Discrete Simulation of Sound Wave Propagation

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Abstract: This paper deals with a proposed method of calculating the impulse response of a two-dimensional space by casting sound cones. This method is partially related to ray-casting used in computer graphics. Sound cones are emitted by sound sources, they are moving through the room and reflecting from walls, which change the cones' characteristics. All cones that hit the listener are added together and are used to determine the impulse response of the room between the source and the listener.

Keywords: sound, wave propagation, simulation

1 Introduction

Modern computer applications often include modelling of real world. So far, modelling was often associated with computer graphics, but it is the time to model real sound now. This paper tries to outline some rather simple methods of creating and using two-dimensional models to get the sound characteristics of a closed room. The major simplification is in using only two dimensions instead of three, but once the 2D methods become well understood, it should not be a big problem to move on to full 3D models.

2 Sound Propagation

Sound waves emitted from a source move at their respective propagation speed c through the medium. The speed depends on the physical characteristics of the medium, e.g. for air at room temperature it is roughly $c \doteq 340\,\mathrm{ms}^{-1}$. These waves are propagated uniformly in all directions from the source.

When a sound wave hits an obstacle it is reflected back at the same angle and another wave can be refracted *into* the obstacle. The combined energy of the reflected and refracted waves can be less than or equal to the energy of the original wave. This paper will not deal with refraction for the sake of simplicity and we will assume that the material of the obstacle will absorb the refracted wave quickly.

The returning reflected wave has lost some of its energy due to the nature of the process, either by dissipation or by refraction. The loss of energy can be expressed by gain $0 \leq g \leq 1$. An obstacle can absorb all the energy in the case of $g = 0$ or it can even reflect the wave ideally without loss if $g = 1$. A typical case would lie somewhere between these two extremes. Gains greater than one are not allowed since they could result in an unstable system (the energy would build up indefinitely with every reflection).

The phase of the reflected wave is inverse to the original, that would mean a phase shift of 180 degrees if the wave carried a periodical signal. This inversion can be viewed as using negative reflection gain $-g$ in all subsequent equations.

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The medium that conveys the wave causes another loss of energy. This loss is proportional to the distance travelled in it. Attenuation a_e describes the loss of energy E in relation to distance d. Since the effects of attenuation multiply with every unit distance travelled, the attenuation results in an exponential curve

$$E_d = E_0 \, \mathrm{e}^{-a_e d}. \tag{1}$$

The energy of the whole wave is spreading as the wave propagates through the medium. In three-dimensional space (3D), the intensity at any given place in the wave falls with the square of distance (d^2). On the contrary, in two-dimensional space (2D), the energy spreads only in one direction (perpendicular to the direction of propagation), and thus the intensity is inversely proportional to distance d. Since we are discussing 2D modelling here, the intensity (amplitude) of a wave that has travelled distance d can be expressed as

$$I_d = I_0 \, d^{-1}. \tag{2}$$

The loss of energy rule applies to the intensity in a similar way, so if we combine equations (1) and (2), we finally get

$$I_d = I_0 \, d^{-1} \, \mathrm{e}^{-a_i d}. \tag{3}$$

3 Simulation Algorithm

This simulation considers the model to behave as a linear system and it tries to calculate its impulse response. This response can be used later to compute the actual output for any desirable set of input signals.

3.1 Impulse Response

Impulse response $h(n)$ uniquely characterizes any discrete linear time-invariant system (LTI). It can be obtained by sending Dirac impulse $\delta(n)$ into system input and watching the response coming from the output. Since any input signal can be represented by a sum of time-shifted weighted impulses, the output of an LTI system can be represented by a sum of time-shifted weighted impulse responses, thus

$$y(n) = \sum_{i=0}^{n} h(i) \, x(n-i) = h(n) * x(t). \tag{4}$$

If the input signal $x(n)$ equals to $\delta(n)$, the system output will be

$$y(n) = \sum_{i=0}^{n} h(i) \, \delta(n-i) = h(n). \tag{5}$$

The principle of superposition which is valid in any LTI system makes it possible to calculate impulse responses of every sound source separately of the others. If all the sources will be driven by the same input signal, their responses can be added together into a single response. If the sources are considered independent of each other, their responses must be kept separate until they are convolved with their input signals.

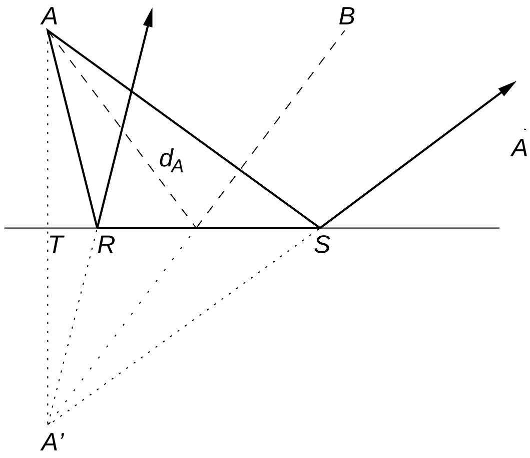

Fig. 1: Cone reflection on a wall

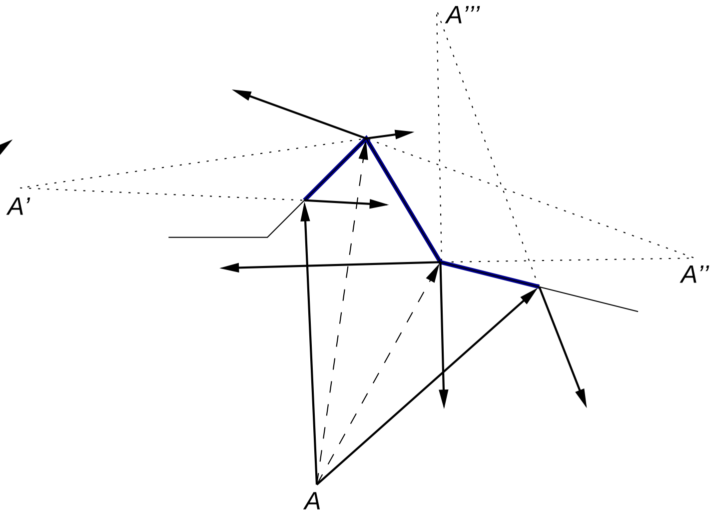

Fig. 2: Cone breaking into three cones

3.2 Reflections on Walls

Main focus of this project is aimed at proper calculation of all sound reflections which take place in the model. The propagation of waves is calculated purely geometrically by the rule of visibility. If a wave can *see* the wall, it can (and will) reflect from it.

Waves are represented by *cones*, which originate in their source and spread with travelled distance. This representation is similar to rays of light which are used in ray-tracing or ray-casting graphical algorithms. Since casting individual *rays* of sound would be too computationally demanding and most rays would miss the listener's ear anyway, we consider the sound to be emitted in cones (groups of rays). Thus if a cone hits listener's ear, the individual hitting ray can be simply determined and accounted for.

The sound sources cast 360 degree wide cones that break into narrower cones once they hit the walls (see fig. 2). The number of reflected cones increases with every wall boundary (corner). If a cone can fully fit on a wall, it will not be broken any further.

When a sound cone passes over a *listener*, exact propagation delay and attenuation level are calculated from the travelled distance and the number of reflections. These two values are added to a list of (delay, level) pairs which can be converted into impulse response later. The impulse responses obtained by simulating all the sound sources can be simply added together to get an *overall* impulse response (assuming that all sources emit the same signal). Every listener will have its own distinct impulse response. Typically, a human head would consist of two listeners (ears) with opposite orientation and about 20cm apart.

The reflection geometry can be calculated simply by moving the source from point A to an imaginary source point A' as shown in fig. 1. The cone will pretend to come from the imaginary source as seen from point B. The points R, S define the reflection line around which the point A is mirrored.

In order to get the position vector of point A', we must first determine the *orthogonal projection* of point A into vector $\overrightarrow{RS}$. Point A will be represented by a vector $\overrightarrow{RA}$. Orthogonal projection T (vector $\overrightarrow{RT}$) can be calculated with *dot point* vector operator as in equation (6) and the position of point A' can be obtained simply by vector subtraction:

$$\overrightarrow{RT} = \overrightarrow{RS}\,\frac{\overrightarrow{RA}\cdot\overrightarrow{RS}}{\|\overrightarrow{RS}\|}, \tag{6}$$

$$\overrightarrow{RA}' = 2\,\overrightarrow{RT} - \overrightarrow{RA}. \tag{7}$$

The impulse response after reflection can be determined by shifting it by the propagation delay d_A/c and by multiplying it by reflection gain $-g_A$. Various walls can have different reflection gains, so the subscript $_A$ denotes the parameters of a specific wall which is hit after passing through point A. Here we consider only one reflection between two points A and B, where d_A is the distance between point A and the center of reflection (length of trajectory).

$$h_B(t) = -g_A\,h_A\left(t - \frac{d_A}{c}\right) \tag{8}$$

In this case, discrete impulse response $h(n)$ is shown as a continuous time quantity $h(t)$ to avoid time quantization errors. The conversion between n and t is determined by sampling interval T, hence $t = nT$.

The overall impulse response caused by multiple reflections can be obtained by chaining simple reflections. Let's suppose that function $\mathcal{F}_X$ denotes single reflection at point X as shown in equation (8), then we get

$$\begin{aligned} h_B &= \mathcal{F}_A(h_A) \\ h_C &= \mathcal{F}_B(h_B) = \mathcal{F}_B(\mathcal{F}_A(h_A)) \\ h_D &= \mathcal{F}_C(h_C) = \mathcal{F}_C(\mathcal{F}_B(\mathcal{F}_A(h_A))) \\ &\cdots. \end{aligned} \tag{9}$$

The impulse response h_A actually does not change its shape, it only changes its amplitude and phase shift with every reflection. Thus the impulse response at point X can also be expressed as

$$h_X(t) = \left(\prod_{i=A}^{X} -g_i\right) h_A\left(t - \sum_{i=A}^{X}\frac{d_i}{c}\right), \tag{10}$$

where d_i means the distance travelled between two adjacent reflections.

It can be seen that in this simple model the impulse response remains actually the same, which means that the resulting $h(t)$ will contain attenuated initial Dirac impulse $\delta(t)$ shifted in time. Since there usually exist more paths in which the cone can reach the listener, the final response will be a sum of these impulses. This result will not suffice, because it will be perceived by a human listener just as a series of echoes.

3.3 Diffusion

Sound diffusion is an important factor which must be taken into account in order to create realistic sounds. If no diffusion were considered, all sound cones would perfectly reflect of mirror-like walls and even after several reflections would sound very sharply. Diffusion can help to add *softness* to reflected sounds and to create better illusion of space. Real surfaces rarely behave as mirrors, so diffusion is needed to increase the level of realism of this model.

Diffusion can be modelled in two different ways:
– convolution with special reflection impulse response $h_r(t)$
– scattering of direction vectors

3.3.1 Reflection Impulse Response

This model assumes you to know the impulse response $h_r(t)$ of sound diffusion. The resulting response after a reflection at point A can be determined by convolving previous response $h_A(t)$ with $h_{r_{A(t)}}$

$$h_B(t) = h_A(t) * h_{r_A}(t). \tag{11}$$

This method is simple, but it requires the knowledge of various reflection impulse responses in order to work properly. The responses can be measured experimentally or another kind of wave propagation simulation can be performed to obtain them. Either way, it is not trivial to determine the values of $h_r(t)$ so that they acceptably match the reality.

3.3.2 Scattering of Direction Vectors

When a sound wave hits uneven surface, it does not reflect perfectly. The individual sound *rays* hit randomly oriented *facets* of the surface and reflect from them in various directions. The majority of the rays will still reflect almost perfectly. Some oppositely oriented rays may cancel out each other's deviations and thus they would count as normally reflected rays. The probability of reflecting a given ray in a given angle can be determined statistically or it can be approximated by Normal (Gaussian) random distribution, where $\mu = 0$ and σ^2 is proportional to the *roughness* of the surface.

The diffusion is modelled by a polar function $D(\varphi)$ whose parameter $\varphi \in (-\pi; \pi)$ determines the deviation from the direction of reflection. Let's assume that $D(0) = 1$, i.e. the signal in the original direction is transferred without a change of intensity. The range of values of φ for which $D(\varphi) \neq 0$ determines the character of diffusion.

The impulse response cannot use the properties of equation (8), that make it possible to *collapse* all reflections into a mere product of gains and a sum of propagation delays to get a simple result as in equation (10). Instead, single *rays* have to be calculated separately what makes this method much more computationally difficult.

Let's assume that we want to know which *rays* will reflect from point A to B. If there were no diffusion, only one ray would satisfy this condition. Because diffusion can change the direction vector of reflected rays, there are more rays that *bend* and reach point B.

$$h_B(t) = -g_A \int_{-\pi}^{\pi} h_A\left(t - \frac{d_A(\varphi)}{c}\right) D(\varphi)\, d\varphi \tag{12}$$

Here unlike in the previous equations distance $d_A(\varphi)$ is not a constant, but it is a function of angle φ. The greater the angle φ, the greater is the difference between $d_A(\varphi)$ and d_A. Obviously, $d_A(0) = d_A$. The shape of $d_A(\varphi)$ function depends on the mutual position of points A and B and also on the position and angle of the reflection plane.

The nature of $d_A(\varphi)$ will in effect *smooth* the resulting impulse response $h_B(t)$ which will in turn positively influence the perceived sound of reflection.

4 Model of the Scene

Modelled scene is described by a simple C-like language. Any number of points, sectors and sources can be declared to create the desired model.

A simple model description is shown here:

```
# points
point A = [5, 5];
point B = [15, 5];
point C = [15, 15];
point D = [20, 20];
point E = [30, 20];
point F = [30, 30];
point G = [15, 30];
point H = [5, 20];
point S1 = [10, 8];

# bounding sector
sector World {
  points [0, 0], [100, 0],
         [100, 100], [0, 100];
  velocity = 330;
  reflection = 0;
  attenuation = 0;
};
```

```
# simulated sector
sector Room {
  inside World;
  points A, B, C, D, E, F, G, H;
  reflection = 0.8;
  attenuation = 3;
};

# sound source
source Src1 {
  position S1;
  direction 120;
  intensity = 50;
};

# listener
listener Mic1 {
  position [25, 25];
  direction 180;
};
```

5 Conclusions

The simulation method presented in this paper seems to be very promising. Despite the complexity of the algorithm, the calculation can still be considered quite effective. Its results tend to reflect the nature of the simulated room quite satisfactorily, even though the method is still in its infancy.

The implementation process is not yet finished. New features will be added as the core of the simulation system stabilizes. So far, the program uses only 2-dimensional sound cones for simplicity. Full 3-dimensional simulation is expected to be developed in the future.

Bibliography

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