ISS – Numerical exercise II.

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Exercise 1 – convolution in discrete time domain

Given a signal in discrete time domain:

$$x[n] = \begin{cases} 2 & \text{for} \quad n = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

and a system's impulse response:

$$h[n] = \begin{cases} -1 & \text{for} \quad n = 0 \\ 0 & \text{for} \quad n = 1 \\ 1 & \text{for} \quad n = 2 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the system's output: using convolution $y[n] = x[n] \star h[n]$.

Exercise 2 – convolution in continuous time domain

Given a signal in continuous time domain:

$$x(t) = \begin{cases} 2 & \text{for} \quad -2 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

and a system's impulse response:

$$h(t) = \begin{cases} -3 & \text{for} \quad 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the system's output: using convolution $y(t) = x(t) \star h(t)$.

Exercise 3 – Fourierova series of a harmonic signal

A harmonic signal $C_1 \cos(\omega_1 t + \phi_1)$ has the following parameters: $C_1 = 10$, $f_1 = 1$ kHz, $\phi_1 = \frac{\pi}{8}$ rad.

a) Estimate coefficients of a Fourier series: $x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk\omega_1 t}$.

b) Draw the spectrum (coefficients' moduls and arguments c_k).

c) Draw the signal and the complex exponentials it is composed of.

Exercise 4 – Fourier series of a periodic signal

Real periodic signal has the following FS coefficients:: $c_1 = 4e^{j\frac{\pi}{4}}$ and $c_{-2} = 2e^{-j\frac{\pi}{2}}$. Express them in terms of cosine functions.

Exercise 5 – Fourier series of a periodic square signal

A periodic square signal is defined as:

$$x(t) = \begin{cases} 10 & \text{for} \quad -1 \text{ ms} < t < 1 \text{ ms} \\ 0 & \text{otherwise} \end{cases}$$

with period $T_1 = 6$ ms. Estimate the FS coefficients and draw graphs of its modules and arguments.

Exercise 6 – Fourier series of a signal with inverse sign

Estimate and plot FS coefficients of a similas signal but with an inverse sign:

$$x(t) = \begin{cases} -10 & \text{for} \quad -1 \text{ ms} < t < 1 \text{ ms} \\ 0 & \text{otherwise} \end{cases}$$

with period $T_1 = 6$ ms.

Exercise 7 – Fourier series of a shifted signal

Estimate and plot FS coefficients of a periodic square signal:

$$x(t) = \begin{cases} 10 & \text{for} \quad -0.5 \text{ ms} < t < 1.5 \text{ ms} \\ 0 & \text{otherwise} \end{cases}$$

with period $T_1 = 6$ ms.

Exercise 8 – Fourier transform

Given a signal $x(t) = 3\delta(t+4)$. Estimate and plot its spectral function $X(j\omega)$.

Exercise 9 – Fourier transform

Square signal is defined as:

$$x(t) = \begin{cases} 0.001 & \text{for} \quad -1 \text{ hodina} < t < 1 \text{ hodina} \\ 0 & \text{otherwise} \end{cases}$$

Estimate and plot its spectral function $X(j\omega)$.

Exercise 10 – Linearity

The examined system will be a normal lake. At one lakeside, fishermen bring carps for breeding. To the other lakeside, schoolbuses bring kids for a picknick and a swim. The input to the system is the number of lorries bringing carps (each carries 1000) and the number of buses with schoolkids (each drives 40). The output is the number of living creatures in the lake.

- Is the system linear?
- Will be the system linear if instead of carps, fishermen start bringing piranhas?