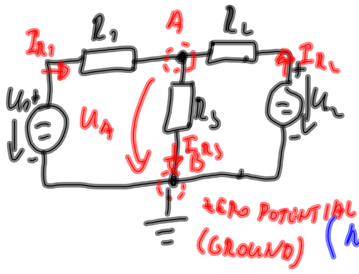
NODE VOLTAGE METHOD

lite
20.10.2017



NODE = THE PLACE IN EL. CIRCUIT WHERE AT LEAST 3 WIRES (COMPONENTS) ARE CONNECTED

(NODES A, B)

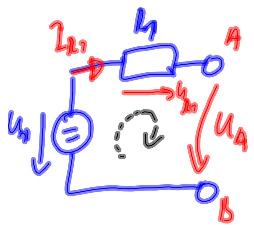
SET ONE NODE AS "REFERENCE" (REFERENTIAL NODE)

SUM OF THE CURRENTS IN THE NODE

$$\sum I = 0 \quad (I.\ K.L.)$$

$$I_{R_1} + I_{R_2} - I_{R_3} = 0$$

ALTERNATIVE EL. CIRCUIT FOR I_{R_1}

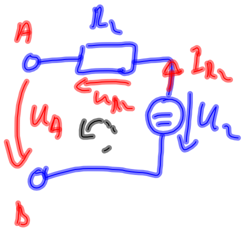


$(\sum U = 0)$ ... II. K.L.

$$R_1 \cdot I_{R_1} + U_A - U_1 = 0$$

$$\boxed{I_{R_1} = \frac{U_1 - U_A}{R_1}}$$ ✓

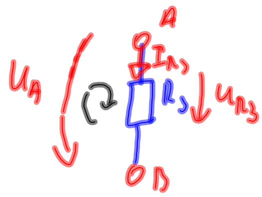
ALTERNATIVE EL. CIRCUIT FOR I_{R2}



$$R_2 \cdot I_{R2} + U_A - U_2 = 0$$

$$\boxed{I_{R2} = \frac{U_2 - U_A}{R_2}}$$ ✓

ALTERNATIVE EL. CIRCUIT FOR I_{R3}



$$R_3 I_{R3} - U_A = 0$$

$$\boxed{I_{R3} = \frac{U_A}{R_3}} = \frac{U_{R3}}{R_3}$$ ✓

NOTE: 'REVERSE' LOOP (↺)

$$U_A - R_3 I_{R3} = 0 \rightarrow I_{R3} = \frac{-U_A}{-R_3}$$

SUBSTITUTE I_{R1}, I_{R2}, I_{R3} INTO

$$I_{R1} + I_{R2} - I_{R3} = 0$$

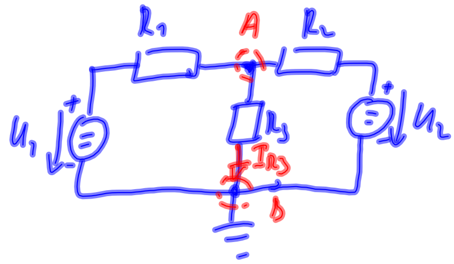
$$\frac{\overbrace{U_1 - U_A}^{U_{R1}}}{R_1} + \frac{\overbrace{U_2 - U_A}^{U_{R2}}}{R_2} - \frac{\overbrace{U_A}^{U_{R3}}}{R_3} = 0$$

$\Rightarrow U_A = \ldots$

OF COURSE WE KNOW CURRENTS (I_{R1}, I_{R2}, I_{R3})

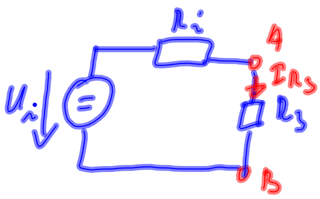
AND VOLTAGES (U_{R1}, U_{R2}, U_{R3})

THÉVENIN'S THEOREM



$I_{R_3} = ?$

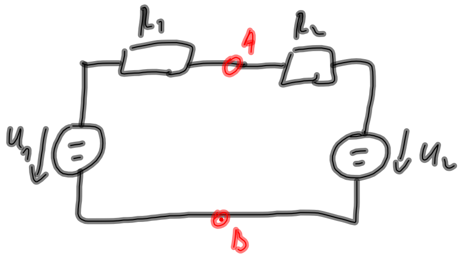
⇒ EQUIVALENT EL. CIRCUIT



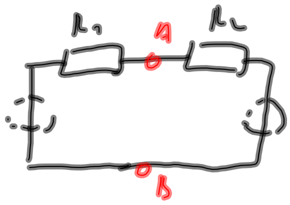
$$I_{R_3} = \frac{U_i}{\underbrace{R_i + R_3}_{R_{EQV}}}$$

$U_i, R_i = ?$

$R_i = ...?$ 1) REDRAW THE EL. CIRCUIT WITHOUT $\underline{R_3}$

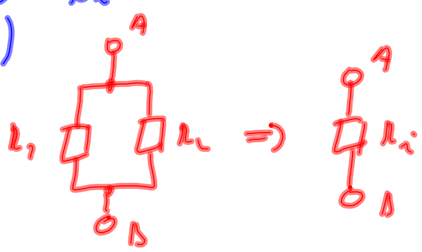


↳ REPLACE VOLTAGE SOURCES BY A "SHORT CIRCUIT"



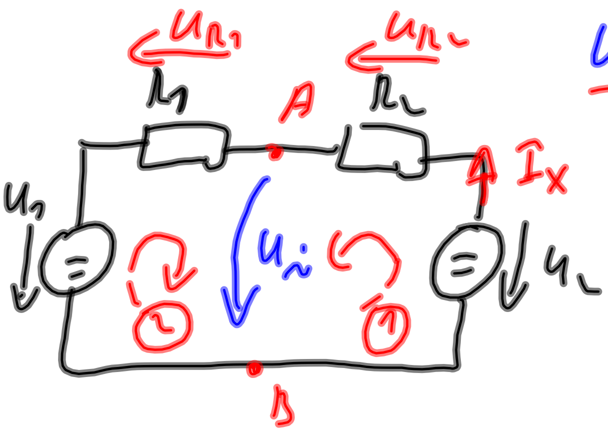
→ COMPUTE RESISTANCE BETWEEN TERMINALS A,B

$R_{EQV} \equiv R_i$
(R_{AB})



$$\boxed{R_i = \frac{R_1 \cdot R_2}{R_1 + R_2}}$$ ... PARALLEL CONNECTION R_1, R_2

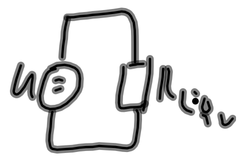
$U_i = ???$ → REDRAW THE EL. CIRCUIT WITHOUT R_3 AND SOLVE THE VOLTAGE U_i AS "OPEN CIRCUIT" A,B



COMPUTE I_x (II. K.L.)

(1) $R_1 I_x + R_2 I_x + U_1 - U_2 = 0$

$$I_x = \frac{\overbrace{U_2 - U_1}^{U}}{\underbrace{R_1 + R_2}_{R_{EQU}}} \equiv$$

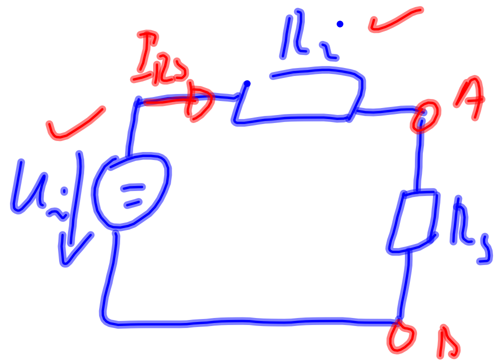


II. K.L.

(2) $-R_2 I_x + U_i - U_1 = 0$

$$\boxed{U_i = U_1 + R_2 I_x}$$

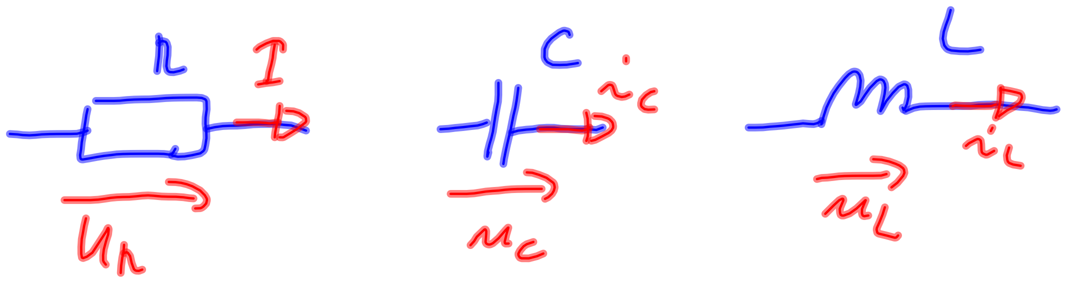
→ LAST STEP



$$\boxed{I_{R_3} = \frac{U_i}{R_i + R_3}}$$

$$\boxed{U_{R_3} = R_3 I_{R_3}}$$

OHM'S LAW

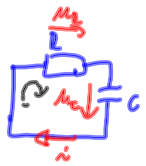


$$I = \frac{U_R}{R} \qquad u_C' = \frac{i_C}{C} \qquad i_L' = \frac{u_L}{L}$$

$$u_C' = \frac{du_C}{dt} \qquad i_L' = \frac{di_L}{dt}$$

TRANSIENT STATE IN RLC ELECT. CIR.

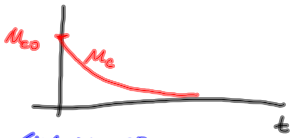
1) EXAMPLE – RC CIRCUIT (DISCHARGING OF THE CAPACITY)



PARAMETERS: R, C,

$u_c(0) = u_{c0} \neq 0$

CONSTANT (INITIAL VOLTAGE ON CAPACITANCE)



$u_c = f(t) = ?$

SYSTEM OF EQUATIONS:

1) $u_R = R \cdot i$ ... OHM'S LAW

2) $u_R + u_c = 0$ ... II. K. L.

3) $u_c' = \frac{i}{C}$, $u_c(0) = u_{c0}$

SET UP SOME EQUATION WITH UNKNOWNS u_c', u_c

→ SUBSTITUTE 1) INTO 2)

$\underbrace{R \cdot i}_{u_R} + u_c = 0 \rightarrow i = \frac{-u_c}{R}$

→ SUBSTITUTE i INTO 3) EQ.

$u_c' = -\frac{u_c}{R \cdot C}$

→ FIND ANALYTIC SOLUTION OF THE 1st ORDER ORDINARY

(*) DIFF. EQ.

$\boxed{u_c' + \frac{1}{RC} \cdot u_c = 0}, \; u_c(0) = u_{c0}$

HOMOGENOUS

→ GENERAL SOLUTION:

$u_c(t) = K(t)\, e^{\lambda t}$

→ CHARACTERISTIC EQUATION

$\lambda + \frac{1}{RC} = 0$ $\quad (u_c' = \lambda, \; u_c = 1, \; \cancel{u_c'' = \lambda^2}, \; \cancel{u_c''' = \lambda^3})$

$\lambda = -\frac{1}{RC}$ ... τ ... TIME CONSTANT OF RC CIRCUIT

→ SUBSTITUTE λ INTO GENERAL SOLUTION

$u_c(t) = K(t)\, e^{-\frac{t}{RC}}$

??