Formal Analysis and Verification
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Some introductory notes

❖ These slides are intended for the course of Formal Analysis and Verification at FIT BUT.

❖ Students are assumed to have a basic knowledge of (and, perhaps even more importantly, to like) the theory of automata, graphs, logics, algebra, and modelling.

❖ Students are expected to themselves actively search and study further information available in recommended textbooks and on the Internet.

❖ Students are also strongly advised to experiment with available tools for formal analysis and verification in order to get a deeper understanding of the capabilities and limits of the presented techniques.

❖ Note that many of the presented subjects are relatively new, still under a very active research, they are rapidly developing, and also the terminology is not always completely uniform and stable.
References, Inspiration

❖ Textbooks, overview articles include:


❖ Related courses include:

Basic Terminology
Analysis and Verification

Verification is a process of checking whether a given system (a real system or a model) satisfies a given correctness specification (property).

- For example, given a concurrent program, we ask whether it is deadlock-free.
- A verification method provides (ideally) yes/no answers having the sense that the system is/is not correct wrt a given specification.
- The yes/no answer may be complemented by some diagnostic information (e.g., when a deadlock is possible, a run leading to some deadlock situation is provided).
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- **Analysis** finds answers to **more general questions** about a given system—either questions which are not Boolean, or even if they are, they do not necessarily directly speak about correctness of the system.
  - For example, given a concurrent system, we ask whether there is some ordering over the shared resources in which they are always locked.
  - Answers provided by analysis may be an **input for a further reasoning** leading to the verification of a system. However, they can also be used for completely **different purposes**, e.g., for optimisation, synthesis or code generation, etc.
Formal Methods

- Verification and analysis methods include:
  - **bug hunting**: simulation, testing and dynamic analysis, some forms of static analysis, bounded model checking, ...
  - **formal analysis and verification**: model checking, some forms of static analysis, theorem proving, ...

- Formal methods are naturally based on **formal, mathematical roots**—*this does not imply that they are necessarily manual and not usable by ordinary users* (!).

- Unlike other approaches, **formal verification** is (at least potentially) capable of proving correctness of a given system wrt a given specification—and not just disprove the property based on some observed behaviour.

- Similarly, **formal analysis** is capable of providing answers **universally (conservatively)** covering all possible behaviours of a given system (not just giving existential answers—though perhaps extrapolated—based on some observed behaviours).
The Ideal of Formal Verification

- **Full automation**: no human help needed.
- **Soundness**: if a verification method claims that a system is correct wrt a given specification, it is indeed correct.
- **Completeness**: if a verification method announces that a system is not correct, there is indeed an error in the system—i.e., no false alarms (false positives).
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- **Hard to achieve in general**:
  - the state explosion problem—i.e., an exponential growth of the number of reachable states wrt the source description of a finite-state system:
    - `int n`; can have $2^{32}$ (or $2^{64}$, ...) possible values,
    - $m$ such variables: $2^{m \cdot 32}$ (or $2^{m \cdot 64}$, ...) possible values,
    - $n$ concurrent processes each with $m$ states can generate $m^n$ states,
  - undecidability: it suffices to have two unbounded integer variables, operations `++` and `--`, and branching according to equality with zero.
Relaxing the Ideal

- A verification method needs not guarantee termination and/or can produce false alarms.

- Alternatively, a method is allowed to stop with a “don’t know” answer and/or becomes not fully automated (i.e., some human help is required).

- Sometimes, even soundness is sacrificed to efficiency leading to an error detection method with formal verification roots.

- Note that even if full formal verification of a system fails, it may still be useful as it can find some errors in the mean time.

- The errors found by formal methods can differ from those found by other methods due to other principles on which the methods work.
  - Hence, it is often a good idea to use as many different approaches as possible.
Systems and Properties
To Be Checked
Systems To Be Checked

- Systems to be verified/analysed can be classified in many different ways:
  - Dealing with **real systems** (programs in common programming languages, hardware described in VHDL, Verilog, ...) or **models** (based on process algebras, Petri nets, specialised modelling languages like SMV or Promela, ...).
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  - **Sequential and concurrent systems**—in the latter case, asynchronous or synchronous, with interleaving or true concurrency, ...
  - **Control-intensive and data-intensive systems**.
  - **Transformational and reactive systems**:
    - **Transformational systems** are designed to transform a certain input to a certain output (compilers, database queries, ...).
    - **Reactive systems** typically transform an infinite sequence of inputs to an infinite sequence of outputs (control systems, operating systems and their sub-systems, ...).
  - ...

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Properties To Be Checked

- Properties to be verified/analysed can again be classified in various ways:
  - The specification language:
    - the property may be fixed for a given method/tool,
    - a choice out of a list,
    - various kinds of labels/inscriptions (assertions, end-state labels, progress labels, invariants, ...),
    - automata (FSA, Büchi automata, ...), temporal logic formulae (LTL, CTL, CTL*, μ-calculus, ...),
    - various specialised textual or graphical specification formalisms (PSL/Sugar),
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  - logical vs. physical,
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- The underlying **notion of time**:
  - logical vs. physical,
  - linear vs. branching.

- The form of **counterexamples** (finite or infinite):
  - safety vs. liveness (or their mixture).
Notions of Time

- Logical time imposes a (partial) ordering on states/events in the behaviour of a system.
- Physical time allows one to measure how much time passed between two states/events.
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- **Physical time** allows one to measure how much time passed between two states/events.

- **Linear time** allows one to speak about particular linear traces of the state space of a given system only.
  - In all traces, \( b \) must happen immediately followed by \( c \). (1)
  - In all traces, \( b \) must happen immediately followed by \( c \) or \( d \). (2)

- **Branching time** allows one to quantify (existentially and universally) over the possible futures of a given state. We view the state space unfolded into an infinite tree.
  - \( b \) must happen, and immediately after, \( c \) may happen and \( d \) may happen. (3)

\[
\begin{array}{ccc}
(a) & \text{a} & (b) \\
& \text{b} & \\
& \text{c} & \text{d} \\
\end{array}
\]

\[
\begin{array}{ccc}
(a) & (b) \\
(1) & \text{No} & \text{No} \\
(2) & \text{Yes} & \text{Yes} \\
(3) & \text{Yes} & \text{No} \\
\end{array}
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Linear Time Safety and Liveness

- **Safety** properties require that something bad never happens.
  - Their violation always has a **finite witness**. In other words, if there is a counterexample, there is a finite counterexample.
  - **Examples**:
    - *Processes mutually exclude each other when accessing a shared variable.*
    - *A program never returns a wrong result.* (partial correctness)
    - *No null pointer exception can arise.*
    - *A signal $s_1$ never arrives before a signal $s_2$.***
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- **Liveness** properties require that something good eventually happens.
  - Their violation can only have an **infinite witness**, or a **finite, but complete witness**, i.e., a witness that cannot be extended any more.
  - **Examples:**
    - A program terminates for any input. (total correctness)
    - A process can never starve.
    - After a signal $s_1$ is received, a signal $s_2$ is eventually sent.
  - Liveness is typically **harder to verify** than safety.

- **General properties** can mix (conjoin) safety and liveness.
The situation is more complicated: a computation tree can be infinite and incomplete at the same time!

One can distinguish [Manolios, Trefler – LICS 2001]:

- **Universally safe properties**: linear time safety over all computations.
- **Existentially safe properties**: guarantee at least one safe computation.
- **Universally live properties**: linear time liveness properties over all computations.
- **Existentially live properties**: linear time liveness for at least one computation.

General properties are intersections of some of the above kinds of properties.
In order to exclude practically infeasible, trivial counterexamples to liveness properties, one usually has to apply some fairness assumptions.

- For example, assume that we have two concurrent, infinitely looping processes, not requesting anything from their environment.

- We ask whether each of them can always eventually do a step.
- We somehow naturally expect the answer to be yes, but this is based on that we implicitly assume that the processes are scheduled by some fair scheduler.
- However, if we do not build the scheduler into the system, the answer will be no.
- Alternatively, we can perform the verification under a suitable fairness assumption—either expressed as a part of the property being checked or hard-wired into the verification algorithm.
**Fairness**

- Fairness limits sources of non-determinism in a system (e.g., in the scheduling or in reading random data from the environment).

- **Two most common kinds of fairness:**
  - **Weak fairness:**
    - an action that is eventually always enabled must always eventually be taken,
    - enough for asynchronous processes.
  - **Strong fairness:**
    - an action that is always eventually enabled must always eventually be taken,
    - needed when dealing with fair resource allocation,
    - more complicated to handle.
Different Approaches to Formal Verification and Analysis
Model Checking

[Clarke, Emerson 81], [Quielle, Sifakis 81]

❖ An algorithmic approach of checking whether a given system satisfies a given property through a systematic exploration of the state space of the system.

- The system may be a real system or a model.
- Usually, finite-state systems are considered—dealing with infinite-state systems is relatively recent and much less standard.
- Properties classically specified using temporal logics (LTL, CTL, CTL*, ...); other forms of specification are also possible.
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- **Advantages** include:
  - a (relatively) high degree of automation (fully automated up to the need of modelling the system, its parts, and/or its environment),
  - (relatively) easy to use,
  - quite general (as for the systems and their properties that can be checked),
  - provides counterexamples.
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❖ Disadvantages:

• the state explosion problem (and dealing with infinite-state spaces),
• a need to model the environment of the checked (sub-)system.
Model Checking

- Dealing with the state explosion problem—just a brief overview, we will get back to (at least some of) the mentioned approaches later on:
  - Efficient storage of state spaces (hierarchical storage of states, BDDs, ...).
  - State space reductions (symmetries, partial-order reduction, ...).
  - Abstraction, counterexample-guided abstraction refinement (CEGAR).
  - Compositional methods, assume-guarantee reasoning.
  - [Bounded model checking: exploring the state space up to some bound only (sacrificing completeness), may leverage advances in SAT/SMT solving.]

- Especially suitable for reactive, concurrent, control-intensive systems.

- Supported by many tools, including industrial-strength tools:
  - Spin, Divine, Blast, CPAchecker, CBMC, JBMC, Smack, Ultimate Automizer, JPF (NASA), NuXMV, ABC, Incisive Formal Analysis (Cadence), RuleBase (IBM), Questa ( Mentor Graphics), VC Formal (Synopsys), Jasper, Uppaal, Prism, ...
**Static Analysis**

- Collects some information about the behaviour of a system from its **source code** without actually executing it under its original semantics.

- This description is very general, it can include even model checking and theorem proving (at least in some of their forms), which is sometimes (but not usually) done.

- Static analyses **range** from simple syntactic checks to iterative fixpoint computations over an abstraction of the examined system (e.g., into a form of equations that need to be solved).

- Different **forms of static analysis**:
  - bug pattern (anti-pattern) searching,
  - dataflow analysis,
  - constraint-based analysis,
  - (extended) type analysis (type and effect systems),
  - abstract interpretation,
  - symbolic execution, ..., mixtures of the above.
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- Disadvantages:
  - can produce many false alarms
    - construction of more and more precise analyses leads to similar efficiency problems like in model checking,
    - often resolved by accepting unsoundness,
  - various analyses are often specialised just for a certain specific task,
  - even more general analyses can have restricted applicability for different properties of interest (e.g., not everything can be simply expressed by a bug pattern).
Static Analysis

There exist many tools for static analysis—often commercial:

- FindBugs/SpotBugs, Coverity, Klocwork, CodeSonar (GrammaTech), PolySpace, Code Analysis in VisualStudio, AbsInt/Astrée, Clang Static Analyser, Cppcheck, cppclean, Sparse, Facebook Infer, Predator, KLEE, Speed, Loopus, TAJS, Weverca, Symbiotic, ...
Theorem Proving

- Deductive verification often similar to the classical mathematical way of proving theorems starting with axioms and inferring further facts using rules of correct inference.

- Advantages and disadvantages:
  - Very general.
  - Usually semi-automated, requires a significant manual effort.
  - Problems with diagnostic information for incorrect systems.

- There exist many interactive theorem provers including:
  - PVS, Coq, Hol, Isabelle, ACL2, Forte, ...

- Used, e.g., by Intel, AMD to verify complex designs of various arithmetic units (sometimes combined with model checking). Ongoing attempts to arrive to fully verified operating system kernels, compilers, ...
Theorem Proving

Recently there has been a lot of progress on fully automated decision procedures for various decidable theories:

- propositional logic (SAT solving),
- various first-order theories (SMT solving)
  - uninterpreted functions,
  - linear integer/real arithmetic,
  - theories of arrays, bitvectors, ...
- WS1S, WS2S, ...

There exist many tools implementing various decision procedures:

- SAT solvers: MapleSAT, COMiniSatPS, glucose, Lingeling, ...
- SMT solvers: Z3, MathSat, CVC4, Yices2, Boolector, SMTinterpol, ...
- WS1S/WS2S: Mona, Gaston, ...

There exist fully automated theorem provers for undecidable logics as well: e.g. Vampire for predicate logic (no guarantee of termination).
Theorem Proving

- Decision procedures often serve as a back-end for other verification approaches, e.g.:
  - Predicate abstraction for model checking.
  - Deductive verification based on discharging verification conditions.
    - Code is annotated by loop invariants, pre-/post-conditions of the program and procedures, and/or various assumptions.
    - Annotations can be (partially) obtained automatically (e.g., from analysing runs of the system and generalising them, solving constraints describing loops, etc.).
    - Due to having annotations, verification can concentrate on loop-free fragments—verification conditions: checking that when one starts under some preconditions and executes such a code fragment, the postcondition will be met.
    - Used, e.g., in attempts to verify operating system kernels—PikeOS, Microsoft’s Hypervisor (see the tool VCC).
  - Symbolic execution – iterating over longer and longer program paths possibly leading to an error, encoding them as formulae, checking satisfiability.
    - Klee, Symbiotic, ...