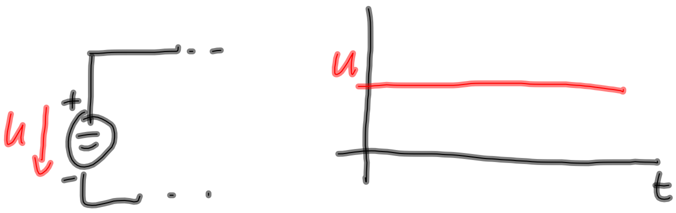
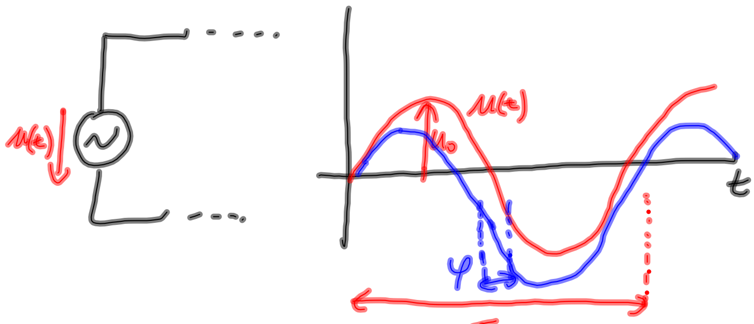
# RLC CIRCUITS WITH AC POWER SUPPLY

1.12.2017
8.12.2...

– DC POWER SUPPLY → CONSTANT VOLTAGE
(DIRECT CURRENT)




– AC POWER SUPPLY → HARMONIC SIGNAL
(ALTERNATING CURRENT)




$u(t) = U_0 \sin(\omega t + \varphi)$ ... PHASE SHIFT

$U_0$ ... PEAK VOLTAGE [V]

$\omega$ ... ANGULAR VELOCITY $\left[\frac{rad}{s}\right]$

$\omega = 2\pi f \rightarrow f$ .. FREQUENCY [Hz]

$f = \frac{1}{T} \rightarrow T = \frac{1}{f}$ ... PERIOD OF THE SIGNAL [

→ STEADY-STATE ANALYSIS

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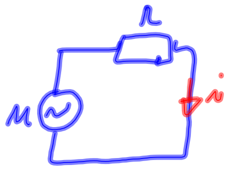
| AMPLITUDES AND PHASE )

1) RESISTOR IN AC CIRCUIT

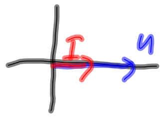



$u = U_0 \sin(\omega t)$

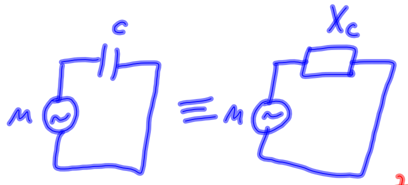
$i = ?$ CURRENT IS IN THE PHASE WITH POWER VOLTAGE ($\varphi = 0°$)

$i = I_0 \sin(\omega t)$

$I_0 = \frac{U_0}{R}$ $\quad i = \frac{u}{R}$




2) CAPACITOR IN AC CIRCUIT




$X_C$ ... CAPACITIVE REACTANCE [Ω]

$X_C = \frac{1}{\omega C}$

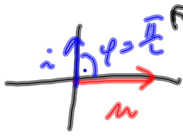
$Z_C$ ... IMPEDANCE [Ω]

$Z_C = \frac{1}{j\omega C}$ $\quad X_C$

$i = \frac{u}{Z_C} = \frac{u}{\frac{1}{j\omega C}} = j\omega C \cdot u$

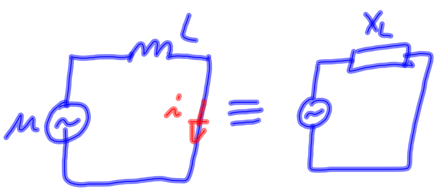
$= \omega C U_0 \sin(\omega t + \frac{\pi}{2})$

ROTATION ⇒ IN TIME




3) INDUCTOR IN AC CIRCUIT

$X_L$ ... INDUCTIVE REACTANCE [Ω]




$X_L = \omega L$

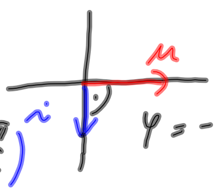
$Z_L = j X_L$
: IMPEDANCE

$i = \frac{u}{jX_L} = \frac{u}{j\omega L} \cdot \frac{-j\omega L}{-j\omega L} = \frac{-j\cancel{\omega L}\,u}{-j^2\,\omega^{\cancel{2}} L^{\cancel{2}}}$

$-j^2 = -(-1)$

$= \frac{-j\,u}{\omega L}$

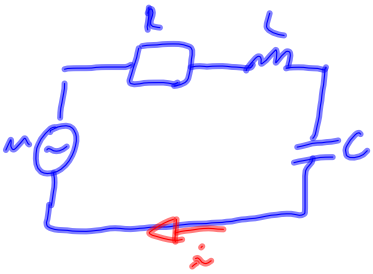
$i = \frac{U_0}{\omega L} \sin(\omega t - \frac{\pi}{2})$




$\varphi = -\frac{\pi}{2} = -90°$

# 4) RLC IN SERIES

$u, R, C, L, \omega, U_0$
$u = U_0 \sin(\omega t)$



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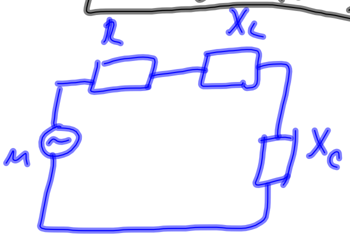
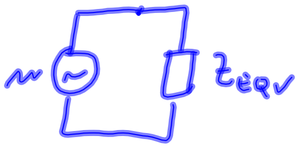


$i = ?$

$$i = \underset{?}{\underline{\underline{I_0}}} \sin(\omega t + \underset{?}{\underline{\underline{\varphi}}})$$

→ METHOD OF SIMPLIFICATION



$Z_{EQV}$ ... IMPEDANCE: $[\Omega]$

$$Z_{EQV} = R + jX$$

$R$: RESISTANCE (REAL PART)

$X$: REACTANCE (IMAGINARY PART)

$$Z_{EQV} = R + (\overbrace{X_L + X_C}^{X})\, j$$

$$Z_{EQV} = R + j\omega L - \frac{j}{\omega C} \quad \ldots \quad \frac{1}{j\omega C} \cdot \frac{-j}{-j} = \frac{-j}{\omega C}$$

$$Z_{EQV} = \underbrace{R}_{Re} + j\underbrace{\left(\omega L - \frac{1}{\omega C}\right)}_{Im}$$

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COMPUTING AMPLITUDES AND PHASE SHIFTS

$$I_0 = \frac{U_0}{Z_{EQV}} \quad \ldots \text{ OHM'S LAW}$$

$$I_0 = A + Bj$$

$$|I_0| = \sqrt{A^2 + B^2}$$

$$\varphi = \mathrm{arctg}\left(\frac{B}{A}\right)$$

$$U_R = I_0 \cdot R$$

$$U_C = I_0 |Z_C| \qquad Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

NOTE: BE CAREFUL OF

→ CAN BE $|U_C| = |U_L|$

YES, WHEN $|X_C| = |X_L| \Rightarrow \mathrm{Im}(Z_{EQV}) = \emptyset$

$$\omega_{RES} L - \frac{1}{\omega_{RES} C} = \emptyset \quad / \cdot \omega_{RES} C$$

RESONANCE EFFECT

$$\omega_{RES}^2 LC - 1 = \emptyset$$

$$\omega_{RES} = \sqrt{\frac{1}{LC}}$$

$$2\pi f_{RES} = \sqrt{\frac{1}{LC}}$$

$$f_{RES} = \frac{1}{2\pi\sqrt{LC}}$$

... RESONANCE FREQUENCY

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→ $f = f_{RES}$ ... ($X = 0$) RESISTIV CHARACTER (REACTANCE)

$U_0$ IS ON RESISTOR $R$

$f < f_{RES}$ ... $X < 0$ ... CAPACITIVE CHARACTER

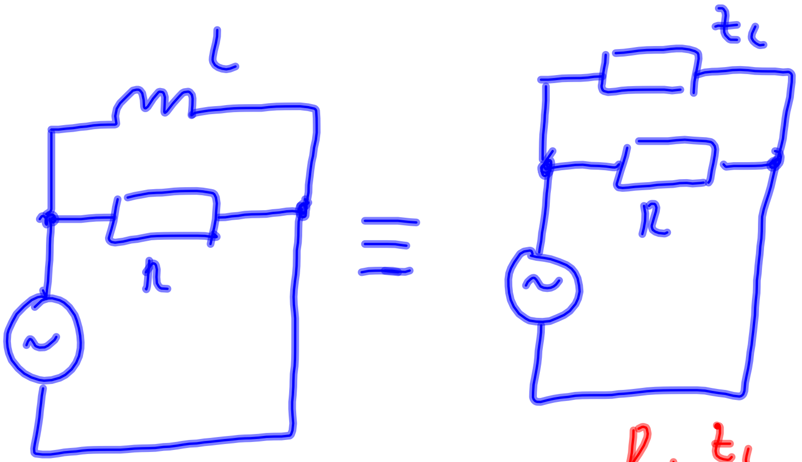
$f > f_{RES}$ ... $X > 0$ ... INDUCTIVE CHARACTER

→ PRACTICAL USAGE: EG. FILTERS

LOW PASS FILTERS (CAPACITOR OUTPUT VOLTAGE)

HIGH PASS (INDUCTOR OUT. VOLT.)

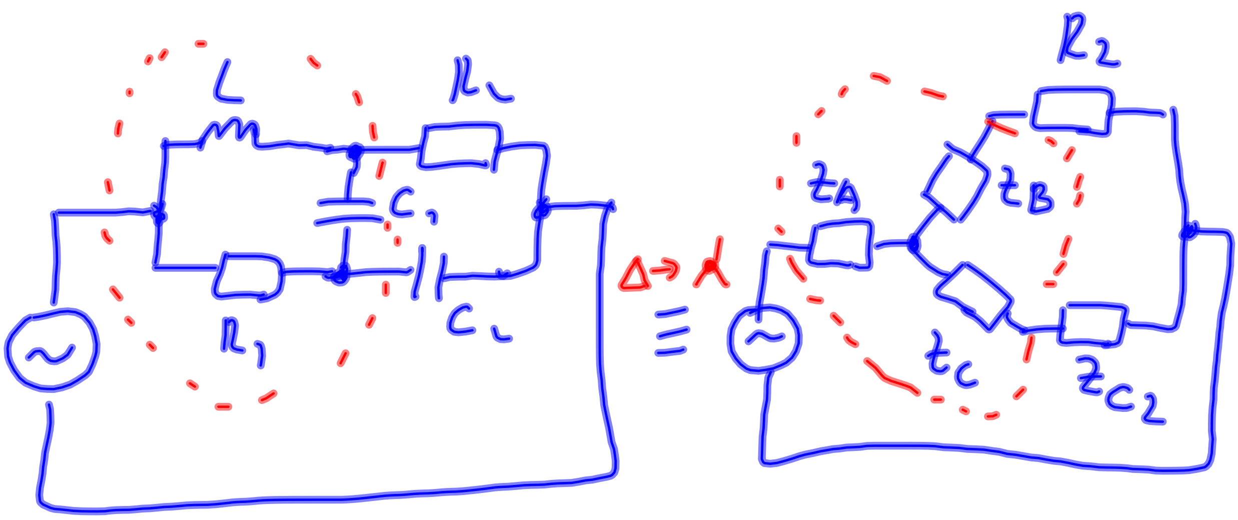
# DON'T AFRAID OF OTHER COMBINATIONS




PARALLEL COMBINATION

$$Z_{eqv} = \frac{R \cdot Z_L}{R + Z_L} = \frac{R \cdot j\omega L}{R + j\omega L}$$

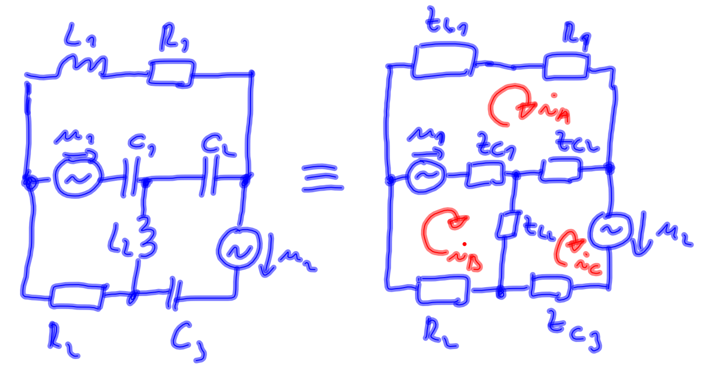
$$Z_{eqv} = \underbrace{A}_{\mathrm{Re}} + j \cdot \underbrace{B}_{\mathrm{Im}} \in \mathbb{C}$$




$$Z_A = \frac{Z_L \cdot R_1}{Z_L + R_1 + Z_{C_1}} \in \mathbb{C}$$

$\vdots$

→ OR CIRCUITS WITH MORE POWER SUPPLIES




$$i_A: (Z_{L_1}+R_1)\, I_A + Z_{C_2}(I_A - I_C) + Z_{C_1}(I_A - I_B) - U_1 = 0$$

$$i_B: Z_{C_1}(I_B - I_A) + Z_{L_2}(I_B - I_C) + R_2 I_B + U_1 = 0$$

$$i_C: Z_{C_3} I_C + Z_{L_2}(I_C - I_B) + Z_{C_2}(I_C - I_A) + U_2 = 0$$

$$\underbrace{\begin{pmatrix} Z_{L_1}+R_1+Z_{C_2}+Z_{C_1} & -Z_{C_1} & -Z_{C_2} \\ -Z_{C_1} & Z_{C_1}+Z_{L_2}+R_2 & -Z_{L_2} \\ -Z_{C_2} & -Z_{L_2} & Z_{C_3}+Z_{L_2}+Z_{C_2} \end{pmatrix}}_{A}$$

NOTE: MATRIX $A$ IS SYMMETRIC

$A = A^T$

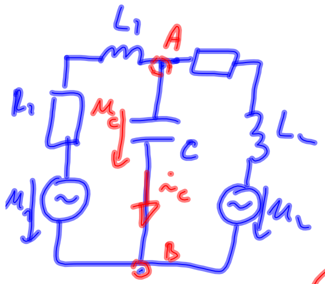
SOLVE SYSTEM (SLAE)

$A\vec{x} = \vec{b}$ ... CRAMER'S RULE, MATLAB INVERSE MATRIX ...

$$\vec{x} = \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} U_1 \\ -U_1 \\ -U_2 \end{pmatrix}$$

$A, \vec{x}, \vec{b} \in \mathbb{C}$ .. COMPLEX NUMBERS

# THÉVENIN'S THEOREM




$I_c, U_c = ?$

$(\varphi_{I_c}, \varphi_{U_c})$

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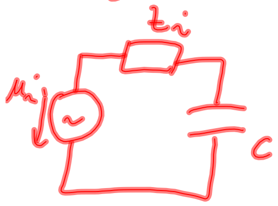
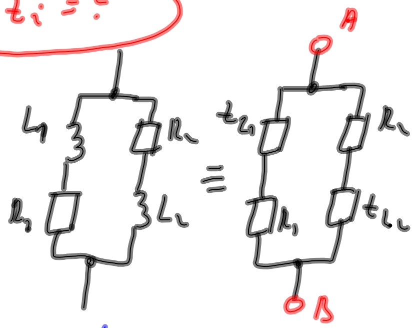
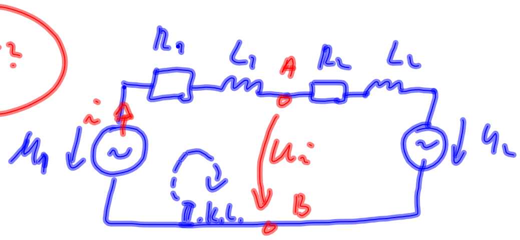



$Z_i = ?$




$$Z_i = (Z_{L_1} + R_1) \,\|\, (R_2 + Z_{L_2})$$

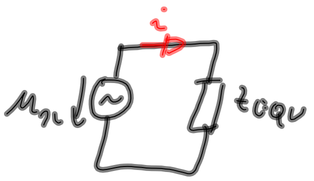
$U_i = ?$




⇓ METHOD OF SIMPLIFICATION

II. Kirch. Law

$$U_{R_1} + U_{L_1} + U_i - U_1 = 0$$




$$U_i = U_1 - U_{R_1} - U_{L_1}$$

$$\underline{U_i = U_1 - R_1 I - I \underbrace{Z_{L_1}}_{j\omega L_1}}$$

$$Z_{EQV} = R_1 + R_2 + j\omega(L_1 + L_2)$$

$$U_{12} = U_1 - U_2 \in \mathbb{R}$$

$$I = \frac{U_{12}}{Z_{EQV}} \in \mathbb{C}$$

$$\underline{I_c} = \frac{U_i}{Z_i + \underbrace{Z_c}_{\frac{-j}{\omega C}}} \in \mathbb{C} \quad ✓ \qquad \underline{U_c} = I_c \cdot Z_c \in \mathbb{C} \quad ✓$$

COMPUTE AMPLITUDES $|I_c|, |U_c|$