METHODS FOR SOLVING ELECTRICAL CIRCUITS

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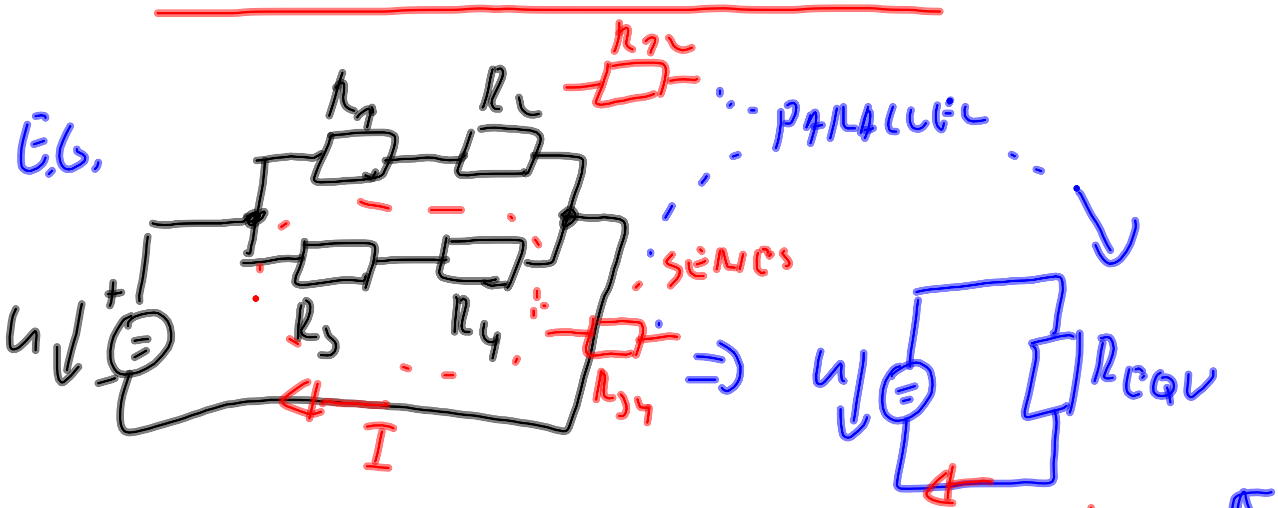
6.10.2017

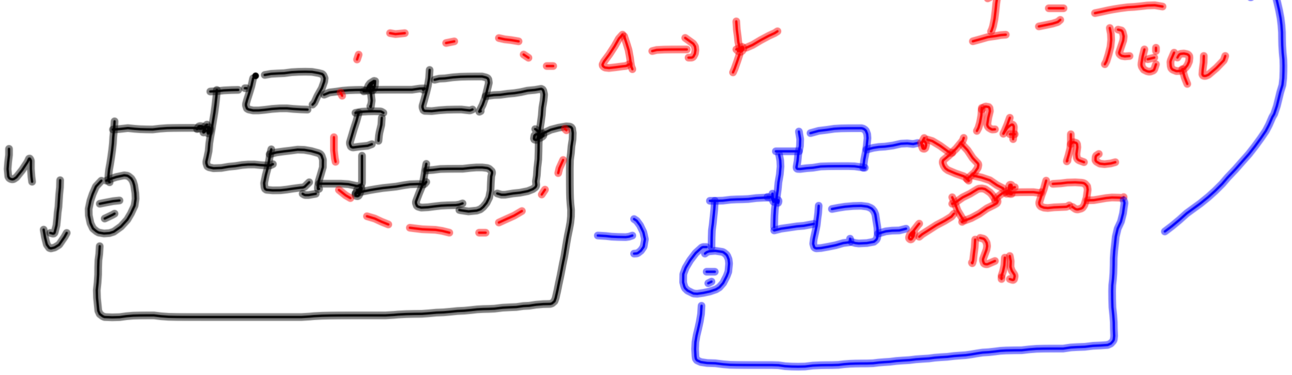
a) METHODS FOR SOLVING THE ELECTR. CIRCUITS WITH 1 POWER SUPPLY

→ METHOD OF SIMPLIFICATION

E.G.



$$I = \frac{U}{R_{EQV}}$$



b) METHODS FOR SOLVING THE EL. CIRCUITS WITH MORE POWER SUPPLIES

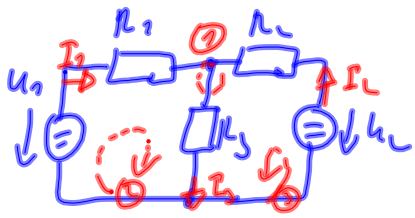
→ LOOP CURRENT METHOD

→ NODE VOLTAGE METHOD

→ THEVENIN'S THEOREM

SOLVING OF EL. CIRCUITS USING SLAE (SYSTEM OF LINEAR ALGEBRAIC EQUATIONS)

EX Solve the el. circuit using Kirchhoff's laws



PARAMETERS:

$U_1 = 5V$
$U_2 = 20V$
$R_1 = 20\Omega$
$R_2 = 20\Omega$
$R_3 = 40\Omega$

CONSTRUCTION OF SLAE:

① $I_1 + I_2 - I_3 = 0$ ... I. Kirchh. law

② $\overbrace{R_1 \cdot I_1}^{U_{R1}} + R_3 I_3 - U_1 = 0$ ... II. K. L.

③ $R_2 I_2 + R_3 I_3 - U_2 = 0$

$I_1, I_2, I_3 = ?$

MATRIX-VECTOR NOTATION:

$$\begin{array}{c} \begin{matrix} I_1 & I_2 & I_3 \end{matrix} \\ \begin{pmatrix} 1 & 1 & -1 \\ R_1 & 0 & R_3 \\ 0 & R_2 & R_3 \end{pmatrix} \end{array} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ U_1 \\ U_2 \end{pmatrix}$$

→ SOLVING SLAE, E.G. CRAMER'S RULE

SOLING DETERMINANTS USING SARRUS RULE

$$D_S = \begin{vmatrix} 1 & 1 & -1 \\ R_1 & 0 & R_3 \\ 0 & R_2 & R_3 \end{vmatrix} = (1 \cdot 0 \cdot R_3) + (1 \cdot R_3 \cdot 0) + (-1 \cdot R_1 \cdot \ldots$$

$$- (0 \cdot 0 \cdot (-1)) - (R_2 \cdot R_3 \cdot 1) - (R_3 \cdot R \ldots$$

$$D_{I_1} = \begin{vmatrix} 0 & 1 & -1 \\ U_1 & 0 & R_3 \\ U_2 & R_2 & R_3 \end{vmatrix} = R_3 U_2 - U_1 R_2 - R_3 U_1 =$$

$$= 800 - 100 - 200 = \underline{\underline{500}}$$

$$D_{I_2} = \begin{vmatrix} 1 & 0 & -1 \\ R_1 & U_1 & R_3 \\ 0 & U_2 & R_3 \end{vmatrix} = U_1 R_3 - R_1 U_2 - U_2 R_3 =$$

$$= 200 - 400 - 800 =$$

$$= -\underline{\underline{1000}}$$

$$D_{I_3} = \begin{vmatrix} 1 & 1 & 0 \\ R_1 & 0 & U_1 \\ 0 & R_2 & U_2 \end{vmatrix} = -R_2 U_1 - U_2 R_1 =$$

$$= -100 - 400 = \underline{\underline{-500}}$$

$$I_1 = \frac{D_{I_1}}{D_S} = \frac{500}{-2000} = \underline{\underline{-0.25\,A}}$$

$$I_2 = \frac{D_{I_2}}{D_S} = \frac{-1000}{-2000} = \underline{\underline{0.5\,A}}$$

$$I_3 = \frac{D_{I_3}}{D_S} = \frac{-500}{-2000} = \underline{\underline{0.25\,A}}$$

CONTROL OF OUR RESULTS:

→ CHECKING CALCULATION BY SUBSTIT. THE CALCULATED VALUES TO THE ORIGINAL SLAE

① $I_1 + I_2 - I_3 = 0 \Rightarrow -0{,}25 + 0{,}5 - 0{,}25 = 0$ ✓

② $\underbrace{R_1 I_1}_{U_{R1}} + \underbrace{R_3 I_3}_{U_{R3}} - U_1 = 0$

$-5 + 10 - 5 = 0$ ✓

③ $\underbrace{R_2 I_2}_{U_{R2}} + \underbrace{R_3 I_3}_{U_{R3}} - U_2 = 0$

$10 + 10 - 20 = 0$ ✓

→ TEST (COMPARE) OUR SOLUTION IN MATLAB

MULTIPLY THE SYSTEM WITH INVERSE MATRIX A^{-1}

$$A\vec{x} = \vec{b}$$

$$\underbrace{A^{-1}A}_{E}\vec{x} = A^{-1}\vec{b}$$

$$\vec{x} = A^{-1}\vec{b}$$

$$A^{-1}A = AA^{-1} = E \quad \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

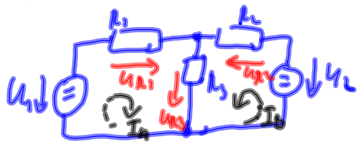
IDENTITY MATRIX

$$\vec{x} = A \backslash \vec{b}$$

GAUSS ELIMINATION METHOD

LOOP CURRENT METHOD

EX PARAMETERS : U_1, U_2, R_1, R_2, R_3



IN EVERY LOOP HOLDS:
$\sum U = 0$ (II. K.L.)

OHM'S LAW:
$U_{R1} = R_1 I_A$
$U_{R3} = R_3 \cdot (I_A + I_B)$
$U_{R2} = R_2 \cdot I_B$

IN THE DIRECTION OF LOOP (I_A) :

$U_{R1} + U_{R3} - U_1 = 0$

$$\boxed{R_1 \cdot I_A + R_3 (I_A + I_B) = U_1}$$

IN THE DIRECTION OF LOOP (I_B) : $U_{R2} + U_{R3} - U_2 = 0$

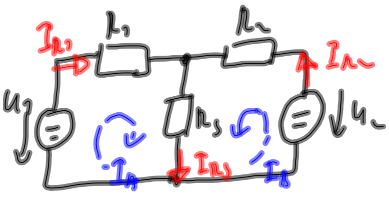
$$\boxed{R_2 I_B + R_3 (I_A + I_B) = U_2}$$

2 EQUATIONS FOR 2 UNKNOWNS $(I_A, I_B$

→ SOLVE SLAG

$\Rightarrow$ $I_A = \ldots$
$I_B = \ldots$

COMPUTATION OF THE CURRENTS IN OUR EL. CIRCUIT: I_{R1}, I_{R2}, I_{R3} !



WE KNOW I_A

WE LOOK AT THE ORIENTATIO (DIRECTION) OF I_A, I_B, I_{R1}, I_{R}

AND COMPUTE:

$$\boxed{\begin{array}{l} I_{R1} = I_A, \; I_{R2} = I_B \\ I_{R3} = I_A + I_B \end{array}}$$

FINALLY WE CAN COMPUTE VOLTAGE

$U_R = R \cdot I$