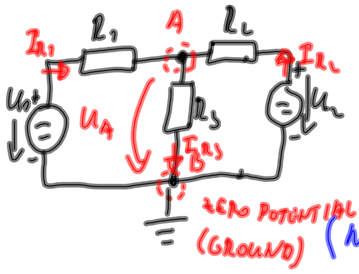
NODE VOLTAGE METHOD

IEL
20.10.2017



ZERO POTENTIAL (GROUND)

NODE = THE PLACE IN EL. CIRCUIT WHERE AT LEAST 3 WIRES (COMPONENTS) ARE CONNECTED

(NODES A, B)

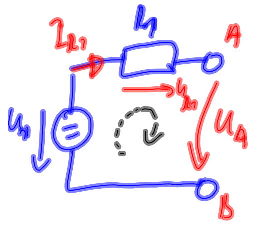
SET ONE NODE AS "REFERENCE" (REFERENTIAL NODE)

SUM OF THE CURRENTS IN THE NODE

$$\sum I = 0 \quad (I.K.L.)$$

$$I_{R1} + I_{R2} - I_{R3} = 0$$

ALTERNATIVE EL. CIRCUIT FOR I_{R1}

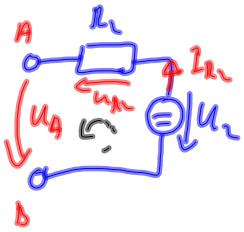


$(\sum U = 0)$ ... II. K.L.

$$R_1 \cdot I_{R1} + U_A - U_1 = 0$$

$$\boxed{I_{R1} = \frac{U_1 - U_A}{R_1}}$$ ✓

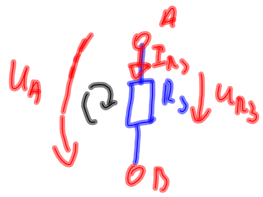
ALTERNATIVE EL. CIRCUIT FOR I_{R2}



$$R_2 \cdot I_{R2} + U_A - U_2 = 0$$

$$\boxed{I_{R2} = \frac{U_2 - U_A}{R_2}}$$ ✓

ALTERNATIVE EL. CIRCUIT FOR I_{R3}



$$R_3 I_{R3} - U_A = 0$$

$$\boxed{I_{R3} = \frac{U_A}{R_3}} = \frac{U_{R3}}{R_3}$$ ✓

NOTE: 'REVERSE' LOOP (↺)

$$U_A - R_3 I_{R3} = 0 \rightarrow I_{R3} = \frac{-U_A}{-R_3}$$

SUBSTITUTE I_{R1}, I_{R2}, I_{R3} INTO

$$I_{R1} + I_{R2} - I_{R3} = 0$$

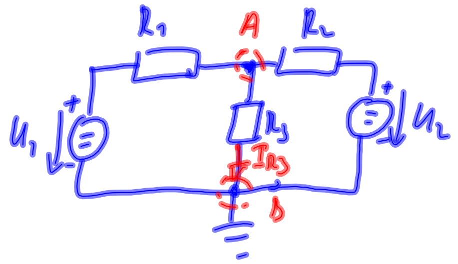
$$\frac{\overbrace{U_1 - U_A}^{U_{R1}}}{R_1} + \frac{\overbrace{U_2 - U_A}^{U_{R2}}}{R_2} - \frac{\overbrace{U_A}^{U_{R3}}}{R_3} = 0$$

$\Rightarrow U_A = \dots$

OF COURSE WE KNOW CURRENTS I_{R1}, I_{R2}, I_{R3}

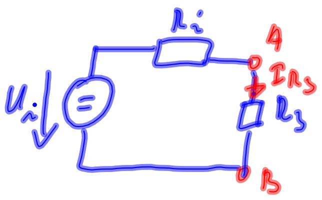
AND VOLTAGES U_{R1}, U_{R2}, U_{R3}

THÉVENIN'S THEOREM



$I_{R_3} = ?$

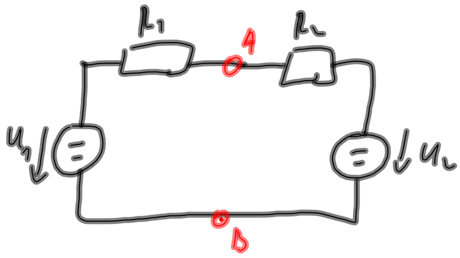
⇒ EQUIVALENT EL. CIRCUIT



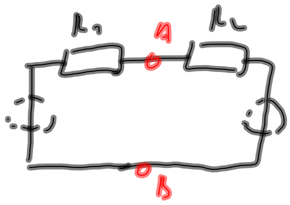
$$I_{R_3} = \frac{U_i}{\underbrace{R_i + R_3}_{R_{EQV}}}$$

$U_i, R_i = ?$

$R_i = \ldots$ 1) REDRAW THE EL. CIRCUIT WITHOUT R_3

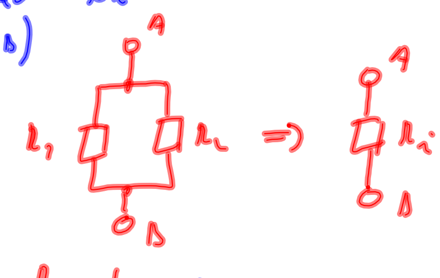


↳ REPLACE VOLTAGE SOURCES BY A "SHORT CIRCUIT"



→ COMPUTE RESISTANCE BETWEEN TERMINALS A,B

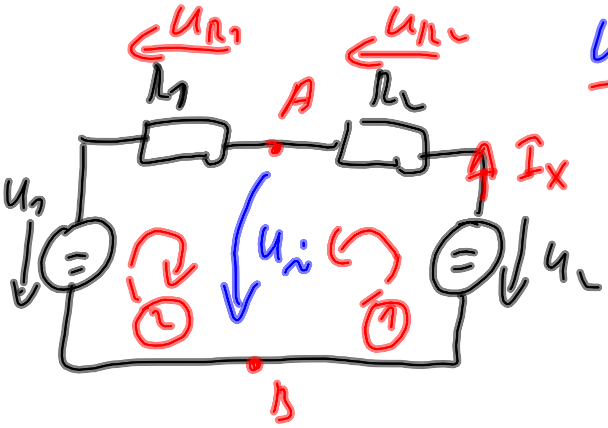
$R_{EQV} \equiv R_i$
(R_{AB})



$$\boxed{R_i = \frac{R_1 \cdot R_2}{R_1 + R_2}}$$ ... PARALLEL CONNECTION R_1, R_2 ✓

$U_i = ???$ → REDRAW THE EL. CIRCUIT WITHOUT R_3

AND SOLVE THE VOLTAGE U_i AS "OPEN CIRCUIT" A,B



COMPUTE I_x (II. K.L.)

(1) $R_1 I_x + R_2 I_x + U_1 - U_2 = 0$

$$I_x = \frac{\overbrace{U_2 - U_1}^{U}}{\underbrace{R_1 + R_2}_{R_{EQV}}}$$

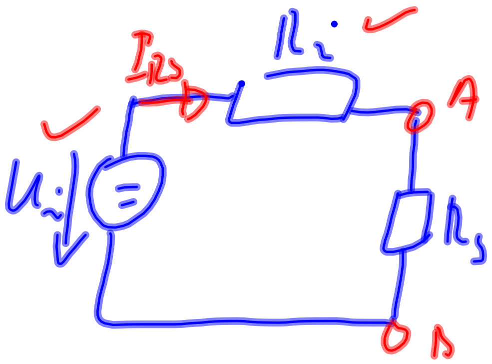
≡

II. K.L.

(2) $-R_2 I_x + U_i - U_1 = 0$

$$\boxed{U_i = U_1 + R_2 I_x}$$

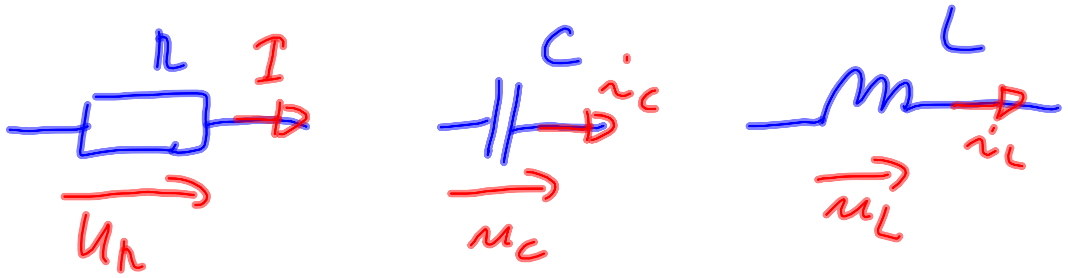
→ LAST STEP



$$\boxed{I_{R3} = \frac{U_i}{R_i + R_3}}$$

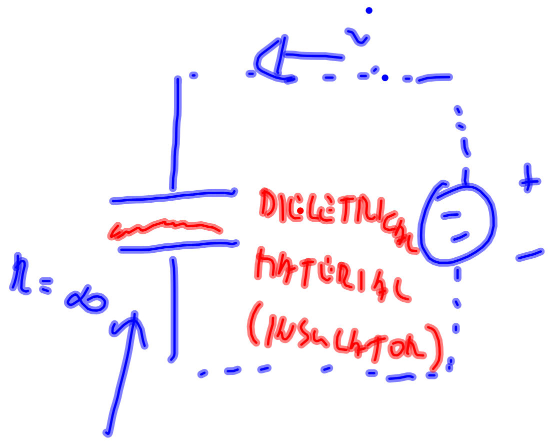
$$\boxed{U_{R3} = R_3 I_{R3}}$$

OHM'S LAW



$I = \frac{U_R}{R}$ $\qquad u_C' = \frac{i_C}{C}$ $\qquad i_L' = \frac{u_L}{L}$

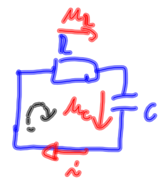
$u_C' = \frac{du_C}{dt}$ $\qquad i_L' = \frac{di_L}{dt}$



EL. COMPONENT CAPACITOR – STORES EL. ENERGY IN AN EL. FIELD

TRANSIENT STATE IN RLC ELECT. CIR.

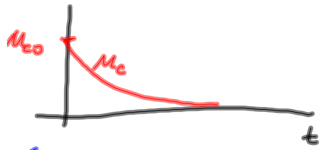
1) EXAMPLE – RC CIRCUIT (DISCHARGING OF THE CAPACITY)



PARAMETERS: R, C,

$u_C(0) = u_{C0} \neq 0$

CONSTANT (INITIAL VOLTAGE ON CAPACITANCE)



$u_C = f(t) = ?$

SYSTEM OF EQUATIONS:

1) $u_R = R \cdot i$ ... OHM'S LAW

2) $u_R + u_C = 0$ ... II. K. L.

3) $u_C' = \frac{i}{C}$, $u_C(0) = u_{C0}$

SET UP SOME EQUATION WITH UNKNOWNS u_C', u_C

→ SUBSTITUTE 1) INTO 2)

$$\underbrace{R \cdot i}_{u_R} + u_C = 0 \rightarrow i = -\frac{u_C}{R}$$

→ SUBSTITUTE i INTO 3) EQ.

$$u_C' = -\frac{u_C}{R \cdot C}$$

→ FIND ANALYTIC SOLUTION OF THE 1st ORDER ORDINARY DIFF. EQ.

(*) $$\boxed{u_C' + \frac{1}{RC} \cdot u_C = 0, \quad u_C(0) = u_{C0}}$$

HOMOGENOUS

→ GENERAL SOLUTION:

$$u_C(t) = K(t)\, e^{\lambda t}$$

→ CHARACTERISTIC EQUATION

$\lambda + \frac{1}{RC} = 0$ $\quad (u_C' = \lambda,\ u_C = 1,\ \cancel{u_C'' = \lambda^2,\ u_C''' = \lambda^3})$

$\lambda = -\frac{1}{RC}$ ... τ ... TIME CONSTANT OF RC CIRCUIT

→ SUBSTITUTE λ INTO GENERAL SOLUTION

$$u_C(t) = K(t)\, e^{-\frac{t}{RC}}$$

– SUBSTITUTE u_c AND u_c' INTO (*)

$$u_c' = K'(t)\, e^{-\frac{t}{RC}} - \frac{K(t)}{RC}\, e^{-\frac{t}{RC}}$$

$$K'(t)\, e^{-\frac{t}{RC}} - \cancel{\frac{K(t)}{RC}\, e^{-\frac{t}{RC}}} + \cancel{\frac{K(t)}{RC}\, e^{-\frac{t}{RC}}} = 0$$

$$K'(t)\, e^{-\frac{t}{RC}} = 0$$

INTEGRATE

$$K'(t) = 0 \quad / \int$$

$$K(t) = \underline{\underline{K}} \quad \dots \text{ INTEGRATION CONSTANT}$$

GENERAL SOLUTION

$$u_c(t) = \underline{\underline{K}}\, e^{-\frac{t}{RC}}$$

→ SUBSTITUTING INITIAL VALUE

$$u_c(0) = u_{c0}$$

$$u_{c0} = K\, e^{-\frac{0}{RC}} = 1$$

$$\underline{K = u_{c0}}$$

PARTICULAR SOLUTION

$$\boxed{u_c(t) = u_{c0}\, e^{-\frac{t}{RC}}}$$

TEST OF THE SOLUTION:

$t = 0$: $u_c = u_{c0}\, e^{-\frac{0}{RC}} = 1 \Rightarrow u_c(0) = u_{c0}$

$t \to \infty$: $u_c = u_{c0}\, e^{-\infty} = 0 \rightarrow u_c(\infty) = 0$

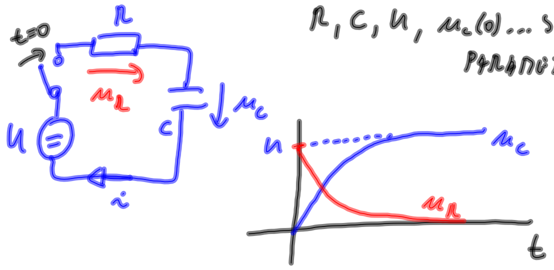
→ SUBSTITUTE u_c AND u_c' INTO (*)

$$u_c' = -\frac{u_{c0}}{RC}\, e^{-\frac{t}{RC}}$$

$$-\frac{u_{c0}}{RC}\, e^{-\frac{t}{RC}} + \frac{1}{RC}\left(u_{c0}\, e^{-\frac{t}{RC}}\right) = 0$$

$$0 = 0 \quad \checkmark$$

CHARGING OF CAPACITOR



$R, C, U, u_C(0)$ ... SET PARAMETERS

II. K. L.

$$i \cdot R + u_C = U$$

(**) $$\boxed{RC\,u_C' + u_C = U} + u_C(0) = u_{C0}$$

NON-HOMOGENEOUS

→ GENERAL SOLUTION

→ CHARACTERISTIC EQ ...

→ SUBSTITUTE u_C AND u_C' INTO (**)

$$RC\,k'(t)\,e^{-\frac{t}{RC}} = U$$

$$\int k'(t) = \frac{U}{RC}\int e^{\frac{t}{RC}} \quad / \int$$

$$k(t) = \frac{U}{\cancel{RC}} \cdot \cancel{RC}\, e^{\frac{t}{RC}} + \ell$$

NOTE:

$$\int e^{\frac{t}{RC}} = RC\,e^{\frac{t}{RC}},$$

$$\left(RC\,e^{\frac{t}{RC}}\right)' = e^{\frac{t}{RC}}$$

$$k(t) = U e^{\frac{t}{RC}} + \ell$$

INTEGRATION CONSTANT

→ SUBSTITUTE $k(t)$ INTO GENERAL SOLU

$$u_C = \left(U e^{\frac{t}{RC}} + \ell\right) \cdot e^{-\frac{t}{RC}} = U + \ell\, e^{-\frac{t}{RC}}$$

→ PARTICULAR SOLUTION: SUBSTITUTE INITIAL VALUE $u_C(0) = u_{C0}$

$$u_{C0} = U + \ell\, e^{0} = 1$$

$$\ell = u_{C0} - U$$

$$\boxed{u_C = U + (u_{C0} - U)\, e^{-\frac{t}{RC}}}$$