Solving a differential equation $y' = \sin(y)$ analytically and by TKSL

Author

November 29, 2013

1 Analytical solution

$$y' = \sin(y) \tag{1}$$

To solve the equation we can use a method of separating variables. Equation can be written in an equivalent form:

$$\frac{dy}{dt} = \sin(y) \tag{2}$$

Solving by gradual adjustment:

$$\frac{dy}{dt} = \sin(y) \tag{3}$$

$$\frac{dy}{\sin(y)} = dt \tag{4}$$

$$\int \frac{dy}{\sin(y)} = \int dt \tag{5}$$

$$\int \frac{1dy}{2\sin(\frac{y}{2})\cos(\frac{y}{2})} = t + C_1 \tag{6}$$

$$\int \frac{\sin^2(\frac{y}{2}) + \cos^2(\frac{y}{2})dy}{2\sin(\frac{y}{2})\cos(\frac{y}{2})} = t + C_1 \tag{7}$$

$$\int \frac{\sin^2(\frac{y}{2})dy}{2\sin(\frac{y}{2})\cos(\frac{y}{2})} + \int \frac{\cos^2(\frac{y}{2})dy}{2\sin(\frac{y}{2})\cos(\frac{y}{2})} = t + C_1 \tag{8}$$

$$\int \frac{\sin(\frac{y}{2})dy}{2\cos(\frac{y}{2})} + \int \frac{\cos(\frac{y}{2})dy}{2\sin(\frac{y}{2})} = t + C_1 \tag{9}$$

$$\frac{1}{2}\left[-2\ln|\cos(\frac{y}{2})| + 2\ln|\sin(\frac{y}{2})|\right] + C_2 = t + C_1 \tag{10}$$

$$-\ln|\cos(\frac{y}{2})| + \ln|\sin(\frac{y}{2})| = t + (C_1 - C_2) = t + C \tag{11}$$

$$\ln|\sin(\frac{y}{2})| - \ln|\cos(\frac{y}{2})| = t + C \tag{12}$$

$$\ln|\tan(\frac{y}{2})| = t + C \tag{13}$$

Expressing y with initial condition $y(0) = 1$ we have:

$$y = 2 * \arctan(e^{t+\ln(\tan(\frac{1}{2}))}) \tag{14}$$

2 Comparsion with TKSL

To compare this solution with TKSL, following code can be used:

```
var y,ya;

const
c=-0.6045824459415915543554179136700715724844695665779l, {ln(tg(0.5))}
eps=1e-20,
tmax=20,
dt=0.1;

system
y' = sin(y) & 1;
ya = 2*arctg(exp(t+c));
sysend.
```

Analytical (YA) and computed (Y) functions are depicted on picture (1). You can see that both functions are identical and aproche π.

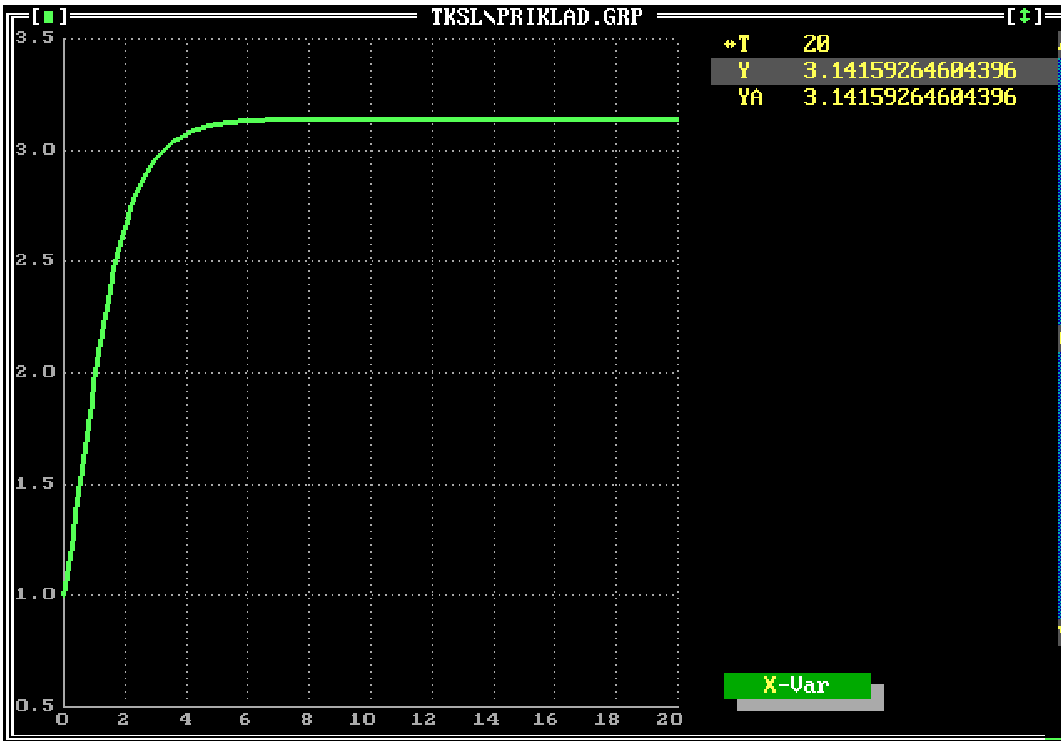

Figure 1: Solution of $y' = \sin(y)$