \in T^*$
- **Output:** Right parse of x if $x \in L(G)$; otherwise, error

---

- **Method:**

```
• Push $ onto the pushdown;
• repeat
    • let a = the current token and
          b = the topmost terminal on the pushdown
    • case Table[b, a] of:
        • = : push(a) & read next a from input string
        • < : replace b with b< on the pushdown &
              push(a) & read next a from input string
        • > : if <y is the pushdown top string and r: A → y ∈ P
              then replace <y with A & write r to output
              else error
        • blank : error
  until a = $ and b = $
• success
```

Operator-Precedence Parser: Example

$G_{expr2} = (N, T, P, E)$, where $N = \{E\}$, $T = \{i, +, *, (,)\}$,
$P = \{\ 1: E \rightarrow E+E,\ 2: E \rightarrow E*E,\ 3: E \rightarrow (E),\ 4: E \rightarrow i\}$

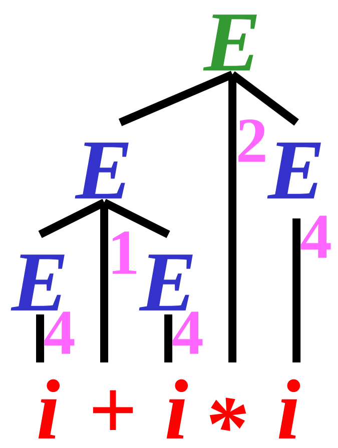
Precedence-table for G_{expr2}:

Input token

Pushdown topmost token	+	*	(	)	*i*	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Note: Operator associativity and precedence rules underlie the precedence table:

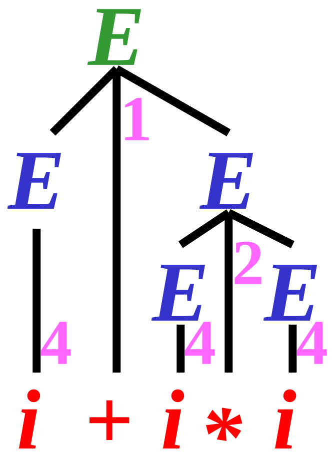
☹ **Wrong tree:**



Right parse:

44142

☺ **Right tree:**



Right parse:

44421

Operator-Precedence Parsing: Example

	+	*	(	)	i	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

Input string: $i + i * i\ \$$

Pushdown	Op	Input	Rule
$\$$	<	$i+i*i\$$	
$\$<i$	>	$+i*i\$$	4: $E \rightarrow i$
$\$E$	<	$+i*i\$$	
$\$<E+$	<	$i*i\$$	
$\$<E+<i$	>	$*i\$$	4: $E \rightarrow i$
$\$<E+E$	<	$*i\$$	
$\$<E+<E*$	<	$i\$$	
$\$<E+<E*<i$	>	$\$$	4: $E \rightarrow i$
$\$<E+<E*E$	>	$\$$	2: $E \rightarrow E*E$
$\$<E+E$	>	$\$$	1: $E \rightarrow E+E$
$\$E$		$\$$	

Rules:

1: $E \rightarrow E+E$

2: $E \rightarrow E*E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Success

Right parse: 44421

Construction of Precedence Table 1/5

- Let $G_{expr} = (N, T, P, E)$, where $N = \{E\}$,

$T = \{(,), \mathbf{id}_1, \mathbf{id}_2, \ldots, \mathbf{id}_m, \mathbf{op}_1, \mathbf{op}_2, \ldots\ \mathbf{op}_n\}$,

$P = \{\ E \rightarrow (E), E \rightarrow \mathbf{id}_1, E \rightarrow \mathbf{id}_2, \ldots, E \rightarrow \mathbf{id}_m,$

$E \rightarrow E\ \mathbf{op}_1\ E, E \rightarrow E\ \mathbf{op}_2\ E, \ldots, E \rightarrow E\ \mathbf{op}_n\ E\ \}$

Note: $\mathbf{id}_1, \mathbf{id}_2, \ldots, \mathbf{id}_m$ are identifiers,

$\mathbf{op}_1, \mathbf{op}_2, \ldots\ \mathbf{op}_n$ are different operators

---

1) Precedence of operators:

- If $\mathbf{op}_i$ has **higher precedence** than $\mathbf{op}_j$ then

$$\mathbf{op}_i > \mathbf{op}_j \text{ and } \mathbf{op}_j < \mathbf{op}_i$$

Example: Precedence-table part derived from the precedence of operators in G_{expr2}

$* > +$

$+ < *$

	+	*
+		<
*	>	

Construction of Precedence Table 2/5

2) Associativity:

Note:

- $\mathbf{op}_i$ is left-associative ⇔ $a\ \mathbf{op}_i\ b\ \mathbf{op}_i\ c = (a\ \mathbf{op}_i\ b)\ \mathbf{op}_i\ c$
- $\mathbf{op}_i$ is right-associative ⇔ $a\ \mathbf{op}_i\ b\ \mathbf{op}_i\ c = a\ \mathbf{op}_i\ (b\ \mathbf{op}_i\ c)$

- Let $\mathbf{op}_i$ and $\mathbf{op}_j$ have **equal precedence**
  - If $\mathbf{op}_i$ and $\mathbf{op}_j$ are **left associative** then

    $\mathbf{op}_i > \mathbf{op}_j$ and $\mathbf{op}_j > \mathbf{op}_i$

  - If $\mathbf{op}_i$ and $\mathbf{op}_j$ are **right associative** then

    $\mathbf{op}_i < \mathbf{op}_j$ and $\mathbf{op}_j < \mathbf{op}_i$

Example: Precedence-table part derived from the associativity of operators in G_{expr2}

+ is left-associative
*** is left-associative**

	+	*
+	>	
*		>

Construction of Precedence Table 3/5

3) Identifiers:

- If $a \in T$ may **precede** $\mathbf{id}_i$, then $a < \mathbf{id}_i$
- If $a \in T$ may **follow** $\mathbf{id}_i$, then $\mathbf{id}_i > a$

Example: Precedence-table part for identifiers

$ *i* * (*i* + *i*) * *i*

$, (, +, * may precede *i*

i * (*i* + *i*)* *i* $

*, +,), $ may follow *i*

	+	*	(	)	*i*	$
+					<	
*					<	
(					<	
)						
i	>	>		>		>
$					<	

Construction of Precedence Table 4/5

4) Parentheses:

- A pair of parentheses:
- Let $a \in T - \{), \$\}$. Then,
- Let $a \in T - \{(, \$\}$. Then,
- Let $a \in T$ and a may **precede** (. Then,
- Let $a \in T$ and a may **follow**). Then,

(=)

(< a

a >)

a < (

) > a

Example: Precedence-table part for parentheses.

$(*i* + ((*i* * (*i* + (*i* + *i*)))))

↓ ↓ ↓ ↓

$, (, *, + may precede (

(((((*i* + *i*)+ *i*)* *i*)) + *i*)$

↓ ↓ ↓ ↓

+, *,), $ may follow)

	+	*	(	)	*i*	$
+			<	>		
*			<	>		
(	<	<	<	=	<	
)	>	>		>		>
i				>		
$			<			

Construction of Precedence Table 5/5

5) End Marker \$

- Let $\mathbf{op}_i$ be any operator. Then:

$$\boxed{\$ < \mathbf{op}_i \text{ and } \mathbf{op}_i > \$}$$

Example: Precedence-table part for end-markers.

$\$ < +$
$\$ < *$
$+ > \$$
$* > \$$

	+	*	$
+			>
*			>
$	<	<	

Summary:

	+	*	(	)	i	$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
$	<	<	<		<	

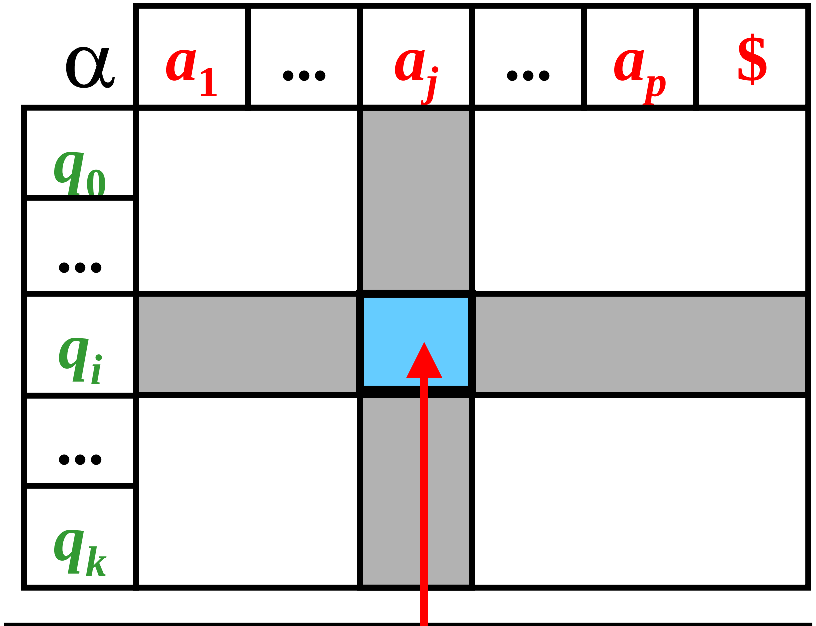
LR-Parser

- Let $G = (N, T, P, S)$ be a CFG, where $N = \{A_1, A_2, \dots , A_n\}$, $T = \{a_1, a_2, \dots , a_m\}$

- LR-parser is a EPDA, M, with states $Q = \{q_0, q_1, \dots, q_k\}$, where q_0 is the start state.
- M is based on LR table that has these two parts
  1) **Action part**
  2) **Go-to part**

Action Part & Go-to Part

Action Part:



α	a_1	…	a_j	…	a_p	$
q_0						
…						
q_i						
…						
q_k						

$\alpha[q_i , a_j]$ = **1** or **2** or **3** or **4**

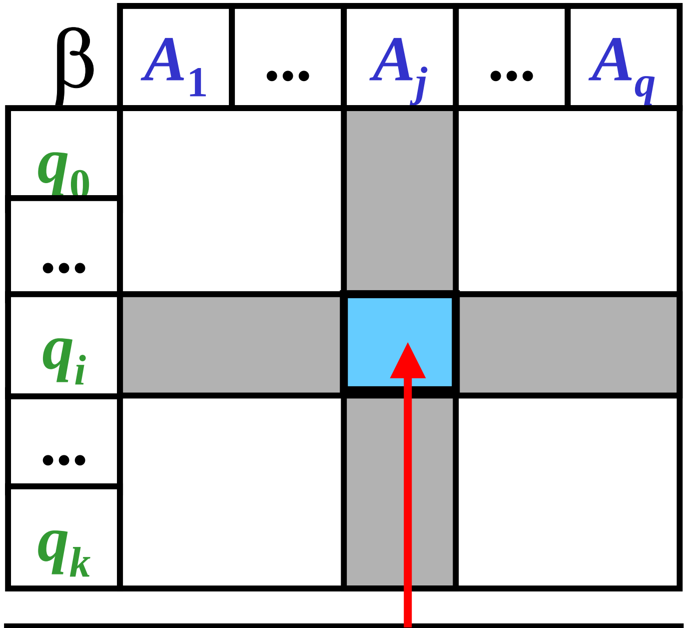
1) **sq**: s = **s**hift, $q \in Q$

2) **rp**: **r** = **r**educe, $p \in P$

3) ☺ **: success**

4) **blank: error**

Go-to Part:



β	A_1	…	A_j	…	A_q
q_0					
…					
q_i					
…					
q_k					

$\beta[q_i , A_j]$ = **1** or **2**

1) **q**: $q \in Q$

2) **blank**

LR-Parser: Algorithm

- **Input:** LR-table for $G=(N, T, P, S)$; $x \in T^*$
- **Output:** Right parse of x if $x \in L(G)$; otherwise, error

---

- **Method:**

```
• push(<$, $q_0$>) onto pushdown; state := $q_0$;
• repeat
    • let a = the current token
    case $\alpha$[state, a] of:
        • sq: push(<a, q>) & read next a from input string &
              state := q;
        • rp: if p: $A \rightarrow X_1X_2\dots X_n \in P$ and
              <?, q><$X_1$,?><$X_2$,?>…<$X_n$,?> is pushdown top
              then state := $\beta$[q, A] &
              replace <$X_1$,?><$X_2$,?>…<$X_n$, ?> with <A, state>
              on the pushdown & write r to output
              else error
        • ☺: success
        • blank: error
until success or error
```

LR-Parser: Example 1/2

$G_{expr1} = (N, T, P, E)$, where $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 1: $E \rightarrow E+T$, 2: $E \rightarrow T$, 3: $T \rightarrow T*F$,

4: $T \rightarrow F$, 5: $F \rightarrow (E)$, 6: $F \rightarrow i$ $\}$

LR-table for G_{expr1}:

α	*i*	+	*	(	)	$
0	s5			s4		
1		s6				☺
2		r2	s7		r2	r2
3		r4	r4		r4	r4
4	s5			s4		
5		r6	r6		r6	r6
6	s5			s4		
7	s5			s4		
8		s6			s11	
9		r1	s7		r1	r1
10		r3	r3		r3	r3
11		r5	r5		r5	r5

Action part for G_{expr1}

Go-to part for G_{expr1}

β	*E*	*T*	*F*
0	1	2	3
1			
2			
3			
4	8	2	3
5			
6		9	3
7			10
8			
9			
10			
11			

LR-Parser: Example 2/2

Rules: 1: *E* → *E*+*T*, 2: *E* → *T*, 3: *T* → *T***F*,
4: *T* → *F*, 5: *F* → (*E*), 6: *F* → *i*

Input string: ***i* * *i* $**

Pushdown	St.	Input	Enter	Rule
<$,0>	0	*i***i*$	α[0, *i*] = s5	
<$,0><*i*,5>	5	**i*$	α[5, *] = r6	6: *F* → *i*
			β[0, *F*] = 3	
<$,0><*F*,3>	3	**i*$	α[3, *] = r4	4: *T* → *F*
			β[0, *T*] = 2	
<$,0><*T*,2>	2	**i*$	α[2, *] = s7	
<$,0><*T*,2><*,7>	7	*i*$	α[2, *i*] = s5	
<$,0><*T*,2><*,7><*i*,5>	5	$	α[5, $] = r6	6: *F* → *i*
			β[7, *F*] = 10	
<$,0><*T*,2><*,7><*F*,10>	10	$	α[10, $]= r3	3: *T* → *T***F*
			β[0, *T*] = 2	
<$,0><*T*,2>	2	$	α[2, $] = r2	2: *E* → *T*
			β[0, *E*] = 1	
<$,0><*E*,1>	1	$	α[1, $] = ☺	

Success

Right parse: 64632

Construction of LR Table: Introduction

- **One parsing algorithm but many algorithms for the construction of LR table.**

Basic algorithms for the construction of LR table:

1) **Simple LR (SLR)**: the least powerful, but simple and few states

2) **Canonical LR**: more powerful, but many states

3) **Lookahead LR (LALR)**: the best because the most powerful and the same number of states as SLR

Extended Grammar with a “Dummy Rule”

Gist: Grammar with special “starting rule”

Definition: Let $G = (N, T, P, S)$ be a CFG, $S' \notin N$.
Extended grammar for G is grammar
$G' = (N \cup \{S'\}, T, P \cup \{S' \rightarrow S\}, S')$.

Why a dummy rule? When $S' \rightarrow S$ is used and the input token is endmarker, then **syntax analysis is successfully completed**.

Example:

$G_{expr1} = (N, T, P, E)$, where $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,
$P = \{$ 1: $E \rightarrow E+T$, 2: $E \rightarrow T$, 3: $T \rightarrow T*F$,
4: $T \rightarrow F$, 5: $F \rightarrow (E)$, 6: $F \rightarrow i$ $\}$

Extended grammar for G_{expr1}:

$G'_{expr1} = (N, T, P, E')$, where $N = \{E', E, F, T\}$, $T = \{i, +, *, (,)\}$,
$P = \{$ 0: $E' \rightarrow E$, 1: $E \rightarrow E+T$, 2: $E \rightarrow T$, 3: $T \rightarrow T*F$,
4: $T \rightarrow F$, 5: $F \rightarrow (E)$, 6: $F \rightarrow i$ $\}$

Construction of LR Table: Items

Gist: Item is a rule of CFG with • in the right side of rule.

Definition: Let $G = (N, T, P, S)$ be a CFG, $\mathbf{A} \rightarrow \mathbf{x} \in P$, $\mathbf{x} = \mathbf{yz}$. Then, $\mathbf{A} \rightarrow \mathbf{y{\bullet}z}$ is an *item*.

Example: Consider $\mathbf{E} \rightarrow \mathbf{E+T}$

All items for $\mathbf{E} \rightarrow \mathbf{E+T}$ are:

$\mathbf{E} \rightarrow \bullet\mathbf{E+T}$, $\mathbf{E} \rightarrow \mathbf{E{\bullet}+T}$, $\mathbf{E} \rightarrow \mathbf{E+{\bullet}T}$, $\mathbf{E} \rightarrow \mathbf{E+T}\bullet$

Meaning: $\mathbf{A} \rightarrow \mathbf{y{\bullet}z}$ means that if $\mathbf{y}$ appears on the pushdown top and a prefix of the input is eventually reduced to $\mathbf{z}$, then $\mathbf{yz}$ (= $\mathbf{x}$) as a handle can be reduced to $\mathbf{A}$ according to $\mathbf{A} \rightarrow \mathbf{x}$.

Closure of Item: Algorithm

Note: Closure(*I*) is the set of items defined by the following algorithm:

- **Input:** $G = (N, T, P, S)$; item I
- **Output:** *Closure*(I)

- **Method:**
- *Closure*(I) := $\{I\}$;
- **Apply the following rule until** *Closure*(I) **cannot be changed:**
- **if** $A \rightarrow y \bullet Bz \in Closure(I)$ **and** $B \rightarrow x \in P$ **then** add $B \rightarrow \bullet x$ to *Closure*(I)

Closure of Item: Example 1/2

$G'_{expr1} = (N, T, P, E')$, where $N = \{E', E, F, T\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ $0: E' \to E$, $1: E \to E+T$, $2: E \to T$, $3: T \to T*F$,
$4: T \to F$, $5: F \to (E)$, $6: F \to i$ $\}$

Task: *Closure*(I) for $I = E' \to \bullet E$

Closure(I) := $\{E' \to \bullet E\}$

---

1) $E' \to \bullet E \in$ *Closure*(I) **and** $E \to E+T \in P$**:**
add $E \to \bullet E+T$ **to** *Closure*(I)

Closure(I) **=** $\{E' \to \bullet E, E \to \bullet E+T\}$

---

2) $E' \to \bullet E \in$ *Closure*(I) **and** $E \to T \in P$**:**
add $E \to \bullet T$ **to** *Closure*(I)

Closure(I) **=** $\{E' \to \bullet E, E \to \bullet E+T, E \to \bullet T\}$

---

3) $E \to \bullet T \in$ *Closure*(I) **and** $T \to T*F \in P$**:**
add $T \to \bullet T*F$ **to** *Closure*(I)

Closure(I) **=** $\{E' \to \bullet E, E \to \bullet E+T, E \to \bullet T, T \to \bullet T*F\}$

Closure of Item: Example 2/2

$G'_{expr1} = (N, T, P, E')$, where $N = \{E', E, F, T\}$, $T = \{i, +, *, (,)\}$,
$P = \{$ 0: $E' \rightarrow E$, 1: $E \rightarrow E+T$, 2: $E \rightarrow T$, 3: $T \rightarrow T*F$,
4: $T \rightarrow F$, 5: $F \rightarrow (E)$, 6: $F \rightarrow i$ $\}$

---

4) $E \rightarrow \bullet T \in Closure(I)$ **and** $T \rightarrow F \in P$**:**
add $T \rightarrow \bullet F$ **to** $Closure(I)$

$Closure(I) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T*F,$
$T \rightarrow \bullet F\}$

---

5) $T \rightarrow \bullet F \in Closure(I)$ **and** $F \rightarrow (E) \in P$**:**
add $F \rightarrow \bullet (E)$ **to** $Closure(I)$

$Closure(I) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T*F,$
$T \rightarrow \bullet F, F \rightarrow \bullet (E)\}$

---

6) $T \rightarrow \bullet F \in Closure(I)$ **and** $F \rightarrow i \in P$**:**
add $F \rightarrow \bullet i$ **to** $Closure(I)$

Summary:
$Closure(I) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T*F,$
$T \rightarrow \bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i\}$

Set Θ_G for Grammar G 1/2

Gist: Θ_G is the set of all prefixes of the right-hand sides of rules from G.

Definition: Let $G = (N, T, P, S)$ be CFG.

$$\Theta_G = \{\langle y\rangle : A \rightarrow y\bullet z \text{ is an item in } G\}$$

Example:

$G'_{expr1} = (N, T, P, E')$, where $N = \{E', E, F, T\}$, $T = \{i, +, *, (,)\}$,
$P = \{$ 0: $E' \rightarrow E$, 1: $E \rightarrow E+T$, 2: $E \rightarrow T$, 3: $T \rightarrow T*F$, 4: $T \rightarrow F$, 5: $F \rightarrow (E)$, 6: $F \rightarrow i$ $\}$

Task: $\Theta_{G'expr1}$

---

1) Members of $\Theta_{G'expr1}$ of length 0: $\langle\varepsilon\rangle \in \Theta_{G'expr1}$

---

2) Members of $\Theta_{G'expr1}$ of length 1:

$$\underbrace{E' \rightarrow \underline{E},\ E \rightarrow \underline{E}+T}_{\langle E\rangle \in \Theta_{G'expr1}},\ \underbrace{E \rightarrow \underline{T},\ T \rightarrow \underline{T}*F}_{\langle T\rangle \in \Theta_{G'expr1}},\ \underbrace{T \rightarrow \underline{F},\ F \rightarrow \underline{(}E),\ F \rightarrow \underline{i}}_{\langle F\rangle,\ \langle(\rangle,\ \langle i\rangle \in \Theta_{G'expr1}}$$

Set Θ_G for Grammar G 2/2

$G'_{expr1} = (N, T, P, E')$, where $N = \{E', E, F, T\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 0: $E' \rightarrow E$, 1: $E \rightarrow E+T$, 2: $E \rightarrow T$, 3: $T \rightarrow T*F$,
4: $T \rightarrow F$, 5: $F \rightarrow (E)$, 6: $F \rightarrow i$ $\}$

---

3) Members of $\Theta_{G'expr1}$ of length 2:

$$E' \rightarrow E,\ \underbrace{E \rightarrow \underline{E+}T}_{\langle E+\rangle \in \Theta_{G'expr1}},\ E \rightarrow \underline{T},\ \underbrace{T \rightarrow \underline{T*}F}_{\langle T*\rangle \in \Theta_{G'expr1}},\ T \rightarrow F,\ \underbrace{F \rightarrow \underline{(E})}_{\langle (E\rangle \in \Theta_{G'expr1}},\ F \rightarrow i$$

---

4) Members of $\Theta_{G'expr1}$ of length 3:

$$E' \rightarrow E,\ \underbrace{E \rightarrow \underline{E+T}}_{\langle E+T\rangle \in \Theta_{G'expr1}},\ E \rightarrow \underline{T},\ \underbrace{T \rightarrow \underline{T*F}}_{\langle T*F\rangle \in \Theta_{G'expr1}},\ T \rightarrow F,\ \underbrace{F \rightarrow \underline{(E)}}_{\langle (E)\rangle \in \Theta_{G'expr1}},\ F \rightarrow i$$

Summary:

$$\Theta_{G'expr1} = \{ \langle\varepsilon\rangle, \langle E\rangle, \langle T\rangle, \langle F\rangle, \langle (\rangle, \langle i\rangle, \langle E+\rangle, \langle T*\rangle, \langle (E\rangle, \langle E+T\rangle, \langle T*F\rangle, \langle (E)\rangle \}$$

Contents(x): Algorithm

Note: For all $x \in \Theta_G$,*Contents*(x) is the set of items defined by the following algorithm:

- **Input:** Extended $G = (N, T, P, S')$; Θ_G
- **Output:** *Contents*(x) for all $x \in \Theta_G$
- **Method:**
- *Contents*($<\varepsilon>$) := *Closure*($S' \to \bullet S$);
- for each $x \in \Theta_G - \{<\varepsilon>\}$: *Contents*($x$) := $\emptyset$
- **Apply the following rule until no *Contents* set can be changed:**

  if $A \to y\bullet Xz \in$ *Contents*($<x>$), where $X \in N \cup T$
  and $<xX> \in \Theta_G$ **then**
  add *Closure*($A \to yX\bullet z$) to *Contents*($<xX>$)

Contents(*x*): Example 1/9

$G'_{expr1} = (N, T, P, E')$, where $N = \{E', E, F, T\}$, $T = \{i, +, *, (,)\}$,

$P = \{$ 0: $E' \to E$, 1: $E \to E+T$, 2: $E \to T$, 3: $T \to T*F$,
4: $T \to F$, 5: $F \to (E)$, 6: $F \to i$ $\}$

$\Theta_{G'expr1} = \{ \langle\varepsilon\rangle, \langle E\rangle, \langle T\rangle, \langle F\rangle, \langle (\rangle, \langle i\rangle, \langle E+\rangle,$
$\langle T*\rangle, \langle (E\rangle, \langle E+T\rangle, \langle T*F\rangle, \langle (E)\rangle \}$

0) *Contents*(<ε>) := *Closure*($E' \to \bullet E$) =

$\{E' \to \bullet E \checkmark, E \to \bullet E+T \checkmark, E \to \bullet T \checkmark, T \to \bullet T*F, T \to \bullet F, F \to \bullet (E), F \to \bullet i \}$

$E' \to \bullet E \in$ *Contents*(<ε>) & $\langle \varepsilon E\rangle = \langle E\rangle \in \Theta_{G'expr1}$:
add *Closure*($E' \to E\bullet$) = $\{E' \to E\bullet\}$ to *Contents*(<E>)

$E \to \bullet E+T \in$ *Contents*(<ε>) & $\langle \varepsilon E\rangle = \langle E\rangle \in \Theta_{G'expr1}$:
add *Closure*($E \to E\bullet+T$) = $\{E \to E\bullet+T\}$ to *Contents*(<E>)

$E \to \bullet T \in$ *Contents*(<ε>) & $\langle \varepsilon T\rangle = \langle T\rangle \in \Theta_{G'expr1}$:
add *Closure*($E \to T\bullet$) = $\{E \to T\bullet\}$ to *Contents*(<T>)

Contents(x): Example 2/9

⋮

Contents($\langle\varepsilon\rangle$) =
$\{E' \checkmark\to \bullet E,\ E \checkmark\to \bullet E+T,\ E \checkmark\to \bullet T,\ T \checkmark\to \bullet T{*}F,\ T \checkmark\to \bullet F,\ F \checkmark\to \bullet(E),\ F \checkmark\to \bullet i\ \}$

$T \to \bullet T{*}F \in$ Contents($\langle\varepsilon\rangle$) & $\langle\varepsilon T\rangle = \langle T\rangle \in \Theta_{G'expr1}$:
add *Closure*($T \to T\bullet{*}F$) = {$T \to T\bullet{*}F$} to *Contents*($\langle T\rangle$)

$T \to \bullet F \in$ Contents($\langle\varepsilon\rangle$) & $\langle\varepsilon F\rangle = \langle F\rangle \in \Theta_{G'expr1}$:
add *Closure*($T \to F\bullet$) = {$T \to F\bullet$} to *Contents*($\langle F\rangle$)

$F \to \bullet(E) \in$ Contents($\langle\varepsilon\rangle$) & $\langle\varepsilon(\rangle = \langle(\rangle \in \Theta_{G'expr1}$:
**add *Closure*($F \to (\bullet E)$) = {$F \to (\bullet E),\ E \to \bullet E+T,\ E \to \bullet T,$
$T \to \bullet T{*}F,\ T \to \bullet F,\ F \to \bullet(E),\ F \to \bullet i$ } to *Contents*($\langle(\rangle$)**

$F \to \bullet i \in$ Contents($\langle\varepsilon\rangle$) & $\langle\varepsilon i\rangle = \langle i\rangle \in \Theta_{G'expr1}$:
add *Closure*($F \to i\bullet$) = {$F \to i\bullet$} to *Contents*($\langle i\rangle$)

Contents(*x*): Example 3/9

✓ *Contents*(<ε>) = $\{E' \to \bullet E,\ E \to \bullet E+T,\ E \to \bullet T,\ T \to \bullet T*F,\ T \to \bullet F,\ F \to \bullet (E),\ F \to \bullet i\ \}$

Contents(<E>) = $\{E' \to E\bullet,\ E \to E\bullet+T\}$

Contents(<T>) = $\{E \to T\bullet,\ T \to T\bullet*F\}$

Contents(<F>) = $\{T \to F\bullet\}$

Contents(< $($>) = $\{F \to (\bullet E),\ E \to \bullet E+T,\ E \to \bullet T,\ T \to \bullet T*F,\ T \to \bullet F,\ F \to \bullet (E),\ F \to \bullet i\ \}$

Contents(<i>) = $\{F \to i\bullet\}$

Contents(<$E+$>) = $\emptyset$

Contents(<$T*$ >) = $\emptyset$

Contents(< $(E$>) = $\emptyset$

Contents(<$E+T$>) = $\emptyset$

Contents(<$T*F$>) = $\emptyset$

Contents(< (E) >) = $\emptyset$

Contents(*x*): Example 4/9

1) *Contents*(<*E*>) = {✓*E’* → *E*•, ✓*E* → *E*•+*T*}:

E → ***E*•** : *nothing*

E → ***E*•+*T*** ∈ *Contents*(<*E*>) & <*E*+> ∈ $\Theta_{G'expr1}$:
add *Closure*(*E* → *E*+•*T*) = {*E* → *E*+•*T*, *T* → •*T*F*, *T* → •*F*,**
F **→ •(*E*), *F* → •*i* } to *Contents*(<*E*+>)**

---

2) *Contents*(<*T*>) = {✓*E* → *T*•, ✓*T* → *T*•**F*}:

E → ***T*•** : *nothing*

T → ***T*•**F*** ∈ *Contents*(<*T*>) & <*T**> ∈ $\Theta_{G'expr1}$:
add *Closure*(*T* → *T•*F*) = {*T* → *T**•*F*, *F* → •(*E*), *F* → •*i* }**
to *Contents*(<*T>)**

---

3) *Contents*(<*F* >) = {✓*T* → *F*•}:

T → ***F*•** : *nothing*

Contents(*x*): Example 5/9

4) *Contents*(< (>) =

$\{\checkmark F \to (\bullet E), \checkmark E \to \bullet E+T, \checkmark E \to \bullet T, \checkmark T \to \bullet T*F, \checkmark T \to \bullet F, \checkmark F \to \bullet(E), \checkmark F \to \bullet i\,\}$

$F \to (\bullet E) \in$ *Contents*(< (>) & < (E > $\in \Theta_{G'expr1}$:
add ***Closure***$(F \to (E\bullet)) = \{F \to (E\bullet)\}$ **to** ***Contents***(< (E >)

$F \to \bullet E+T \in$ *Contents*(< (>) & < (E > $\in \Theta_{G'expr1}$:
add ***Closure***$(F \to E\bullet+T) = \{F \to E\bullet+T\}$ **to** ***Contents***(< (E >)

$E \to \bullet T \in$ *Contents*(< (>) **but** < (T > $\notin \Theta_{G'expr1}$: *nothing*
$T \to \bullet T*F \in$ *Contents*(< (>) **but** < (T > $\notin \Theta_{G'expr1}$: *nothing*
$T \to \bullet F \in$ *Contents*(< (>) **but** < (F > $\notin \Theta_{G'expr1}$: *nothing*
$F \to \bullet(E) \in$ *Contents*(< (>) **but** < ((> $\notin \Theta_{G'expr1}$: *nothing*
$T \to \bullet i \in$ *Contents*(< (>) **but** < (i > $\notin \Theta_{G'expr1}$: *nothing*

5) *Contents*(<i>) = $\{\checkmark F \to \bullet i\,\}$:

$F \to i\bullet$: *nothing*

Contents(*x*): Example 6/9

✓ *Contents*(<ε>) = $\{E' \to \bullet E,\ E \to \bullet E+T,\ E \to \bullet T,\ T \to \bullet T*F,\ T \to \bullet F,\ F \to \bullet(E),\ F \to \bullet i\ \}$

✓ *Contents*(<E>) = $\{E' \to E\bullet,\ E \to E\bullet+T\}$

✓ *Contents*(<T>) = $\{E \to T\bullet,\ T \to T\bullet*F\}$

✓ *Contents*(<F>) = $\{T \to F\bullet\}$

✓ *Contents*(< (>) = $\{F \to (\bullet E),\ E \to \bullet E+T,\ E \to \bullet T,\ T \to \bullet T*F,\ T \to \bullet F,\ F \to \bullet(E),\ F \to \bullet i\ \}$

✓ *Contents*(<i>) = $\{F \to i\bullet\}$

Contents(<$E+$>) = $\{E \to E+\bullet T,\ T \to \bullet T*F,\ T \to \bullet F,\ F \to \bullet(E),\ F \to \bullet i\ \}$

Contents(<$T*$ >) = $\{T \to T*\bullet F,\ F \to \bullet(E),\ F \to \bullet i\ \}$

Contents(< $(E$ >) = $\{F \to (E\bullet),\ E \to E\bullet+T\ \}$

Contents(<$E+T$>) = $\emptyset$

Contents(<$T*F$>) = $\emptyset$

Contents(< (E) >) = $\emptyset$

Contents(x): Example 7/9

6) *Contents*($<E+>$) =

$\{E \checkmark\rightarrow E+\bullet T,\ T \checkmark\rightarrow \bullet T*F,\ T \checkmark\rightarrow \bullet F,\ F \checkmark\rightarrow \bullet(E),\ F \checkmark\rightarrow \bullet i\ \}$

$E \rightarrow E+\bullet T \in$ *Contents*($<E+>$) & $<E+T> \in \Theta_{G'expr1}$:

add *Closure*($E \rightarrow E+T\bullet$) = $\{E \rightarrow E+T\bullet\}$ to *Contents*($<E+T>$)

$T \rightarrow \bullet T*F \in$ *Contents*($<E+>$) & $<E+T> \in \Theta_{G'expr1}$:

add *Closure*($T \rightarrow T\bullet *F$) = $\{T \rightarrow T\bullet *F\}$ to *Contents*($<E+T>$)

$T \rightarrow \bullet F \in$ *Contents*($<E+>$) **but** $<E+F> \notin \Theta_{G'expr1}$: *nothing*

$F \rightarrow \bullet(E) \in$ *Contents*($<E+>$) **but** $<E+(> \notin \Theta_{G'expr1}$: *nothing*

$T \rightarrow \bullet i \in$ *Contents*($<E+>$) **but** $<E+i> \notin \Theta_{G'expr1}$: *nothing*

7) *Contents*($<T*>$) = $\{T \checkmark\rightarrow T*\bullet F,\ F \checkmark\rightarrow \bullet(E),\ F \checkmark\rightarrow \bullet i\ \}$

$T \rightarrow T*\bullet F \in$ *Contents*($<T*>$) & $<T*F> \in \Theta_{G'expr1}$:

add *Closure*($T \rightarrow T*F\bullet$) = $\{T \rightarrow T*F\bullet\}$ to *Contents*($<T*F>$)

$F \rightarrow \bullet(E) \in$ *Contents*($<T*>$) **but** $<T*(> \notin \Theta_{G'expr1}$: *nothing*

$T \rightarrow \bullet i \in$ *Contents*($<T*>$) **but** $<T*i> \notin \Theta_{G'expr1}$: *nothing*

Contents(x): Example 8/9

8) Contents(<(E>) = {F ✓→ (E•), E ✓→ E•+T }

F → (E•) ∈ Contents(<(E>) & <(E)> ∈ $\Theta_{G'expr1}$:
add Closure(E → (E)•) = {F → (E)•} to Contents(<(E)>)

E → E•+T ∈ Contents(<(E>) **but** <(E+ > ∉ $\Theta_{G'expr1}$: nothing

---

9) Contents(<E+T>) = {E ✓→ E+T•, T ✓→ T•*F }

E → E+T• : nothing

T → T•*F ∈ Contents(<E+T>) **but** <E+T*> ∉ $\Theta_{G'expr1}$: nothing

---

10) Contents(<E+T>) = {T ✓→ T*F• }

T → T*F• : nothing

---

11) Contents(<(E)>) = {F ✓→ (E)• }

F → (E)• : nothing

Contents(*x*): Example 9/9

✓ Contents($<\varepsilon>$) = $\{E' \to \bullet E,\ E \to \bullet E+T,\ E \to \bullet T,\ T \to \bullet T*F,\ T \to \bullet F,\ F \to \bullet (E),\ F \to \bullet i\ \}$

✓ Contents($<E>$) = $\{E' \to E\bullet,\ E \to E\bullet +T\}$

✓ Contents($<T>$) = $\{E \to T\bullet,\ T \to T\bullet *F\}$

✓ Contents($<F>$) = $\{T \to F\bullet\}$

✓ Contents($<\ (>$) = $\{F \to (\bullet E),\ E \to \bullet E+T,\ E \to \bullet T,\ T \to \bullet T*F,\ T \to \bullet F,\ F \to \bullet (E),\ F \to \bullet i\ \}$

✓ Contents($<i>$) = $\{F \to i\bullet\}$

✓ Contents($<E+>$) = $\{E \to E+\bullet T,\ T \to \bullet T*F,\ T \to \bullet F,\ F \to \bullet (E),\ F \to \bullet i\ \}$

✓ Contents($<T*\ >$) = $\{T \to T*\bullet F,\ F \to \bullet (E),\ F \to \bullet i\ \}$

✓ Contents($<\ (E>$) = $\{F \to (E\bullet),\ E \to E\bullet +T\ \}$

✓ Contents($<E+T>$) = $\{E \to E+T\bullet,\ T \to T\bullet *F\ \}$

✓ Contents($<T*F>$) = $\{T \to T*F\bullet\ \}$

✓ Contents($<\ (E)\ >$) = $\{F \to (E)\bullet\ \}$

Construction of LR-table: Algorithm

- **Input:** Extended $G = (N, T, P, S')$; Θ_G;
  Contents(x) for all $x \in \Theta_G$; *Follow*(A) for all $x \in A$
- **Output:** LR-table for G (α = Action part, β = Go-to part)

- **Method:**

```
• $StatesOfTable := \Theta_G$; $StartingState := <\varepsilon>$
• for each $<x> \in \Theta_G$ do
• for each $I \in Contents(<x>)$ do
    • case $I$ of
        • $I = A \rightarrow y \bullet X z$, where $X \in N$:
          if $A \rightarrow yX \bullet z \in Contents(<q>)$ then $\beta[<x>, X] := <q>$
        • $I = A \rightarrow y \bullet X z$, where $X \in T$:
          if $A \rightarrow yX \bullet z \in Contents(<q>)$ then $\alpha[<x>, X] := s<q>$
        • $I = S' \rightarrow S \bullet$: $\alpha[<x>, \$] :=$ ☺
        • $I = A \rightarrow y \bullet$ $(A \neq S')$:
          for each $a \in Follow(A)$ do $\alpha[<x>, a] := rp$,
          where $p$ is a label of rule $A \rightarrow y$
```

Construction of LR-table: Example 1/5

Task: LR-table for G_{expr1}

	α						β		
	i	+	*	(	)	$	*E*	*T*	*F*
<ε>							<*E*>	<*T*>	<*F*>
<*E*>									
<*T*>									
<*F*>									
<(>									
<*i*>									
<*E*+>									
<*T**>									
<(*E*>									
<*E*+*T*>									
<*T***F*>									
<(*E*)>									

Contents(<ε>):

$I = E' \to \bullet E \in Contents(<\varepsilon>)$:

$E' \to E\bullet \in Contents(<E>)$: $\beta[<\varepsilon>, E] := <E>$

$I = E \to \bullet E+T \in Contents(<\varepsilon>)$:

$E \to E\bullet+T \in Contents(<E>)$: $\beta[<\varepsilon>, E] := <E>$

$I = E \to \bullet T \in Contents(<\varepsilon>)$:

$E \to T\bullet \in Contents(<T>)$: $\beta[<\varepsilon>, T] := <T>$

$I = E \to \bullet T*F \in Contents(<\varepsilon>)$:

$E \to T\bullet*F \in Contents(<T>)$: $\beta[<\varepsilon>, T] := <T>$

$I = E \to \bullet F \in Contents(<\varepsilon>)$:

$E \to F\bullet \in Contents(<F>)$: $\beta[<\varepsilon>, F] := <F>$

Construction of LR-table: Example 2/5

Task: LR-table for G_{expr1}

	α						β		
	i	**+**	*****	**(**	**)**	**$**	***E***	***T***	***F***
<ε>	s<*i*>			s<(>			<*E*>	<*T*>	<*F*>
<*E*>									
<*T*>									
<*F*>									
<(>									
<*i*>									
<*E*+>									
<*T**>									
<(*E*>									
<*E*+*T*>									
<*T***F*>									
<(*E*)>									

Contents(<ε>):

$I = F \to \bullet(E) \in Contents(<\varepsilon>)$:
$F \to (\bullet E) \in Contents(<(>)$: $\alpha[<\varepsilon>, (] := s<(>$

$I = F \to \bullet i \in Contents(<\varepsilon>)$:
$F \to i\bullet \in Contents(<i>)$: $\alpha[<\varepsilon>, E] := s<i>$

Construction of LR-table: Example 3/5

Task: LR-table for G_{expr1}

	α						β		
	i	+	*	(	)	$	*E*	*T*	*F*
<ε>	s<*i*>			s<(>			<*E*>	<*T*>	<*F*>
<*E*>		s<*E*+>				☺			
<*T*>									
<*F*>									
<(>									
<*i*>									
<*E*+>									
<*T**>									
<(*E*>									
<*E*+*T*>									
<*T***F*>									
<(*E*)>									

Contents(<*E*>):

I = *E’* → *E*• ∈ *Contents*(<*E*>): α[<*E*>, $] := ☺

I = *E* → *E*•+*T* ∈ *Contents*(<*E*>):
E → *E*+•*T* ∈ *Contents*(<*E*+>): α[<*E*>, +] = s<*E*+>

Construction of LR-table: Example 4/5

Task: LR-table for G_{expr1}

	α						β		
	i	+	*	(	)	$	*E*	*T*	*F*
<ε>	s<*i*>			s<(>			<*E*>	<*T*>	<*F*>
<*E*>		s<*E*+>				☺			
<*T*>		r2	s<*T**>		r2	r2			
<*F*>									
<(>									
<*i*>									
<*E*+>									
<*T**>									
<(*E*>									
<*E*+*T*>									
<*T***F*>									
<(*E*)>									

Contents(<*T*>):

$I = \boldsymbol{E} \to \boldsymbol{T}\bullet \in$ *Contents*(<*T*>): *Follow*(***E***) = {+,), $}

α[<*T*>, +] = α[<*T*>,)] = α[<*T*>, $] := r2

Note: $\boldsymbol{E} \to \boldsymbol{T}$ is rule with label 2

$I = \boldsymbol{T} \to \boldsymbol{T}\bullet{*}\boldsymbol{F} \in$ *Contents*(<*T*>):

$\boldsymbol{T} \to \boldsymbol{T}{*}\bullet\boldsymbol{F} \in$ *Contents*(<*T**>): α[<*E*>, +] = s<*T**>

Construct the rest analogically.

Construction of LR-table: Example 5/5

Final LR-table for G_{expr1}

	α						β		
	i	+	*	(	)	$	*E*	*T*	*F*
<ε>	s<*i*>			s<(>			<*E*>	<*T*>	<*F*>
<*E*>		s<*E*+>				☺			
<*T*>		r2	s<*T**>		r2	r2			
<*F*>		r4	r4		r4	r4			
<(>	s<*i*>			s<(>			<(*E*>	<*T*>	<*F*>
<*i*>		r6	r6		r6	r6			
<*E*+>	s<*i*>			s<(>				<*E*+*T*>	<*F*>
<*T**>	s<*i*>			s<(>					<*T***F*>
<(*E*>		s<*E*+>			s<(*E*)>				
<*E*+*T*>		r1	s<*T**>		r1	r1			
<*T***F*>		r3	r3		r3	r3			
<(*E*)>		r5	r5		r5	r5			

Renaming the states

Rename the states:

Old	New
<ε>	0
<E>	1
<T>	2
<F>	3
<(>	4
<i>	5
<E+>	6
<T*>	7
<(E>	8
<E+T>	9
<T*F>	10
<(E)>	11

LR table for G_{expr1} with the renamed states:

α	*i*	+	*	(	)	$
0	s5			s4		
1		s6				☺
2		r2	s7		r2	r2
3		r4	r4		r4	r4
4	s5			s4		
5		r6	r6		r6	r6
6	s5			s4		
7	s5			s4		
8		s6			s11	
9		r1	s7		r1	r1
10		r3	r3		r3	r3
11		r5	r5		r5	r5

β	*E*	*T*	*F*
0	1	2	3
1			
2			
3			
4	8	2	3
5			
6		9	3
7			10
8			
9			
10			
11			