Static Analysis and Verification

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Data Flow Analysis

Introduction

❖ Data flow analysis is a special form of static analysis.

❖ Given a pre-defined set of properties of program states denoted as data flow values, needed for some particular reason, a data flow analysis (DFA) tracks how these values propagate between neighbouring control points of the control flow graph (CFG) of a program.

❖ The propagation of data flow values is tracked in a way consistent with all feasible paths through the CFG but without directly executing the program.

❖ The most common approach to DFA is the lattice-theoretic iterative DFA (pioneered by Kildall in 1973).

- The set of data flow values typically forms a complete lattice.
- The meet operation is used to merge analysis results flowing to a certain control point through several different controls paths.
- Program statements are modelled using monotone transfer functions, giving the analysis the name of monotone DFA.
- Forward or backward data flow equations describe how data flow values flow between control locations through the transfer functions and meets starting from an initial data flow associated with the entry or exit control point, respectively.
- An iterative solution of the data flow equations is used to perform the analysis.

Introduction

❖ The data flow values to be tracked for some particular application, the lattice over them, and the appropriate transfer functions are created manually.

❖ Data flow analyses can have a different scope:

- Local data flow analysis: performed within maximal sequences of statements with no control transfer into their middle/from their middle (basic blocks).
- Global data flow analysis: performed within a single function/procedure.
- Interprocedural data flow analysis: performed across functions/procedures.

❖ Possible applications of data flow analysis include:

- optimisation;
- verification – either directly or as a means for gathering some information about the behaviour that can be used to
  - refine some further analyses, e.g., based on looking for erroneous code patterns (Coverity, ...), by allowing some infeasible program paths to be ignored;
  - simplify some more costly analyses, e.g., dynamic analyses looking for data races (where cheap but less precise static analysis can be used to see which variables are for sure safe and not needed to be monitored);
- program understanding, e.g., for debugging.

A Motivating Example: Code Optimisation

A Motivating Example: Code Optimisation

❖ Consider the following program:

```
**Procedure:** quicksort($int\ m$,$int\ n$)
if $n < m$ then return;
$i = m - 1; j = n; v = a[n];$                         /* $v$ is the pivot. */
while $true$ do                                       /* Move values smaller */
    repeat $i = i + 1$ until $a[i] < v$;              /* than $v$ to the left */
    repeat $j = j - 1$ until $a[j] > v$;              /* of the split point (sp) */
    if $i \geq j$ then break;                         /* and other values */
    $x = a[i]; a[i] = a[j]; a[j] = x;$                /* to the right of sp. */
end
$x = a[i]; a[i] = a[n]; a[n] = x;$                    /* Move pivot to sp, and sort */
quicksort($m$,$i$);quicksort($i$+$1$,$n$);            /* the partitions independently. */
```

Intermediate Code

❖ When the program is compiled in a naïve way, many redundancies can arise (the initial test of $n < m$ is left out):

```
 1:  i = m – 1
 2:  j = n
 3:  t1 = 4 x n
 4:  t6 = a[t1]
 5:  v = t6
 6:  i  = i + 1
 7:  t2 =  4 x i
 8:  t3 = a[t2]
 9:  if t3 < v goto 6
10:  j = j – 1
11:  t4 = 4 x j
12:  t5 = a[t4]
13:  if t5 > v goto 10
14:  if i >= j goto 25
15:  t2 = 4 x i
16:  t3 = a[t2]
17:  x = t3
18:  t2 = 4 x i
19:  t4 = 4 x j
20:  t5 = a[t4]
21:  a[t2] = t5
22:  t4 = 4 x j
23:  a[t4] = x
24:  goto 6
25:  t2 = 4 x i
26:  t3 = a[t2]
27:  x = t3
28:  t2 =  4 x i
29:  t1 = 4 x n
30:  t6 = a[t1]
31:  a[t2] = t6
32:  t1 = 4 x n
33:  a[t1] = x
```

❖ Data flow analyses—e.g., deriving available expressions, copy propagation, computing dead variables—can help us detect such redundancies (and then get rid of them).

Basic Blocks and CFGs

❖ To decrease the granularity of the code to be analysed, one often works with the so called basic blocks instead of particular statements.

❖ A basic block is a maximal sequence of statements that is

- always entered via its first statement and
- always left via its last statement,

i.e., there is no transfer of control into the middle/from the middle of the sequence of statements.

❖ A control flow graph (CFG) is an oriented graph where nodes are basic blocks, and edges follow the transfer of control.

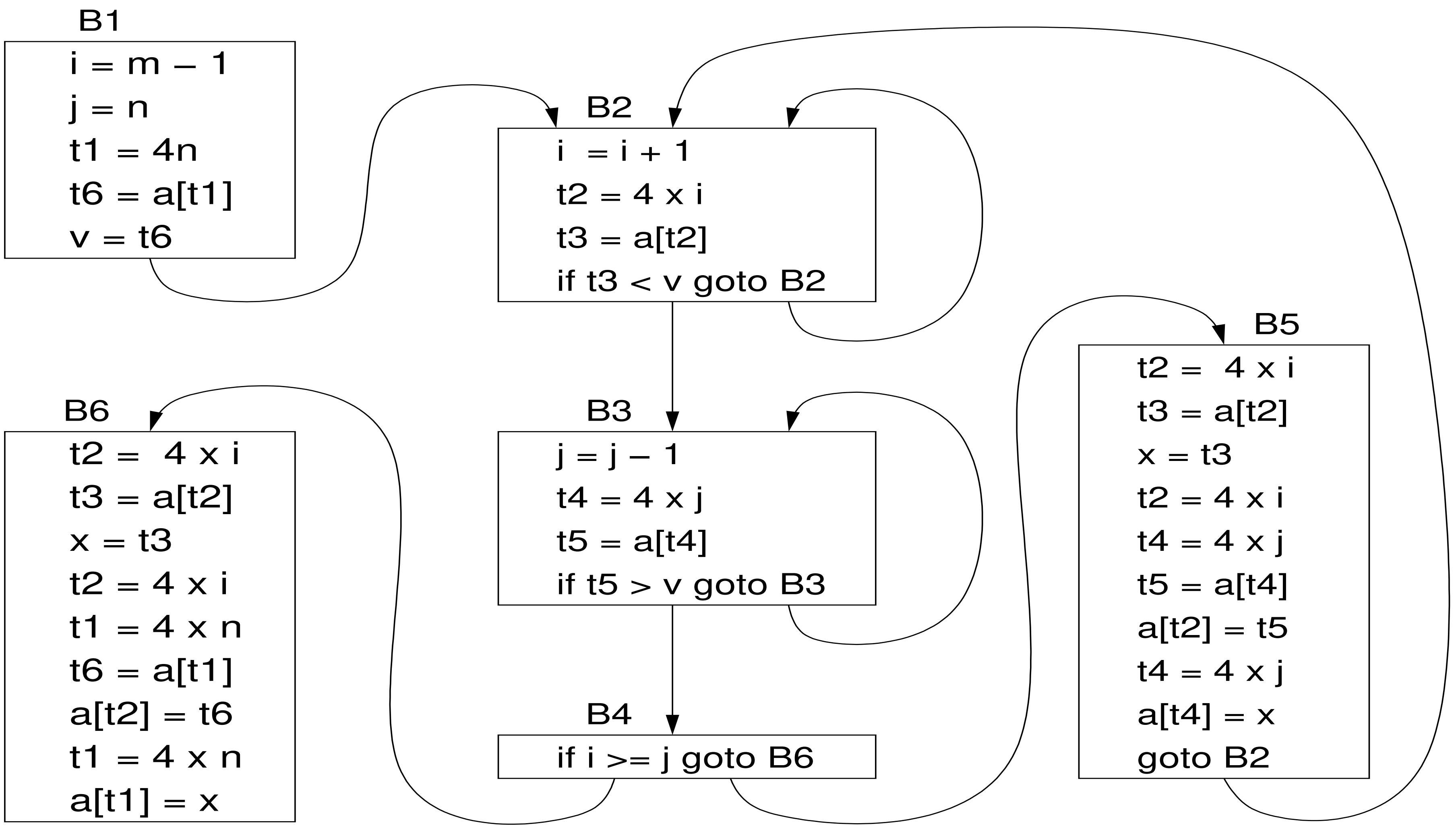
❖ Assume statements can be used at the beginning of blocks that are targets of conditional jumps to reflect the result of evaluating the condition.

❖ For simplicity, we assume that a control flow graph always contains

- a $Start$ block with no incoming edges and
- an End block with no outgoing edges.

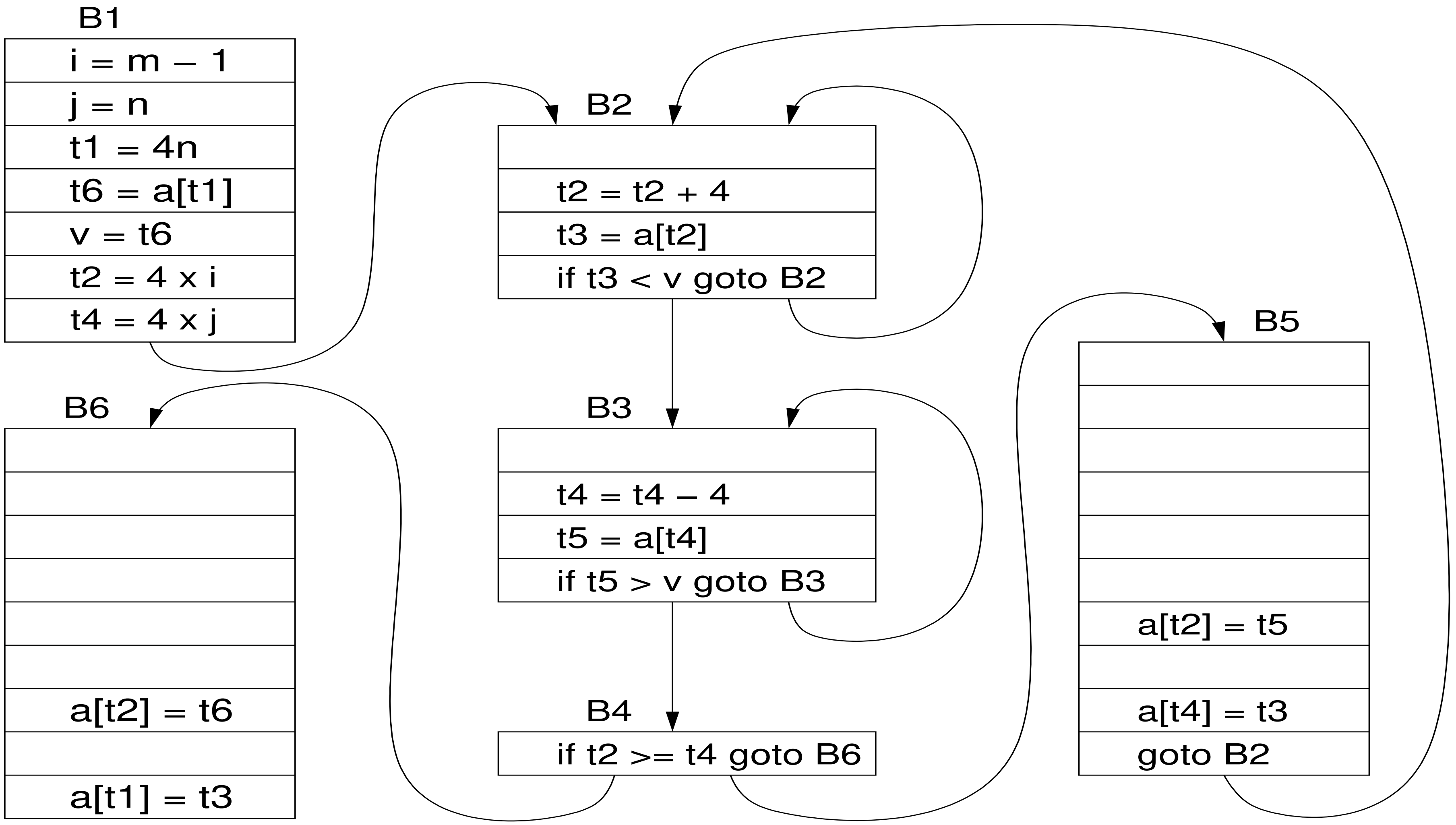
An Example of a CFG

- The CFG of the quicksort example (without testing $n < m$, without assume statements):



The Same CFG after Simplification

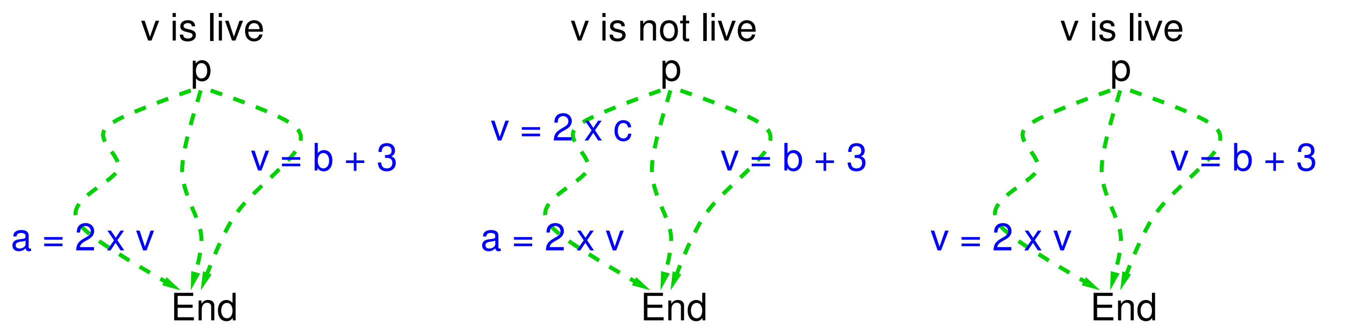
❖ The CFG of the quicksort examples after simplification by using available expressions, copy propagation, elimination of dead variables, elimination of inductive variables:



Classical Data Flow Analyses for Code Optimisation

Live Variables

❖ A variable v is live at a program point p iff some path from p to End contains an r-value occurrence (i.e., a read) of v which is not preceded by an l-value occurrence (i.e., a write).



❖ Usage:

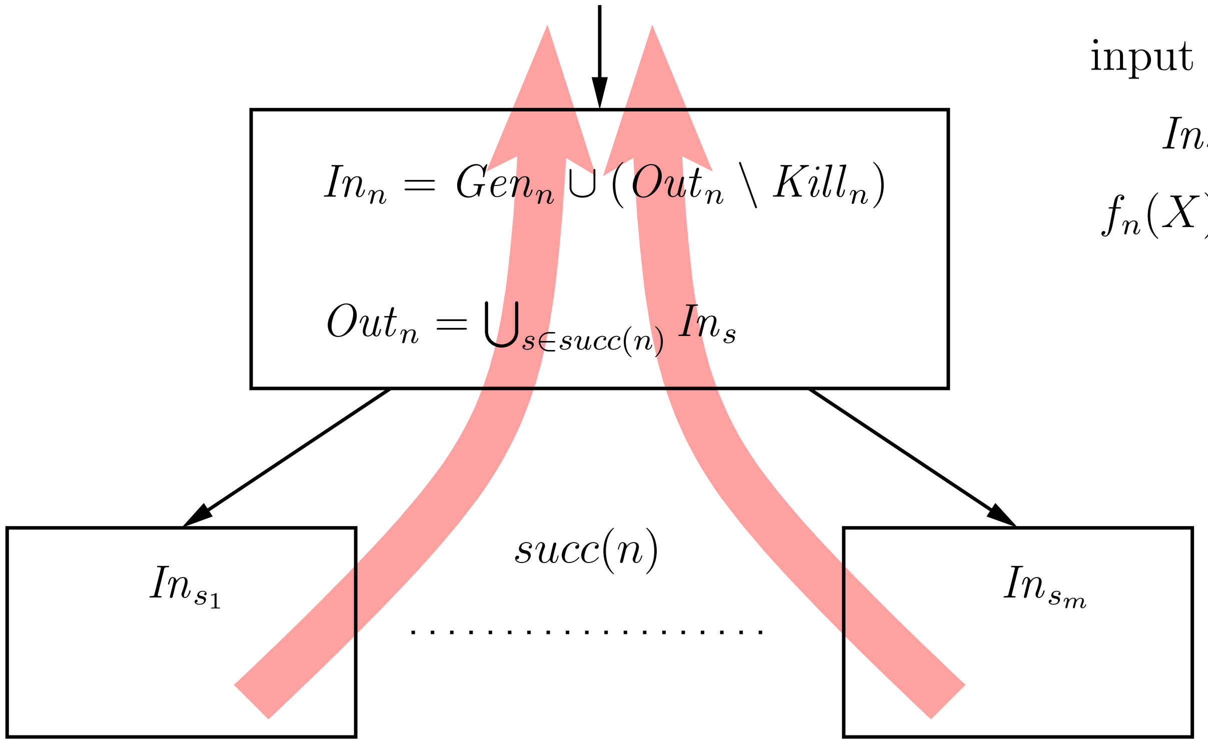
- Live variables are candidates for register allocation.
- An assignment to v at a program point where v is not live can be eliminated.

❖ How to compute live variables?

- Define data flow values: sets of variables (estimated) to be live.
- Direction: backward (intuitively: nothing is live at the exit).
- Define local constraints that restrict data flow values within blocks.
- Define global equalities that relate data flow values across different blocks.
- Compute the most general solution that satisfies all equalities and local constraints.
- The obtained liveness at the block boundaries is locally propagated into them.

Data Flow Analysis for Live Variables

- Local constraints are given by two sets of variables defined for each block n:
  - Gen_n: variables used in n such that their use is not preceded by their definition.
  - $Kill_n$: variables (re)defined in n.
- Let In_n and Out_n represent sets of variables live at the input/output of each basic block n.
- Global flow equations define how information is propagated backwards through the CFG:


$In_n = Gen_n \cup (Out_n \setminus Kill_n)$
$Out_n = \bigcup_{s \in succ(n)} In_s$
In_{s_1}
$succ(n)$
In_{s_m}

input is a function of output:

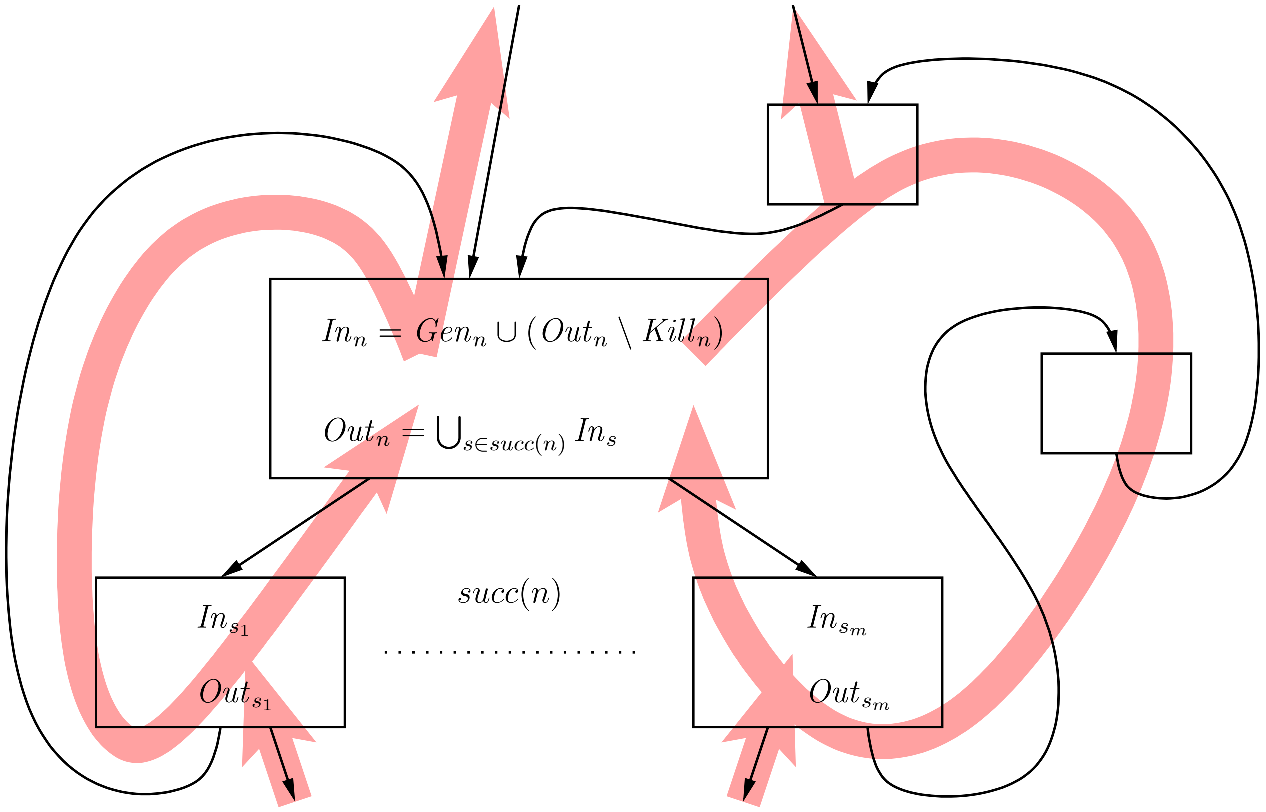
$$In_n = f_n(Out_n) \text{ where}$$

$$f_n(X) = Gen_n \cup (X \setminus Kill_n)$$

Computing Live Variables

❖ A simple algorithm:

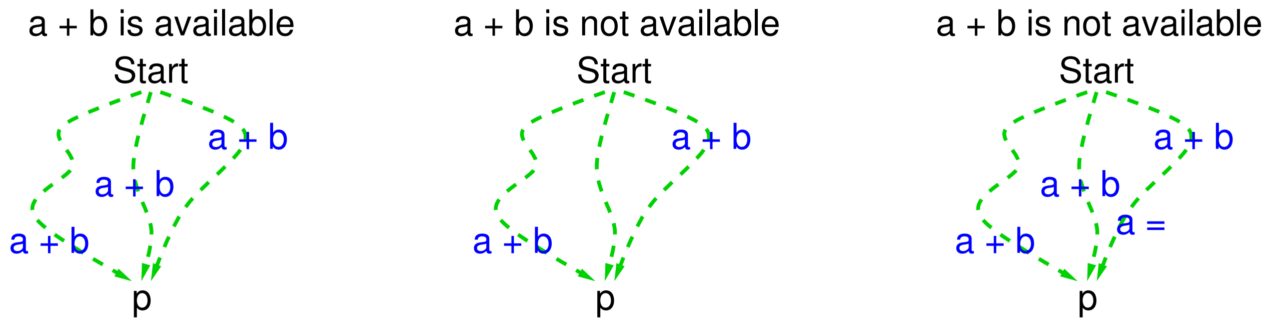
1. Compute $Kill$ and Gen sets.
2. Start with all Out and In sets initialised to $\emptyset$.
3. Update In sets according to the current values of Out sets.
4. If anything changed, update Out sets according to the new values In and go to 3.



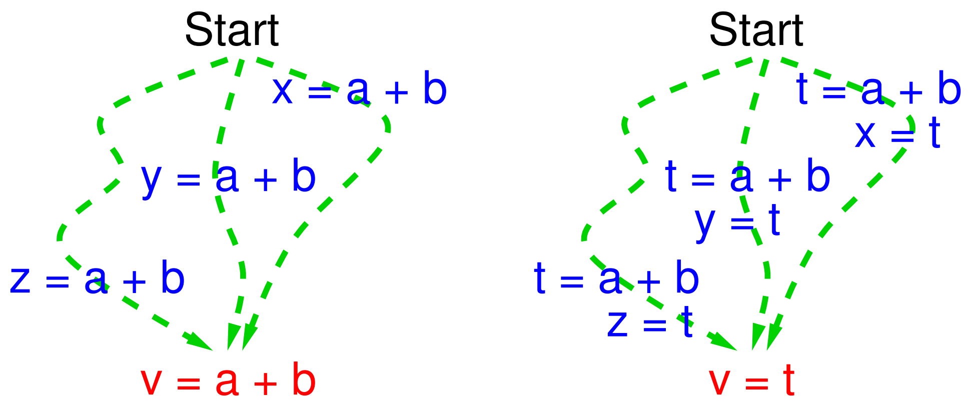
$In_n = Gen_n \cup (Out_n \setminus Kill_n)$
$Out_n = \bigcup_{s \in succ(n)} In_s$
In_{s_1}
Out_{s_1}
$succ(n)$
In_{s_m}
Out_{s_m}

Available expressions

❖ An expression e is available at a program point p if every path from the program entry to p contains an evaluation of e which is not followed by a definition of any operand of e.



❖ Usage: common subexpression elimination.



Data Flow Analysis for Available Expressions

❖ Data flow values: sets of expressions (estimated to be available).

❖ Direction: forward (intuitively: nothing is available at the entry).

❖ Local constraints:

- $Gen_n = \{e \mid$ the expression e is evaluated in the basic block n, and this evaluation is not followed by a definition of any operand of $e\}$.
- $Kill_n = \{e \mid$ the basic block n contains a definition of an operand of $e\}$.

❖ Flow equations:

$$In_n = \begin{cases} \emptyset & \text{if } n \text{ is the } Start \text{ block} \\ \bigcap_{p \in pred(n)} Out_p & \text{otherwise} \end{cases}$$

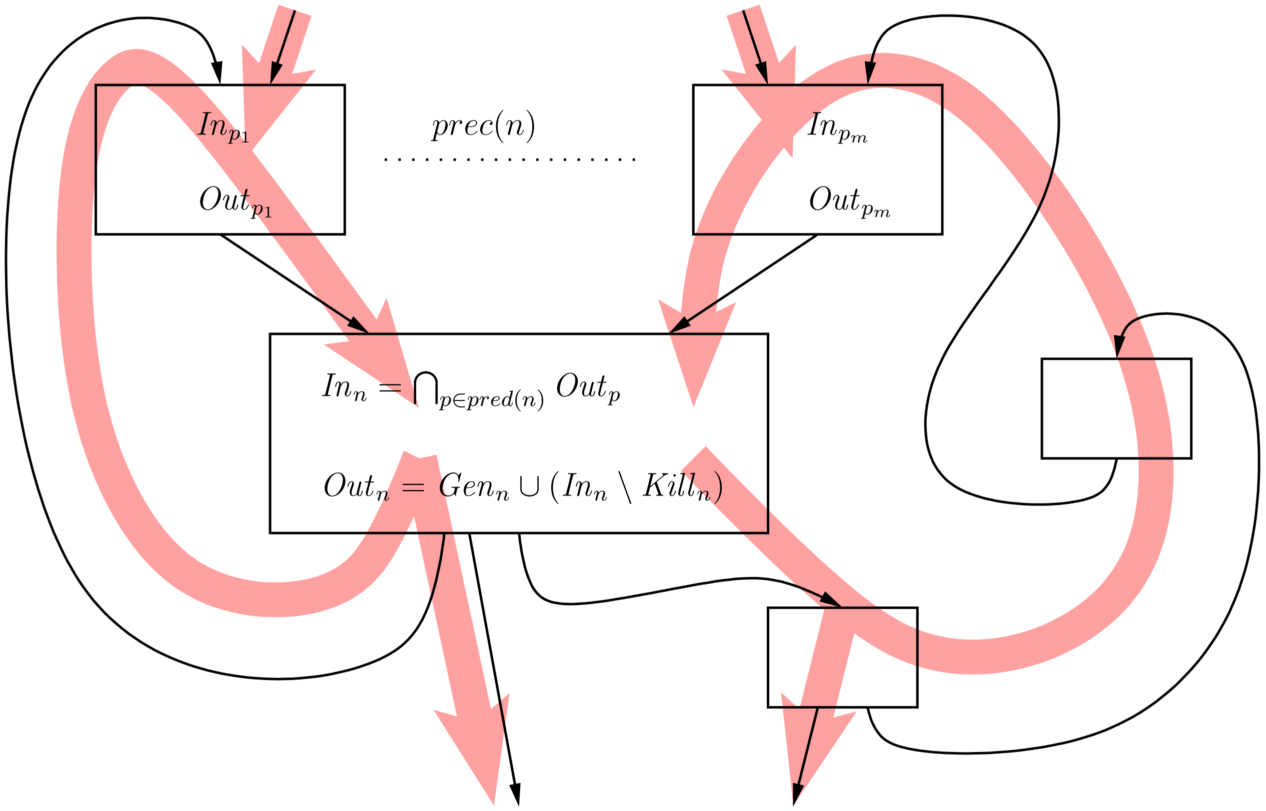
output is a function of input:

$$Out_n = f_n(In_n) \text{ where}$$
$$f_n(X) = Gen_n \cup (X \setminus Kill_n)$$

Computing Available Expressions

❖ A simple algorithm (let $Exps$ be the set of all expressions in the program at hand):

1. Compute $Kill$ and Gen sets.
2. Start with all Out and In sets initialised to $Exps$ (In of the start block is initialised to $\emptyset$).
3. Update Out sets according to the current values of In sets.
4. If anything changed, update In sets according to the new values Out and go to 3.


In_{p_1}
Out_{p_1}
$prec(n)$
In_{p_m}
Out_{p_m}
$In_n = \bigcap_{p \in pred(n)} Out_p$
$Out_n = Gen_n \cup (In_n \setminus Kill_n)$

Towards a General Framework

❖ We are going to formulate data flow analyses in a more abstract and generic way in terms of lattices and monotone functions.

❖ Sets of data flow values that may be associated with blocks as their $Gen/Kill/In/Out$ values can be seen as partially ordered sets wrt. some ordering $\sqsubseteq$.

❖ The order $\sqsubseteq$ reflects the information content of the particular values depending on how the computed information is to be used.

- Live variables:
  - The fewer variables are live, the more assignments we can eliminate.
  - Hence, $V_1 \subseteq V_2 \iff V_1 \sqsupseteq V_2$.
  - Use the complete lattice $(2^{Vars}, \supseteq)$.
- Available expressions:
  - The more expressions are available, the more common subexpressions we can eliminate.
  - Hence, $E_1 \subseteq E_2 \iff E_1 \sqsubseteq E_2$.
  - Use the complete lattice $(2^{Exps}, \subseteq)$.

Towards a General Framework

- Local propagation of information within blocks:
  - Live variables—bottom up, against the control flow:
    - $In_n = f_n(Out_n)$ where $f_n(X) = Gen_n \cup (X \setminus Kill_n)$,
    - f_n is monotone in $(2^{Vars}, \supseteq)$ $(V_1 \supseteq V_2 \implies f_n(V_1) \supseteq f_n(V_2))$.
  - Available expressions—top down, along the control flow:
    - $Out_n = f_n(In_n)$ where $f_n(X) = Gen_n \cup (X \setminus Kill_n)$,
    - f_n is monotone in $(2^{Exps}, \subseteq)$ $(E_1 \subseteq E_2 \implies f_n(E_1) \subseteq f_n(E_2))$.

- Confluence—merging information of two or more flow paths and computing the maximal information consistent with all the incoming edges:
  - Live variables—exists some path . . . :
    - $Out_n = \bigcup_{s \in succ(n)} In_s$,
    - $\cup$ is the meet in $(2^{Vars}, \supseteq)$.
  - Available expressions—for all paths . . . :
    - $In_n = \bigcap_{p \in pred(n)} Out_p$ (up to In_{Start}),
    - $\cap$ is the meet $(2^{Exps}, \subseteq)$.

Monotone Framework

❖ Ingredients:
- A complete lattice $(L, \sqsubseteq)$ of flow values with explicitly stated $\top$, $\bot$, and $\sqcap$. It has to satisfy the descending chain condition: every descending chain $x_1 \sqsupseteq x_2 \sqsupseteq \ldots$ eventually stabilises (after a finite number of steps).
- A set $\mathcal{F}$ of flow functions $L \rightarrow L$. Every basic block n is mapped to its flow function f_n. $\mathcal{F}$ is closed under composition, the functions are monotone.
- A data flow F: against/along the CFG.
- An extreme flow variable E $(In/Out_{Start/End})$ and its value val_E.

	Available Expressions	Live Variables
L	2^{Exps}	2^{Vars}
$\sqsubseteq$	$\subseteq$	$\supseteq$
$\sqcap$	$\cap$	$\cup$
$\top, \bot$	$Exps, \emptyset$	$\emptyset$, $Vars$
E	In_{Start}	Out_{End}
val_E	$\emptyset$	$\emptyset$
F	along control flow	against control flow
$\mathcal{F}$	$\{f : L \rightarrow L \mid \exists l_k, l_g : f(l) = (l \setminus l_k) \cup l_g\}$	
f_n	$f_n(X) = Gen_n \cup (X \setminus Kill_n)$	

Performing the Analysis

- The Goal: The most exhaustive/informative assignment of flow values to blocks that is safe (consistent with all equations) called as the maximum fixpoint (MFP) solution.

- Initialise all values to $\top$, set the value of the extremal label E to val_E.

- Round Robin—simple, not efficient:
  - Traverse all blocks in a fixed order.
  - Always apply the flow function (f_n) and propagate along the flow F using $\sqcap$.
  - Terminate when all values stabilise.

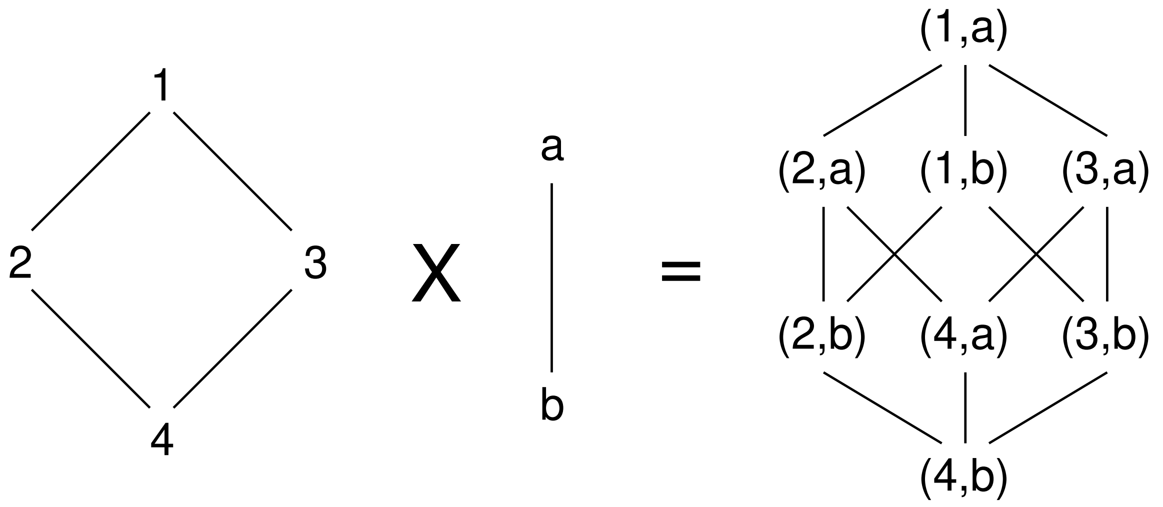
- Use a worklist—faster, but with a memory overhead:
  - Maintain a worklist of blocks with changed values (initially, put all blocks there).
  - Keep processing elements of the worklist until it is empty:
    - Update a block n from the list locally using f_n and erase it from the list.
    - If its value changed, propagate the change to its successors wrt. F using $\sqcap$.
    - Add the updated successors to the worklist.

Cartesian Product of Lattices

❖ In order to sketch correctness of the iterative solution of flow equations, we need to be able to deal with flow values for each control block, i.e., with vectors of flow values. This leads us to a need of Cartesian product lattices.

❖ A Cartesian product of lattices $(L_1, \sqsubseteq_1), \ldots, (L_n, \sqsubseteq_n)$ is defined as the pair $(L, \sqsubseteq)$ where:

- $L = L_1 \times \ldots \times L_n$ and
- $\sqsubseteq \subseteq L \times L$ is defined as $(x_1, \ldots, x_n) \sqsubseteq (y_1, \ldots, y_n)$ iff $x_1 \sqsubseteq_1 y_1, \ldots, x_n \sqsubseteq_n y_n$.



❖ Then:

- $(L, \sqsubseteq)$ is a lattice where $(x_1, \ldots, x_n) \sqcap (y_1, \ldots, y_n) = (x_1 \sqcap_1 y_1, \ldots, x_n \sqcap_n y_n)$.
- The product of complete lattices is a complete lattice.
- The product of lattices satisfying the descending chain condition satisfies the descending chain condition.

Correctness of Round Robin

❖ To simplify, assume that Round Robin always first computes all meets and then applies all flow functions.

❖ Statement: Round Robin always terminates with the maximum fixpoint solution (MFP).

❖ Idea of the proof:

- Consider k basic blocks, giving $2k$ In/Out flow variables. A global state of the whole analysis is a $2k$-tuple of flow values, an element of the classical Cartesian product of $2k$ lattices of flow values.
- The product lattice inherits the important properties of the flow value lattice. It is also complete and satisfies the descending chain condition.
- Consider a function $\mathbf{f} : L^{2k} \to L^{2k}$ applying all the flow functions. It is monotone.
- Consider a function $\mathbf{f}_{\sqcap} : L^{2k} \to L^{2k}$ computing all the meets. It is also monotone.
- Now, notice that:
  - $\mathbf{f}_{\circ} = \mathbf{f} \circ \mathbf{f}_{\sqcap}$ is monotone too. It corresponds to one iteration of the algorithm.
  - Apparently, solutions of the analyses are the fixpoints of $\mathbf{f}_{\circ}$.
  - Take Knaster-Tarski with the descending chain condition.
  - And you shall see the light . . .

Very Busy Expressions/Anticipable

❖ An expression e is very busy/anticipable at a program point p if every path from p to exit contains an evaluation of e which is not preceded by a redefinition of an operand of e.

❖ Usage: safety of code hoisting—moving code that occurs in all branches above the branching.

❖ The corresponding data flow analysis:

- Lattice of data flow values $(2^{Exps}, \subseteq)$ with $(\top, \bot, \sqcap) = (Exps, \emptyset, \cap)$.
- Flow F: against the control flow.
- flow functions: $f_n(X) = Gen_n \cup (X \setminus Kill_n)$.
  - $Gen_n = \{e \mid$ the expression e is evaluated in the basic block n, and this evaluation is not preceded (within n) by a definition of any operand of $e\}$.
  - $Kill_n = \{e \mid$ the basic block n contains a definition of an operand of $e\}$.
- Extreme flow variable: Out_{End}.
- Initial value of the extreme flow variable: $\emptyset$.

Reaching Definitions

❖ A definition $L : x = y$ reaches a program point p if it appears (without a redefinition of x) on some path from the program entry to p.

❖ Usage: copy propagation—a use of a variable x at a program point p can be replaced by y if $L : x = y$ is the only definition of x that reaches p, and y is not modified between L and p.

❖ The corresponding data flow analysis:

- Lattice of data flow values: $(2^{Labs \times Vars}, \supseteq)$ with $(\top, \bot, \sqcap) = (\emptyset, Labs \times Vars, \cup)$.
- Flow F: along the control flow.
- Flow equations: $f_n(X) = Gen_n \cup (X \setminus Kill_n)$.
  - $Gen_n = \{(L, v) \mid$ the variable v is defined on line L in the basic block n, and this definition is not followed (within n) by a definition of $v\}$.
  - $Kill_n = \{(L, v) \mid$ the basic block n contains a definition of $v\}$.
- Extreme flow variable: In_{Start}.
- Initial value of the extreme flow variable: $\{(0, v) \mid v \in Vars\}$ where $(0, v)$ means that v is undefined at the start.

Summary of the Presented Analyses

	Available Expressions	Live Variables	Very Busy Expressions	Reaching Definitions
L	2^{Exps}	2^{Vars}	2^{Exps}	$2^{Labs \times Vars}$
$\sqsubseteq$	$\subseteq$	$\supseteq$	$\subseteq$	$\supseteq$
$\sqcap$	$\cap$	$\cup$	$\cap$	$\cup$
$\top, \bot$	$Exps, \emptyset$	$\emptyset, Vars$	$Exps, \emptyset$	$\emptyset, Labs \times Vars$
E	In_{Start}	Out_{End}	Out_{End}	In_{Start}
val_E	$\emptyset$	$\emptyset$	$\emptyset$	$\{(0, v) \mid v \in Vars\}$
F	control flow	reversed control flow	reversed control flow	control flow
$\mathcal{F}$	$\{f : L \to L \mid \exists l_k, l_g : f(l) = (l \setminus l_k) \cup l_g\}$			
f_n	$f_n(X) = Gen_n \cup (X \setminus Kill_n)$			

❖ These analyses are often referred to as bit-vector analyses since the flow values, which are subsets of a finite set, can be represented as bit-vectors (for each control point). Note that each of the bits is handled independently of the others.

- Do not confuse with the vectors that one obtains when speaking about flow values for all control points—then we get vectors of vectors.

General Data Flow Analysis

Data Flow Equations For General Flows

❖ So far, flow functions based on constant Gen and $Kill$ sets, independent of the input of the flow functions (and hence of the current values of the flow variables), were considered.

❖ However, in general, Gen and $Kill$ can be functions that have a flow independent part but that can also have a flow dependent part:

$$\begin{aligned} Gen_n(X) &= ConstGen_n \cup DepGen_n(X) \\ Kill_n(X) &= ConstKill_n \cup DepKill_n(X) \\ f_n(X) &= Gen_n(X) \cup (X \setminus Kill_n(X)) \end{aligned}$$

❖ Note that the data flow values associated with the program entities whose properties are tracked by an analysis (variables, expressions, ...) may depend on the old flow values associated with them only—the so-called separable case, but also on any other entities—the non-separable case.

- All the so-far considered analyses were separable.
- Examples of non-separable analyses include faint variable analysis (transitive liveness of variables), possibly uninitialised variable analysis (again with transitive dependencies), or constant propagation analysis.
- For non-separable analyses, flow equations are often defined for particular statements (instead of basic blocks).

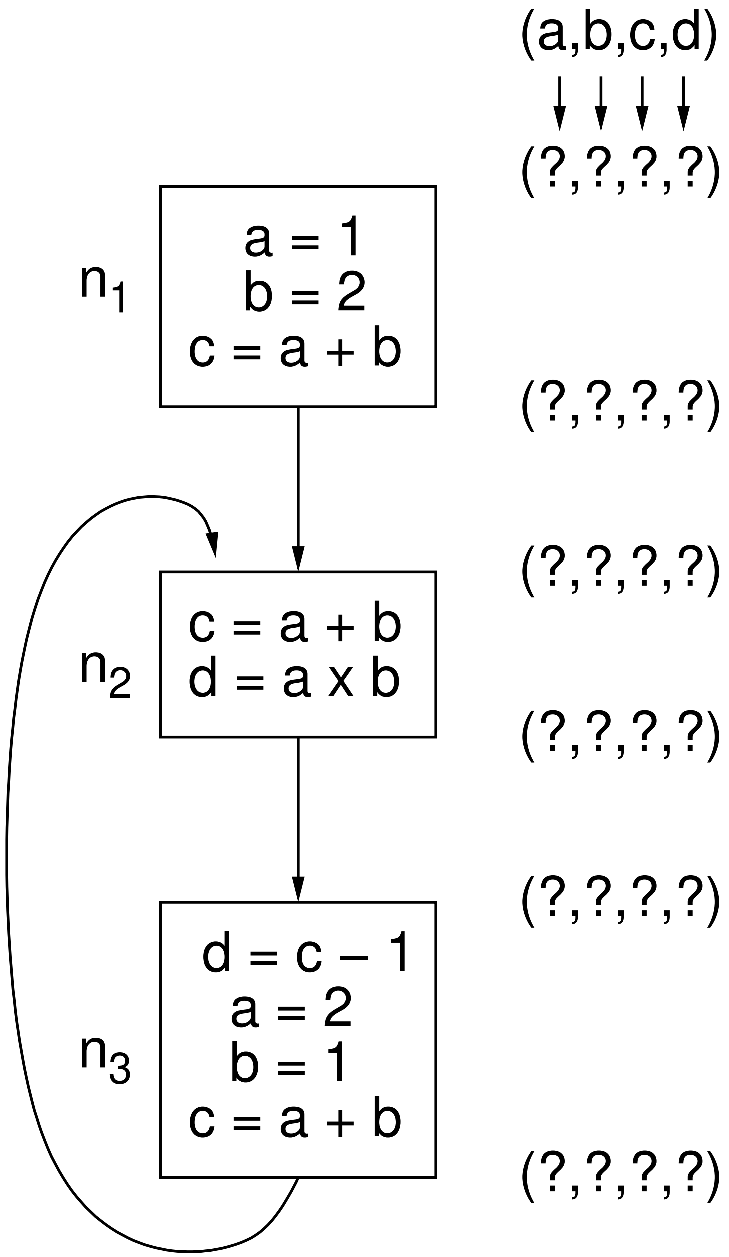
Constant Propagation

- A value of a variable v is constant at a program point p if the value of the variable v will be the same at this point regardless of the program path taken to p.

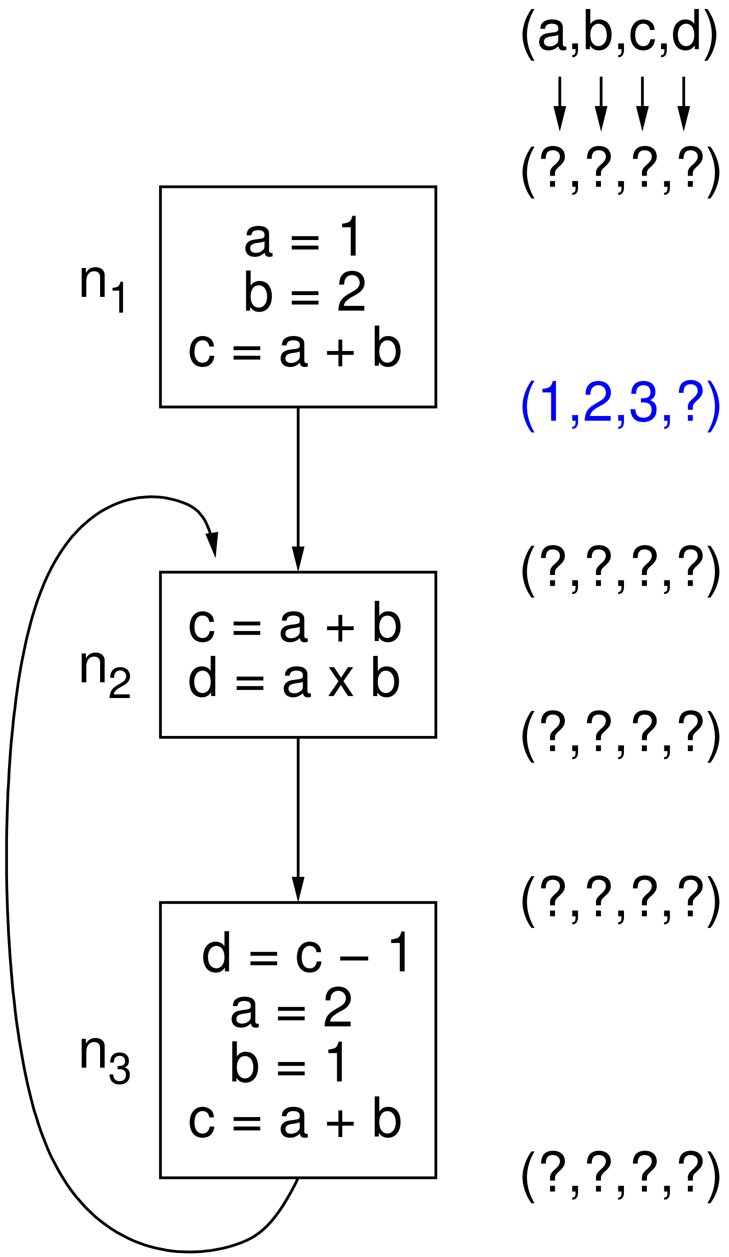
- Usage: Replacing constant variables with constants.

- The analysis is interesting from several points of view:
  - non-separable: clearly, to compute the value of an expression assigned to some variable, we need values of the variables that appear in the expression,
  - not a bit-vector analysis: at least when considering a set of infinitely many different constant values, we have infinitely many possible flow values,
  - non-distributive analysis: the MFP solution needs not be the most precise one (to be discussed later on).

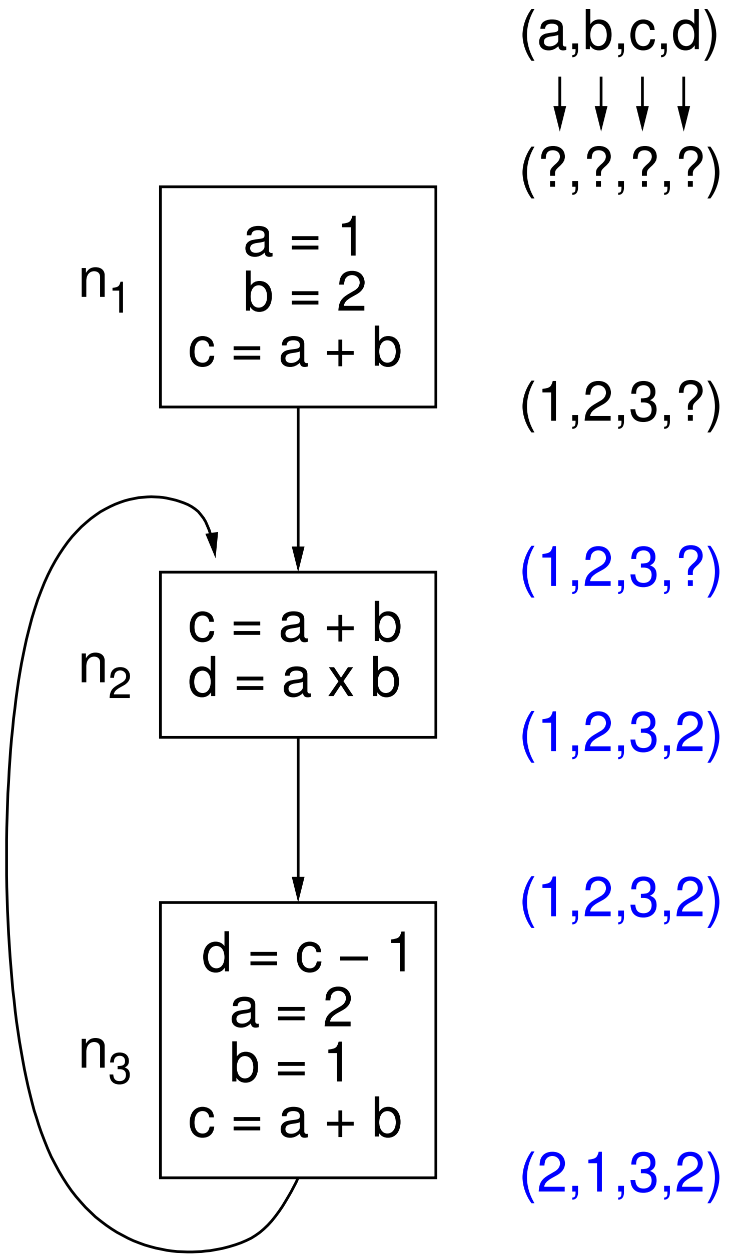
Constant Propagation Movie



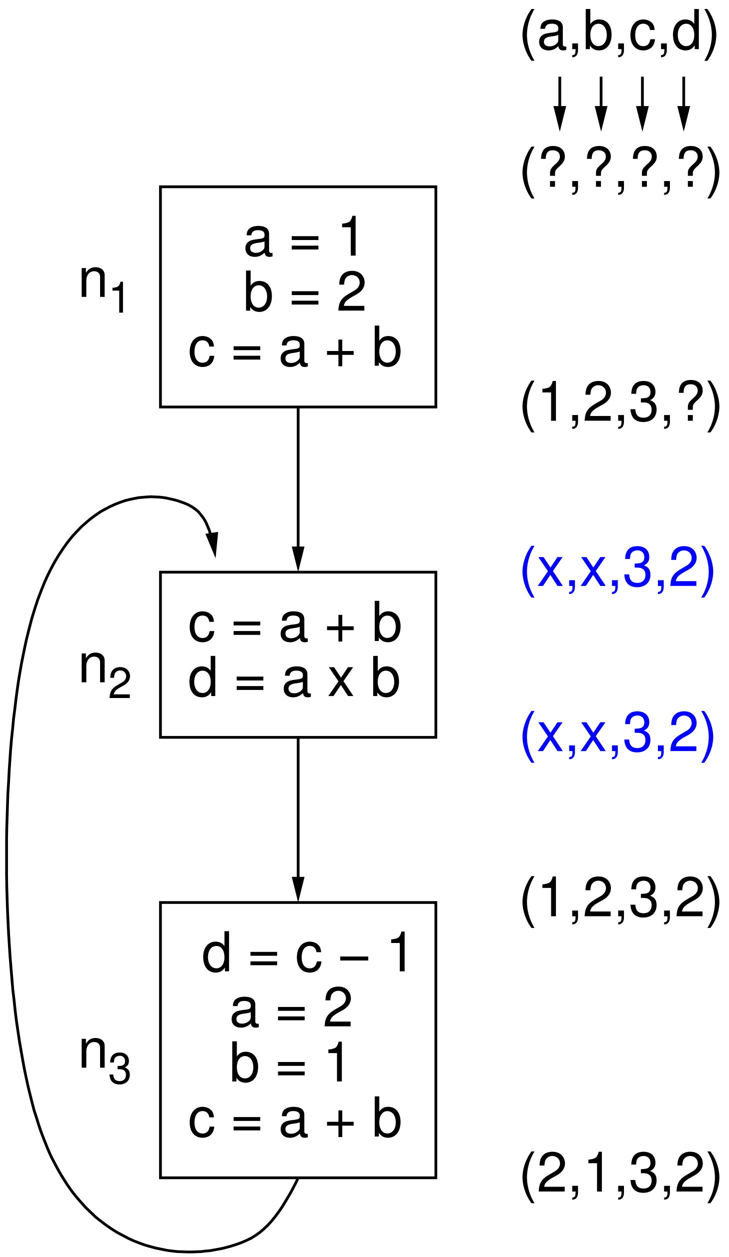
Constant Propagation Movie



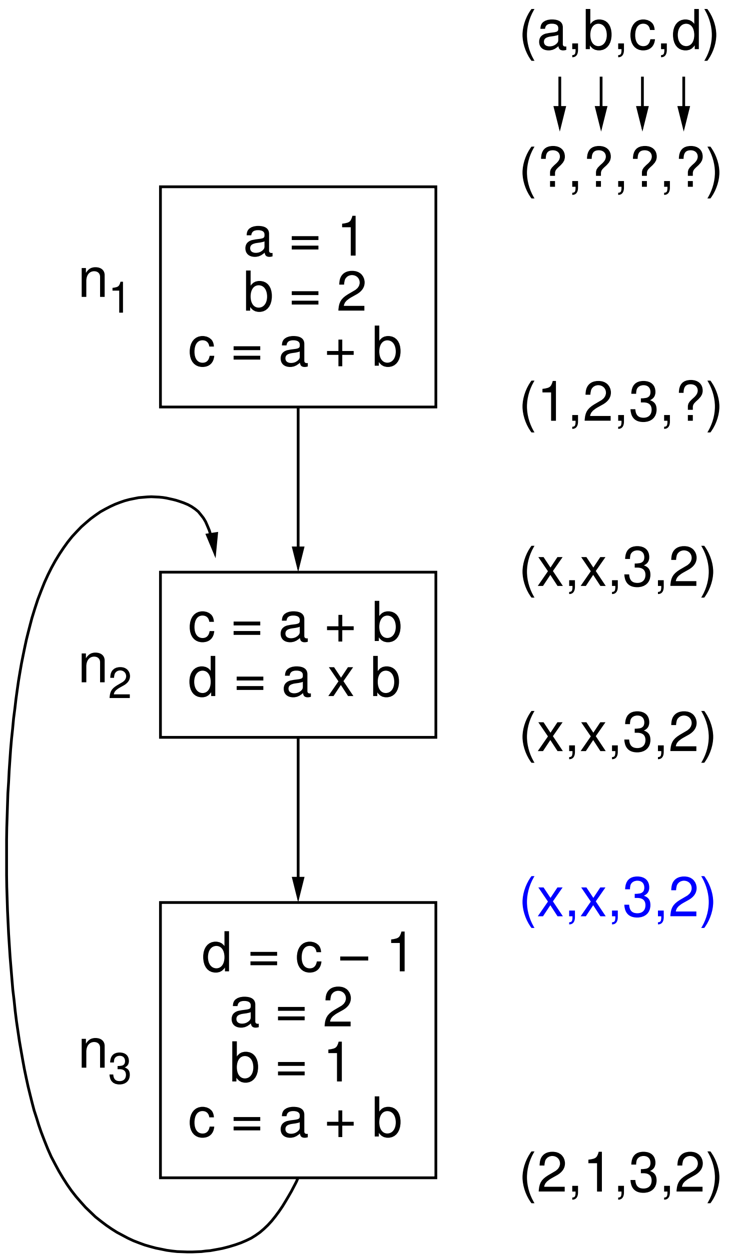
Constant Propagation Movie



Constant Propagation Movie



Constant Propagation Movie



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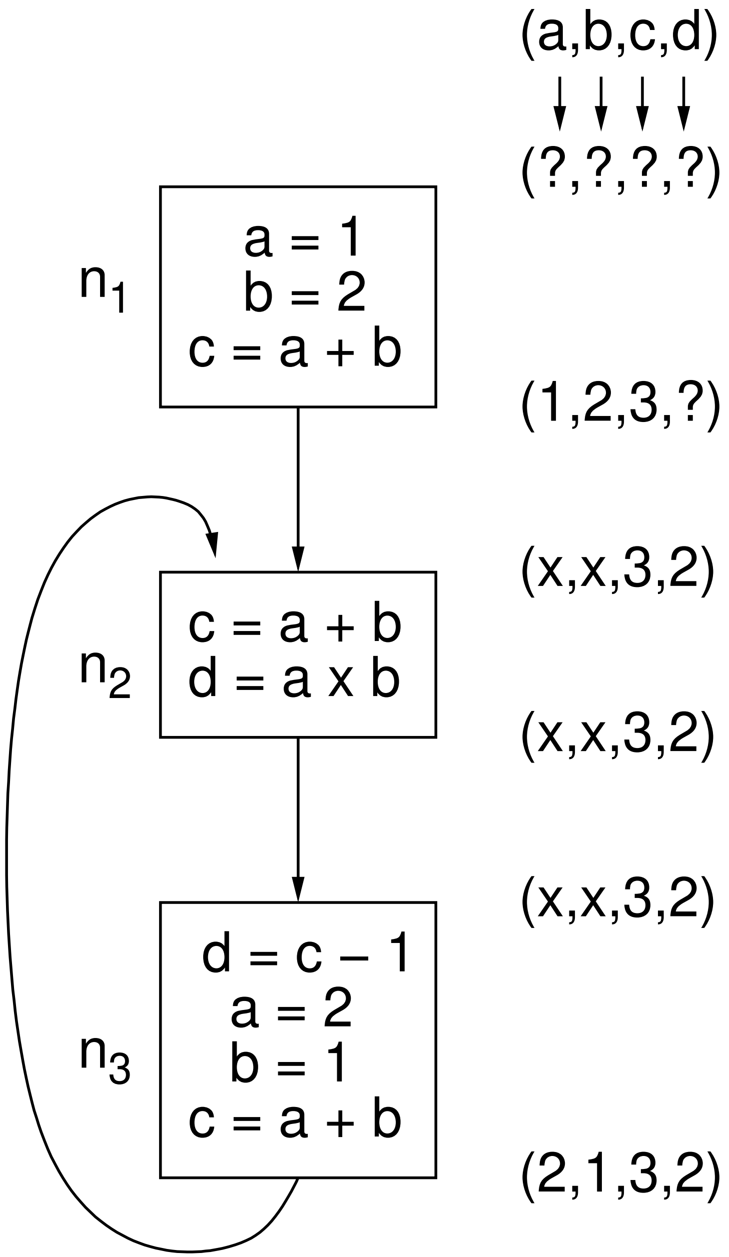
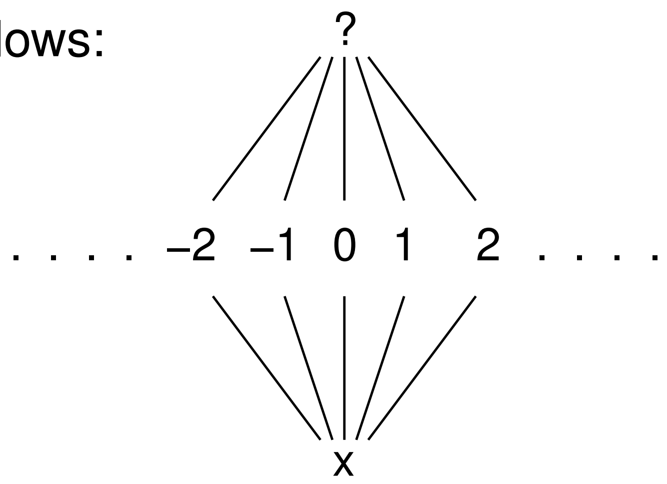


Flow Lattice for (Integer) Constant Propagation

❖ In order to track flow values associated with particular variables, the component lattice $(\hat{L} = \mathbb{Z} \cup \{?, \mathrm{x}\}, \hat{\sqsubseteq})$ can be used where

- the maximum $\hat{\top} = ?$ is the undefined value, i.e., "anything", and
- the minimum $\hat{\bot} = \mathrm{x}$ is a an indication that the given variable is non-constant, and
- the ordering $\hat{\sqsubseteq}$ is as follows:



❖ The data flow value lattice $(L, \sqsubseteq)$ can then be obtained as the product of k component lattices where k is the number of variables ($\sqsubseteq, \sqcap$ are defined by $\hat{\sqsubseteq}, \hat{\sqcap}$).

❖ In order to fit the set-based $Gen/Kill$ framework, $(L, \sqsubseteq)$ can be replaced by a lattice $(L', \sqsubseteq')$ where $L' \subseteq 2^{Vars \times \hat{L}}$ s.t. $\forall X \in L' \; \forall v \in Vars \; \exists! a \in \hat{L} : (v, a) \in X$.

Flow Equations for Constant Propagation

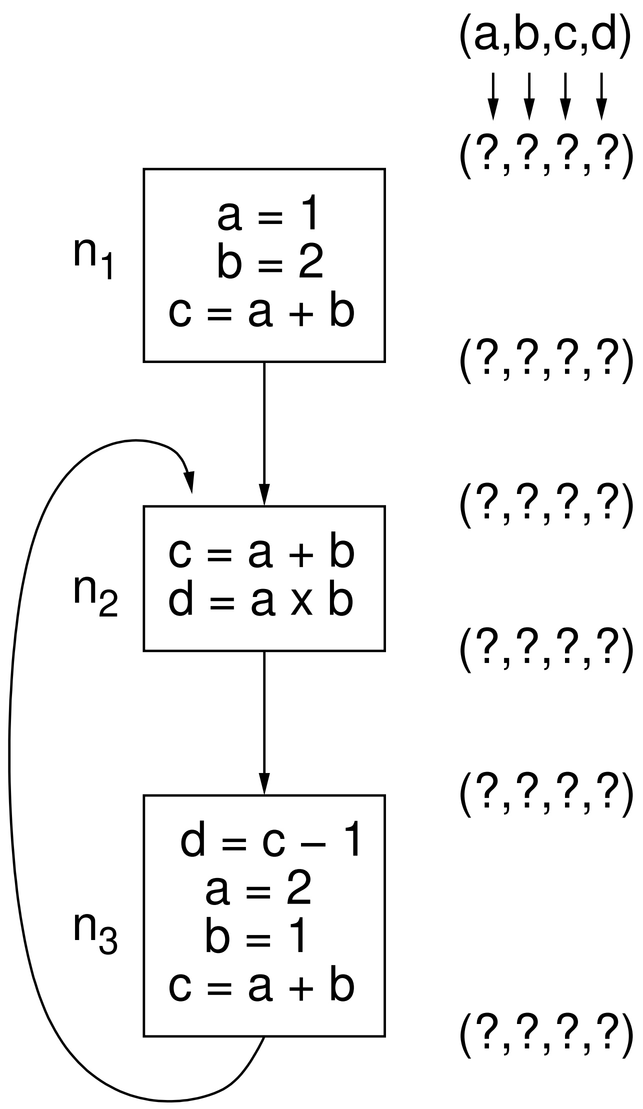
	$ConstGen_n$	$DepGen_n(X)$	$ConstKill_n$	$DepKill_n(X)$
$v = c$, $c \in Const$	$\{(v, c)\}$	$\emptyset$	$\emptyset$	$\{(v, d) \mid (v, d) \in X\}$
$v = e$, $e \in Exps$	$\emptyset$	$\{(v, eval(e, X))\}$	$\emptyset$	$\{(v, d) \mid (v, d) \in X\}$
$read(v)$	$\{(v, \mathsf{x})\}$	$\emptyset$	$\emptyset$	$\{(v, d) \mid (v, d) \in X\}$
other	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

$eval(a_1\ \mathsf{op}\ a_2, X)$	$(a_1, ?) \in X$	$(a_1, \mathsf{x}) \in X$	$(a_1, c_1) \in X$
$(a_2, ?) \in X$	$?$	x	$?$
$(a_2, \mathsf{x}) \in X$	x	x	x
$(a_2, c_2) \in X$	$?$	x	$c_1\ \mathsf{op}\ c_2$

Meet over Paths Solution and Distributivity

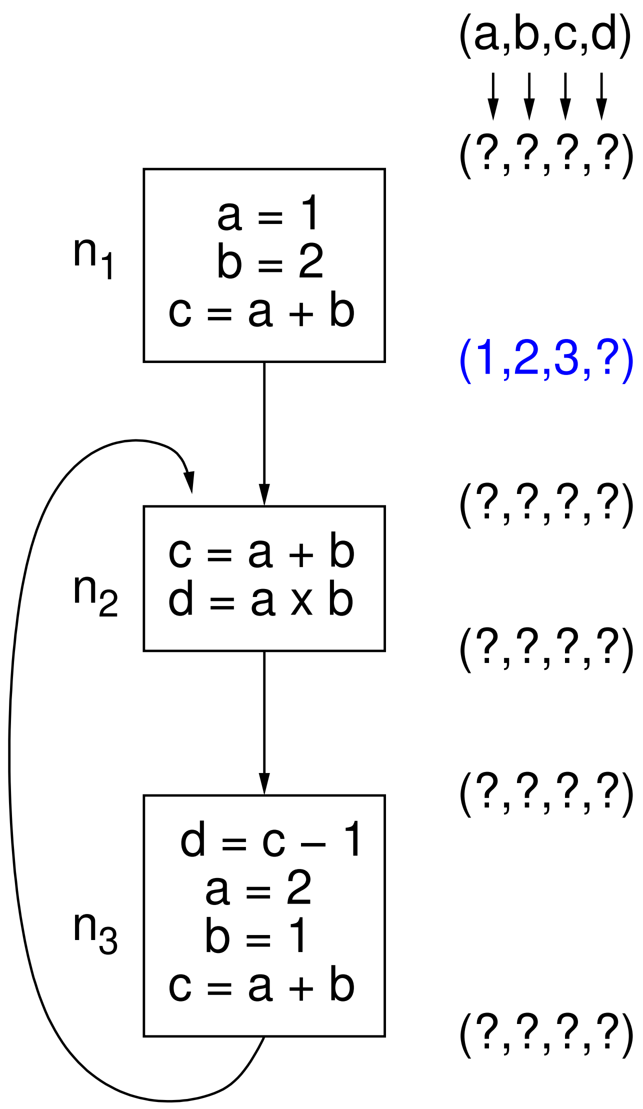
Constant Propagation Movie 2

We proceed carefully according to the defined framework.



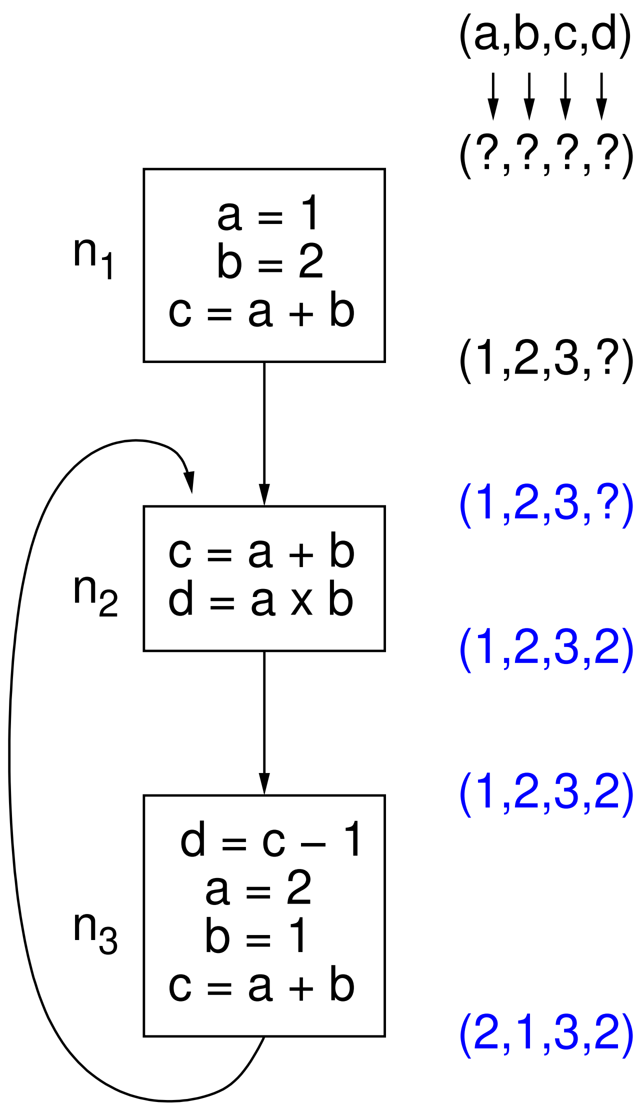
Constant Propagation Movie 2

We proceed carefully according to the defined framework.



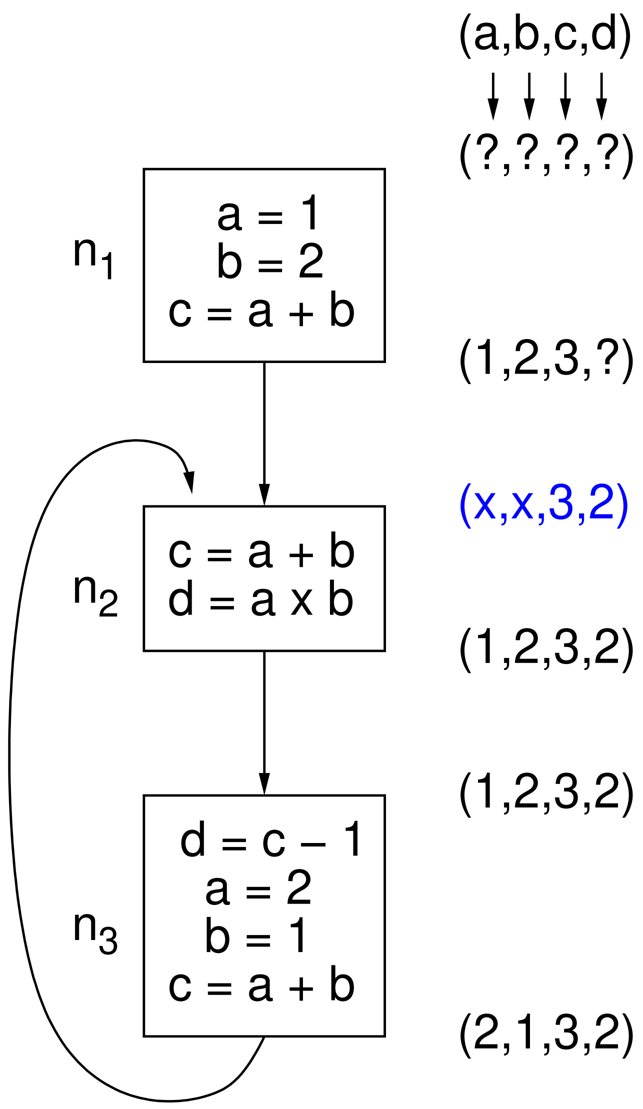
Constant Propagation Movie 2

We proceed carefully according to the defined framework.



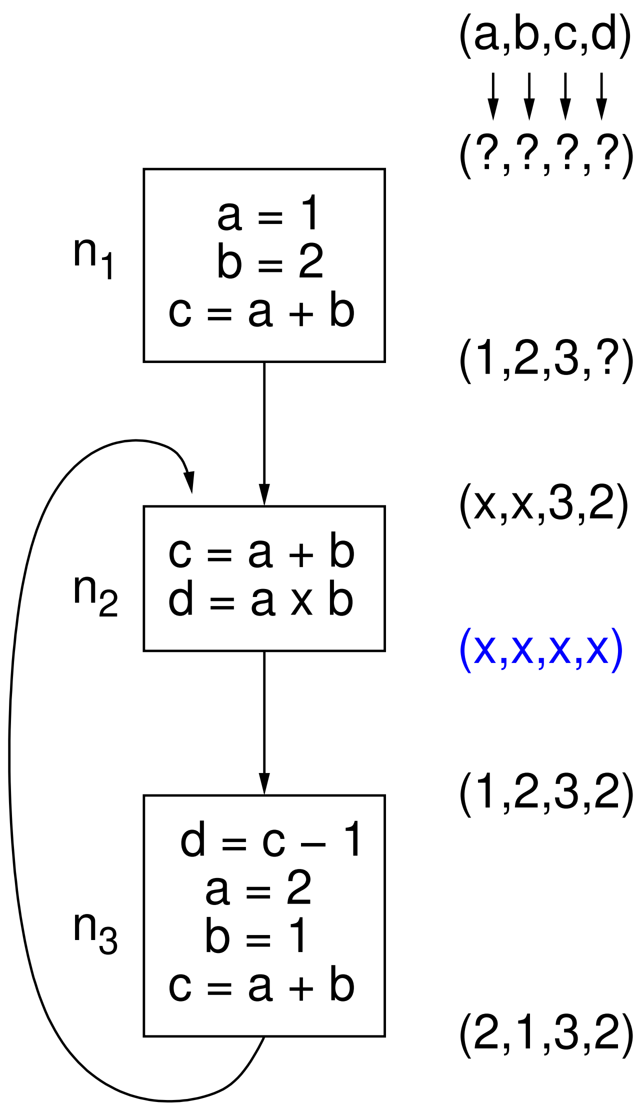
Constant Propagation Movie 2

We proceed carefully according to the defined framework.



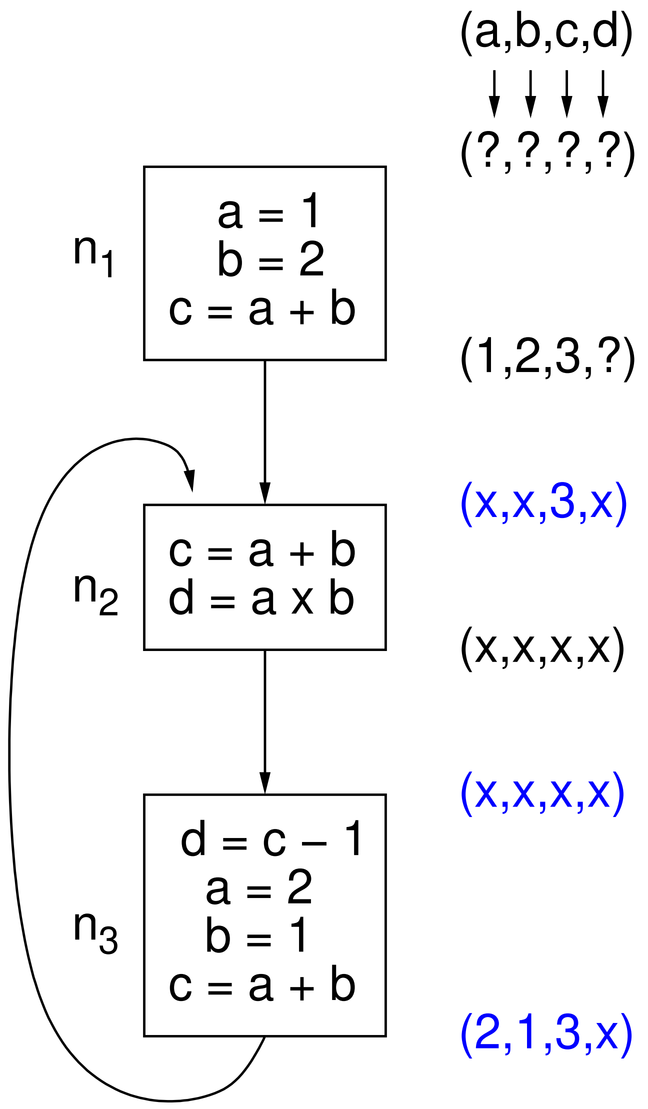
Constant Propagation Movie 2

We proceed carefully according to the defined framework.



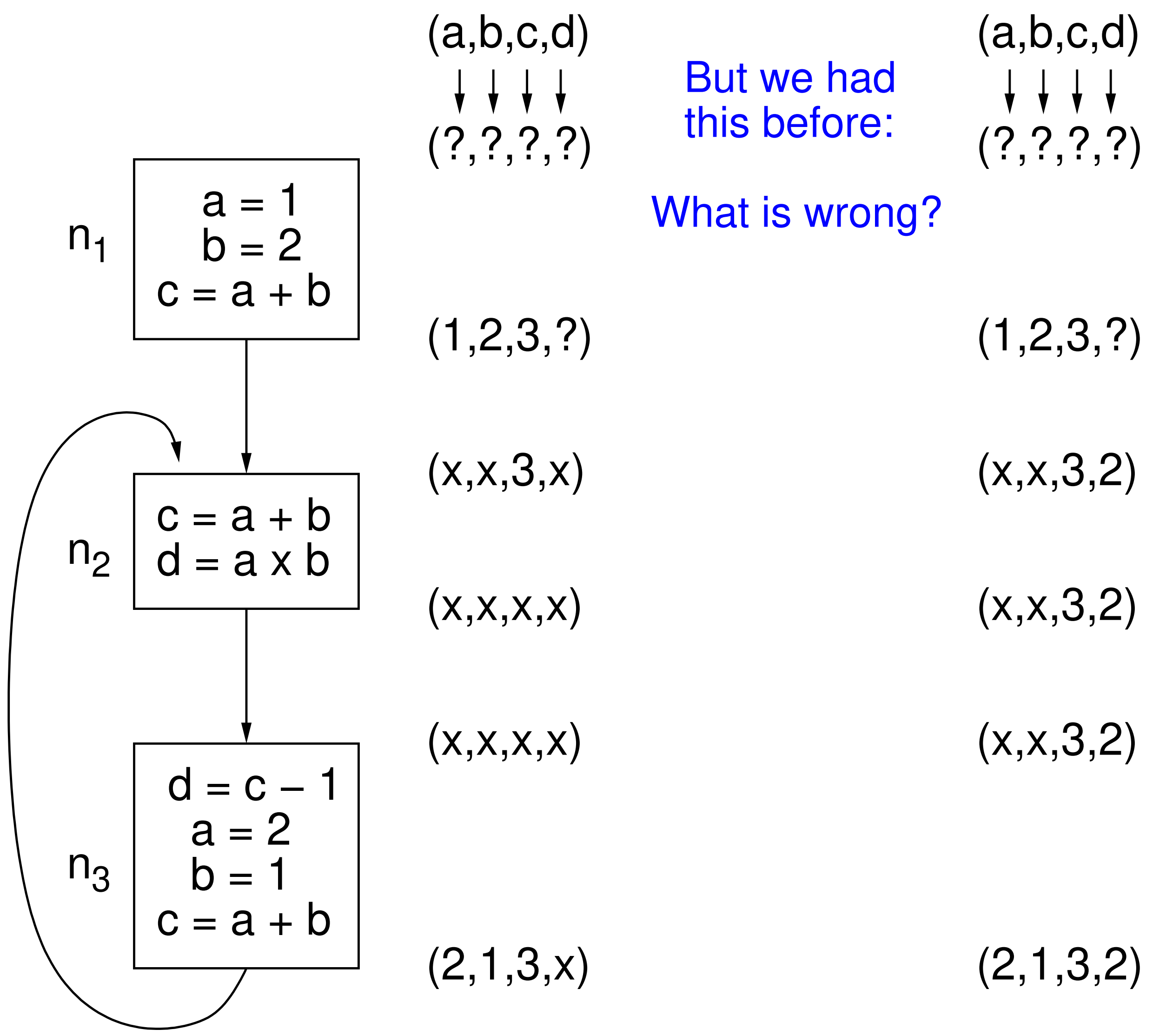
Constant Propagation Movie 2

We proceed carefully according to the defined framework.



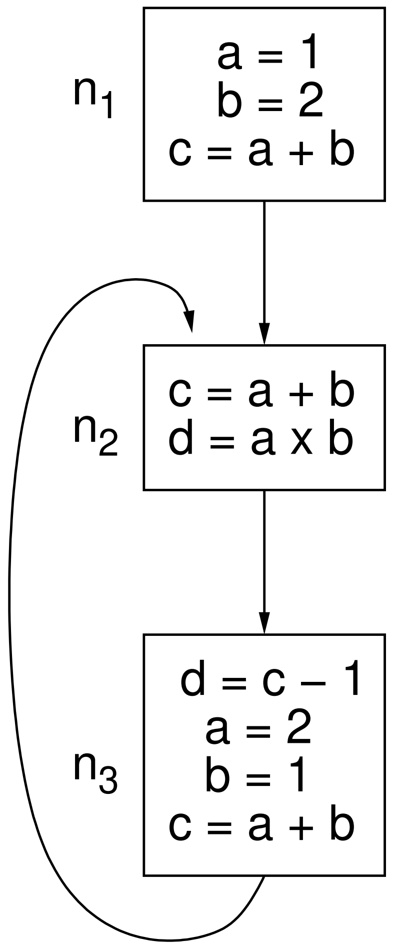
Constant Propagation Movie 2

We proceed carefully according to the defined framework.



Constant Propagation Movie 2

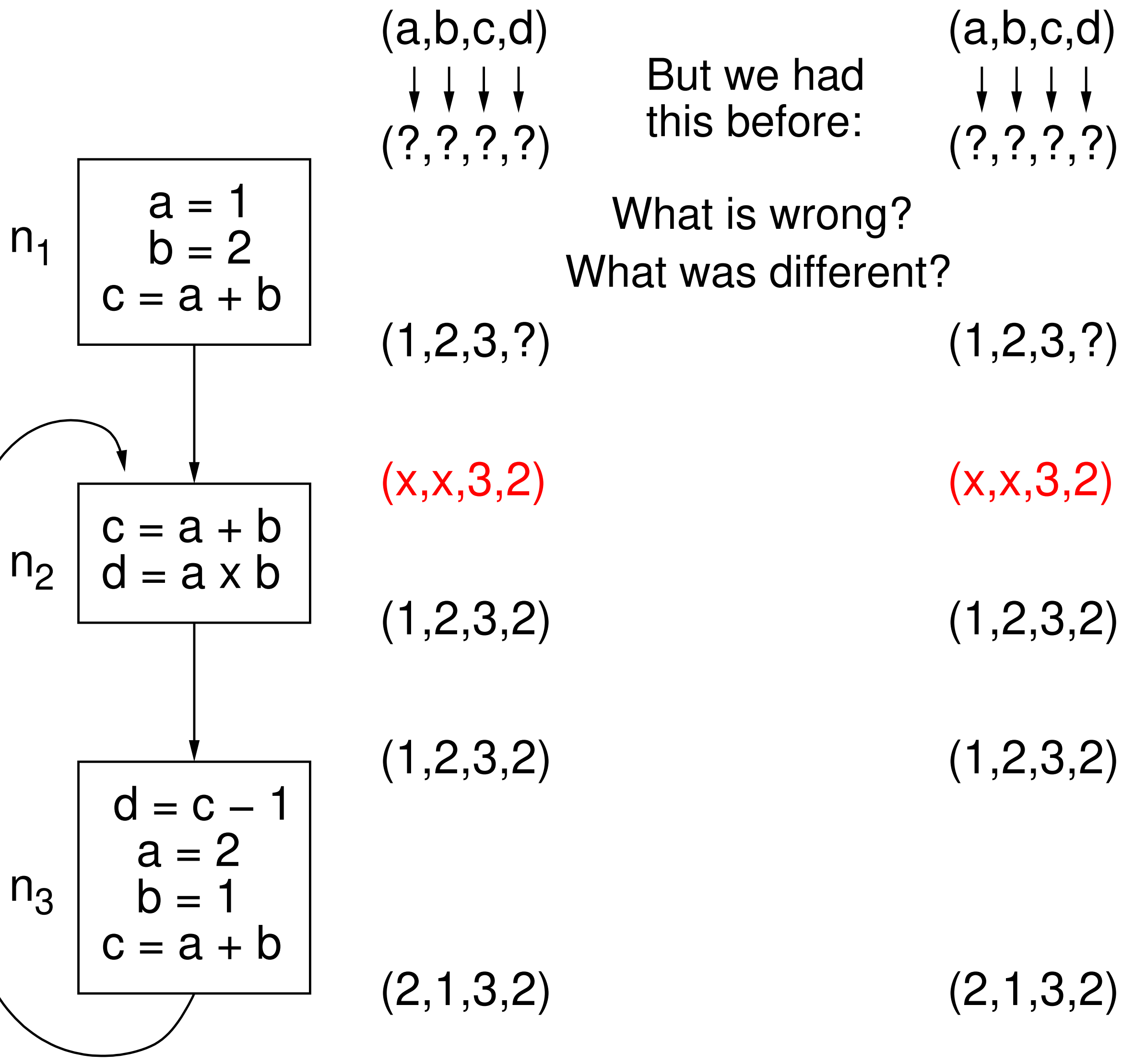
Let us compare the two computations from the last point they were the same.



(a,b,c,d) $\downarrow\downarrow\downarrow\downarrow$ (?,?,?,?)	But we had this before:	(a,b,c,d) $\downarrow\downarrow\downarrow\downarrow$ (?,?,?,?)
	What is wrong?	
	What was different?	
(1,2,3,?)		(1,2,3,?)
(1,2,3,?)		(1,2,3,?)
(1,2,3,2)		(1,2,3,2)
(1,2,3,2)		(1,2,3,2)
(2,1,3,2)		(2,1,3,2)

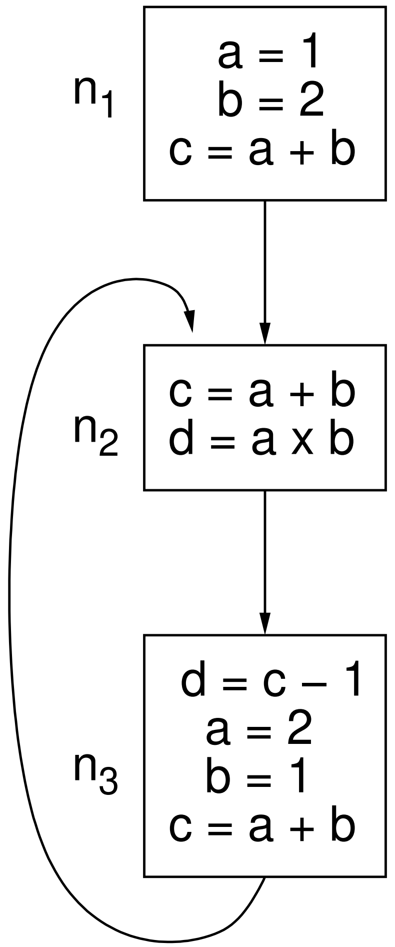
Constant Propagation Movie 2

Still the same.



Constant Propagation Movie 2

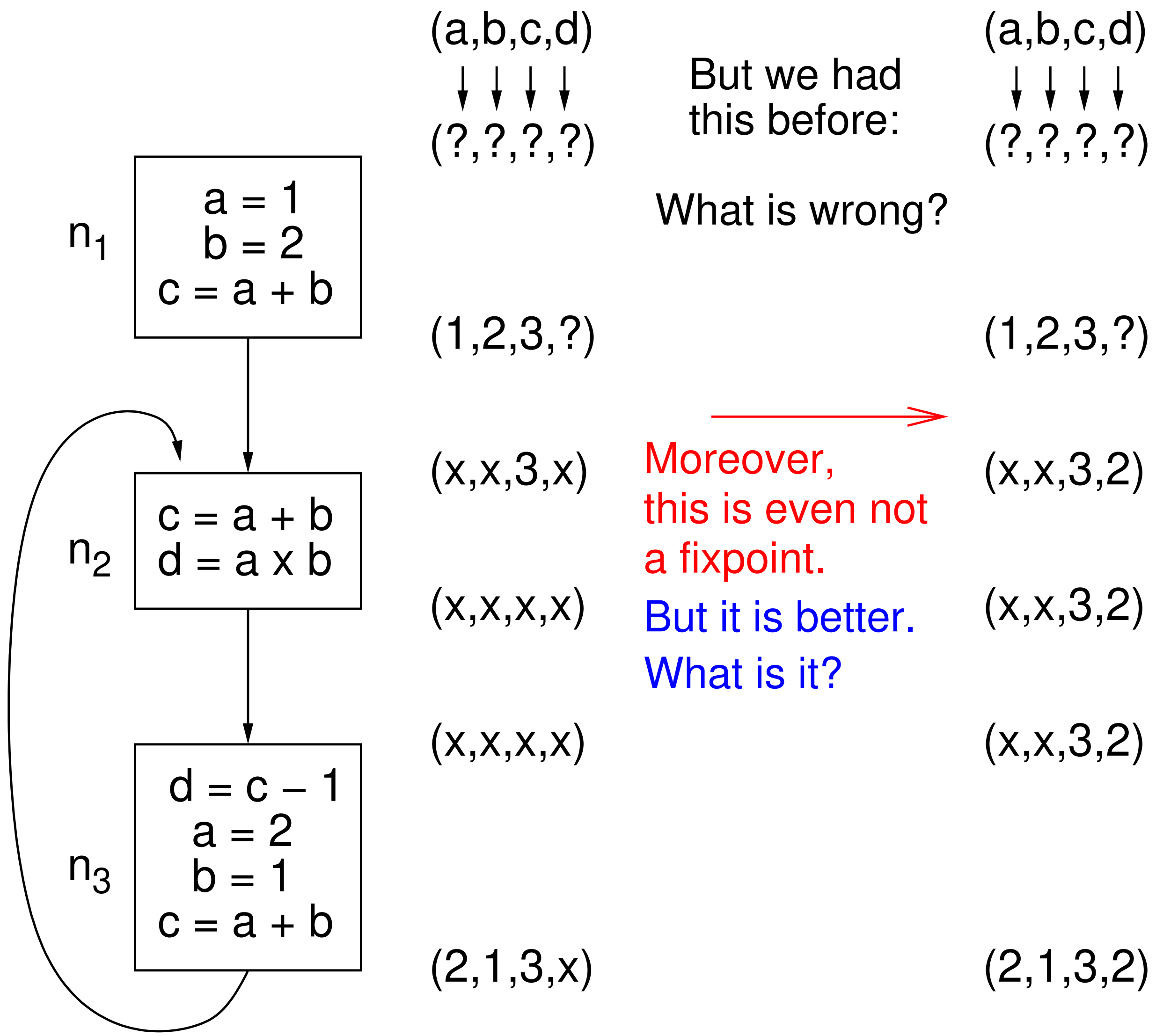
The difference is here: applying the flow function to merged input values X merging the results of the flow function applied to input values.



(a,b,c,d) ↓↓↓↓ (?,?,?,?)	But we had this before:	(a,b,c,d) ↓↓↓↓ (?,?,?,?)
	What is wrong? What was different?	
(1,2,3,?)		(1,2,3,?)
(x,x,3,2)		(x,x,3,2)
(x,x,x,x)		(x,x,3,2)
(1,2,3,2)		(1,2,3,2)
(2,1,3,2)		(2,1,3,2)

Constant Propagation Movie 2

The previous computation does not follow the algorithm and yields a better solution that is not a solution of the fixpoint framework. It is the so called Meet over Paths (MOP) solution.



But we had this before:

What is wrong?

Moreover, this is even not a fixpoint.

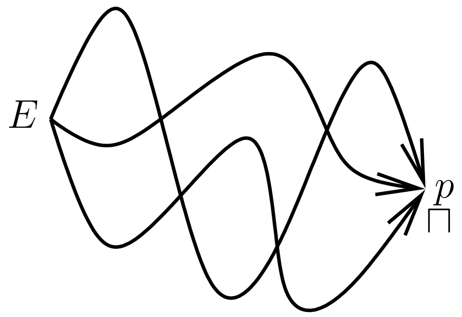
But it is better.

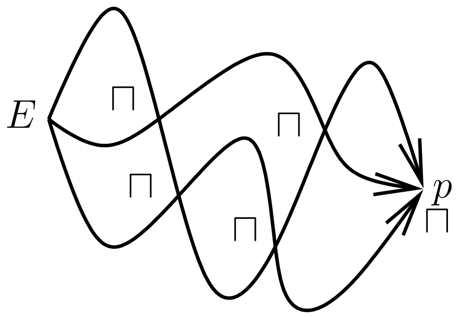
What is it?

Meet over Paths (MoP) Solution

❖ Let us make more precise what we want:

- For each program point p, we want the maximum flow information that is consistent with all flow values that we could get when evaluating all flow paths leading to p.
- This is the so called Meet over Paths solution (MoP).
- More formally: For each program path $\rho = p_1 p_2 \ldots p_k$, $p_1 = Start$, $p_k = p$ leading through the CFG to p, $val_\rho = f_k \circ \cdots \circ f_2 \circ f_1(val_{In_{Start}})$. We want to compute $val_p = \bigsqcap\{val_\rho \mid \rho$ leads to $p\}$. A similar definition applies for the backward case.
- This is a path-based specification, contrary to the specification of MFP, which is edge-based.





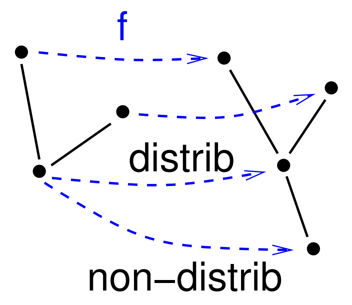
❖ However, the MoP solution is not algorithmically computable already for constant propagation. Notice that, in presence of loops, there is an infinite number of paths.

❖ Fortunately, the MFP solution can still be used. It can be less precise than the MoP solution, but it is safe.

Distributivity of Monotone Frameworks

- A monotone framework is distributive iff

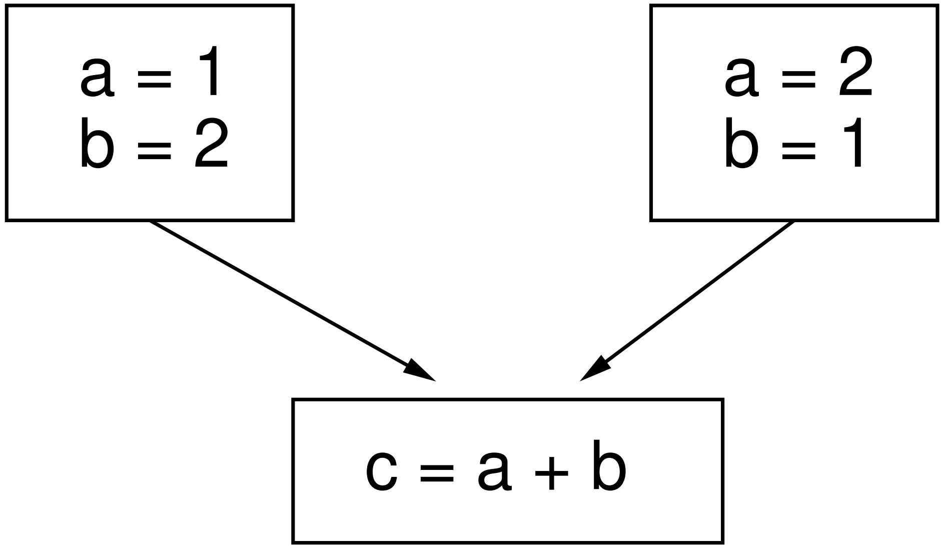
$$\forall f \in \mathcal{F}, \forall x, y \in L : f(x \sqcap y) = f(x) \sqcap f(y)$$



- Why is it important?
  - In a distributive framework, the MFP solution equals the MoP solution.
  - Otherwise, the MoP solution can be better than the MFP solution (however, the MFP solution is still safe).

- The bit-vector frameworks are distributive, constant propagation is not.

Nondistributivity of Constant Propagation



- We have that:
  - $f(\{(a,1),(b,2),(c,?)\} \sqcap \{(a,2),(b,1),(c,?)\}) = \{(a,\mathsf{x}),(b,\mathsf{x}),(c,\mathsf{x})\}$, but
  - $f(\{(a,1),(b,2),(c,?)\}) \sqcap f(\{(a,2),(b,1),(c,?)\}) = \{(a,\mathsf{x}),(b,\mathsf{x}),(c,3)\}$.

A Note on
Interprocedural Data Flow Analysis

Interprocedural Data Flow Analysis

❖ One possible approach to interprocedural data flow analysis is to use flow values in the form of functions (mapping an input to an output).

❖ Another approach is to use procedure/function summaries having basically a form of a cache saying whether a certain procedure/function has already been analysed for some input and, if so, giving the result for that input.

- The cache can be searched wrt. equality of flow values or the $\sqsubseteq$ ordering: one can take a summary for a less informative flow value: more reuse, less precision.

❖ Interprocedural analyses may differ in whether and how much context-sensitive they are, i.e., in how much they track the context of the procedure/function calls (giving the values of the global variables at the call time, the contents of the stack, etc.).

- A trade-off between precision and cost.

❖ For more information, see literature.

- E.g., the books that are mentioned at the end of the slides.

Pointer Analyses

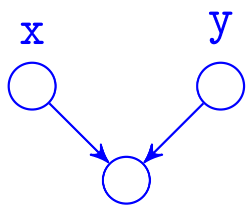
May and Must Aliasing

❖ Let us consider static variables only, no heap, no unbounded data structures, ...

❖

```
      int*x;
      int*y;
        ⋮
p1 : *x = 1;
     *y = 2;
p2 : i = *x + 3;
        ⋮
```

If $\texttt{x} == \texttt{y}$, then x and y alias (point to the same address).



- If x and y certainly alias at p_1, then the statement at p_2 may be replaced by $\texttt{i} = 5$.
- If x and y certainly do not alias at p_1 (i.e., it is not that they may alias), then the statement may be replaced by $\texttt{i} = 4$.

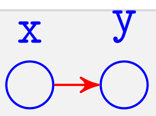
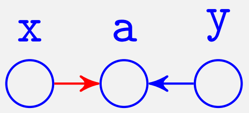
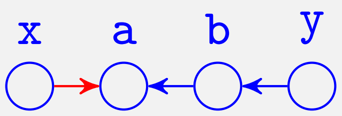
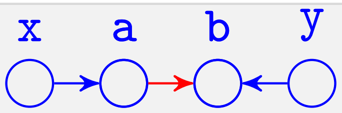
❖ We are interested in the may and must point-to relations on program variables, which may be viewed as directed graphs where nodes are program variables.

❖ These points-to relations contain information about aliasing.

- The must point-to relation gives a must alias equivalence relation on variables.
- The may point-to relation gives a may alias similarity relation on variables.

Flow Sensitive May Point-to Analysis

- Lattice of data flow values $(2^{Vars \times Vars'}, \supseteq)$ with $(\top, \bot, \sqcap) = (\emptyset, Vars \times Vars', \cup)$ where $Vars' = Vars \cup \{\texttt{null}\}$.
- Flow functions: $f_n(X) = Gen_n(X) \cup (X \setminus Kill_n(X))$.
- Flow F: along the control flow.
- Extreme flow variable E: $Start_{In}$.
- Initial value of E: $\{\texttt{x} \rightarrow \texttt{y} \mid \texttt{x} \in Vars, \texttt{y} \in Vars'\}$.

	operation		$Kill_n(X)$	$Gen_n(X)$
Strong upds.	$\texttt{x} = \&\texttt{y}$		$\{\texttt{x} \rightarrow \texttt{a} \mid \texttt{a} \in Vars'\}$	$\{\texttt{x} \rightarrow \texttt{y}\}$
	$\texttt{x} = \texttt{y}$		$\{\texttt{x} \rightarrow \texttt{a} \mid \texttt{a} \in Vars'\}$	$\{\texttt{x} \rightarrow \texttt{a} \mid \texttt{y} \rightarrow \texttt{a} \wedge a \in Vars'\}$;
	$\texttt{x} = {}^*\texttt{y}$		$\{\texttt{x} \rightarrow \texttt{a} \mid \texttt{a} \in Vars'\}$	$\{\texttt{x} \rightarrow \texttt{a} \mid \texttt{y} \rightarrow \texttt{b} \rightarrow \texttt{a} \wedge \texttt{a}, \texttt{b} \in Vars'\}$
Weak upd.	${}^*\texttt{x} = \texttt{y}$		$\emptyset$	$\{\texttt{a} \rightarrow \texttt{b} \mid \texttt{x} \rightarrow \texttt{a} \wedge \texttt{y} \rightarrow \texttt{b} \wedge \texttt{a}, \texttt{b} \in Vars'\}$

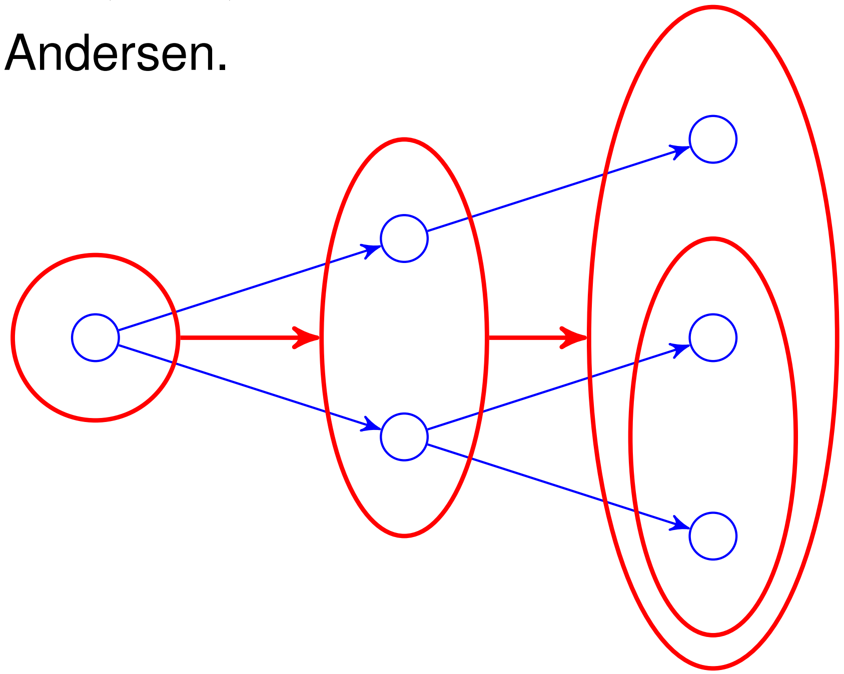
Flow Insensitive Analysis, Andersen’s Algorithm

- Computing a different points-to graph at each program point is expensive.
- An alternative is a flow insensitive analysis.
- The Andersen’s algorithm:
  - Ignore control-flow, i.e., assume that every block is connected with every block.
  - Replace all strong updates by weak updates $(Kill = \emptyset)$.
  - Compute a single points-to relation that holds regardless of the order in which assignment statements are actually executed.
  - The computed points-to relation is the greatest lower bound ($\sqcap$/$\cup$) of all the points-to relations computed by the flow sensitive analysis.
  - Less but still comparably accurate than the flow sensitive analysis.
  - Much cheaper.
  - Requires $\mathcal{O}(n^3)$ time where n is the number of pointer statements.

Flow Insensitive Analysis, Steensgaard's Algorithm

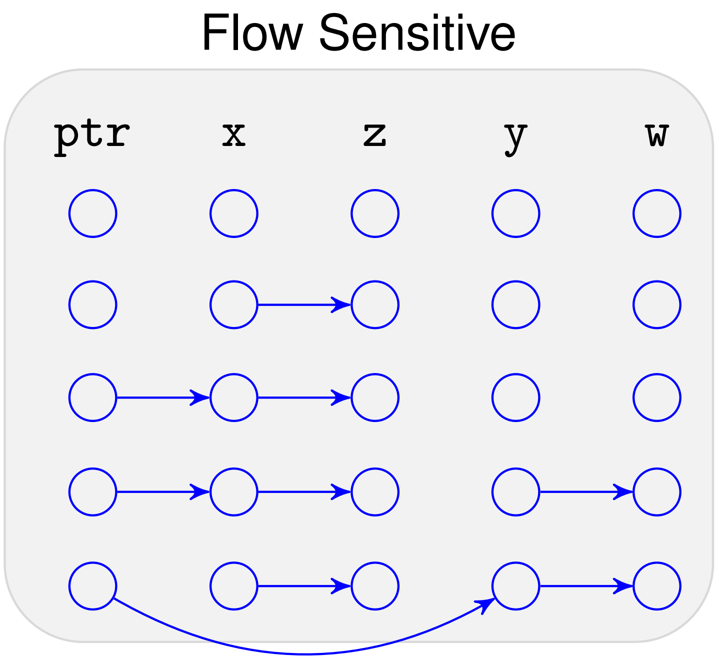
❖ The Steensgaard's algorithm is essentially the Andersen's algorithm simplified by merging nodes that may be pointed-to by the same node.

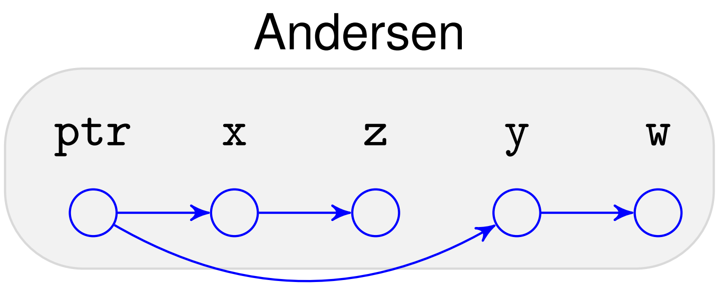
- Start with singleton equivalence classes.
- Merge two classes if a class may point-to both of them.
- Relate a new class with all the classes that its components were related with.
- May be implemented as a single pass through a program, which gives $\mathcal{O}(n\alpha(n))$ time (i.e., almost linear time) algorithm (n is the number of pointer statements and $\alpha(n) = f^{-1}(n)$ where $f(m) = A(m,m)$ for the Ackermann function A).
- Faster but less precise than Andersen.

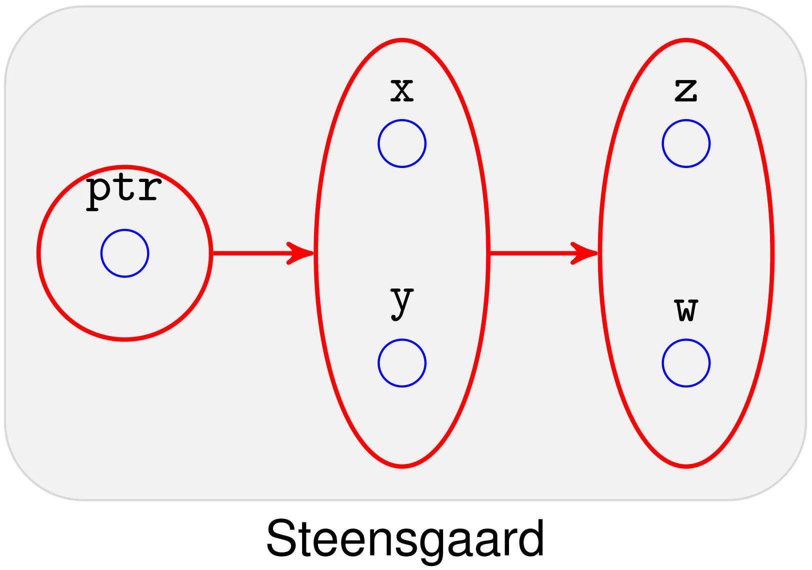


An Example

```
x = &z;
ptr = &x;
y = &w;
ptr = &y;
```







❖ Experience says that flow/context sensitivity does not add that much accuracy so that it would be worth the additional computational cost.

Horowitz and Shapiro

❖ The alias analysis algorithm of Horowitz and Shapiro combines the (usually) faster Steensgaard's and (usually) more accurate Andersen's algorithm:

- Divide variables into k different classes.
- Merge only variables contained in the same class.
- Roughly, an $\mathcal{O}(k^2 n)$ algorithm.
- $k = 1$ gives Steensgaard, $k = |Vars|$ gives Andersen.
- One can iterate over different numbers of classes and different assignments of variables to classes and then take the intersection of the results.
- A good strategy is to consider assignments of variables to classes such that for every pair of variables, it is guaranteed that they belong to different classes for at least one run of the basic algorithm.

Shape Analysis

❖ Unlike alias analysis, shape analysis tracks the shapes of the points-to links, including deep chains of next pointers.

❖ Can be used to check for memory safety (absence of invalid dereferences, double free operations, memory leaks, etc.) as well as for shape invariance (e.g., does one really maintain an unshared, acyclic list somewhere?).

❖ Much more demanding.

❖ Many different approaches based on

- separation logic used in tools like Meta Infer, Space Invader, Slayer, ...,
- symbolic memory graphs used, e.g., in Predator (FIT BUT),
- tree automata used, e.g., in the Forester tool (FIT BUT),
- three valued predicate logic with transitive closure (TVLA),
- WSkS (PALE),
- graph grammars, ...

Literature

Literature

- The slides are mostly based on the book Data Flow Analysis: Theory and Practice by U.P. Khedker, A. Sanyal, and B. Karkare:
  - www.cse.iitb.ac.in/˜uday/dfaBook-web

- Some parts were also taken from the book Principles of Program Analysis by F. Nielson, H. R. Nielson, and C. Hankin.