RC Circuit

1 Problem 1 - Discharging Capacity

Let's find a solution of u_C in RC circuit in Fig. 1.

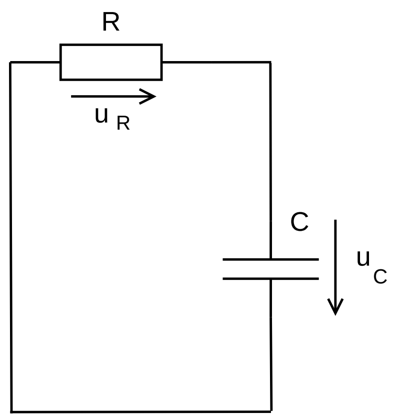

Figure 1: RC Circuit

$$
\begin{array}{rcl}
u_R & = & Ri \quad \dots \quad \text{Ohm's law} \\
u_C + Ri & = & u_C + u'_C RC = 0 \quad \dots \quad 2.\ \text{Kirchhoff's law} \\
u'_C & = & \frac{i}{C} \quad \Rightarrow \quad i = u'_C C \\
\hline
\end{array}
$$

$$u_C + RCu'_C = 0$$

$$u'_C + \frac{u_C}{RC} = 0, \quad u_C(0) = u_{C_0} \tag{1}$$

1.1 Analytic solution

Finding solution of homogenous differential equation

$$u_C' + \frac{1}{RC} \cdot u_C = 0$$

that leads to **characteristic equation**

$$\begin{aligned} \lambda + \tfrac{1}{RC} &= 0 \\ \lambda &= -\tfrac{1}{RC} \end{aligned}$$

General solution of homogenous differential equation

$$u_C = K(t)e^{\lambda t} \quad \Rightarrow \quad u_C = K(t)e^{-\frac{1}{RC}t}$$

We have to find $K(t) = ???$ so we must derive u_C

$$u_C' = K'(t)e^{-\frac{1}{RC}t} - K(t)\frac{1}{RC}e^{-\frac{1}{RC}t}$$

and substitute u_C and u_C' into (1)

$$K'(t)e^{-\frac{1}{RC}t} - K(t)\frac{1}{RC}e^{-\frac{1}{RC}t} + \frac{1}{RC}e^{-\frac{1}{RC}t} = 0$$

$$K'(t)e^{-\frac{1}{RC}t} = 0$$

to obtain $K(t)$ we have to integrate :-(

$$\begin{aligned} \int K'(t) &= \int 0 \\ K(t) &= k \quad \ldots \quad \text{constant of integration} \end{aligned}$$

General solution of homogenous differential Eq. (1)

$$u_C = ke^{-\frac{1}{RC}t}$$

Substituting initial value $u_C(0) = u_{C_0}$

$$\begin{aligned} u_{C0} &= ke^{-\frac{1}{RC}\cdot 0} \\ k &= u_{C_0} \end{aligned}$$

we obtain **particular solution**

$$u_C = u_{C_0}e^{-\frac{1}{RC}t} \tag{2}$$

1.2 TKSL

```
var ur, uc, ucanal;
const dt=0.1, eps=1e-10, tmax=10, C=1e-3, R=1000, uc0=10;

system
 uc'= (-1/(R*C))*uc &uc0;
 ur = -uc;
 ucanal = uc0*exp(-(1/(R*C))*t);
sysend.
```

1.3 Homework

Compare analytic solution (2) with numerical computation of differential equation u'_C in TKSL 386.

2 Problem - Charging of Capacitance

Let's find a solution of u_C in RC circuit in Fig. 2.

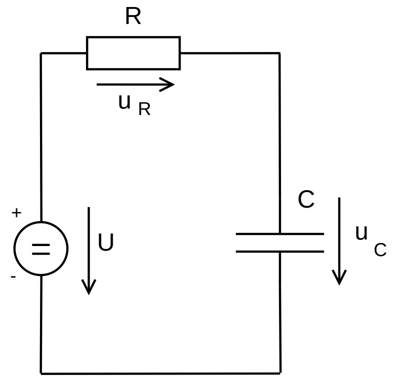

Figure 2: RC Circuit with Supply Voltage U

$$\begin{array}{rcl} u_R & = & Ri \quad \ldots \quad \text{Ohm's law} \\ u_C + Ri & = & u_C + u'_C RC = U \quad \ldots \quad \text{2. Kirchhoff's law} \\ u'_C & = & \frac{i}{C} \quad \Rightarrow \quad i = u'_C C \end{array}$$

$$u_C + RCu'_C = U$$

$$u'_C + \frac{u_C}{RC} = \frac{U}{RC}, \quad u_C(0) = u_{C_0} \tag{3}$$

2.1 Analytic solution

Finding solution of non-homogenous differential equation

$$u'_C + \frac{1}{RC} \cdot u_C = \frac{1}{RC} U$$

that leads to **characteristic equation**

$$\begin{array}{rcl} \lambda + \frac{1}{RC} & = & 0 \\ \lambda & = & -\frac{1}{RC} \end{array}$$

General solution of homogenous differential equation

$$u_C = K(t)e^{\lambda t} \quad \Rightarrow \quad u_C = K(t)e^{-\frac{1}{RC}t}$$

We have to find $K(t) =???$ so we must derive u_C

$$u'_C = K'(t)e^{-\frac{1}{RC}t} - K(t)\frac{1}{RC}e^{-\frac{1}{RC}t}$$

and substitute u_C and u'_C into (3)

$$K'(t)e^{-\frac{1}{RC}t} - K(t)\frac{1}{RC}e^{-\frac{1}{RC}t} + \frac{1}{RC}e^{-\frac{1}{RC}t} = \frac{1}{RC}U$$

$$K'(t)e^{-\frac{1}{RC}t} = \frac{1}{RC}U$$

to obtain $K(t)$ we have to integrate :-(

$$\begin{array}{rcl} \int K'(t) & = & \int \frac{1}{RC} U \cdot e^{\frac{1}{RC}t} \\ K(t) & = & Ue^{\frac{1}{RC}t} + k \quad \ldots \quad \text{constant of integration} \end{array}$$

General solution of homogenous differential Eq. (3)

$$
\begin{aligned}
u_C &= (Ue^{\frac{1}{RC}t} + k)e^{-\frac{1}{RC}t} \\
u_C &= U + ke^{-\frac{1}{RC}t}
\end{aligned}
$$

Substituting initial value $u_C(0) = u_{C_0}$

$$
\begin{aligned}
u_{C0} &= U + ke^{-\frac{1}{RC}0} = U + k \\
k &= u_{C_0} - U
\end{aligned}
$$

we obtain **particular solution**

$$
\begin{aligned}
u_C &= (Ue^{\frac{1}{RC}t} + u_{C_0} - U) \cdot e^{-\frac{1}{RC}t} \\
u_C &= U + (u_{C_0} - U)e^{-\frac{1}{RC}t} \qquad (4)
\end{aligned}
$$

2.2 TKSL

```
var u, ur, uc, ucanal;
const dt=0.1, eps=1e-10, tmax=10, C=1e-3, R=1000;

system
 u = 1;
 uc'= (u-uc)*(1/(R*C)) &0;
 ur = u-uc;
 ucanal = u-u*exp(-(1/(R*C))*t);
sysend.
```

2.3 Homework

Compare analytic solution (4) with numerical computation of differential equation u'_C in TKSL 386.