1 Chap5: Ohm's law, resistance

Electromotive force, E, is responsible for creating the current, I. In 1827 George Simon Ohm stated that the exact relationship between E and I was linear. Under the law which bears Ohm's name, the current flowing through a conductor (such as a very long length of thin wire made from silver, copper, aluminium, etc.) is *directly proportional* to the EMF applied across the conductor. This occurs under constant physical conditions of temperature, humidity, and pressure. This means that if we triple the voltage, the current will also be tripled and if we halve the voltage, the current will also be divided by 2. Whatever we do to the voltage, the same will happen to the current. However, please do not think that Ohms law is obviously true. Most electronic components within their operating range obey Ohm's law, but others do not. For example, if you double the forward voltage across a semiconductor diode, the current will increase, but will not double. In general, solid-state devices and tubes do not obey Ohm's law.

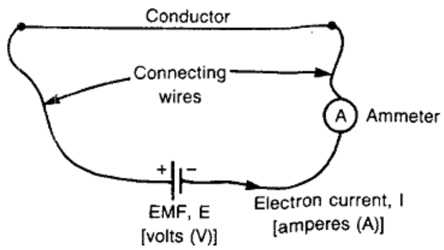

Figure 1.1: Voltage source

In figure 1.1 a voltage source whose EMF can be varied, is connected across a long length of copper wire, which acts as the conductor. An ammeter is placed in the path of the current, I, and will record the current value in amperes. A voltmeter is connected across the battery and will read the value of the EMF. E. To start with, it is obvious that if the applied voltage is zero, the current is also zero. We next assume that when the initial EMF is $12\,\mathrm{V}$, the recorded current is $2\,\mathrm{A}$. If this EMF is now doubled to $24\,\mathrm{V}$, the current will also double to $4\,\mathrm{A}$. When the voltage is again doubled to $48\,\mathrm{V}$, the new current is $8\,\mathrm{A}$. If the EMF is multiplied by 10 to a value of $10 \cdot 12\,\mathrm{V} = 120\,\mathrm{V}$, the accompanying current is $10 \cdot 2\,\mathrm{A} = 20\,\mathrm{V}$ Finally, if the initial EMF is halved to $\frac{12\,\mathrm{V}}{2} = 6\,\mathrm{V}$, the current drops to $\frac{2\,\mathrm{A}}{2} = 1\,\mathrm{A}$. These corresponding values of E and I are illustrated in table 1.1..

EMF (E) volts	**Current (I) amperes**	**Ratio $\frac{E}{I}$**
6 V	1 A	6
12 V	2 A	6
24 V	4 A	6
48 V	8 A	6
120 V	20 A	6

Table 1.1: Example of constant ratio of $\frac{E}{I}$

In table 1.1, the ratio of $\frac{E}{I}$ is calculated for each corresponding voltage and current. In every case the answer is the same which is a constant for the circuit of figure 1.1. Ohm's law may therefore be restated as:

Under constant physical conditions, the ratio of the voltage applied across a conductor to the current flowing through the conductor is a constant.

This constant measures the conductor's opposition to current flow and is called its resistance. The letter symbol for resistance is R while its unit is the ohm, which is denoted by the Greek capital letter, Omega Ω. Therefore in figure 1.1 the conductor's resistance is 6 Ω.

Resistance is that property of an electrical circuit that opposes or limits the flow of current. The component possessing this property is called a resistor.

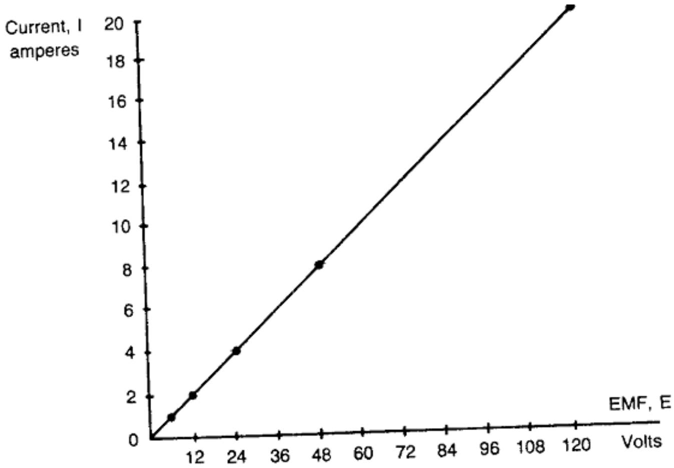

Figure 1.2: Graph of E versus I

If the graph of E versus I is plotted for the values of table 1.1, the result is the straight line illustrated in figure 1.2. This means that there is a linear relationship between E and I and the straight line is the graphical way of showing that the current is directly proportional to the voltage.

1.1 Potential difference across a resistor

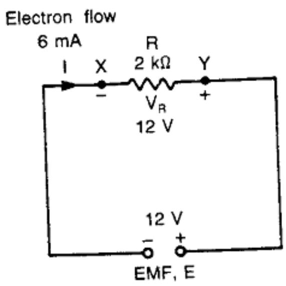

Figure 1.3: Circuit

In figure 1.3 a source whose EMF is 12 V is connected across a resistor and the measured (electron flow) current is 6 mA. Therefore, the value of the resistor is $\frac{E}{I} = \frac{12\,\mathrm{V}}{6\,\mathrm{mA}} = 2\Omega$. The electron flow from X to Y develops a voltage, V_R, across the resistor. This voltage is called the potential difference (PD) or difference of potential (DP).

2 Chap9: Resistors in the series arrangement

Resistors in series are joined end-to-end so that there is only a single path for the current. Starting at the negative battery terminal (where a surplus of electrons exists) there is an electron flow through the connecting wires (which are assumed to possess zero resistance) and the three resistors. Finally this flow reaches the positive terminal where there is a deficit of electrons. The battery through its chemical energy is then responsible for maintaining the surplus of electrons at the negative terminal and the deficit at the positive terminal. It follows from the above discussion that the current is the same throughout the circuit and that the ammeters (2.1) A1,A2,A3 and A4 all have the same reading.

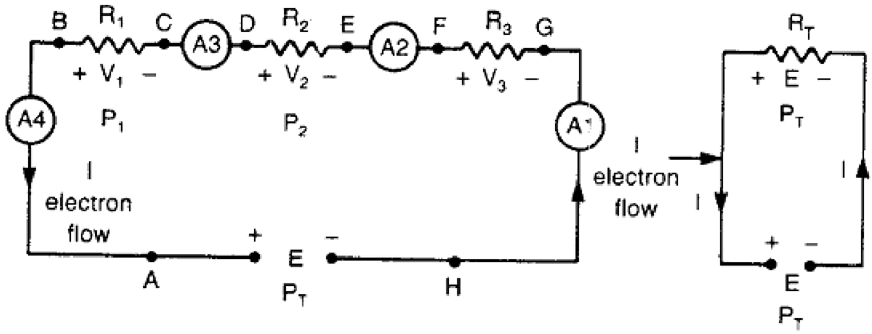

Figure 2.1: Circuit with resistors in series arrangement

Across each resistor is developed a difference of potential (DP) [sometimes referred to as the potential difference (PD)]; in this sense "potential" is another word for "voltage". For example, V_1 is the amount of voltage that is required to drive the current, I, through the resistor, $R1$. Since $V_1 = I \cdot R_1$ the voltage is often called the "IR drop".

The sum of the voltages across the resistors must exactly balance the source voltage, E; this is an example of **Kirchhoff's Voltage Law (KVL)** which we will fully explore in chapter 11. Since the current through each resistor is the same, the highest value resistor will develop the greatest voltage drop and the lowest value resistor will have the smallest voltage drop. In the extreme case of the connecting wires, which theoretically have zero resistance, there will be no voltage drop so that if, for example, a voltmeter were connected between points A and B in figure 2.1, its reading would be zero.

Notice that other voltages exist in the circuit apart from V_1, V_2, V_3, and E. If a voltmeter were connected between points B and E, its reading will be the sum of the voltages V_1 and V_2, while between the points D and G the voltage is equal to $V_2 + V_3$. It is often said that the order in which series resistors are connected is immaterial. This is true as far as the current and the individual voltage drops are concerned, but not in terms of the other voltages that exist in the circuit (see example 2.2).

In each resistor a certain amount of power is dissipated in the form of heat. (Care must be taken that the amount of power dissipated does not exceed the resistor's wattage rating). The sum of the individual powers dissipated is equal to the total power derived from the source.

Finally, what is the purpose of connecting resistors in series? One obvious reason is to increase the total equivalent resistance, R_T, and thereby limit the current to a safe value; however this could also be achieved by using a higher value single resistor. More importantly, adding resistors enables you to increase the overall wattage rating and to obtain non-standard resistance values by using standard components. In addition, series resistors may be connected across a source voltage to provide a voltage divider circuit (chapter 3).

2.1 Mathematical derivations

Voltage drop across the resistor $R1$:

$$V_1 = V_{AB} = I \cdot R_1 \tag{2.1}$$
$$I = \frac{V_1}{R_1}, R_1 = \frac{V_1}{I}$$

Voltage drop across the resistor $R2$:

$$V_2 = V_{CD} = I \cdot R_2 \tag{2.2}$$
$$I = \frac{V_2}{R_2}, R_2 = \frac{V_2}{I}$$

Voltage drop across the resistor $R3$:

$$V_3 = V_{FG} = I \cdot R_3 \tag{2.3}$$
$$I = \frac{V_3}{R_3}, R_3 = \frac{V_3}{I}$$

Source voltage:

$$E = V_1 + V_2 + V_3 \tag{2.4}$$
$$= I \cdot R_1 + I \cdot R_2 + I \cdot R_3$$
$$= I \cdot (R_1 + R_2 + R_3)\,\text{volts} \tag{2.5}$$

If the total equivalent resistance is R_T then source voltage:

$$E = I \cdot R_T\,\text{volts} \tag{2.6}$$

Comparing equations 2.4 and 2.5, Total equivalent resistance:

$$R_T = R_1 + R_2 + R_3\,\text{ohms} \tag{2.7}$$

If N resistors are connected in series, Total equivalent resistance:

$$R_T = R_1 + R_2 + R_3 + \cdots + R_N\,\text{ohms} \tag{2.8}$$

Total equivalent resistance:

$$R_T = NR\,\text{ohms} \tag{2.9}$$

Power dissipated in the R1 resistor:

$$P_1 = I \cdot V_1 = I^2 R_1 = \frac{V_1^2}{R_1}\,\text{watts} \tag{2.10}$$

Power dissipated in the R2 resistor:

$$P_2 = I \cdot V_2 = I^2 R_2 = \frac{V_2^2}{R_2}\,\text{watts} \tag{2.11}$$

Power dissipated in the R3 resistor:

$$P_3 = I \cdot V_3 = I^2 R_3 = \frac{V_3^2}{R_3}\,\text{watts} \tag{2.12}$$

Total power dissipated:

$$P_T = P_1 + P_2 + P_3\,\text{watts} \tag{2.13}$$

Power derived from the source voltage:

$$P_T = I \cdot E = I^2 R_T = \frac{E^2}{R_T}\,\text{watts} \tag{2.14}$$

2.2 Example

In figure 2.1, calculate the values of R_T, I_1, V_1, V_2, V_3,V_4, P_1, P_2,P_3,P_4,P_T. What are the voltages between the points PA, PB, PC, PD, PE, PF, PG, PH, PN, AB, AC, AD, AE, AF, AG, AH, AN, BC, BD, BE, BF, BG, BH, BN, CD, CE, CF, CG, CH, CN, DE, DF, DG, DH, DN, EF, EG, EH, EN, FG, FH, FN, GH, GN, HN?

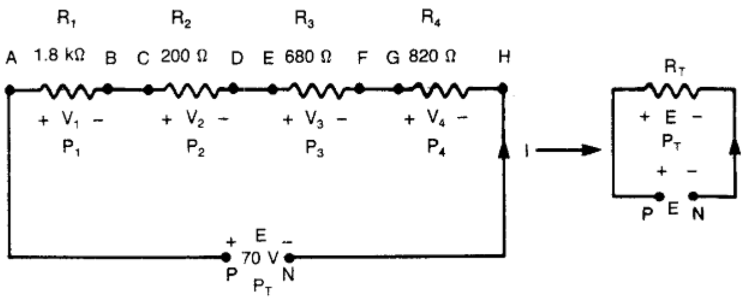

Figure 2.2: Example circuit

2.2.1 Solution

We must first convert the 1.8 kΩ into ohms, so that 1.8 kΩ = 1800 Ω. Total equivalent resistance:

$$R_T = R_1 + R_2 + R_3 + R_4 = 1800 + 200 + 680 + 820 = 3500\,\Omega = 3.5\,\text{k}\Omega \tag{2.15}$$

Current:

$$I = \frac{E}{R_T} = \frac{70V}{3.5\,\text{k}\Omega} = 20\,\text{mA} \tag{2.16}$$

Voltage drop V_1:

$$V_1 = V_{AB} = I \cdot R_1 = 20\,\text{mA} \cdot 1.8\,\text{k}\Omega = 36\,\text{V} \tag{2.17}$$

Voltage drop V_2:

$$V_2 = V_{CD} = I \cdot R_2 = 20\,\text{mA} \cdot 200\,\Omega = 4\,\text{V} \tag{2.18}$$

Voltage drop V_3:

$$V_3 = V_{EF} = I \cdot R_3 = 20\,\text{mA} \cdot 680\,\Omega = 13.6\,\text{V} \tag{2.19}$$

Voltage drop V_4:

$$V_4 = V_{GH} = I \cdot R_4 = 20\,\text{mA} \cdot 820\,\Omega = 16.4\,\text{V} \tag{2.20}$$

Notice that the highest value resistor ($R_1 = 1800\,\Omega$) carries the greatest voltage drop ($V_1 = 36\,\text{V}$) while the lowest value resistor ($R_2 = 200\,\Omega$) has the least voltage drop ($V_2 = 4\,\text{V}$).

Voltage check for source voltage:

$$E = V_1 + V_2 + V_3 + V_4 = 36 + 4 + 13.6 + 16.4 = 70\,\text{V} \tag{2.21}$$

Power dissipated in the R1 resistor:

$$P_1 = I \cdot V_1 = 20\,\text{mA} \cdot 36\,\text{V} = 720\,\text{mW} \tag{2.22}$$

Power dissipated in the R2 resistor:

$$P_2 = I \cdot V_2 = 20\,\text{mA} \cdot 4\,\text{V} = 80\,\text{mW} \tag{2.23}$$

Power dissipated in the R3 resistor:

$$P_3 = I \cdot V_3 = 20\,\text{mA} \cdot 13.6\,\text{V} = 272\,\text{mW} \tag{2.24}$$

Power dissipated in the R4 resistor:

$$P_4 = I \cdot V_4 = 20\,\text{mA} \cdot 16.4\,\text{V} = 328\,\text{mW} \tag{2.25}$$

Total power dissipated:

$$\begin{aligned} P_T &= P_1 + P_2 + P_3 + P_4 && (2.26)\\ &= 720 + 80 + 272 + 328 \\ &= 1400\,\text{mW} && (2.27) \end{aligned}$$

The highest value resistor ($R1 = 1800\,\Omega$) dissipates the greatest power ($P_1 = 720\,\text{mW}$) while the lowest value resistor ($R_2 = 200\,\Omega$) has the least power dissipation ($P_2 = 80\,\text{mW}$).

Notice that a $\frac{1}{2}$-watt resistor would be adequate for R_2 and R_3. By contrast, R_1 would need to be a 1-watt resistor, while R_4 would require a 2-watt resistor.

Total power derived from the source:

$$P_T = I \cdot E = 20\,\text{mA} \cdot 70\,\text{V} = 1400\,\text{mW} \tag{2.28}$$

Voltage between the points PA. $V_{PA} = 0\,\text{V}$ since PA is a connecting wire of zero resistance.

$$
\begin{aligned}
V_{PB} &= V_{PA} + V_{AB} = 0\,\mathrm{V} + 36\,\mathrm{V} = 36\,\mathrm{V}\\
V_{PC} &= V_{PA} + V_{AB} + V_{BC} = 0\,\mathrm{V} + 36\,\mathrm{V} + 0\,\mathrm{V} = 36\,\mathrm{V}\\
V_{PD} &= V_{PA} + V_{AB} + V_{BC} + V_{CD} = 0\,\mathrm{V} + 36\,\mathrm{V} + 0\,\mathrm{V} + 4\,\mathrm{V} = 40\,\mathrm{V}\\
V_{PE} &= V_{PA} + V_{AB} + V_{BC} + V_{CD} + V_{DE} = 0\,\mathrm{V} + 36\,\mathrm{V} + 0\,\mathrm{V} + 4\,\mathrm{V} + 0\,\mathrm{V} = 40\,\mathrm{V}\\
V_{PF} &= V_{PA} + V_{AB} + V_{BC} + V_{CD} + V_{DE} + V_{EF}\\
&= 0\,\mathrm{V} + 36\,\mathrm{V} + 0\,\mathrm{V} + 4\,\mathrm{V} + 0\,\mathrm{V} + 13.6\,\mathrm{V} = 53.6\,\mathrm{V}\\
V_{PG} &= V_{PA} + V_{AB} + V_{BC} + V_{CD} + V_{DE} + V_{EF} + V_{FG}\\
&= 0\,\mathrm{V} + 36\,\mathrm{V} + 0\,\mathrm{V} + 4\,\mathrm{V} + 0\,\mathrm{V} + 13.6\,\mathrm{V} + 0\,\mathrm{V} = 53.6\,\mathrm{V}\\
V_{PH} &= V_{PA} + V_{AB} + V_{BC} + V_{CD} + V_{DE} + V_{EF} + V_{FG} + V_{GH}\\
&= 0\,\mathrm{V} + 36\,\mathrm{V} + 0\,\mathrm{V} + 4\,\mathrm{V} + 0\,\mathrm{V} + 13.6\,\mathrm{V} + 0\,\mathrm{V} + 16.4\,\mathrm{V} = 70\,\mathrm{V}\\
V_{PN} &= V_{PA} + V_{AB} + V_{BC} + V_{CD} + V_{DE} + V_{EF} + V_{FG} + V_{GH} + V_{HN}\\
&= 0\,\mathrm{V} + 36\,\mathrm{V} + 0\,\mathrm{V} + 4\,\mathrm{V} + 0\,\mathrm{V} + 13.6\,\mathrm{V} + 0\,\mathrm{V} + 16.4\,\mathrm{V} + 0\,\mathrm{V} = 70\,\mathrm{V}
\end{aligned}
$$

Notice that if for example, the resistors $R2$ and $R3$ were interchanged, V_{PD} would become $36\,\mathrm{V} + 13.6\,\mathrm{V} = 49.6\,\mathrm{V}$. Since P and A are only joined by a connecting wire of zero resistance $V_{AB} = V_{PB}$, $V_{AC} = V_{PC}$, etc.

$$
\begin{aligned}
V_{BC} &= 0\,\mathrm{V}\\
V_{BD} &= V_{BC} + V_{CD} = 0\,\mathrm{V} + 4\,\mathrm{V} = 4\,\mathrm{V}\\
V_{BE} &= V_{BC} + V_{CD} + V_{DE} = 0\,\mathrm{V} + 4\,\mathrm{V} + 0\,\mathrm{V} = 4\,\mathrm{V}\\
V_{BF} &= V_{BC} + V_{CD} + V_{DE} + V_{DF} = 0\,\mathrm{V} + 4\,\mathrm{V} + 0\,\mathrm{V} + 13.6\,\mathrm{V} = 17.6\,\mathrm{V}\\
V_{BG} &= V_{BC} + V_{CD} + V_{DE} + V_{DF} + V_{FG} = 0\,\mathrm{V} + 4\,\mathrm{V} + 0\,\mathrm{V} + 13.6\,\mathrm{V} + 0\,\mathrm{V} = 17.6\,\mathrm{V}\\
V_{BH} &= V_{BC} + V_{CD} + V_{DE} + V_{DF} + V_{FG} + V_{GH}\\
&= 0\,\mathrm{V} + 4\,\mathrm{V} + 0\,\mathrm{V} + 13.6\,\mathrm{V} + 0\,\mathrm{V} + 16.4\,\mathrm{V} = 34\,\mathrm{V}\\
V_{BN} &= V_{BC} + V_{CD} + V_{DE} + V_{DF} + V_{FG} + V_{GH} + V_{HN}\\
&= 0\,\mathrm{V} + 4\,\mathrm{V} + 0\,\mathrm{V} + 13.6\,\mathrm{V} + 0\,\mathrm{V} + 16.4\,\mathrm{V} + 0\,\mathrm{V} = 34\,\mathrm{V}
\end{aligned}
$$

Between B and C there is a connecting wire of zero resistance so that $V_{HD} = V_{CD}$, $V_{BE} = V_{CE}$, etc.

$$
\begin{aligned}
V_{DE} &= 0\,\mathrm{V}\\
V_{DF} &= V_{DE} + V_{EF} = 0\,\mathrm{V} + 13.6\,\mathrm{V} = 13.6\,\mathrm{V}\\
V_{DG} &= V_{DE} + V_{EF} + V_{FG} = 0\,\mathrm{V} + 13.6\,\mathrm{V} + 0\,\mathrm{V} = 13.6\,\mathrm{V}\\
V_{DH} &= V_{DE} + V_{EF} + V_{FG} + V_{GH} = 0\,\mathrm{V} + 13.6\,\mathrm{V} + 0\,\mathrm{V} + 16.4\,\mathrm{V} = 30\,\mathrm{V}\\
V_{DN} &= V_{DE} + V_{EF} + V_{FG} + V_{GH} + V_{HN} = 0\,\mathrm{V} + 13.6\,\mathrm{V} + 0\,\mathrm{V} + 16.4\,\mathrm{V} + 0\,\mathrm{V} = 30\,\mathrm{V}
\end{aligned}
$$

Between D and E there is a connecting wire of zero resistance $V_{DF} = V_{EF}$, $V_{DG} = V_{EG}$, etc.

$$V_{FG} = 0\,\mathrm{V}$$
$$V_{FH} = V_{FG} + V_{GH} = 0\,\mathrm{V} + 16.4\,\mathrm{V} = 16.4\,\mathrm{V}$$
$$V_{FN} = V_{FG} + V_{GH} + V_{HN} = 0\,\mathrm{V} + 16.4\,\mathrm{V} + 0\,\mathrm{V} = 16.4\,\mathrm{V}$$
$$V_{HN} = 0\,\mathrm{V}$$

3 Chap11: Voltage division rule - voltage divider circuit

We have already learned that, in a series string of resistors, the highest voltage drop is developed across the largest value resistor while the lowest voltage drop appears across the smallest value resistor. In fact the voltage drops across all the resistors are directly proportional to the resistor values so that we may use the method of proportion to solve a problem in which, for example, we are given the values of R_1, R_2, V_1 and then asked to find the value of V_2 (figure 3.1). The source voltage is divided between the series resistors in a proportional manner that is determined by the resistor values. The fraction of the source voltage developed across a particular resistor is equal to the ratio of the resistor's value to the circuit's total resistance.

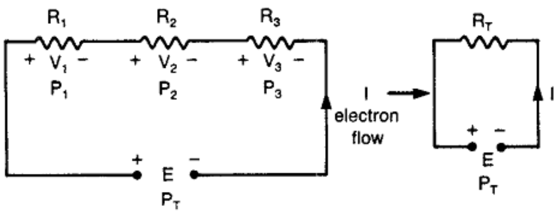

Figure 3.1: Example circuit

This result is known as the **voltage division rule (VDR)**. In the same manner the total power derived from the source divides between the resistors to produce the individual powers dissipated.

3.1 Mathematical derivations

Voltage drops:

$$V_1 = IR_1 \text{ volts} \tag{3.1}$$
$$V_2 = IR_2 \text{ volts} \tag{3.2}$$

Therefore current:

$$I = \frac{V_1}{R_1} = \frac{V_2}{R_2} \tag{3.3}$$

and

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} \tag{3.4}$$

Voltage drop:

$$V_1 = V_2 \cdot \frac{R_1}{R_2} \qquad V_2 = V_1 \cdot \frac{R_2}{R_1} \text{ volts} \tag{3.5}$$

Power dissipated:

$$P_1 = I^2 R_1 \,\text{watts} \tag{3.6}$$

$$P_2 = I^2 R_2 \,\text{watts} \tag{3.7}$$

$$\tag{3.8}$$

Then

$$I^2 = \frac{P_1}{R_1} = \frac{P_2}{R_2} \tag{3.9}$$

and

$$\frac{P_1}{P_2} = \frac{R_1}{R_2} \tag{3.10}$$

4 Chap12: Sources connected in series-aiding and in series-opposing

Two sources are in series-aiding when their polarities are such as to drive the current in the same direction around the circuit. This is illustrated in figure 4.1A where the negative terminal of E_1 is directly connected to the positive terminal of E_2. Both voltages will then drive the electrons around the circuit in the counter-clockwise direction. If both voltage sources are reversed, they will still be connected in series-aiding but the electron flow will now be in the clockwise direction.

The normal purpose of connecting sources in series-aiding is to increase the amount of voltage applied to a circuit. A good example is the insertion of two $1\frac{1}{2}$-V "D" cells into a flashlight to create a total of $2 \cdot 1\frac{1}{2}\,\mathrm{V} = 3\,\mathrm{V}$. In the case of the flashlight the positive center terminal of one cell must be in contact with the negative casing of the other cell to provide the series-aiding connection.

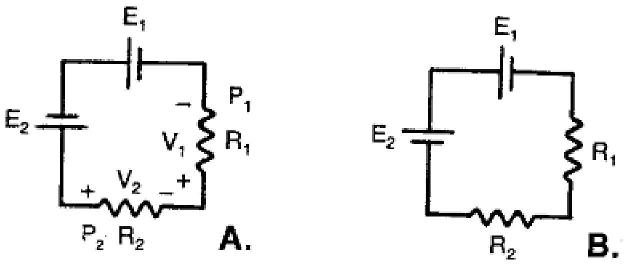

Figure 4.1: Example circuit

If the E_1 cell is reversed as in figure 4.1B the polarities of the sources will be such as to drive currents in opposite direction around the circuit. The connection is therefore series-opposing and the total voltage available is the difference of the individual EMFs. The greater of the two EMFs will then determine the actual direction of the current flow. In the particular case when two identical voltage sources are connected in series-opposing, the total voltage is zero. Consequently the current is also zero, as are the individual voltage drops and the powers dissipated in the resistors.

4.1 Mathematical derivations in figure 4.1A

Total equivalent EMF:

$$E_T = E_1 + E_2 \,\text{volts} \tag{4.1}$$

Total equivalent resistance:

$$R_T = R_1 + R_2 \,\text{ohms} \tag{4.2}$$

Current:

$$I = \frac{E_1 + E_2}{R_1 + R_2} \,\text{amperes} \tag{4.3}$$

Voltage drops:

$$V_1 = IR_1 \text{ volts} \tag{4.4}$$
$$V_2 = IR_2 \text{ volts} \tag{4.5}$$

5 Chap13: The potentiometer and the rheostat

The potentiometer (or POT) is another practical application of the voltage division rule. Essentially it consists of a length of resistance wire (for example, nichrome wire) or a thin carbon track along which a moving contact or slider may be set at any point (figure 5.1A). The wire is wound on an insulating base that may either be straight or, more conveniently, formed into a circle.

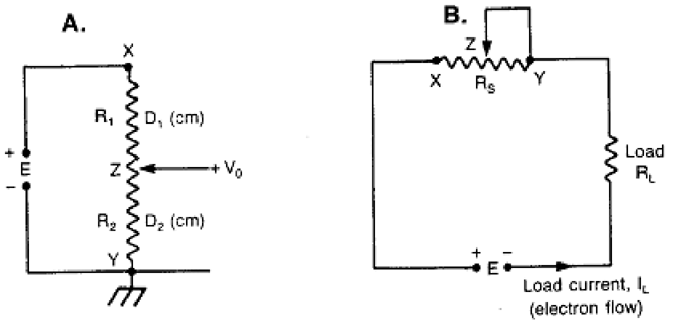

Figure 5.1: Example circuit

The purpose of the potentiometer is to obtain an output voltage, V_0, which may be varied from zero up to the full value of the source voltage, E. There are two terminals, X and Y, at the ends of the potentiometer while the slider is connected to the third terminal, Z. The source voltage is then applied between the end terminals, X and Y, while the output voltage appears between the slider terminal, Z, and the end terminal, Y, which is commonly grounded. A practical example of the potentiometer is a receiver's volume control. In this case the source is an audio signal but the principles of the potentiometer apply equally well to both dc and ac. In either case care must be taken not to exceed the potentiometer's power rating.

While a potentiometer varies an output voltage, the rheostat is used for controlling the current in a series circuit (figure 5.1B). The construction of a rheostat is similar to that of a potentiometer except that the rheostat needs only one end terminal, X, and the terminal, Z, which is connected to the moving contact. The third terminal, Y, is then directly joined to the terminal, Z. A practical example of a rheostat is the dimmer control for the lights on a cars dashboard.

5.1 Mathematical derivations

In figure 5.1A, the total resistance of the potentiometer is $R_1 + R_2$, while the resistance between Z and Y is R_2. Then, by the voltage division rule (VDR):

Output voltage:

$$V_0 = E \cdot \frac{R_2}{R_1 + R_2} \text{ volts} \tag{5.1}$$

If the slider is moved to the terminal Y:

$$R_2 = 0\,\Omega \,\text{and}\, V_0 = 0\,\text{V} \tag{5.2}$$

When the slider is moved to the terminal X:

$$R_1 = 0\,\Omega \,\text{and}\, V_0 = E\,\text{volts} \tag{5.3}$$

If the terminal Z is at the center position:

$$R_1 = R_2 \,\text{and}\, V_0 = \frac{E}{2}\,\text{volts} \tag{5.4}$$

The results of equations 5.1, 5.2, 5.3 and 5.4 are only true if no load is connected across the output voltage. Once a load is connected, there would be an additional current flow through Rh increasing its voltage drop so that the output voltage would fall.

If the ratio of R_2 to the total resistance of the potentiometer is equal to the ratio of the slider's travel,D_2 (from its starting position), to the total travel available,$D_1 + D_2$, the potentiometer is said to be "linear". However, if the resistance and the travel ratios are connected by a logarithmic relationship, we have a "log pot". Whether a potentiometer is linear or logarithmic depends on the winding of the resistance wire or the construction of the carbon track.

For a linear potentiometer, we have output voltage:

$$V_0 = E \cdot \frac{D_2}{D_1 + D_2}\,\text{volts} \tag{5.5}$$

where D_1 and D_2 may be conveniently measured in centimetres. In figure 5.1B, the slider is moved to terminal X. None of the rheostat's resistance is now included in the circuit so that the current has its maximum value given by maximum current:

$$I_{max} = \frac{E}{R_L}\text{amperes} \tag{5.6}$$

When the slider is subsequently moved to the terminal Y, the whole of the rheostat's resistance, R_S, is in series with the load R_L, so that a minimum current will flow. Minimum current:

$$I_{min} = \frac{E}{R_S + R_L}\,\text{amperes} \tag{5.7}$$

The rheostat can therefore control any level of current between the maximum and minimum values. Total power delivered from source:

$$\begin{aligned} P_T &= P_L + P_D \\ &= I_L \cdot (V_L + V_D) \\ &= I_L^2 (R_L + R_D) \\ &= \frac{E^2}{R_L + R_D}\text{watts} \end{aligned}$$

5.2 Example

A relay, whose coil resistance is $400\,\Omega$ is designed to operate when the voltage across the relay is $80\,\text{V}$. If the relay is operated from a $230\,\text{V}$ supply, calculate the ohmic value of the required series dropping resistor and its power dissipation.

Voltage across the dropping resistor:

$$
\begin{aligned}
V_D &= E - V_L \\
&= 230\,\mathrm{V} - 80\,\mathrm{V} \\
&= 150\,\mathrm{V}
\end{aligned}
$$

Relay current:

$$
I_L = \frac{V_L}{R_L} = \frac{230\,\mathrm{V}}{400\,\Omega} = 0.58\,\mathrm{A} \tag{5.8}
$$

Ohmic value of the dropping resistor:

$$
\begin{aligned}
R_D &= \frac{V_D}{I_L} \\
&= \frac{40\,\mathrm{V}}{0.58\,\mathrm{A}} \\
&= 69\,\Omega
\end{aligned}
$$

Power dissipation of the dropping resistor:

$$
\begin{aligned}
P_D &= I_L \cdot V_D \\
&= 0.58\,\mathrm{A} \cdot 150\mathrm{V} \\
&= 116\,\mathrm{W}
\end{aligned}
$$

6 Chap15: Resistors in parallel

All loads (lights, vacuum cleaner, toaster, etc.) in your home are normally stamped with a 230 V rating. It follows that all these loads must be connected directly across the household supply because although each load has its own power rating, its required voltage value in every case is the same. This type of arrangement is known as parallel and is illustrated in figure 6.1 A and B in which the three resistors are connected between two common points X, Y (figure 6.1A) or two common lines (figure 6.1B).

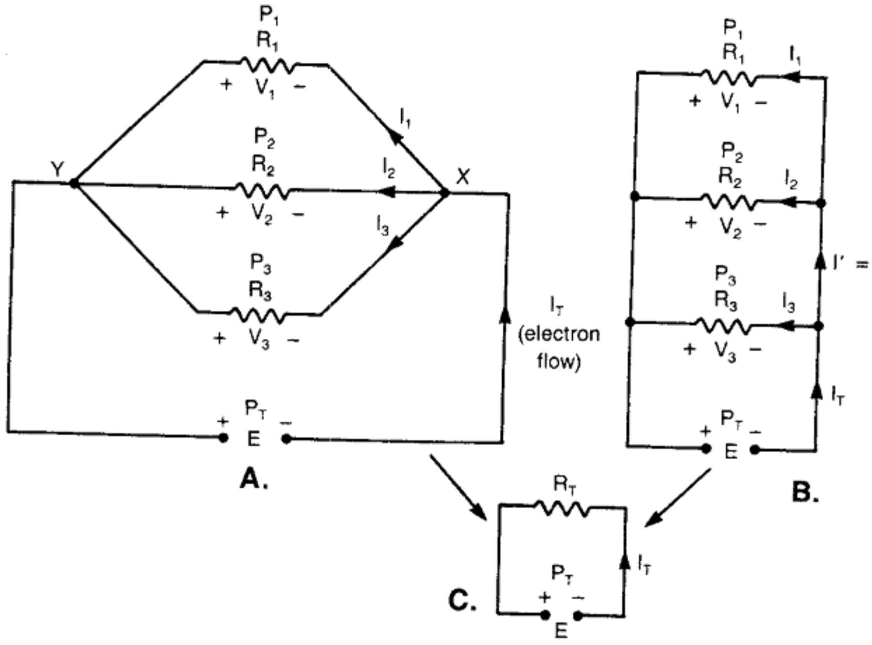

Figure 6.1: Example circuit

The points or lines are then directly connected to the voltage source. It follows that the voltage drop across each parallel resistor is the same and equal to the source voltage. This is in contrast with the series arrangement where voltage division occurred.

Each of the resistors in a parallel circuit is a path for current flow and each path is called a branch. Because the voltage across each branch is the same, each branch wilt carry its own current and the individual branch currents will all be different (unless the branch resistances are the same in which case their currents would also be equal). Notice that the branch currents are independent of one another. In other words, if one branch current is switched off, the other branch currents are unaffected. This would be equivalent to saying that, if you switch off the light in the kitchen, the TV set in the lounge continues to operate.

The total source current, IT, splits at the point X into individual branch currents. The current in a particular branch is inversely proportional to the value of the resistance in that branch. Consequently the lowest-value resistor will carry the greatest current, while the smallest current will flow through the highest-value resistor. In terms of electron flow the branch currents, I_1, I_2, I_3 are all leaving the junction point X, while the total current I_T is entering the same junction point. Recombination of the branch currents then occurs at the other junction point, Y. It follows that the total current is the sum of the individual branch currents. This is an expression of **Kirchhoff's Current Law (KCL)**, which is further explored in chapter 11.

The total equivalent resistance, R_T, is defined as the ratio of the source voltage, E, to the source current, I_T. If an additional branch is added to a parallel arrangement of resistors, the total current must increase by the amount of the new branch current and therefore the total equivalent resistance must *decrease*. For parallel resistors the value of R_T must always be less than the lowest-value resistor in the arrangement.

Each parallel resistor dissipates its own power and the sum of the individual powers dissipated is equal to the total power delivered from the source. The greatest power is dissipated by the lowest-value resistor, which carries the .greatest branch current. For example, the resistance of a nickel-iron 1000 W heater element is only one-tenth that of the tungsten filament for a 100 W electric light bulb.

6.1 Mathematical derivations

Source voltage:

$$E = V_1 = V_2 = V_3 \,\text{volts} \tag{6.1}$$

Branch currents:

$$\begin{aligned} I_1 &= \frac{V_1}{R_1} = \frac{E}{R_1} \\ I_2 &= \frac{V_2}{R_2} = \frac{E}{R_2} \\ I_3 &= \frac{V_3}{R_3} = \frac{E}{R_3} \end{aligned} \tag{6.2}$$

Total source current:

$$\begin{aligned} I_T &= I_1 + I_2 + I_3 \\ &= \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} \\ &= E \cdot (\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}) \,\text{amperes} \end{aligned} \tag{6.3}$$

Notice that the current I', which exists in the circuit of figure 6.1B, cannot be measured in the circuit of figure 6.1A.

Total equivalent resistance:

$$R_T = \frac{E}{R_T} = E \cdot \frac{1}{R_T} \,\text{ohms} \tag{6.4}$$

Comparing equations 6.3 and 6.4:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{6.5}$$

For N resistors in parallel:

$$I_T = I_1 + I_2 + I_3 + \cdots + I_N \tag{6.6}$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N} \tag{6.7}$$

This is known as the "reciprocal formula" since $\frac{1}{R_T}$, $\frac{1}{R_2}$, etc., are the reciprocals of R_T, R_1, etc.. Then:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \text{ ohms} \tag{6.8}$$

7 Chap17: Voltage sources in parallel

Figure 7.1 illustrates n identical cells, each of EMF E, which are parallel-connected across a number (M) of resistive loads. All positive terminals are joined together so that any one of these terminals may be chosen as the positive output terminal; the negative terminals are likewise connected. If a voltmeter V is connected across any one of the cells, it must also be in parallel with all the other cells, so that the voltmeter reading is only E volts (the EMF of one cell). The purpose of connecting cells in parallel is therefore not to increase the total voltage. However, the current capability has been increased since the total load current, I?, will be shared between the batteries.

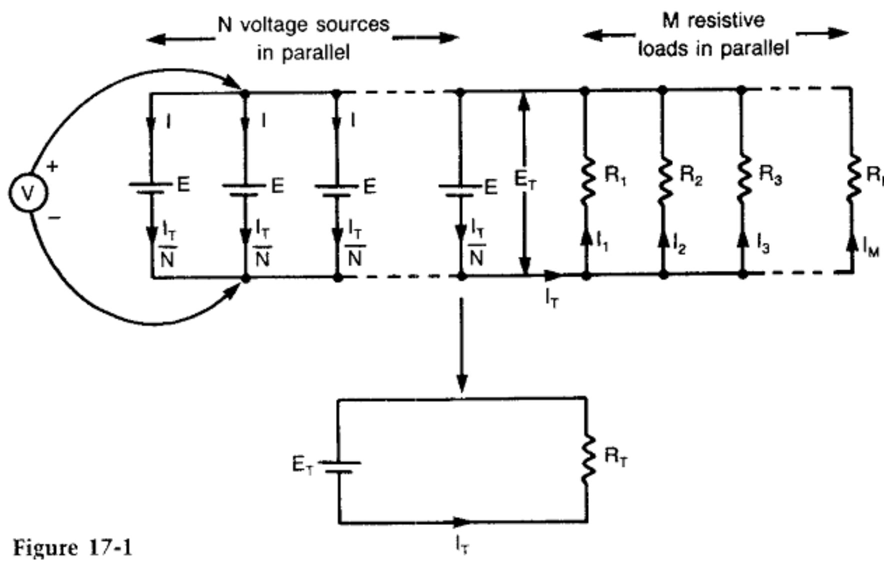

Figure 7.1: Example circuit

An everyday example of parallel-connected batteries occurs when you give someone a "jump start" for his/her car. You use the cables to join the positive terminals of the two 12 V batteries together and make the same type of connection with the negative terminals. However, with this parallel arrangement the total voltage available is still only equal to 12 V (the EMF of one battery).

7.1 Mathematical derivations

For N identical cells in parallel. Total voltage of the parallel combination:

$$E_T = E\,\text{volts} \tag{7.1}$$

Total equivalent resistance of the parallel loads:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \tag{7.2}$$

Total load current:

$$I_T = \frac{E_T}{R_T}\,\text{amperes} \tag{7.3}$$

Current supplied by each cell:

$$I = \frac{I_T}{N}\,\text{amperes} \tag{7.4}$$

7.2 Example

Four identical 6 V cells are connected in parallel across two loads whose resistances are 20 Ω and 5Ω. Calculate the value of the current supplied by each cell.

7.2.1 Solution

Total voltage applied to the parallel loads:

$$E_T = 6\,\text{V (EMF of one cells} \tag{7.5}$$

Total load:

$$R_T = \frac{20 \cdot 5}{20 + 5} = 4\,\Omega \tag{7.6}$$

Total load current:

$$I_T = \frac{E_T}{R_T} = \frac{6\,\text{V}}{4\,\Omega} = 1.5\,\text{A} \tag{7.7}$$

Current supplied by each cell:

$$I = \frac{I_T}{N} = \frac{1.5\,\text{A}}{4} = 0.375\,\text{A} \tag{7.8}$$

8 Chap18: The current division rule

The current division rule is used if you are given the values of the parallel resistors R_1, R_2, R_3, $\ldots$, , R_N (figure 8.1) and the value of the total current, I_r, and are then asked to determine the individual branch currents I_1, I_2, I_3, $\ldots$, I_N. Because the voltage across each branch is the same, the individual branch currents are in inverse proportion to the values of the resistors. In other words, the branch with the lowest resistance will carry the highest current while the smallest branch current will flow in the branch with the greatest resistance.

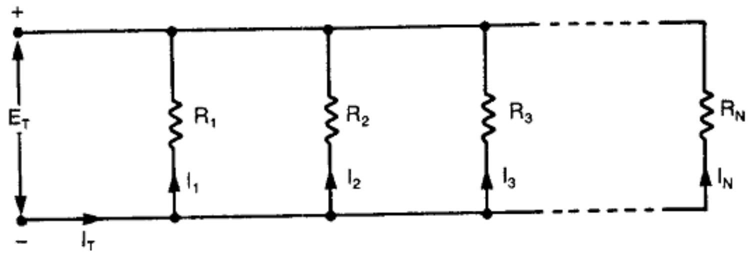

Figure 8.1: Example circuit

Because the circuit's total resistance R_T, is equal to the source voltage E_T, divided by the total current I_T, the fraction of the source current flowing through a particular branch is equal to the ratio of the total equivalent resistance to the branch resistance. This relationship is called the current division rule.

8.1 Mathematical derivations

Source voltage $E_T = I_1R_1 = I_2R_2 = I_3R_3 = \cdots = I_NR_N = I_TR_T$ volts. Then:

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \text{ etc.}$$

Therefore:

$$I_1 = I_2 \cdot \frac{R_2}{R_1} \, I_2 = I_1 \cdot \frac{R_1}{R_2} \cdots \text{etc., amperes} \tag{8.1}$$

Also

$$\frac{I_1}{I_T} = \frac{R_T}{R_1} \tag{8.2}$$

This yields

$$I_1 = I_T \cdot \frac{R_T}{R_1} \, I_2 = I_T \cdot \frac{R_T}{R_2} \cdots \text{etc., amperes} \tag{8.3}$$

and

$$I_T = I_1 \cdot \frac{R_1}{R_T} = I_2 \cdot \frac{R_2}{R_T} \text{ amperes} \tag{8.4}$$

Equations 8.3 represents the *current division rule.* If there are only two resistors in parallel:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}\,\text{ohms} \tag{8.5}$$

9 Chap19:Series-parallel arrangements of resistors

In chapter 2 we learned about the properties related to the series strings of resistors while in chapter 6 we studied the results of connecting resistors in parallel banks. Briefly summarizing, the single current was the same throughout the series circuit and the source voltage was divided among the resistors in proportion to their resistances. By contrast, the (same) source voltage was applied across each resistor in parallel and the (total) source current was divided between the various branches in inverse proportion to their resistances. However, series strings and parallel banks may be combined to create so-called series-parallel circuits of which the two simplest are shown in figure 9.1A and B.

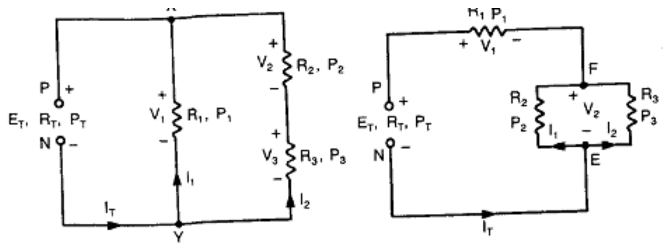

Figure 9.1: Example circuit

In figure 9.1A, the resistors $R2$ and $R3$ are pined end-to-end so that the same current must flow through each of these resistors. Consequently, the resistors $R2$ and $R3$ are in series. However, the (electron flow) current splits at the point Y and recombines at the point X. Therefore $R1$ is in parallel with the series combination of $R2$ and $R3$.

The (electron flow) current splits at the point, E, in figure 9.1B and recombines at the point, F, so that $R2$ and $R3$ are in parallel. However, this parallel combination is joined to one end of Rl so that this resistor is in series with $R_{2,3}$ ($R_{2,3}M$ is the equivalent resistance of the $R2$, $R3$ parallel combination).

The equations for the circuits of figures 9.1A, B are fully developed in the mathematical derivations. However, there is an infinite variety of complex series-parallel networks of resistors. For example, we might be faced with the circuit of figure 9.1A and be asked to find the total equivalent resistance presented to the source. This problem is solved in a number of steps.

9.1 Step 1

Identify all points that are electrically different in the circuit. These are the points P, N, X, Y, and Z. The circuit can then be redrawn in the more conventional manner of figure 9.2B.

Corresponding points are labeled in the two drawings and you must make certain that the same resistors are connected between any two such points.

9.2 Step 2

Combine all obvious series strings and parallel banks. Redraw the resulting simplified circuit. The general practice is to start furthest away from the source and then work your way towards the source. In our example $R4$ and $R5$ are in parallel and their equivalent resistance is:

$$R_{4,5} = \frac{R_4 \cdot R_5}{R_4 + R_5} = \frac{4.7 \cdot 2.7}{4.7 + 2.7} = 1.715\,\mathrm{k}\Omega \tag{9.1}$$

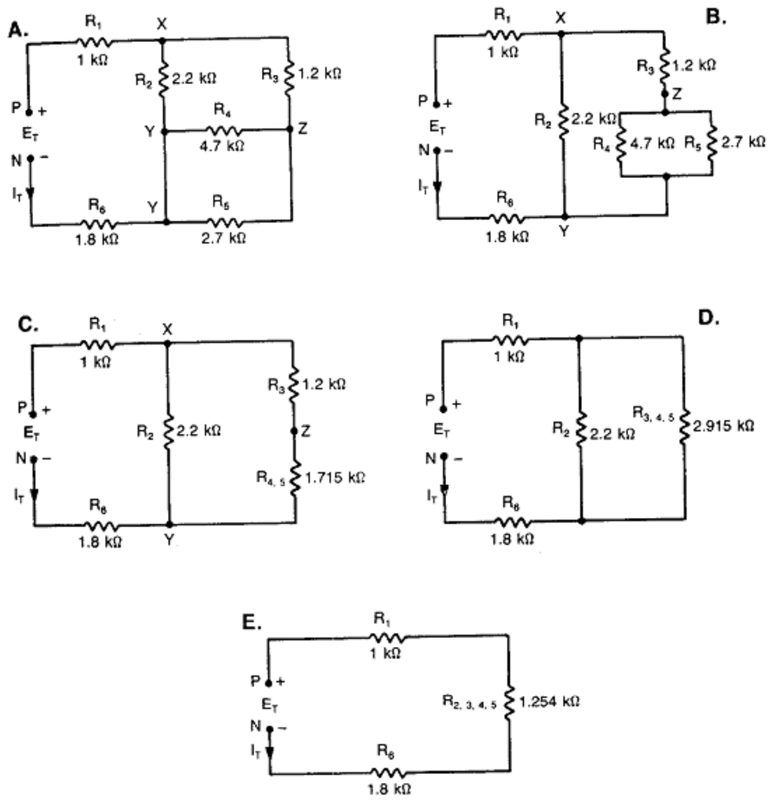

Figure 9.2: Solution steps

The circuit may then be redrawn as shown in figure 9.2C.

9.3 Step 3

Add together the series equivalent resistances and again draw the circuit. In figure 9.2C, $R3$ is in series with $R_4, 5$ so that the equivalent resistance of the combination is $R_{3,4,5} = 1.2 + 1.715 = 2.915\mathrm{k}\Omega$. The circuit when redrawn is shown in figure 9.2D.

9.4 Step 4

Use the product-over-sum or reciprocal formula, to combine the equivalent parallel resistances. In our example $R2$ is in parallel with $R_{3,4,5}$ and the equivalent resistance of this combination is:

$$R_{2,3,4,5} = \frac{2.915 \cdot 2.2}{2.915 + 2.2} = 1.254\,\mathrm{k}\Omega \tag{9.2}$$

When the circuit is again redrawn, its appearance is as in figure 9.2E.

9.5 Step 5

Repeat steps 3 and 4 alternatively and redraw the circuit as many times as necessary (note that, with experience, it is possible to eliminate most, if not all, of the intermediate drawings).

The redrawn circuit of figure 9.2E is clearly a simple series arrangement and therefore the total equivalent resistance presented to the source is $R_T = R_1 + R_{2,3,4,5} + R_6 = 1 + 1.254 + 1.8 = 4.054\text{k}\Omega$.

9.6 Step 6

Knowing the value of the total equivalent resistance you can determine the total source current, I_T, and then work away from the source and calculate in turn the branch currents and the voltage drops across the individual resistors.

10 Chap24:The constant current source

To represent an electrical source, ve can use a constant voltage model. This consisted of the constant EMF, E, in series with the internal resistance, (figure 10.1A). But is this the only model we can devise? As far as the load is concerned, it is also possible to have a *constant current source* that is capable of generating a variable EMF. The value of the constant current is the short-circuit terminal value. Across this current generator is a parallel internal resistance that has the same value as the series internal resistance of the constant voltage model (figure 10.1B).

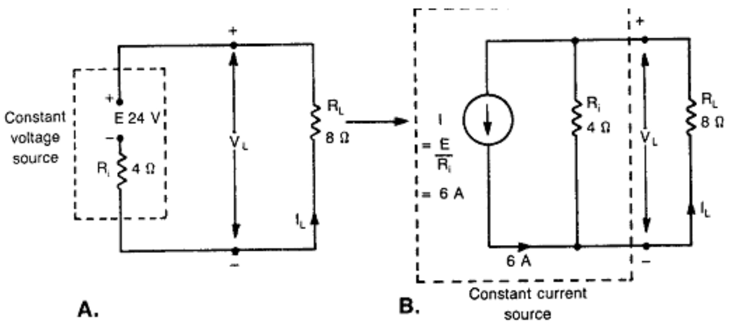

Figure 10.1: Example

Let us compare the two models by an example in which an electrical source has an open-circuit voltage of 24 V and a short circuit current of 6 A. Then the internal resistance is $\frac{24\,\mathrm{V}}{6\,\mathrm{A}} = 4\Omega$. The two comparable models are illustrated in figure 10.1A and B and the arrow convention in the constant current source indicates the same direction of electron flow as produced by the constant voltage source.

If we now connect an 8-ohm load across the terminals of each source, the load current for the constant voltage generator is:

$$I_L = \frac{24\,\mathrm{V}}{4\Omega + 8\Omega} = 2\,\mathrm{A} \tag{10.1}$$

For the constant current model the load current by the CDR rule is:

$$I_L = 6\,\mathrm{A} \cdot \frac{4\Omega}{4\Omega + 8\Omega} = 2\,\mathrm{A} \tag{10.2}$$

Consequently, as far as the load is concerned, both models are equally valid. However, the generators themselves are not exact equivalents because under open-circuit load conditions, there is power dissipated in the internal resistance of the current generator but not in the internal resistance of the voltage generator.

One question has not been answered. Why do we need the concept of the constant current source when we already have the constant voltage generator? We shall see that in the analytical methods of chapters
refchap11 through 32, one model is sometimes preferable to the other. Moreover some active devices approximate more to the constant voltage model (figure 10.2A) while others tend to behave like constant current sources (figure 10.2B).

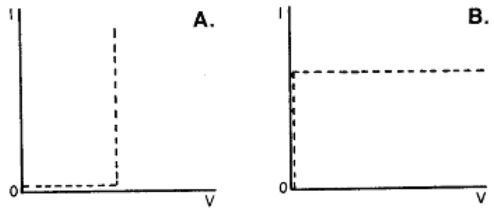

Figure 10.2: Voltage models

10.1 Mathematical derivations

10.1.1 Constant voltage source

Constant voltage EMF:

$$E = \text{open-circuit terminal voltage (volts)} \tag{10.3}$$

Series internal resistance:

$$R_i = \frac{\text{open circuit voltage}\, E}{\text{short-circuit current}\, I}\ \text{ohms} \tag{10.4}$$

Load current:

$$I_L = \frac{E}{R_1 + R_L}\ \text{amperes} \tag{10.5}$$

Load voltage:

$$V_L = I_L R_L = \frac{E R_L}{R_i + R_L}\ \text{volts} \tag{10.6}$$

10.1.2 Constant current source

Constant current:

$$I_L = \text{short-circuit termial current (amperes)} \tag{10.7}$$

Parallel internal resistance:

$$R_i = \frac{\text{open-cirtuit voltage}\, E}{\text{short-circuit current}\, I}\ \text{ohms} \tag{10.8}$$

Load current:

$$I_L = I \cdot \frac{R_i}{R_i + R_L} = \frac{E}{R_i \text{ffl} R_L}\ \text{amperes} \tag{10.9}$$

11 Chap26: Kirchhoff's voltage and current laws

In previous sections we have loosely refereed to Kirchhoff's voltage law (KVL) as the requirement for a voltage balance around any closed circuit or loop while Kirchhoff's current law (KCL) fulfilled the need for the current balance that must exist at any junction joint. These laws are commonly used to analyze circuits that are too difficult to be solved by Ohm's law alone.

A more formal KVL statement would be: " *The algebraic sum of the constant voltage EMFs and the voltage drops around any closed electrical loop is always zero.*"

Because the algebraic sum is involved, we must have a convention that distinguishes between positive and negative voltages. Normally we start at any point in a loop and move around in the clockwise direction. A voltage is then positive if the negative polarity of that voltage is first encountered. In the circuit of figure 11.1 there are three loops ($XYZX$, $XWYX$, and $XWYZX$) so that three KVL equations can be obtained.

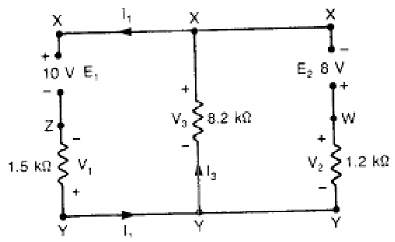

Figure 11.1: Voltage models

Initially the separate currents (electron flow) must be specified in the circuit. Sometimes the direction of a particular current is doubtful, such is the case with I_3 because the voltages E_1 and E_2 are tending to drive currents in opposite directions through the $8.2\,\mathrm{k}\Omega$ resistor. However, this is not a serious problem because if you choose the wrong direction, the mathematical sign of the current will be revealed as negative.

11.1 Mathematical derivations

If the currents are measured in miliamperes:

11.1.1 Loop XYZX (starting at X)

The first KVL equation is:

$$\begin{aligned} +(-V_3) + (-V_1) + (+E_1) &= 0 \\ -I_3 \cdot 8.2\,\mathrm{k}\Omega - I_1 \cdot 1.5\,\mathrm{k}\Omega + 12\,\mathrm{V} &= 0 \\ 1.5 I_1 + 8.2 I_3 &= 12 \end{aligned} \tag{11.1}$$

12 Chap27: Mesh analysis

A closed voltage loop may also be referred to as a mesh. In mesh analysis, each of the mesh currents i_1,i_2,i_3 (figure 12.1) flow around a complete loop although an individual resistor may carry one or more mesh currents. For example, the mesh currents i_1 and i_2 flow in the opposite direction through the resistor $R1$.

The normal convention is to assign clockwise mesh (electron flow) currents in each of the loops and then write down the KVL equation for each loop. The main advantages of this method are:

- we will not require any KCL equations
- the KVL equations can be written by inspection and do not require precise voltage conventions

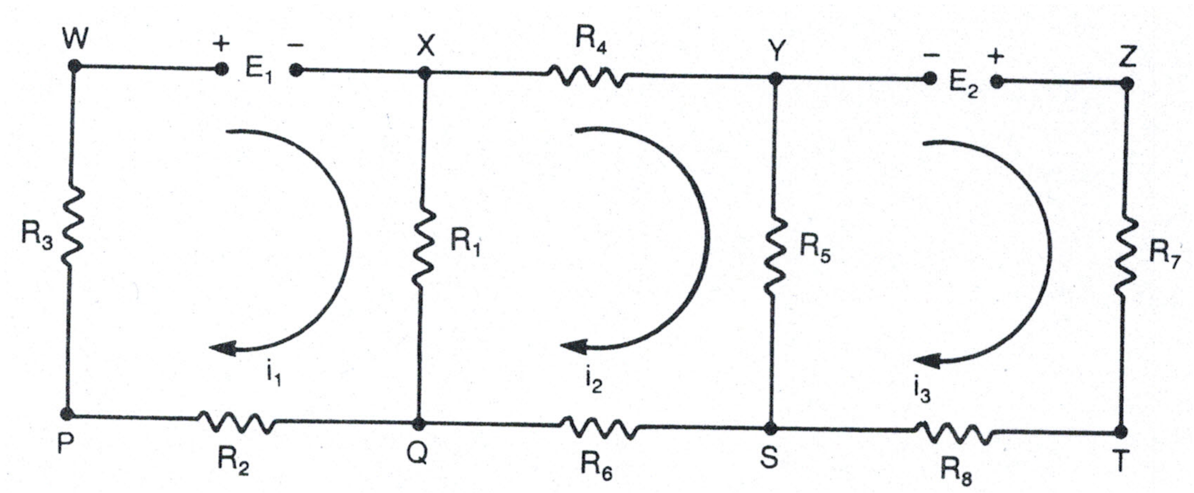

Figure 12.1: Mesh analysis

12.1 Mathematical derivations in 12.1

KVL equations for the mesh $PWXQP$ (starting at P):

$$
\begin{aligned}
i_i \cdot R_3 - E_1 + i_1 \cdot R_1 + i_1 \cdot R_2 - i_2 \cdot R1 = 0 \\
-i_2 \cdot R_1 + i_1 (R_1 + R_2 + R_2) = E_1
\end{aligned}
$$

KVL equation for the mesh $QXYSQ$ (starting at Q):

$$
\begin{aligned}
i_2 \cdot R_1 - i_1 \cdot R_1 + i_2 \cdot R_4 + i_2 \cdot R_5 - i_3 \cdot R_5 + i_2 \cdot R_6 = 0 \\
-i_1 \cdot R_1 - i_3 \cdot R_5 + i_2 \cdot (R_1 + R_4 + R_5 + R_6) = 0
\end{aligned}
$$

KVL equation for the mesh $SYZTS$ (starting at S):

$$i_3 \cdot R_5 - i_2 \cdot R_5 + E_2 + i_3 \cdot R_7 + i_3 \cdot R_8 = 0$$
$$-i_2 \cdot R_5 + i_3 \cdot (R_5 + R_7 + R_8) = -E_2$$

From the pattern of these results it follows that the KVL equations can be written down from inspection if we observe the following rules:

1. Add together all the resistances in the loop and multiply the results by that loop's mesh current. This applies to the terms $i_1 \cdot (R_1 + R_2 + R_3)$, $i_2 \cdot (R_1 + R_4 + R_5 + R_6)$, and $i_3 \cdot (R_5 + R_7 + R_8)$.
2. If a resistor in a particular mesh carries a current assigned to another mesh, the corresponding IR drop is given a negative sign, This refers to the terms $-i_2 \cdot R_1$, $-i_1 \cdot R_1$, $-i_3 \cdot R_5$, and $-i_2 \cdot R_5$.
3. The signs of the voltage sources are determined by the normal KVL convention.

12.2 Example

In figure 12.2, calculate the value of the potential a the point X.

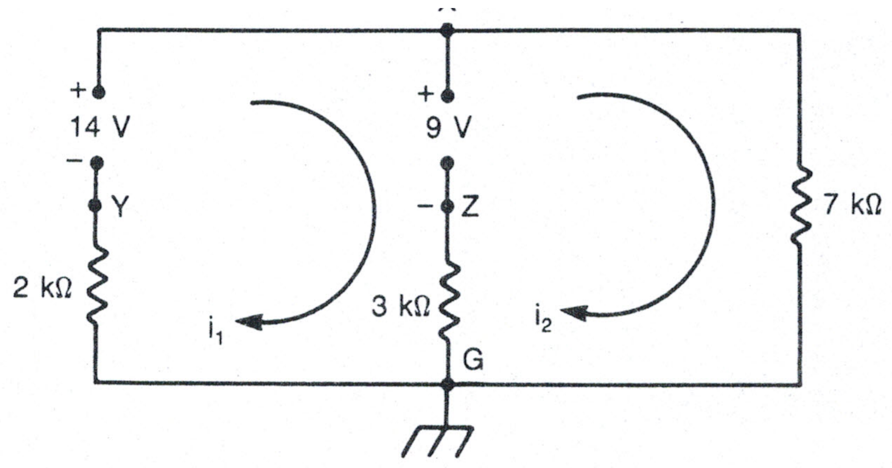

Figure 12.2: Example

For mesh $XZGYX$:

$$i_1 \cdot (2\,\mathrm{k\Omega} + 3\,\mathrm{k\Omega}) - i_2 \cdot 3\,\mathrm{k\Omega} - 9\,\mathrm{V} + 14\,\mathrm{V} = 0 \tag{12.1}$$
$$5 \cdot i_1 - 3 \cdot i_2 = -5$$

For mesh $XGZX$:

$$i_1 \cdot 3\,\mathrm{k\Omega} + i_2 \cdot (3\,\mathrm{k\Omega} + 7\,\mathrm{k\Omega}) + 9\,\mathrm{V} = 0 \tag{12.2}$$
$$-3 \cdot i_1 + 10 \cdot i_2 = -9$$

Multiply equation 12.1 by 3 and 12.2 by 5

$$\begin{aligned} 15 \cdot i_1 - 9 \cdot i_2 &= -15 \\ -15 \cdot i_1 + 50 \cdot i_2 &= -45 \end{aligned}$$

Therefore

$$\begin{aligned} 41 \cdot i_2 &= -60 \\ i_2 &= \frac{-60}{41}\,\mathrm{mA} \end{aligned}$$

Potential at point X is $i_2 \cdot R_L = -(-\frac{60}{41}\,\mathrm{mA}) \cdot 7\,\mathrm{k}\Omega = 10.24\,\mathrm{V}$.

13 Chap28: Nodal analysis

Nodal analysis is another method for solving circuits, but this time, only current sources are involved. By contrast, those circuits that were solved by mesh analysis or Kirchhoff's laws contained voltage sources and required the use of simultaneous equations. However, on many occasions it is possible to convert the voltage sources into their equivalent current sources and subsequently solve the problem with a single nodal equation rather than with two or more simultaneous equations.

A node is another term for a junction point at which two or more electrical currents exist. Therefore the junction N is a node (figure 13.1) and the purpose of the analysis is to find the value of the potential at N. As a convention we shall assume that the potential at N is negative. The true polarity will be shown by the sign of the value for V_L. If the potential at N is negative, the electrons through the resistors must flow from the nodal point to ground. The generators that force (electron flow) currents into the node are then positive while those driving currents out of the node are negative.

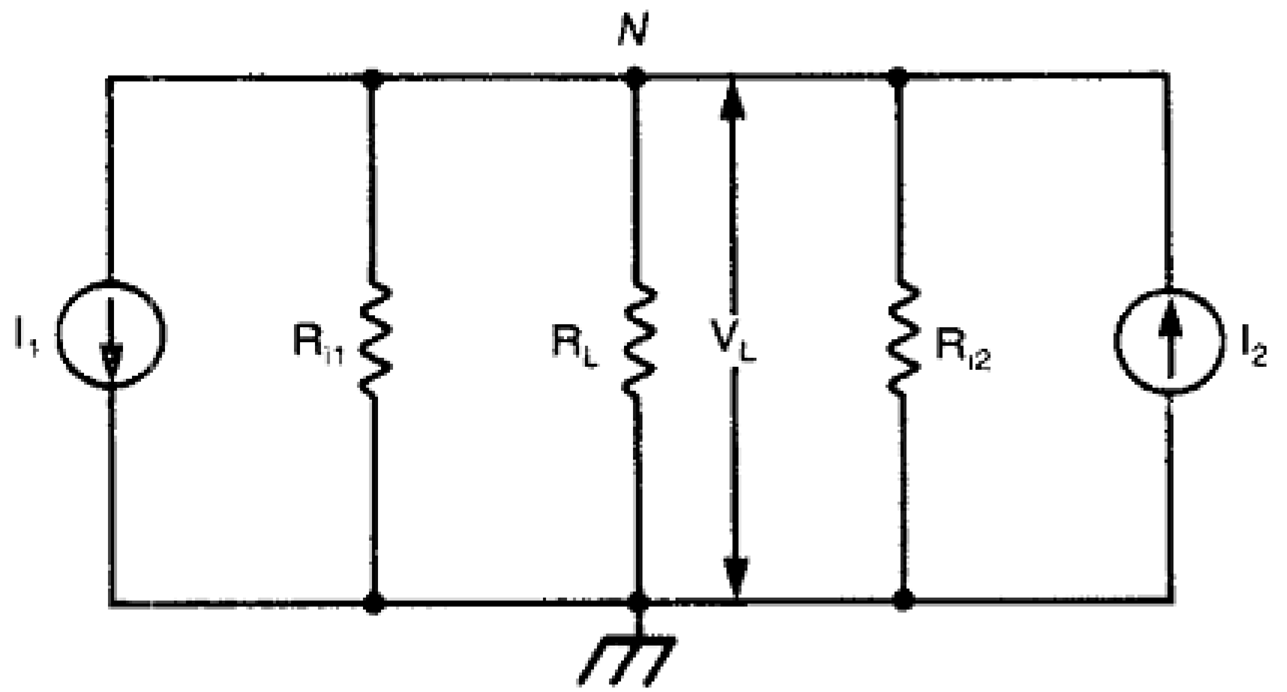

Figure 13.1: Voltage models

13.1 Mathematical derivations in figure 13.1

Total electron flow currents leaving node N through resistors $= \frac{V_L}{R_{i1}} + \frac{V_L}{R_L} + \frac{V_L}{R_{i2}}$ amperes.

Total of the generator currents $= -(I_1) + (+I_2)$ amperes. Consequently the nodal equation is:

$$(-I_1) + (+I_2) = \frac{V_L}{R_{i1}} + \frac{V_L}{R_L} + \frac{V_L}{R_{i2}} \tag{13.1}$$

13.2 Example

In figure 13.2A, calculate the value of the potential at the point N.

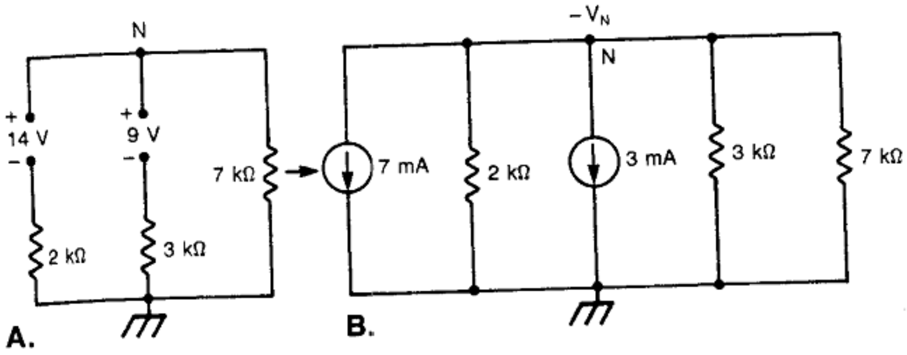

Figure 13.2: Voltage models

13.2.1 Solution

Convert the voltage sources of figure 13.2A into their constant current equivalents (figure 13.2B).

Total electron flow currents leaving node N through resistors $= \frac{V_N}{2\,\mathrm{k}\Omega}+\frac{V_N}{3\,\mathrm{k}\Omega}+\frac{V_N}{7\,\mathrm{k}\Omega} = 0.9762V_N\,\mathrm{mA}$.

Total of generator currents $= (-7) + (-3) = -10\,\mathrm{mA}$. Therefore

$$\begin{aligned} 0.9762\,V_N &= -10\,\mathrm{mA} \\ V_N &= -\frac{10}{0.9762} \\ &= -10.24\,\mathrm{V} \end{aligned}$$

The potential at N is therefore 10.24 V.

14 Chap29: The superposition theorem

The principle of superposition may be used to solve a number of problems that contain more than one source and only linear resistances. The method requires you to consider the currents and voltages created by each source in turn and then finally combine (superimpose) the results from all the sources. A formal statement of this theorem follows.

In a network of linear resistances containing more than one source, the resultant current flowing at any one point is the algebraic sum of the currents that would flow at that point if each source is considered separately while all other sources are replaced by their equivalent internal resistances. This last step is earned out by short-circuiting all sources of constant voltage and open-circuiting all sources of constant current.

This theorem has the advantage of allowing each source to be considered separately so that only Ohm's law equations are required in the solution. However, each time one of the sources is applied to the circuit, a different voltage will normally appear across a particular resistor. The superposition theorem therefore requires the resistance to be linear so that its resistance does not change with the amount of the voltage drop but remains at a constant value. In other words, the voltage drop is always directly proportional to the current.

If a large number of sources are involved, use of the superposition theorem is not recommended because the circuit has to be solved separately for each source before combining the results. The final analysis would probably be far more tedious than if we used either the mesh or nodal methods.

15 Chap31:Thévenin's theorem

Thévenin's theorem is the most valuable analytical tool when dealing with complex networks. It enables you to focus your attention on a particular component or any circuit part that is connected between two terminals and is regarded as the load. The remainder of the circuit is then represented by a single generator with a constant voltage, E_{TH}, which is in series with an internal resistance, R_{TH}. The purpose of Thévenin's theorem is then to obtain the values of E_{TH} and R_{TH} from the components and sources in the original circuit. Stated formally. Thévenin's theorem is:

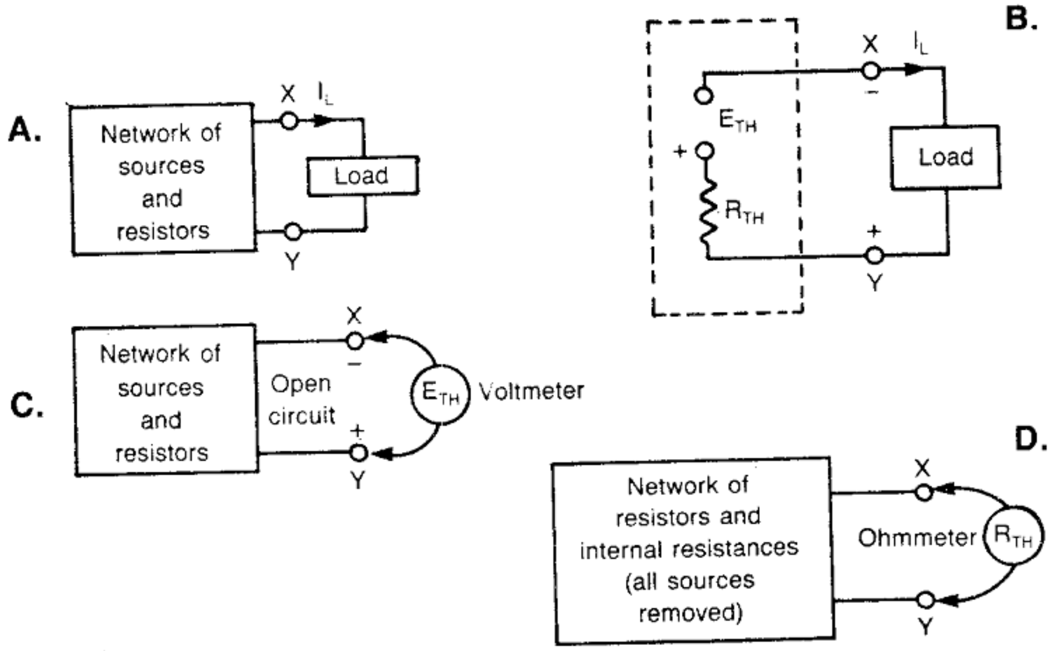

Figure 15.1: Examle of Thévein's Theorem

The current in a load connected between two output terminals, X and Y a network of resistors and electrical sources (figure 15.1A) is no different than it that same load were connected across a simple constant voltage generator (figure 15.1B) whose EMF. E_{TH}, is the open circuit voltage: measured between X and Y (figure 15.1C) and whose series interned resistance, R_{TH}, is the resistance of the network looking back into the terminals X and Y with all sources replaced by resistances equal to their internal resistances (figure 15.1D). This last step involves short-circuiting all sources of constant voltage and open-circuiting all sources of constant current.

The process of Thevenizing the circuit is illustrated in figure 15.1. As an example let us Thevenize the same circuit (figure 15.2A).

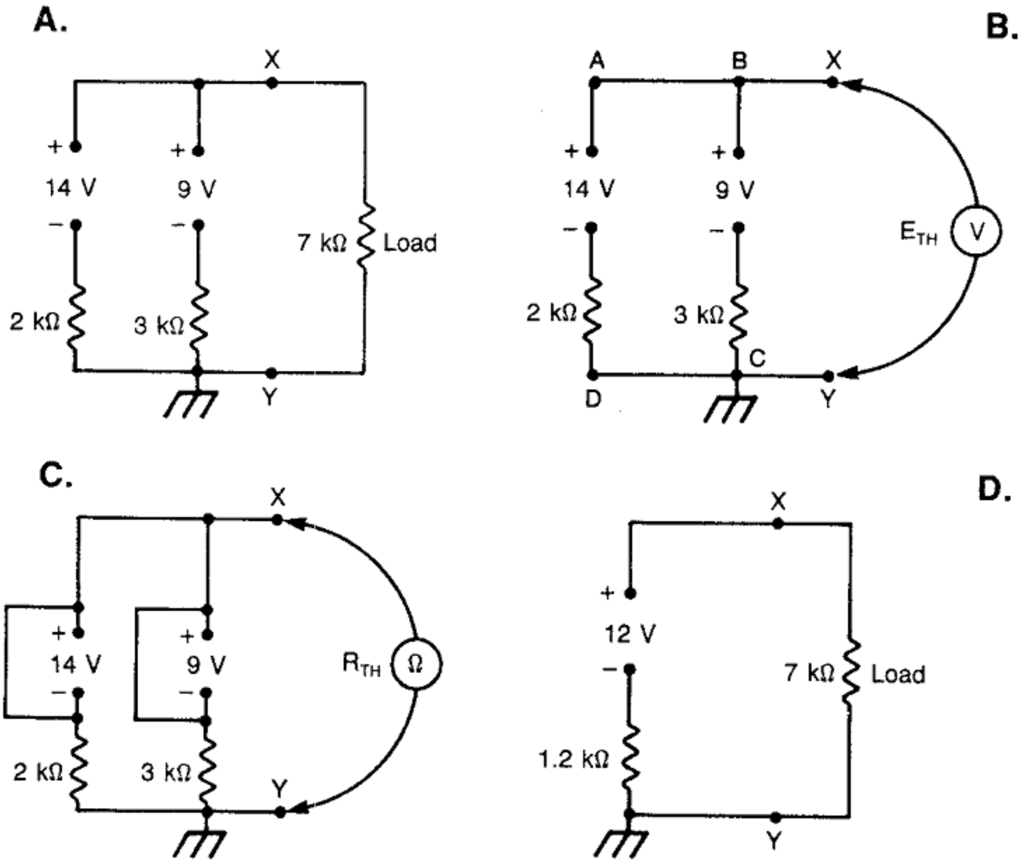

Figure 15.2: Example

The $7\,\mathrm{k\Omega}$ resistor is regarded as the load. When this load is removed, we consider that a voltmeter is connected between X and Y to record the value of E_{TH}. In the loop $ABCD$ the sources are in series-opposing and the current, I, is:

$$\frac{14\,\mathrm{V} - 9\,\mathrm{V}}{2\,\mathrm{k\Omega} + 3\,\mathrm{k\Omega}} = 1\,\mathrm{mA}$$

in the direction shown. The value of E_{TH} can be thought of as either $14\,\mathrm{V} - (1\,\mathrm{mA} \cdot 2\,\mathrm{k\Omega} = 12\,\mathrm{V}$ or $9\,\mathrm{V} + (1\,\mathrm{mA} \cdot 3\,\mathrm{k\Omega}) = 12\,\mathrm{V}$ (figure 15.2B).

To find the value R_{TH}, the 14-V and 9-V constant voltage sources are shorted out and we visualize that an ohmmeter is connected between X and Y (figure 15.2C). The ohmmeter reading will then be $2\,\mathrm{k\Omega} | 3\,\mathrm{k\Omega} = \frac{2 \cdot 3}{2+3} = 1.2\,\mathrm{k\Omega}$. The load is now replaced across the Thévenin equivalent generator (figure 15.2D) and by the VDR, the potential at X will be $+122\,\mathrm{V} \cdot \frac{7\,\mathrm{k\Omega}}{1.2\,\mathrm{k\Omega} + 7\,\mathrm{k\Omega}} = 10.24\,\mathrm{V}$.

You are probably not impressed with the performance of Thévenin's theorem in solving this particular problem. The solution certainly seemed much more involved that the Millman treatment. However, with other types of problems Thévenin's theorem has a number of advantages and nowhere is this better shown than in the case of the unbalanced bridge circuit of figure 15.3A. Using mesh or nodal analysis such a circuit would require three algebraic simultaneous equations while the Thévenin method does not require algebra at all.

15.1 Example

In figure 15.3A, calculate the value of the load current I_L.

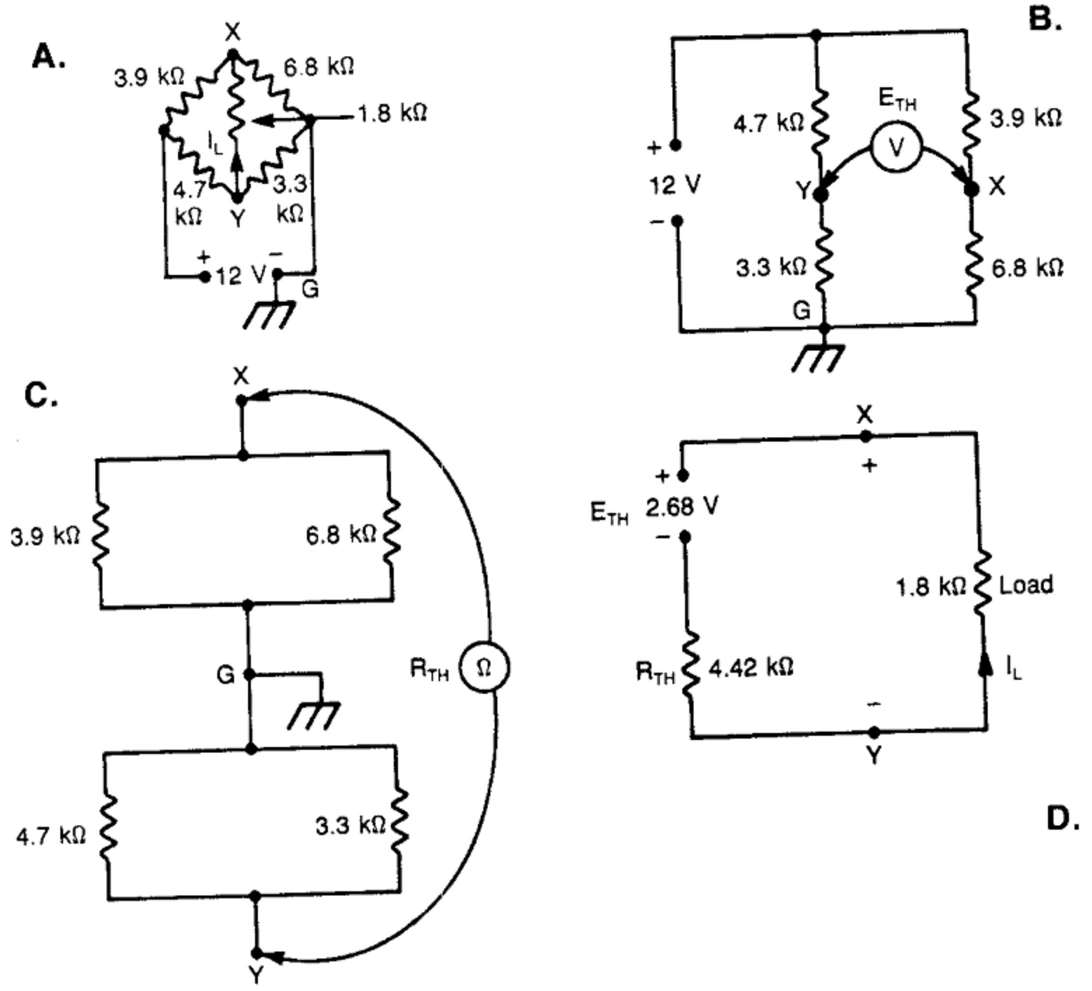

Figure 15.3: Example - bridge cirtuit

Regard the $1.8\,\mathrm{k\Omega}$ resistor as the load. When this load is removed and replaced by a voltmeter, the remaining circuit is as shown in figure 15.3B. By the voltage division rule, the potential at Y is:

$$+12\,\mathrm{V} \cdot \frac{3.3\,\mathrm{k\Omega}}{3.3\,\mathrm{k\Omega} + 4.7\,\mathrm{k\Omega}} = +4.95\,\mathrm{V}$$

16 Chap 33: Delta-wye and wye-delta transformations

We have already examined the unbalanced bridge circuit, which cannot be regarded as a series-parallel arrangement because the center arm bears no simple series or parallel relationship to the other four resistors. However, on many occasions it is possible to convert such circuits into simpler arrangements by means of a delta-wye (or a wye-delta) transformation. A triangular arrangement of resistors (figure 16.1A) is known as a delta (Δ, the Greek capital letter) connection. However, the same arrangement can be changed to resemble another Greek capital letter, pi, Π (figure 16.1B) so that the two names are interchangeable. The same applies to a second configuration, which is referred to as a Wye (letter Y) connection (figure 16.2A). This can also be modified to look like the letter tee (T), so that either tee or wye is used to describe identical networks of resistors.

If the delta-wye networks are to be equivalent, the resistances between the points, X, Y, Z in both configurations must be the same. The required equations for the two transformations ($\Delta \to Y$ and $Y \to A$) follow.

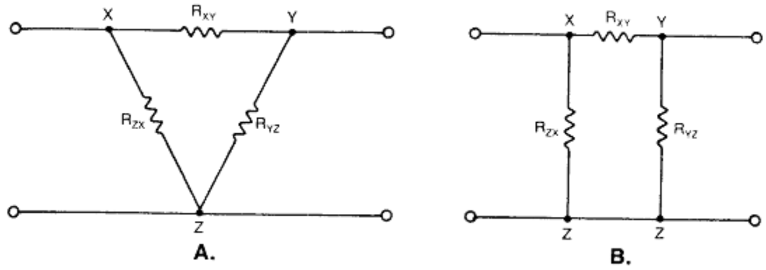

Figure 16.1: Examle

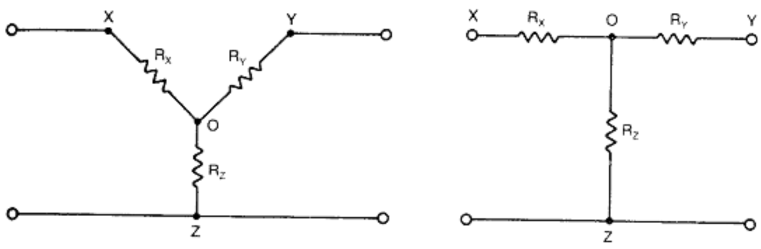

Figure 16.2: Examle

16.1 Mathematical derivations

16.1.1 Figure 16.1

Resistance between the points X and $Y = \frac{R_{XY} \cdot (R_{YZ} + R_{ZX})}{R_{XY} + (R_{YZ} + R_{ZX})}$

16.1.2 Figure 16.2

Resistance between the points X and $Y = R_X + R_Y$. Therefore:

$$R_X + R_Y = \frac{R_{XY}(R_{YZ} + R_{ZN})}{R_{XY} + R_{YZ} + R_{ZN}} \tag{16.1}$$

By symmetry:

$$R_Y + R_Z = \frac{R_{YZ}(R_{ZX} + R_{XY})}{R_{XY} + R_{YZ} + R_{ZX}} \tag{16.2}$$

$$R_Z + R_X = \frac{R_{ZX}(R_{XY} + R_{YZ})}{R_{XY} + R_{YZ} + R_{ZX}} \tag{16.3}$$

Add equations 16.1 and 16.3. From their sum subtract equation 16.2 and divide result by 2. This yields:

$$R_X = \frac{R_{XY} R_{ZX}}{R_{XY} + R_{YZ} + R_{ZX}} \tag{16.4}$$

Similarly:

$$R_Y = \frac{R_{YZ} R_{XY}}{R_{XY} + R_{YZ} + R_{ZX}} \tag{16.5}$$

and

$$R_Z = \frac{R_{ZX} R_{YZ}}{R_{XY} + R_{YZ} + R_{ZX}} \tag{16.6}$$

Equations 16.4, 16.5, 16.6 are used to achieve a $\Lambda \rightarrow Y$ transformation. For the $Y \rightarrow \Lambda$ transformation, the following three equations can be derived:

$$R_{XY} = \frac{R_X R_Y + R_Y R_Z + R_Z R_X}{R_Z} \tag{16.7}$$

$$R_{YZ} = \frac{R_X R_Y + R_Y R_Z + R_Z R_X}{R_X} \tag{16.8}$$

$$R_{ZX} = \frac{R_X R_Y + R_Y R_Z + R_Z R_X}{R_Z} \tag{16.9}$$

These transformations can be used to advantage with the analysis of the unbalanced bridge circuit.

17 Chap41: Energy stored in the magnetic field of an inductor

The inductor is assumed to be ideal in the sense that there are no resistance losses associated with the component. When S is closed, the growth rate of the current is $\frac{E}{L}$ amperes per second so that the counter EMF induced in the inductor exactly balances the source voltage (figure 17.1A). After a period of t seconds the current is equal to I amperes, and because the growth of the current is linear with respect to time, the average current during the period is 7/2 amperes (figure 17.1B). Around the inductor is established a magnetic field that represents the energy derived from the source. If the switch is then opened, the magnetic field collapses and the energy is dissipated in the form of an arc, which occurs between the contacts of the switch.

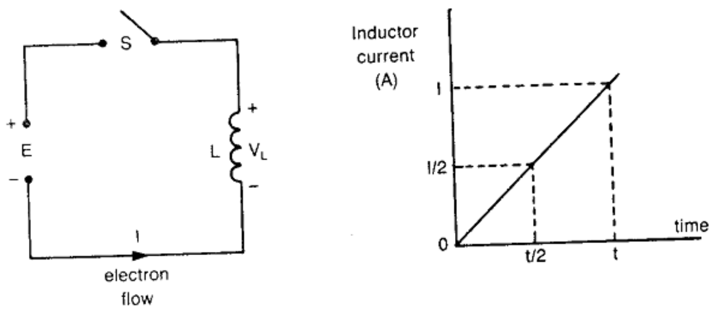

Figure 17.1: Circuit with the inductor, Current graph

18 Chap. 47: Capacitance and the capacitor

Capacitance is that property of an electrical circuit that prevents any sudden change in voltage and limits the rate of change in the voltage. In other words, the voltage across a capacitance, whose letter symbol is C, cannot change instantaneously.

The property of capacitance is possessed by a capacitor, which is a device for storing charge and consists of two conducting surfaces (copper, silver, aluminum, tin foil, etc.) which are separated by an insulator or dielectric (air, mica, ceramic, etc.).

When a dc voltage, E, is switched across the capacitor, C, in figure 18.1, electrons are drawn off the right-hand plate, flow through the voltage source and are then deposited on the left-hand plate. This electron flow is only momentary and ceases when the voltage, V_C, between the plates exactly balances the source voltage, E. The charge then stored by the capacitor directly depends on the value of the applied voltage and also on some constant of the capacitor called the capaci¬tance, which is determined by the component's physical construction. The unit for the capacitance is the farad, F. The capacitance is one farad, if when the voltage applied across the capacitor's plates is one volt, the charge stored is one coulomb. Unfortunately, the farad is far too large a unit for practical purposes so that capacitances are normally either measured in microfarads (μ)F or picofarads (pF).

We already know that a particular electric field intensity (which is inversely proportional to the separation between the plates) produces a certain flux (which is directly proportional to the area of one of the plates). This means that a capacitor's capacitance is directly proportional to the area of the plates, but is inversely proportional to their distance apart. It is true that the capacitance is increased if the conductor plates are brought closer together. However, if the dielectric is made too thin, the voltage between the plates may cause arcing to occur so that the capacitor will be damaged.

Each type of insulator has a certain dielectric strength, which is a measure of the insulator's ability to withstand a high electric field intensity. For example, the dielectric strength of mica is typically 50 kV/mm so that in order to avoid breakdown, no more than 50 kV should be applied across a 1 mm thickness of mica. It follows that each capacitor is rated for a particular dc working voltage (WVdc).

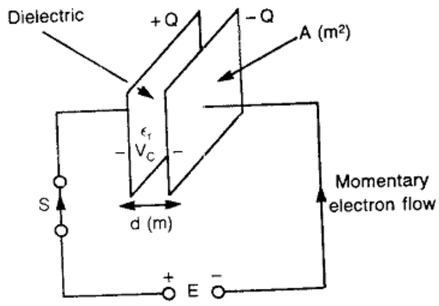

Figure 18.1: Example

19 Chap. 49: Capacitors in series

Because the capacitors are joined end-to-end in figure 19.1, there is only one path for current flow. Therefore connecting capacitors in series reduces the total capacitance so that the total capacitance is less than the value of the smallest capacitance in series. Basically, this is because the series arrangement effectively increases the distance between the end plates, $P1$ and $P6$, connected to the source voltage, and the capacitance is inversely proportional to this distance. Because the capacitor has a dc working voltage (WVdc), the series arrangement may be used to distribute the source voltage between the capacitors so that the voltage across an individual capacitor does not exceed its rating.

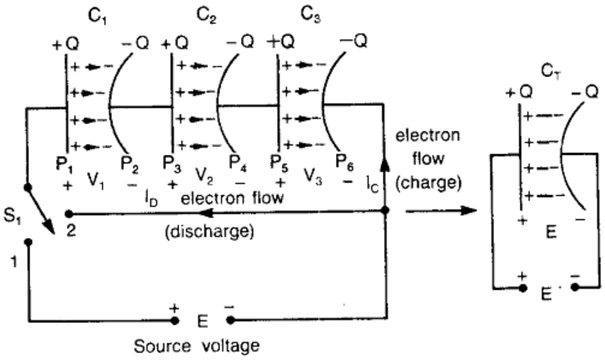

Figure 19.1: Example

19.1 Mathematical derivations

$$V_1 = \frac{Q}{C_1} \qquad Q = C_1 V_1 C_1 = \frac{Q}{V_1} \tag{19.1}$$

$$V_2 = \frac{Q}{C_2} \qquad Q = C_2 V_2 C_2 = \frac{Q}{V_2} \tag{19.2}$$

$$V_3 = \frac{Q}{C_3} \qquad Q = C_3 V_3 C_3 = \frac{Q}{V_3} \tag{19.3}$$

Source voltage:

$$\begin{aligned} E &= V_1 + V_2 + V_3 \qquad (19.4) \\ &= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \\ &= Q \cdot (\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}) \text{ volts} \end{aligned}$$

20 Chap. 50: Capacitors in parallel

Capacitors in parallel are connected across two common points (A, B) or lines so that the source voltage is applied across each of the capacitors (figure 20.1). Because $Q = CV$, it follows that a different charge must be stored by each capacitor. This is due to the different momentary charging currents that exist at the instant the switch, S, is closed. The total charge is then equal to the sum of the individual charges stored by each capacitor.

As we shall see in the mathematical derivations, the purpose of connecting capacitors in parallel is to increase the total capacitance. In figure 20.1 we have connected together on one side the plates $P1$, $P2$, $P3$, and, on the other side, the plates $P4$, PS, $P6$. The result is to increase the effective surface area to which the capacitance is directly proportional.

Each of the three capacitors in figure 20.1 stores energy in the form of the electric field between its plates. The total energy stored is the sum of the capacitor's individual energies.

The principles of capacitors in series (chapter 19) and capacitors in parallel may be extended to series-parallel arrangements.

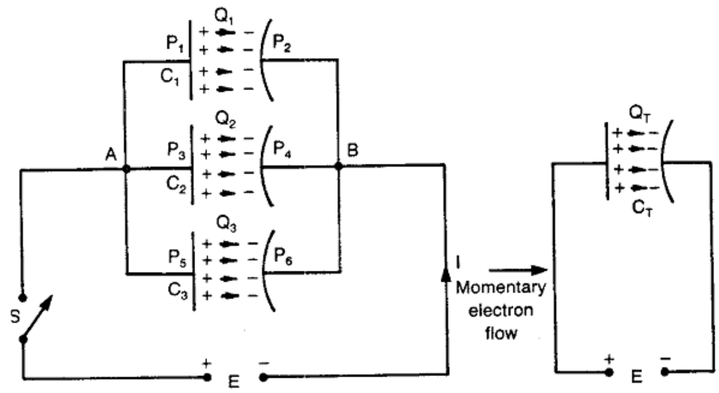

Figure 20.1: Example

20.1 Mathematical derivations

Charges:

$$Q_1 = C_1 E Q_2 = C_2 E \qquad Q_3 = C_3 E \tag{20.1}$$

Total charge stored:

$$\begin{aligned} Q_T &= Q_1 + Q_2 + Q_3 \\ &= C_1 E + C_2 E + C_3 E \\ &= E \cdot (C_1 + C_2 + C_3)\,\text{coulombs} \end{aligned} \tag{20.2}$$

If C_T is the total equivalent capacitance, we can calculate the total charge as

$$Q_T = E \cdot C_T\,\text{coulombs} \tag{20.3}$$

21 Chap. 59: Introduction to alternating current (ac)

Previous chapters have been concerned with direct current in which the electron flow has always been in the same direction although the magnitude of the flow has not necessarily been constant.

With alternating current, the current reverses its direction periodically and has an average value of zero. If the average value of the waveform is not zero, it is regarded as a combination of a dc component and an ac component. The alternating current may assume a variety of wave¬forms some of which are shown in figures 21.1A, B, C, and D. In every case a single complete waveform is known as a cycle and the number of cycles occurring in one second is the frequency, which is therefore the rate at which the waveform repeats.

The letter symbol for the frequency is f and its SI unit is the hertz (Hz) which is equivalent to one cycle per second.

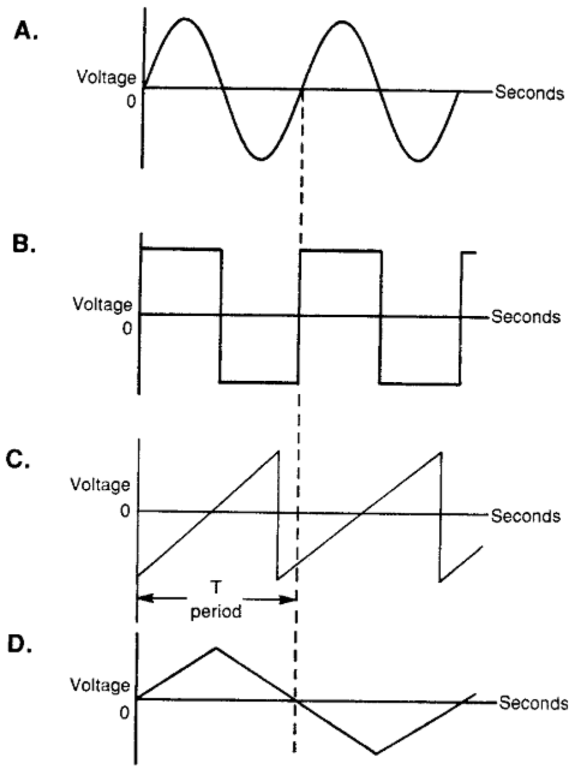

Figure 21.1: Example

The time interval taken by a complete cycle is the period, which is equal to the reciprocal of the frequency. The letter symbol for the period is T and its basic unit is the second (s). As an example the commercial ac line voltage has a frequency of 60 Hz and therefore its period is l/60th of a second.

The sine wave is not only associated with the commercial line voltage but is also widely used for communications. The curve itself is the result of plotting the mathematical sine function, $sin\phi$, versus the angle ϕ, measured in either degrees or radians (figure 21.2).

The vertical axis measures only one of a number of electrical parameters such as voltage, current, and power, while the horizontal axis is a time scale. It follows that, for every sine wave, a certain number of radians on the horizontal scale must correspond to a particular time interval. The angular frequency whose letter symbol is the Greek lowercase omega, ω, is found

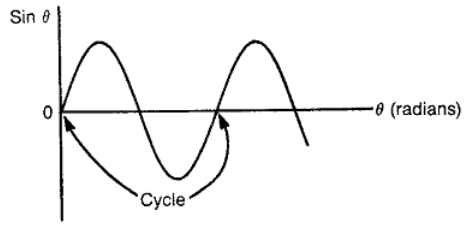

Figure 21.2: Example

by dividing the number of radians by the equivalent time interval and is therefore measured in radians per second (rad/s).

One cycle of a sine wave consists of a "positive" alternation and a "negative" alternation (figure 21.3). The maximum excursion from its average or zero line is called the peak or maximum (E_{max}) value while the distance between the wave's crest and trough is a measure of the peak-to-peak (E_{p-p}) value.

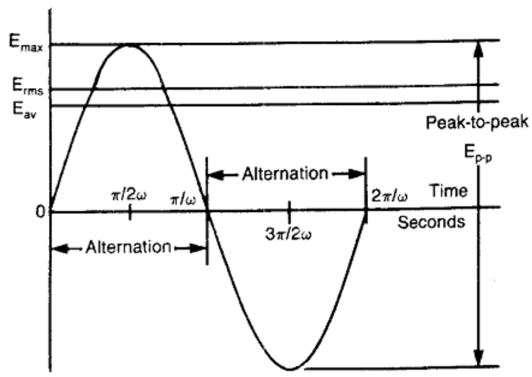

Figure 21.3: Example

Sine waves are of particular importance because Fourier analysis allows the waveforms of figure 21.1B, C, and D to be broken into a series of sine waves that consist of a fundamental component together with its harmonics (figure 21.4). The harmonic of a sine wave is another sine wave whose frequency is a whole number times the original or fundamental frequency.

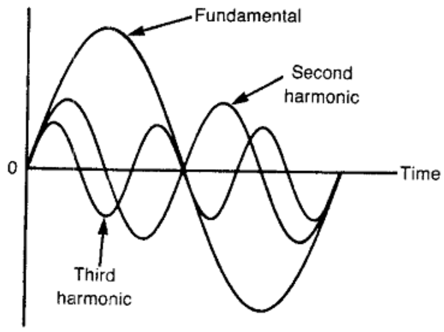

Figure 21.4: Example

For example if the fundamental frequency is 1 kHz (f), the second harmonic has a frequency of 2 kHz ($2f$), the third harmonic frequency is 3 kHz ($3f$) etc. The sine wave syntheses of a square wave and a sawtooth wave are illustrated in figures 21.5A and B.

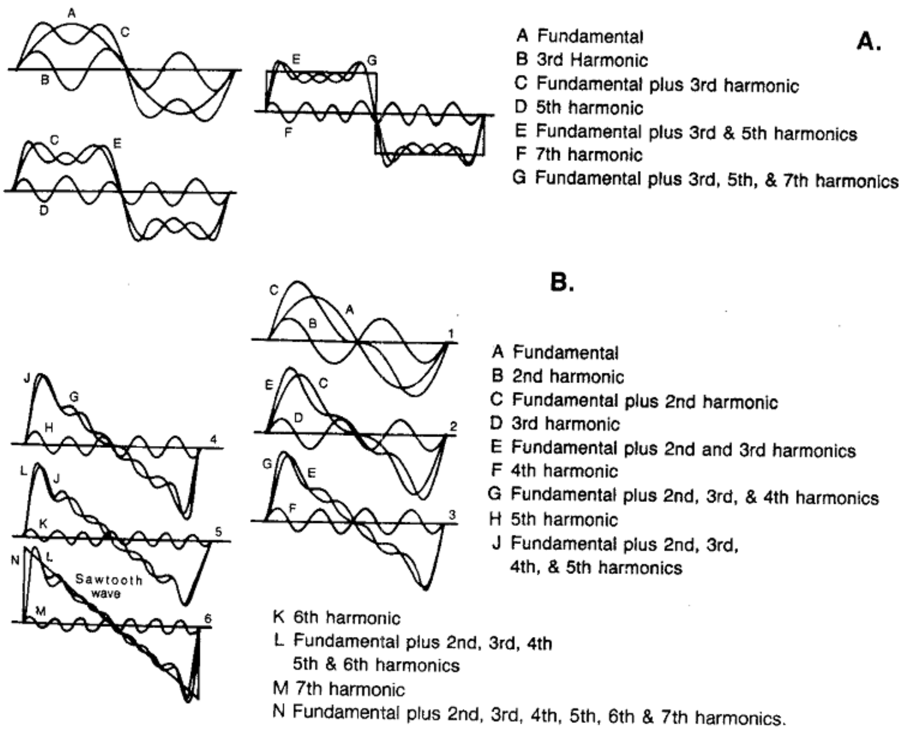

Figure 21.5: Example

21.1 The simple ac generator (alternator)

The ac generator is a machine that is capable of converting mechanical energy into alternating electrical energy. In its simplest form (figure 21.6A), a shaft DD' is driven around by mechanical energy. Attached to the shaft but insulated from it, is a copper loop that rotates between the poles, PP', of a permanent magnet or an electromagnet (energized by direct current).

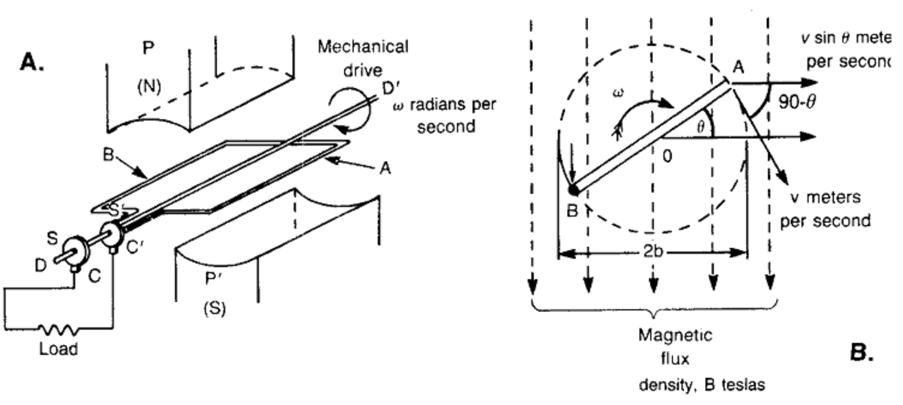

Figure 21.6: Example

The ends of the rotating loop are joined to two separate slip rings, SS' that make continuous contact with the stationary carbon brushes, CC'; these brushes are then connected to the electrical load.

The poles pieces, PP', are especially shaped to provide a constant flux density, B, in which the loop rotates (figure 21.6B). One complete rotation of the loop will generate one cycle of alternating voltage, which is applied across the load. If the output frequency is 60 Hz, the loop's speed of rotation is $60 \cdot 60 = 3600$ revolutions per minute. However, if the alternator has four poles, each complete rotation of the loop generates two cycles of alternating voltage and therefore the speed required for a 60 Hz output is only 1800 rpm.

21.2 Mathematical derivations

Frequency:

$$f = \frac{1}{T}\,\mathrm{Hz} \tag{21.1}$$

Period:

$$T = \frac{1}{f}\,\mathrm{seconds} \tag{21.2}$$

22 Chap. 62: Phasor representation of an ac voltage or current

If two components are in series across an ac source, there will be an alternating voltage across each component. If these two voltages are then added, their resultant exactly balances the source voltage (Kirchhoff's voltage law). In the case of a parallel circuit the individual branch currents must be added to obtain the total current drawn from the source. In previous sections we have expressed a sine wave either as a trigonometrical equation, $e = E_{max} \sin \omega t$ or graphically by a waveform. When we need to add or subtract ac quantities, the trigonometrical equations become difficult to manipulate and the combination of waveforms tends to be tedious.

A third method involves the representation of ac quantities by means of phasors. If the equation of an ac voltage is $e = E_{max} \sin \omega t$, its phasor is a line whose length is a measure of E_{max}. By convention the line phasor is assumed to rotate in the positive or counterclockwise direction with an angular velocity of $\omega = 2\pi f)$ radians per second, where f is the voltage's frequency in hertz. The vertical projection, PN, of the phasor on the horizontal reference line is then equal to the instantaneous value, e, of the ac voltage (figure 22.1). Therefore as the phasor rotates, the extremity of the line can be said to trace out the voltage s sinusoidal waveform with a frequency that is equal to the phasor's speed of rotation in revolutions per second. One complete rotation of the phasor then traces out one cycle of the sine wave. The phasor diagram therefore contains the same information as the waveform presentation but it is obviously easier to work with lines rather than sine waves. Because there is a constant relationship between effective and peak values (effective value = $0.707 \cdot$ peak value), the phasor's length may also be used to indicate the rms value.

Prior to about 1960, the word vector was used rather than a phasor. A vector is a quantity that possesses both magnitude and direction so that a mechanical force is an example of a vector. By contrast a scalar quantity such as mass possesses magnitude only. The rules for adding or subtracting mechanical vectors are the same as those for adding or subtracting ac phasors. However, the vector rules for multiplication and division are totally different from the corresponding phasor rules. For this reason vectors were no longer used to represent ac quantities and instead the word phasor was introduced.

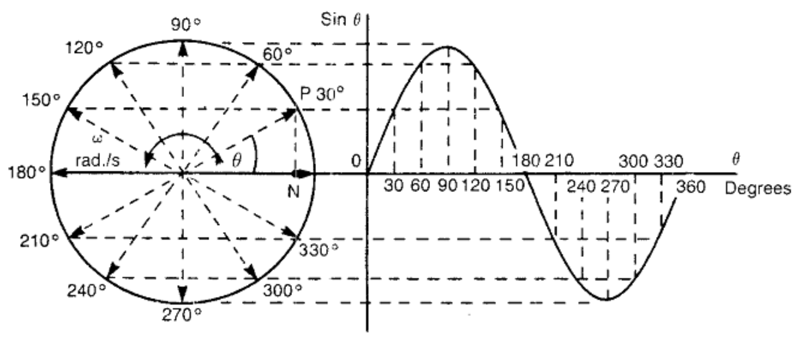

Figure 22.1: Example

23 Chap. 63: Phase relationships

When to dc voltages E_1 and E_2, are in series, they are either in series-aiding or in series-opposing so that only addition or subtraction is necessary in order to obtain their combined voltage (chapter 4). However, two ac sine wave voltages of the same frequency may not reach similar points in their cycles at the same time. As an example (chapter 28), when an inductor and a resistor are in series across an ac source, the voltage across the inductor reaches its peak value when the voltage across the resistor is zero. In other words, the two voltages are a quarter of a cycle apart. The amount by which the two sine waves are out of step is referred to as their phase difference, which is either measured in degrees or radians. In our example the shift of one quarter of a cycle is equivalent to a phase difference of 90° or $\pi/2$ radians.

In figure 23.1 the e_2 waveform reaches its peak, X, earlier in time than the e_1 waveform with its corresponding peak, Y. The difference between the two waveforms is Φ radians with e_2 leading e_1 (or e_1 lagging e_2). Phase differences may therefore be either leading or lagging with their angles usually extending up to 180°. In the particular case where Φ is 180° or π radians, the terms "leading" or "lagging" are not used since 180 deg leading has the same meaning as 180 deg lagging. In most cases there is little point in using angles greater than 180° since, for example, a phase difference of 270° leading is equivalent to 90° lagging.

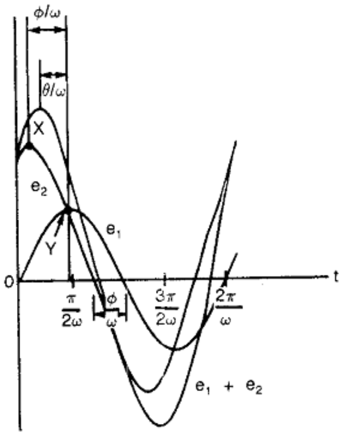

Figure 23.1: Example

It is impossible to add together two or more series ac voltages (or currents if some form of parallel circuit is involved) without knowing their peak values and their phase relationships. Figure 23.2 shows the results of adding two ac voltages, each of 10 V peak, but with phase differences that are in turn 0°, 90°, 120°, 180°; the resultant (sum) voltages have corresponding peak values of 20 V, 14.4 V, 10 V, and zero. Notice that in each case the frequency of $e_1 + e_2$ is the same as the frequency of e_1 and e_2.

As the mathematical derivations will show, the frequency of $e_1 + e_2$ is unchanged. The peak value of $e_1 + e_2$ is obtained from the peak values of e_1,e_2 and their phase relationship. The same factors determine the phase relationships between $e_1 + e_2$ and e_1,e_2.

23.1 Mathematical derivations

In figure 23.1 e_2 leads e_1 by am angle of Φ radians. Therefore instantaneous voltages:

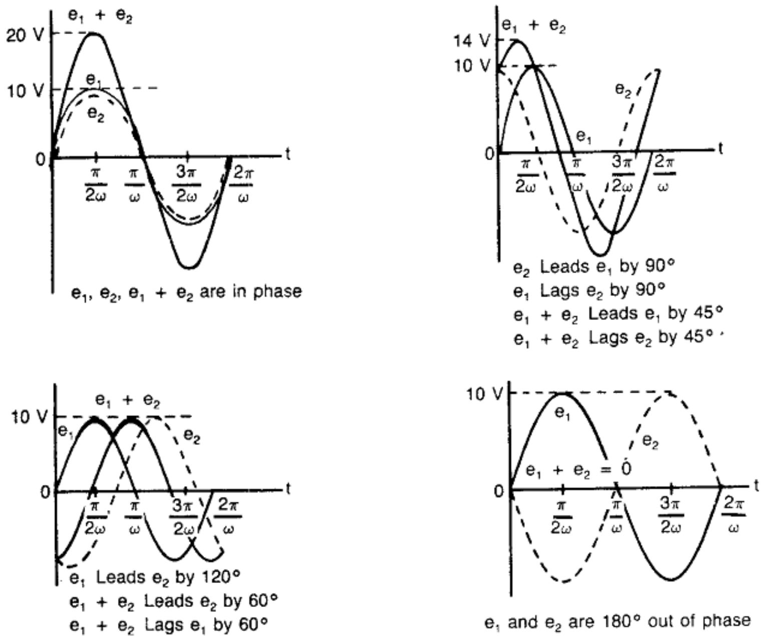

Figure 23.2: Example

$$e_1 = E_{1max} \sin \omega t \,\text{volts} \tag{23.1}$$
$$e_2 = E_{2max} \sin(\omega t + \Phi) \,(volts) \tag{23.2}$$

The sum these two ac voltages is:

$$\begin{aligned} e_1 + e_2 &= E_{1max} \sin \omega t + E_{2max} \sin(\omega t + \Phi) \\ &= E_{1max} \sin \omega t + E_{2max} \sin \omega t \cos \Phi + E_{2max} \cos \omega t \sin \Phi \\ &= (E_{1max} + E_{2max} cos\Phi) \sin \omega t + E_{2max} \sin \Phi \cos \omega t \,\text{volts} \end{aligned} \tag{23.3}$$

Let $(E_{1max} + E_{2max}) \cos \Phi = E_{max} cos\Phi$ volts and $E_{2max} sin\Phi = E_{max} \sin \Phi$ volts

24 Chap. 64: Addition and subtraction of phasors

In the mathematical derivations of chapter 23, we added together two voltage sine waves that were represented by the equations $e_1 = E_{1max}\sin\omega t$ and $e_2 = E_{2max}\sin(\omega t+\Phi)$. In this chapter we are going to show how these same voltages are added together in a phasor diagram. Because e_2 leads e_1 by Φ radians, we can use e_1 as the reference phasor; the e_2 phasor will appear in the first quadrant with its direction inclined at the angle Φ to the horizontal line (figure 24.1).

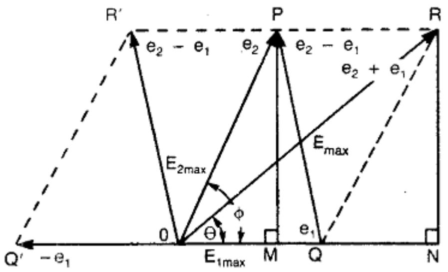

Figure 24.1: Example

The sine waves of e_1 and e_2 are added together to produce the $e_1 + e_2$ sine wave which has a peak value of E_{max} and leads e_1 by the angle Φ. The corresponding $e_1 + e_2$ phasor is found by completing the parallelogram whose sides are OP, OQ; the diagonal OR is then the$e_1 + e_2$ phasor while the other diagonal QP is the $e_2 - e_1$ phasor.

24.1 Mathematical derivations

In figure 24.1

$$RN = E_{2max}\sin\Phi \tag{24.1}$$
$$OM = E_{2max}\cos\Phi \tag{24.2}$$
$$ON = OM + MN = E_{2max}\cos\Phi + E_{max} \tag{24.3}$$

Using the Pythagorean theorem

$$\begin{aligned} OR^2 &= RN^2 + ON^2 \\ E_{max}^2 &= (E_{2max} sin\Phi)^2 + (E_{1max} + E_{2max}\cos\Phi)^2 \\ &= E_{1max}^2 + E_{2max}^2\sin^2\Phi + 2E_{1max}E_{2max}\cos\Phi + E_{2max}^2\cos^2\Phi \end{aligned}$$

because

$$sin^2\Phi + \cos^2\Phi = 1 \tag{24.4}$$

Therefore

$$E_{max} = \sqrt{E_{1max}^2 + E_{2max}^2 + 2E_{1max}E_{2max}\cos\Phi\,\text{volts}} \tag{24.5}$$
$$\tan\Phi = \frac{RN}{ON} = \frac{E_{2max}\sin\Phi}{E_1 + E_{2max}\cos\Phi}$$

25 Chap. 65: Resistance in ac circuit

If a sine wave is connected across a resistor, R (figure 25.1A, Ohm's Law applies throughout the cycle of the alternating voltage, e. When e is instantaneously zero, there is zero current flowing in the circuit but when the applied voltage reaches one of its peaks,X, the current is also a maximum in one direction. When we reach the peak, Y of the other alternation, the current is again at its maximum value but its direction is reversed. At all times $\frac{e}{i} = R$ and it therefore follows that the sine waves of e and i are in phase (figure 25.1B) with a phase difference of zero degrees. This is indicated by the e and i phasors, which lie along the same horizontal line (figure 25.1C).

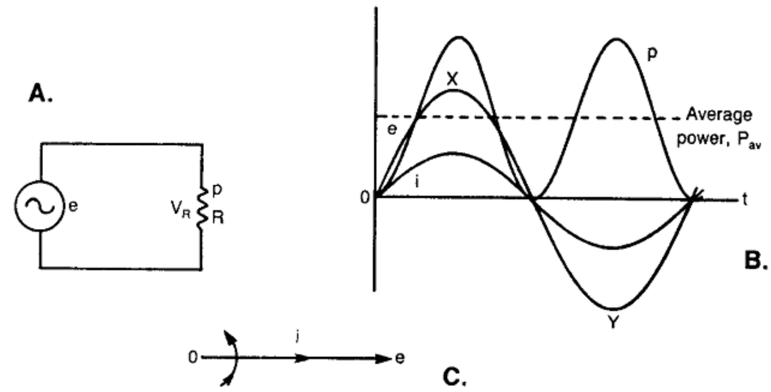

Figure 25.1: Example

The instantaneous power, p, in the circuit is the product of the instantaneous voltage, e, and the instantaneous current, i. When e and i are simultaneously zero, the instantaneous power is also zero, but when both e and i reach their peaks together, the power reaches its peak value of $E_{max} \cdot I_{max}$ watts. When the voltage reverses polarity, the current reverses direction but the resistor continues to dissipate (lost) power in the form of heat. The whole of the instantaneous power curve must be drawn above the zero line and its frequency is twice that of the applied voltage. The mean value of the power curve is a measure of the average power dissipated over the voltage cycle.

26 Chap. 67: Inductive reactance

Let us assume in figure 26.1A that the inductor only possesses the property of inductance and that we can ignore it's resistance. When an alternating current is flowing through the coil, the surrounding magnetic field is continually expanding and collapsing so that there is an induced voltage, v_L whose value must at all times exactly balance the instantaneous value of the source voltage, e. Therefore, when e is instantaneously at its peak value, the value of v, is also at its peak and since:

$$v_L = -L\frac{\mathrm{d}i}{\mathrm{d}t} \tag{26.1}$$

the *rate of change of the current* is at its highest level. This occurs when the slope of the current sine wave is steepest at the point of zero time and zero current. Consequently, when e is at its peak value, i is instantaneously zero and therefore e, i are 90° out of phase (figures 26.1B and C). This may be remembered by the word "*eLi*"; for an inductor, L, the instantaneous voltage, e, leads the instantaneous current, i, by 90°.

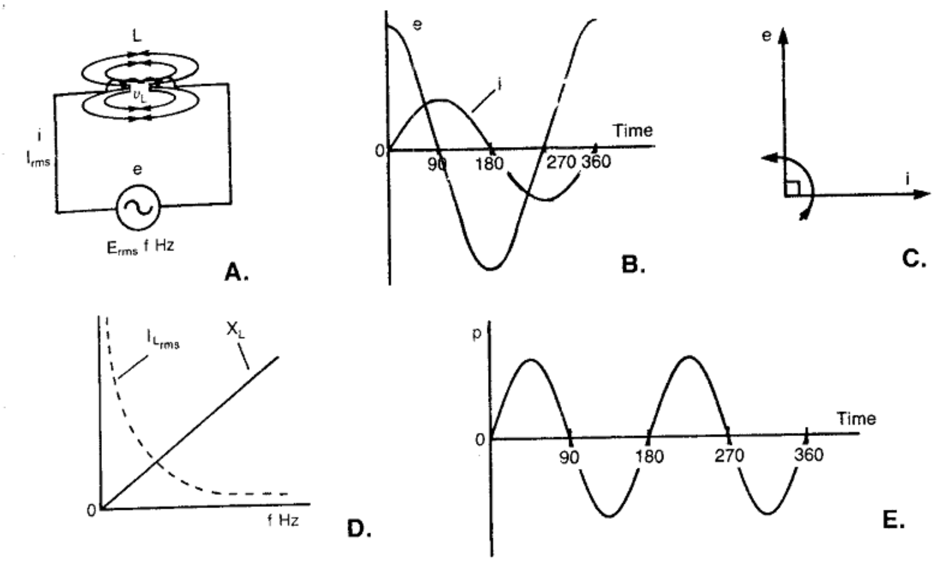

Figure 26.1: Example

We cannot obtain the opposition to the current flow by dividing the instantaneous voltage by the instantaneous current. For example, at the zero degree point, the value of eh is infinite while at the 90° mark, $\frac{e}{i}$ is zero. However, we can attempt to find the value of $\frac{E_{rms}}{I_{rms}}$ because the effective values are derived from the complete cycles of the voltage and the current.

Assume that the source voltage and the inductance are kept constant but the frequency is raised. The voltage induced in the coil must remain the same but the magnetic field is expanding and collapsing more rapidly so that the required magnetic flux is less. As a result, the current is reduced and is inversely proportional to the frequency. For example, if the frequency is doubled, the value of the effective current is halved.

Now we will keep the source voltage and the frequency constant but the inductor is replaced by another coil with a higher inductance. Because the induced voltage, $v_L = -L\frac{\mathrm{d}i}{\mathrm{d}t}$ is the same, the value of $\frac{\mathrm{d}i}{\mathrm{d}t}$ must be less and therefore the effective current is again reduced. Finally by raising the value of the source voltage while the frequency and the inductance are unchanged, the induced voltage must be increased and consequently the effective current is greater.

Summarizing, the effective current is directly proportional to the effective voltage, but it is inversely proportional to the frequency and the inductance. The opposition to the alternating current is therefore determined by the product of the frequency and the inductance. This opposition is called the inductance reactance, $XL = \frac{E_{rms}}{I_{rms}}$, and is measured in ohms. As shown in the mathematical derivation, $X_L = 2\pi f L\Omega$ and because X_L is directly proportional to the frequency, the graph of X_L versus frequency is a straight line (figure 26.1D).

The instantaneous power, p, is equal to the product of e and i. The graph of the instantaneous power, p, versus time is a second harmonic sine wave (figure 26.1E) whose average value is zero over the source voltage's cycle. During the first quarter cycle the magnetic field is established around the inductor and the energy is drawn from the source. However, during the second quarter cycle the magnetic field collapses and the energy is returned to the source. This action is repeated during the third and fourth quarter cycles so that the average power over the complete cycle is zero.

This highlights the difference between resistance and reactance. Both resistance and reactance limit the value of the alternating current but while resistance dissipates (lost) power in the form of heat, reactance does not lose any power at all.

27 Chap. 68: Capacitive reactance

When an alternating current, i, is flowing in the circuit of figure 27.1A, the capacitor is continuously charging and discharging so that the voltage, v_c, across the capacitor must at all times exactly balance the instantaneous value of the source voltage, e. Therefore, when e is momentarily at its peak value, the value of v_c is also at its peak and the capacitor is fully charged. The current is then instantaneously zero and therefore e and i are 90° out of phase (figures 27.1B, C). This may be remembered by the word *iCe*; for an ideal capacitor, C, the instantaneous current, i, leads the instantaneous voltage, e, by 90°. We then combine "*iCe*" with the word "*eLi*" for the inductor and create "*eLi, the iCe man!*"

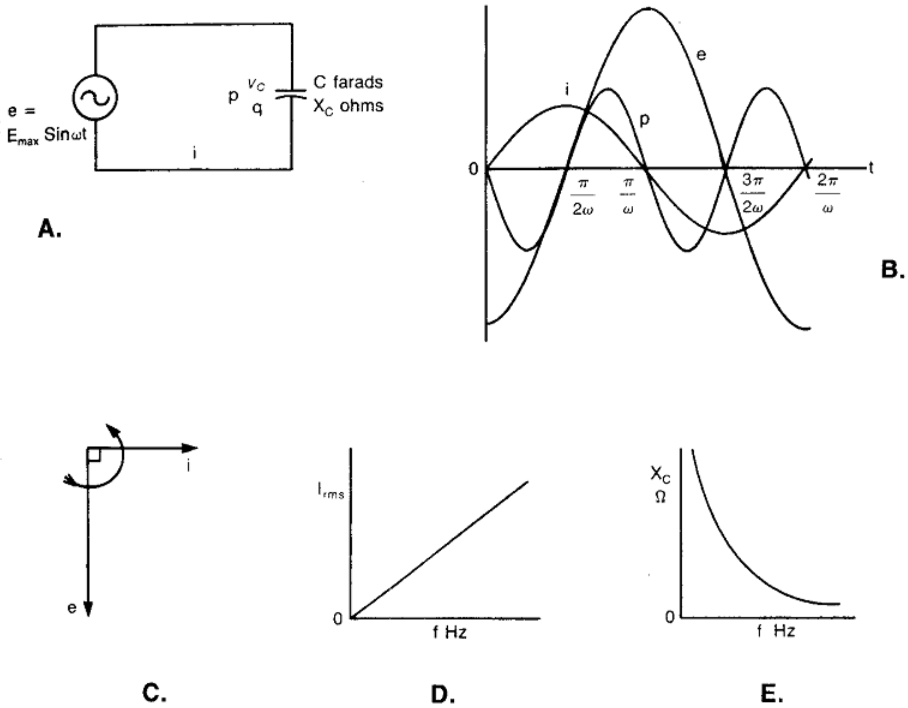

Figure 27.1: Example

Let us now derive the factors that determine the capacitor's opposition to the flow of alternating current. Assume that the source voltage and the capacitance are kept constant but the frequency is raised. This reduces the period so that the capacitor must acquire or lose the same amount of charge in a shorter time. The charging or discharging current must therefore be greater and, in fact, the effective current is directly proportional to the frequency, so that if the frequency is doubled, the effective current is also doubled. By contrast, when the frequency is doubled for an inductor, the effective current is halved (chapter 26).

Now we will keep the source voltage and the frequency constant but we will raise the capacitance. The new capacitor will have to store or lose more charge in the same time so that the effective current is again increased. The values of the capacitance and the effective current are directly proportional so that if the capacitance is halved, the effective current is also halved. Finally by raising the source voltage while the frequency and the capacitance are unchanged, the voltage across the capacitor must be greater, the capacitor must store more charge in the same period of time and consequently the effective current is once more increased.

Summarizing, the value of the effective current is directly proportional to the voltage, the capacitance and the frequency. The opposition to the alternating current flow is therefore inversely determined by the product of the frequency and the capacitance. This opposition is called the capacitive reactance:

$$X_C = \frac{E_{rms}}{I_{rms}}$$

and is measured in ohms. As shown in the mathematical derivations,

$$X_C = \frac{1}{2\pi f C}\Omega$$

and because X_C is inversely proportional to the frequency, the graph of X_C versus frequency is the rectangular hyperbolic curve of figure 27.1E. By contrast, the effective current is directly proportional to the frequency so that the graph of I_{rms} versus frequency is a straight line (figure 27.1D).

The instantaneous power, p, is equal to the product of e and i. The graph of the instantaneous power, p, versus time is a second harmonic sine wave (figure 27.1B) whose average value is zero over the source voltage's cycle. During one quarter cycle the capacitor charges so that energy is drawn from the source and appears in the form of the electric field between the capacitor's plates. However, during the following quarter cycle, ihe capacitor discharges and the energy is returned to the source.

Like the inductor, the importance of the capacitor lies in the fact that its opposition to alternating current depends on frequency; as the frequency is raised, the effective current increases from zero to infinity (figure 27.1D) while the capacitive reactance decreases from infinity towards zero (figure 27.1E). Consequently, for given values of L and C there must always be a (resonant) frequency for which $X_L = X_C$; this fact is used in the L, C tuning circuit which, for example, is capable of selecting a particular station through its ability to distinguish between one frequency and another (Chapters 80 and 82).

28 Chap. 70: Sine-wave input voltage to R and L in series

Let us assume that the value of $X_L = 2\pi f L\Omega$ is greater than the value of R. Since R and L are in series, the same alternating current must flow through each component (figure 28.1A). The instantaneous voltage drop v_R across the resistor and the instantaneous current (i) through the resistor are in phase (figure 28.1B) so that their phasors lie along the same horizontal line (figure 28.1C) By contrast the instantaneous voltage (v_L) across the inductor leads the instantaneous current (i) by 90° and consequently their phasors are perpendicular. The phasor sum of V_R and V_L is the supply voltage e the current,I, then lags the source voltage, e, by the phase angle, Φ. This inductive circuit is then considered to have a lagging power factor (resistance R, impedance Z) and the phase angle is *positive*.

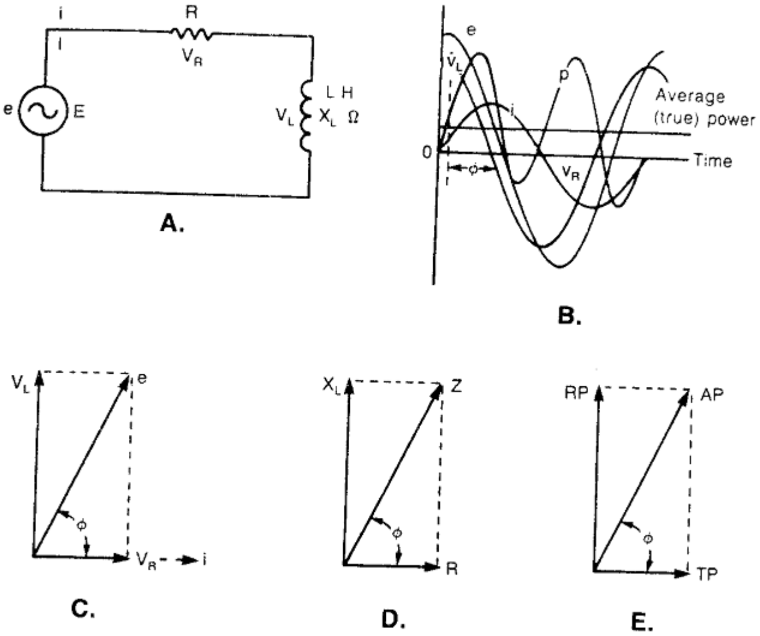

Figure 28.1: Example

The total opposition to the alternating current flow is measured by the impedance phasor, z which is defined as the ratio of the e phasor to the i phasor and is equal to the phasor sum of R and X_L (figure 28.1D).

The true power (TP in watts) is the power dissipated or lost as heat in the resistor; it is also the average value of the instantaneous power curve (figure 28.1B). The reactive, idle or wattless power (RP) in volt-amperes reactive (VAr) is the power stored by the inductor as a magnetic field during one quarter of the ac cycle; this power is subsequently returned to the source during the next quarter cycle. The apparent power (AP) in volt-amperes (VA) is the product of the source voltage and the source current, and is the phasor sum of the true power and the reactive power (figure 28.1E). The power factor is the ratio of the true power to the apparent power and is equal to the cosine of the phase angle.

29 Chap. 71: Sine-wave input voltage to R and C in series

Let us assume that the value of $X_C = \frac{1}{2\pi fC}\Omega$ is greater than the value of R. Since R and C are in series, the same alternating current must flow through each component (figure 29.1A). The instantaneous voltage drop (v_R) across the resistor and the instantaneous current (i) through the resistor are in phase (figure 29.1B) so that their phasors lie along the same horizontal reference line (figure 29.1C). By contrast the instantaneous voltage (v_C) across the capacitor lags the instantaneous current (i) by 90° and consequently their phasors are perpendicular. The phasor sum of v_R and v_C is the supply voltage, e; the current, i, then leads the source voltage, e, by the phase angle, Φ. This capacitive circuit is considered to have a leading power factor (resistance R, impedance Z) and the phase angle is *negative*.

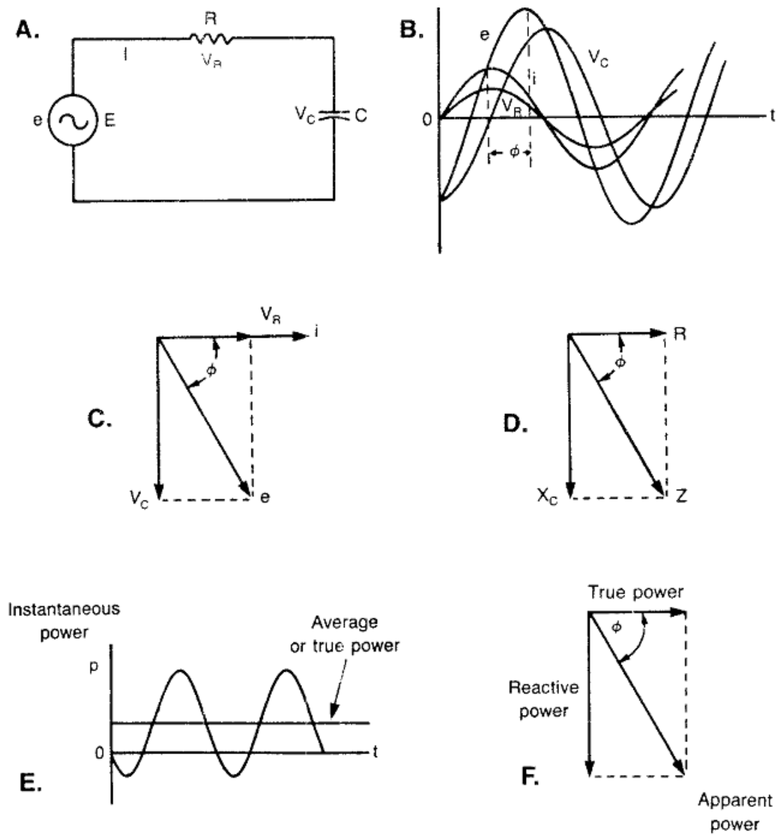

Figure 29.1: Example

The total opposition to the alternating current flow is measured by the impedance phasor, z, which is defined as the ratio of the e phasor to the i phasor, and is equal to the phasor sum of R and X_C (figure 29.1D).

The true power (TP in watts) is the power dissipated or lost as heat in the resistor; it is also the average value of the instantaneous power curve (figure 29.1E). The reactive, idle or wattless power RP) in volt-amperes reactive (VAr; is the power stored by the capacitor as an electric field during one quarter of the ac cycle; this power is subsequently returned to the source during the next quarter cycle. The apparent power (AP) in volt-amperes (VA) is the product of the source voltage and the source current, and is the phasor sum of the true power and the reactive power (figure 29.1F). The power factor is the ratio of the true power to the apparent power and is equal to the cosine of the phase angle.

30 Chap. 72: Sine-wave input voltage to L and C in series

Let us assume that the value of $X_L = 2\pi fL\Omega$ is greater than the value of $X_C = \frac{1}{2\pi fC\Omega}$. Because L and C are in series, the same alternating current must be associated with both the inductor and the capacitor (figure 30.1A). The instantaneous voltage (v_L) across the inductor leads the instantaneous current (i) by 90° while the instantaneous voltage across the capacitor v_C) lags i by 90° (figure 30.1B). Consequently, v_L and v_C are 180° out of phase and their phasors are pointing in opposite directions (figure 30.1C). Since we have assumed that $X_L > X_C$, $v_L > v_C$ so that the supply voltage, e, (which is the phasor sum of v_L and v_C) is in phase with v_L but has a magnitude of $V_L - V_C$. Because i lags e by 90°, the circuit behaves inductively and has a lagging power factor with a phase angle of +90°. The magnitude of the impedance, Z, is found by combining X_L and X_C so that $Z = X_L - X_C\Omega$ (figure 30.1D). Because the inductor and the capacitor are considered to be ideal components, there is no resistance present in the circuit and no true power is dissipated. The inductive reactive power (IRP) is greater than the capacitive reactive power (CRP) and the apparent power (AP) is entirely reactive with a value equal to IRP − CRP (figure 30.1E).

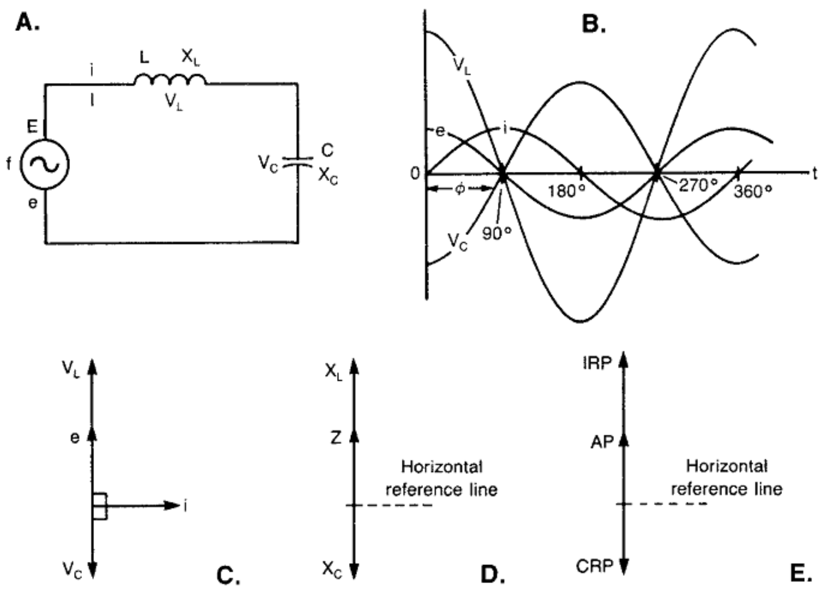

Figure 30.1: Example

If, by contrast, we lowered the frequency to the point where X_C became greater than X_L, E would be equal to $V_C - V_L$ and Z would be $X_C - X_L$. The current, i, would then lead e by 90°, the circuit would behave capacitively with a leading power factor and a phase angle of |90°. The apparent power would equal CRP − IRP.

In the particular case where the frequency is chosen so that X_L is equal to X_C, V_L is equal to V_C and the impedance, Z, is zero. The current would then be theoretically infinite.

30.1 Mathematical derivations

Impedance phasor:

$$z = j(X_L - X_C) = j(2\pi f L - \frac{1}{2\pi f C})\,\text{ohms} \tag{30.1}$$

In terms of the effective values (capital letters)

$$\frac{E}{I} = ZE = I \cdot Z \qquad\qquad I = \frac{E}{Z} \tag{30.2}$$

$\Phi = 90°$ if $X_L > X_C$ and $\Phi = -90°$ if $X_C > X_L$. Impedance $Z = X_L \sim X_C$ ohms.

The sign "$\sim$" means the "difference": you are required to subtract the smaller quantity from the larger quantity so that the result is always positive. This takes into account the two cases, $X_L > X_C$ and $X_C > X_L$.

$$V_L = I \cdot X_L X_L = \frac{V_L}{I} \qquad\qquad I = \frac{V_L}{X_L} \tag{30.3}$$

$$V_C = I \cdot X_C X_C = \frac{V_C}{I} \qquad\qquad I = \frac{V_C}{X_C} \tag{30.4}$$

Source voltage:

$$E = V_L \sim V_C\,\text{volts} \tag{30.5}$$

True power (TP) = 0.
Inductive reactive power:

$$\text{IRP} = I \cdot V_L = I^2 \cdot X_L = \frac{V_L^2}{X_L}\,\text{VArs} \tag{30.6}$$

Capacitive reactive power:

$$\text{CRP} = I \cdot V_C = I^2 \cdot X_C = \frac{V_C^2}{X_C}\,\text{VArs} \tag{30.7}$$

Apparent power:

$$\text{AP} = I \cdot E = I^2 \cdot Z = \frac{E^2}{Z}\,\text{VA(r)s} \tag{30.8}$$

It is permissible to measure the apparent power in VArs because the power is entirely reactive and there is no true power in the circuit.

Apparent power:

$$\text{AP} = \text{IRP} \sim \text{CRPVA(r)s} \tag{30.9}$$

Power factor = 0 and is lagging if $X_L > X_C$, but is leading if $X_C > X_L$. The particular frequency f, for which $X_L = X_C$, $V_L = V_C$, $Z = 0$ and I is theoretically infinite. Then:

$$2\pi f L = \frac{1}{2\pi f C} \tag{30.10}$$

31 Chap. 73: Sine-wave input voltage to R, L, and C in series

This circuit must contain all of the information contained in the previous three chapters (28, 29 and 30). For example, if the capacitor were eliminated, the results would then be the same as for R and L in series.

Let us assume that X_L is greater than X_C and that R is less than $X_L - X_C$. Since all three components are in series, the instantaneous current, i, is the same throughout the circuit. In terms of phase relationships, v_R is in phase with i, v_L leads i by 90° while v_C lags i by 90° (figures 31.1B and C). The combined voltage across the inductor and the capacitor is represented by the phasor v_X, which is in phase with v_L and leads i by 90°. The current i lags the source voltage e which is the phasor sum of v_X and v_R. The circuit is therefore overall inductive, the power factor is lagging and the phase angle, Φ, is positive.

The net reactance, X, is equal to $X_L - X_C$which is then combined with R to produce the impedance, Z (figure 31.1D). The total reactive power is the phasor sum of the inductive reactive power and the capacitive reactive power and when the total reactive power is combined with the true power, the result is the apparent power (figure 31.1E).

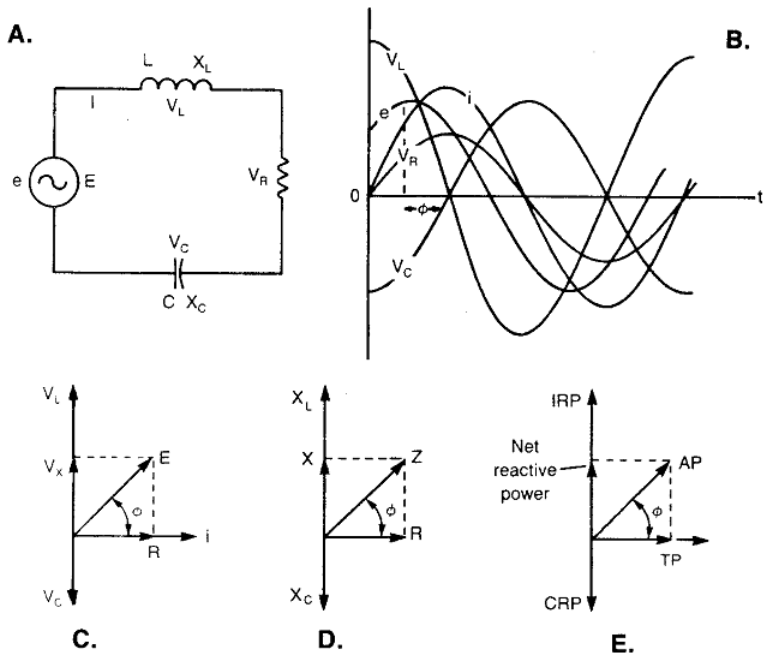

Figure 31.1: Example

If X_C were greater than X_L, v_X would be in phase with v_C, i would lead e by the phase angle, Φ. The power factor would then be leading and Φ would be a negative angle.

At the particular frequency, $f = \frac{1}{2\pi\sqrt{LC}}$ Hz, the reactances are equal and therefore cancel each other. The impedance is then equal to the resistance and the phase angle is zero. As a result the values of the true power and the apparent power are the same.

31.1 Mathematical derivations

Impedance phasor

$$z = R + jX = R + j(X_L - X_C) = R + j(2\pi f L - \frac{1}{2\pi f C})\,\text{ohms} \tag{31.1}$$

In terms of the effective values (capital letters):

$$\frac{E}{I} = Z \qquad E = I \cdot Z I = \frac{E}{Z} \tag{31.2}$$

$$\Phi = \text{inv}\tan(\frac{X_L - X_C}{R})\,\text{degrees} \tag{31.3}$$

Φ is a positive angle, when $X_L > X_C$, but is a negative angle if $X_C > X_L$. When $R = X_L - X_C$, $\Phi = 45°$.

Impedance:

$$Z = \sqrt{R^2 + (X_L \sim X_C)^2} = \sqrt{R^2 + X^2}\,\text{ohms} \tag{31.4}$$

For the meaning of the difference "$\sim$" sign, see chapter 30.

$$V_R = I \cdot R R = \frac{V_R}{I} \qquad I = \frac{V_R}{R} \tag{31.5}$$

$$V_L = I \cdot X_L X_L = \frac{V_L}{I} \qquad I = \frac{V_L}{X_L} \tag{31.6}$$

$$V_C = I \cdot X_C X_C = \frac{V_C}{I} \qquad I = \frac{V_C}{X_C} \tag{31.7}$$

Source voltage:

$$E = \sqrt{V_R^2 + (V_L \sim V_C)^2} = \sqrt{V_R^2 + V_X^2}\,\text{volts} \tag{31.8}$$

True power

$$\text{TR} = I \cdot V_R = I^2 R = \frac{V_R^2}{R}\,\text{watts} \tag{31.9}$$

Inductive reactive power

$$\text{IRP} = I \cdot V_L = I^2 \cdot X_L = \frac{V_L^2}{X_L}\,\text{VArs} \tag{31.10}$$

Capacitive reactive power

$$\text{CRP} = I \cdot V_C = I^2 \cdot X_C = \frac{V_C^2}{X_C}\,\text{VArs} \tag{31.11}$$

32 Chap. 74: Sine-wave input voltage to R and L in parallel

Because R and L are in parallel, the source voltage, e, is across each of the components and the source or supply current, i_S, will be the phasor sum of the two branch currents, i_R and i_L (figures 32.1A and B). If we assume that the value of R is greater than the value X_L, the resistor current ($I_R = \frac{E}{R}$) will be less than the inductor current ($I_L = \frac{E}{X_L}$). Because i_R is in phase with e while i_L lags e by 90°, the source current will lag the source voltage by the phase angle, Φ (figures 32.1B and C). The power factor will be lagging and the phase angle is positive (note that in both the series and parallel combinations of R and L, the source current lags the source voltage so that in each case the phase angle is positive).

The "ease" with which the source current flows, is measured by the admittance,$Y_T = \frac{I_S}{E}$ which is the reciprocal of the total impedance, Z_T, and is measured in siemens. Since $I_R = E \cdot G$, $I_L = E \cdot B_L$ and $I_S = E \cdot Y_T$, the admittance, Y_T, is the phasor sum of the conductance, G, and the inductive susceptance, B_L (figure 32.1D).

The true power, TP, is equal to $\frac{E^2}{R}$ watts and is independent of the frequency. The reactive power, RP, is associated with the inductor while the apparent power (AP) at the source is the phasor sum of the true power and the reactive power (figure 32.1E).

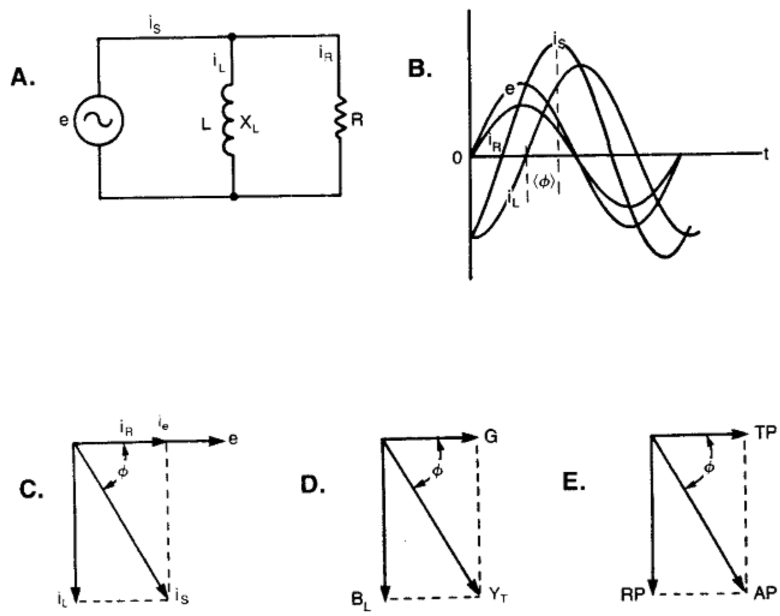

Figure 32.1: Example

32.1 Mathematical derivations

In terms of effective values:

$$I_R = \frac{E}{R} E = I_R \cdot R \qquad R = \frac{E}{I_R} \tag{32.1}$$

$$I_L = \frac{E}{X_L} E = I_L \cdot X_L \qquad R = \frac{E}{I_L} \tag{32.2}$$

$$I_S = \frac{E}{Z_T} E = I_S \cdot Z_T \qquad Z_T = \frac{E}{I_S} \tag{32.3}$$

Source current:

$$I_S = \sqrt{I_R^2 + I_L^2} \tag{32.4}$$

$$I_R = \sqrt{I_S^2 - I_L^2}$$

$$I_L = \sqrt{I_S^2 - I_R^2} \text{ amperes}$$

Because $I_S^2 = I_R^2 + I_L^2$:

$$(\frac{E}{Z_T})^2 = (\frac{E}{R})^2 + (\frac{E}{X_L})^2$$

This yields total impedance:

$$Z_T = \frac{R \cdot X_L}{\sqrt{R^2 + X_L^2}} \text{ ohms} \tag{32.5}$$

Equation 32.5 represents the "product-over-sum" formula when the "Pythagorean sum" is involved. Because the conductance $G = \frac{1}{R}$, the inductive susceptance $B_L = \frac{1}{X_L}$ and the admittance $Y_T = \frac{1}{Z_T}$.

Admittances:

$$Y_T = \sqrt{G^2 + B_L^2} G = \sqrt{Y_T^2 - B_L^2} \qquad B_L = \sqrt{Y_T^2 - G^2} \text{ siemens} \tag{32.6}$$

33 Chap. 75: Chapter 75 Sine-wave input voltage to R and C in parallel

Because R and C are in parallel, the source voltage, E, is across each of the components and the source or supply current, is, will be the phasor sum of the two branch currents i_R, and i_C (figures 33.1A and B). If we assume that the value of R is greater than the value of X_C, the resistor current ($I_R = \frac{E}{R}$) will be less than the capacitor current ($I_C = \frac{E}{X_C}$). Because i_R is in phase with e while i_C leads e by 90°, the source current will lead the source voltage by the phase angle, Φ (figures 33.1B and C). The power factor will then be leading and the phase angle is negative.

The "ease" with which the source current flows is measured by the admittance $Y_T = \frac{I_S}{E}$, which is the reciprocal of the total impedance, Z_T, and is measured in siemens. Since $I_R = E \cdot G$, $I_C = E \cdot B_C$ and $I_S = E \cdot Y_T$, the admittance, Y_T, is the phasor sum of the conductance, G, and the capacitive susceptance, B_C (figure 33.1D).

The true power, TP, is equal to $\frac{E^2}{R}$ watts and is independent of the frequency. The reactive power, RP, is associated with the capacitor while the apparent power, AP, at the source is the phasor sum of the true power and the reactive power (figure 33.1E).

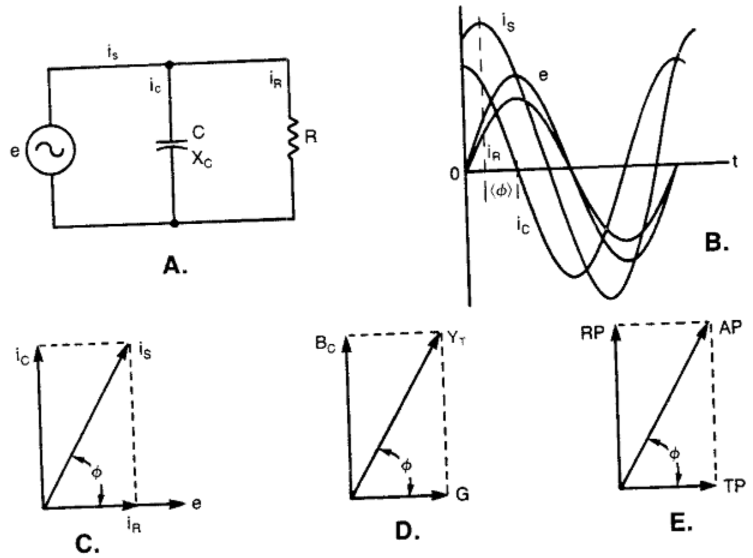

Figure 33.1: Example

33.1 Mathematical derivation

In terms of effective values:

$$I_R = \frac{E}{R} E = I_R \cdot R \qquad R = \frac{E}{I_R} \tag{33.1}$$

$$I_C = \frac{E}{X_C} E = I_C \cdot X_C \qquad X_C = \frac{E}{I_C} \tag{33.2}$$

$$I_S = \frac{E}{Z_T} E = I_S \cdot Z_T \qquad R = \frac{E}{I_S} \tag{33.3}$$

Source currents:

$$I_S = \sqrt{I_R^2 + I_C^2} \tag{33.4}$$

$$I_R = \sqrt{I_S^2 - I_C^2}$$

$$I_C = \sqrt{I_S^2 - I_R^2} \text{ amperes} \tag{33.5}$$

Since $I_S^2 = I_R^2 + I_C^2$

$$(\frac{E}{Z_T})^2 = (\frac{E}{R})^2 + (\frac{E}{X_C})^2$$

This yields total impedance:

$$Z_T = \frac{R \cdot X_C}{\sqrt{R^2 + X_C^2}} \text{ ohms} \tag{33.6}$$

Equation 33.6 represents the "product-over-sum" formula when the "Pythagorean sum" is involved. Because the conductance $G = \frac{1}{R}$, the inductive susceptance $B_C = \frac{1}{X_C}$ and the admittance $Y_T = \frac{1}{Z_T}$:

$$Y_T = \sqrt{G^2 + B_C^2} G = \sqrt{Y_T^2 - B_C^2} \qquad B_C = \sqrt{Y_T^2 - G^2} \text{ siemens} \tag{33.7}$$

34 Chap. 76: Sine-wave input voltage to L and C in parallel

Because L and C are in parallel, the source voltage, E, is across each of the components and the source or supply current, i_S, will be the phasor sum of the two branch currents, i_L and i_C (figures 34.1A and B). If we assume that the value of X_C is greater than the value of X_L, the capacitor current ($I_C = \frac{E}{X_C}$) will be less than the inductor current ($I_L = \frac{E}{X_L}$). Because i_C leads the source voltage by 90° while i_L lags the same source voltage by 90°, i_C and i_L are 180° out-of-phase (figure 34.1B) and their phasors are pointing in opposite directions (figure 34.1C). The phasor sum of i_L and i_C is the source current, is, which will be in phase with iL and will lag the source voltage by 90°. The current therefore behaves inductively, the power factor is lagging and the phase angle is *positive*. Notice that this opposite to the result obtained from the series L, C circuit of chapter 30 where if X_C were greater than X_L, the power factor was leading. This is because, in the parallel circuit, we are concerned with the branch currents, which are inversely proportional to the reactances, while in the series circuit, we considered the component voltages, which were directly proportional to the reactances.

If X_L is greater than X_C, the capacitor current is greater than the inductor current; is is in phase with i_C and leads the source voltage by 90°. The power factor is then leading and the phase angle is negative.

The admittance, $Y_T = \frac{I_S}{E}$ is the reciprocal of the total impedance, Z_T, and is measured in siemens. Because $I_L = E \cdot B_L$, $I_C = E \cdot B_C$ and $I_S = E \cdot Y_T$, the admittance, Y_T, is the phasor sum of the inductive susceptance, B_L, and the capacitive susceptance, B_C (figure 34.1D).

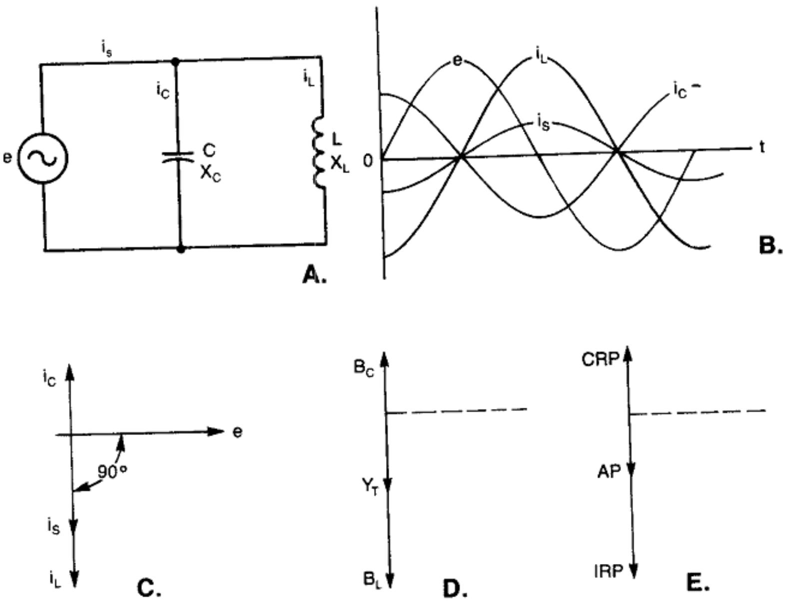

Figure 34.1: Example

Because the components are assumed to be ideal, there are no resistance losses associated with the circuit and the true power, TP, is zero. The apparent power (AP) is the phasor sum of the inductive reactive power (IRP) and the capacitive reactive power (CRP) (figure 34.1E).

Notice at the particular frequency, f for which the reactances are equal, the branch currents are also equal and therefore the supply current is zero. The total impedance is then infinite and the parallel combination behaves as an open circuit. This is theoretically possible because the power in the circuit is entirely reactive and there is no true power dissipated as heat.

34.1 Mathematical derivation

In terms of effective values:

$$I_L = \frac{E}{X_L} E = I_L \cdot X_L \qquad X_L = \frac{E}{I_L} \tag{34.1}$$

$$I_C = \frac{E}{X_C} E = I_C \cdot X_C \qquad X_C = \frac{E}{I_C} \tag{34.2}$$

$$I_S = \frac{E}{Z_T} E = I_S \cdot Z_T \qquad R = \frac{E}{I_S} \tag{34.3}$$

Source current

$$I_S = I_L \sim I_C \,\text{amperes} \tag{34.4}$$

35 Chap. 77: Sine-wave input voltage to R, L and C in parallel

This circuit must contain all the information of chapters 32, 33, and 34 because if, for example, the capacitor were eliminated, we would be left with R and L in parallel (chapter 32). Because all three components are in parallel across the source voltage, the source current, i_S will be the phasor sum of the three branch currents, i_R, i_L and i_C (figures 35.1A and B). If we assume that the inductive reactance is greater than the capacitive reactance and that the value of the resistance is relatively large, the capacitor current ($I_C = \frac{E}{X_C}$) will be greater than the inductor current ($I_L = \frac{E}{X_L}$) and the resistor current ($I_R = \frac{E}{R}$) will be small. Since i_C leads the source voltage by 90° while i_L lags the same source voltage by 90°, i_C and i_L are 180° out-of-phase (figure 35.1B) and their phasors are pointing in opposite directions (figure 35.1C). The phasor sum of i_L and i_C is the total reactive current, i_X, which will be in phase with i_C. The resistor current, i_R, is then combined with i_X to produce the source current, i_S, which leads the source voltage so that the power factor is leading and the phase angle Φ, is negative.

If X_C is greater than X_L, the inductor current is greater than the capacitor current; i_X is in phase with i_L and i_S lags the source voltage by the positive phase angle, Φ. The power factor is then lagging.

The admittance, $Y_T = \frac{I_S}{E}$ is the reciprocal of the total impedance, Z_T, and is measured in siemens. Because $I_R = E \cdot G$, $I_L = E \cdot B_L$, $I_C = E \cdot B_C$ and $I_S = E \cdot Y_T$, the admittance, Y_T, is the phasor sum of the conductance, G, the inductive susceptance, B_L, and the capacitive susceptance, B_C (figure 35.1D). In a similar way the apparent power (AP) is the phasor sum of the true power (TP), the inductive reactive power (IRP) and the capacitive reactive power (CRP) (figure 35.1E).

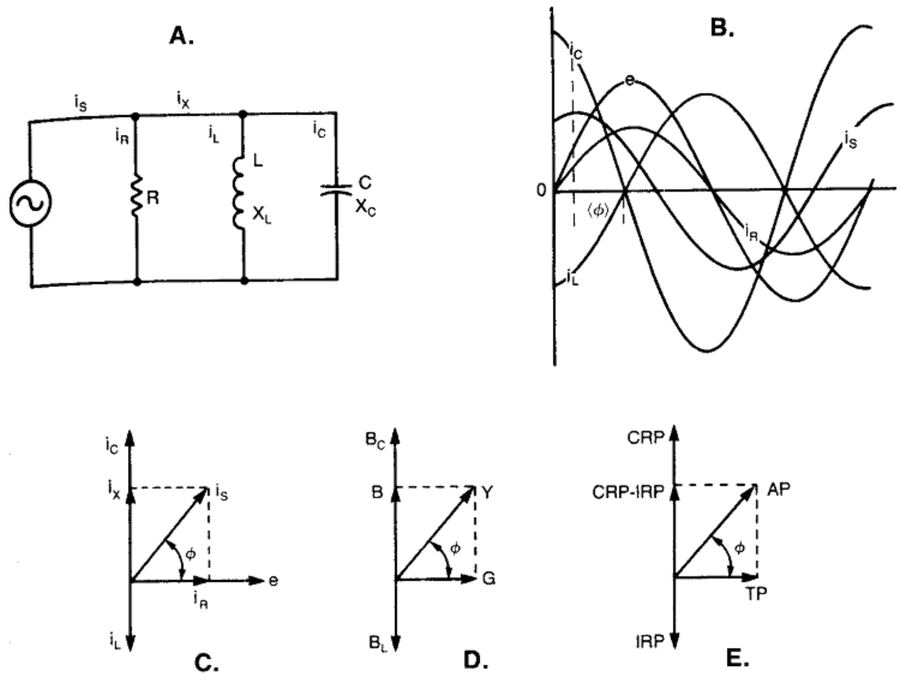

Figure 35.1: Example

At the particular frequency, f, for which the reactances are equal, the inductor and the capacitor currents cancel out (the parallel L, C combination behaves as an open circuit) and the source current is the same as the resistor current. The total impedance is then equal in value to the resistance. At frequencies above and below f, the parallel L, C combination behaves as a certain reactance value which, when placed in parallel with the resistance, produces a total impedance that is less the value of the resistance. When these two statements are combined, it means that the total impedance cannot be higher than the value of the resistance.

35.1 Mathematical derivations

In terms of effective values:

$$I_R = \frac{E}{R} E = I_R \cdot R \qquad R = \frac{E}{I_R} \tag{35.1}$$

$$I_L = \frac{E}{X_L} E = I_L \cdot X_L \qquad X_L = \frac{E}{I_L} \tag{35.2}$$

$$I_C = \frac{E}{X_C} E = I_C \cdot X_C \qquad X_C = \frac{E}{I_C} \tag{35.3}$$

$$I_S = \frac{E}{Z_T} E = I_S \cdot Z_T \qquad R = \frac{E}{I_S} \tag{35.4}$$

Total reactive current $I_X = I_L \sim I_C$ amperes.
Source current:

$$I_S = \sqrt{I_R^2 + (I_L \sim I_C)^2}\,\text{amperes} \tag{35.5}$$

Total resistance:

$$X_T = \frac{X_L \cdot X_C}{X_L \sim X_C}\,\text{ohms} \tag{35.6}$$

Total impedance:

$$Z_T = \frac{R \cdot X_T}{\sqrt{R^2 + X_T^2}}\,\text{ohms} \tag{35.7}$$

Because $I_R = EG$, $I_L = EB_L$, $I_C = EB_C$ and $I_S = EY_T$, we can calculate:
Admittance:

$$Y_T = \sqrt{G^2 + (B_L \sim B_C)^2}\,\text{siemens} \tag{35.8}$$

True power:

$$\begin{aligned} \text{TP} &= E \cdot I_R \\ &= I_R^2 \cdot R \\ &= \frac{E^2}{R} \\ &= E^2 G\,\text{watts} \end{aligned} \tag{35.9}$$

36 Chap. 79: Resonance in a series LCR circuit

The condition of electrical resonance in all circuits is defined as follows: any iwo-terminal (single source) network containing resistance and reactance is said to be in resonance when the source voltage and the current drawn from the source are in phase.

It follows from this definition that a resonant circuit has a phase angle of zero and a power factor of unity.

If the series LCR circuit of figure 36.1A is at resonance, the values of the inductive reactance and the capacitive reactance must be equal. Therefore the phasor sum of v_L and v_C is zero (figures 36.1B and C) so that $e = v_R$ and the circuit is purely resistive. The total impedance of the circuit is equal to the resistance and is at its minimum value (figure 36.1D); for this reason the series resonant LCR combination is sometimes referred to as an acceptor circuit. It follows that, at resonance, the circuit current is at its maximum value which is equal to $\frac{E}{R}$ amperes.

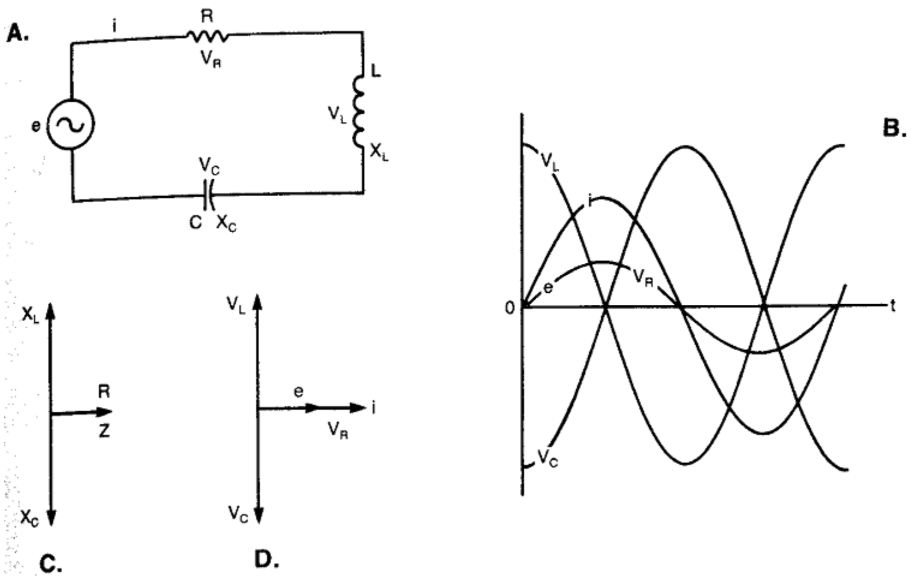

Figure 36.1: Example

Because the values of the inductive reactance and the capacitive reactance are both dependent on the frequency, there must be a particular resonant frequency for which the two reac¬tances are equal. The manner in which the behavior of the series LCR circuit varies with frequency, is illustrated by means of response curves (figure 36.2A and B). These are the graphs of certain variables (such as impedance, current, voltages across inductor, capacitor, etc.) versus frequency. Such response curves are important because they show the circuit's ability to distinguish between one frequency and another.

Tuning a series LCR circuit means adjusting the value of the inductor or the capacitor until the resonant frequency is the same as the desired signal frequency. Let us take the amplitude modulation (AM) broadcast band as an example. Each station is assigned a particular operating frequency while the frequencies of the nearest stations either side are 10 kHz away. Induced in the antenna (figures 36.3A and B) of an AM receiver are literally hundreds of signals from all the radio waves in the vicinity. The purpose of the tuned circuit is to provide maximum response at the frequency of the wanted signal but much smaller responses at the other unwanted signals. This is achieved by adjusting the capacitor until the resonant frequency is equal to the assigned frequency of the desired station.

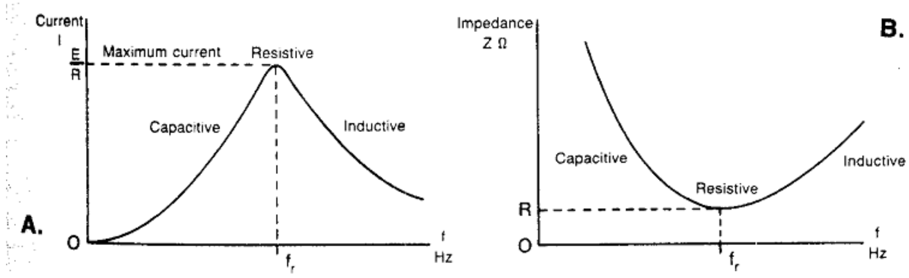

Figure 36.2: Example

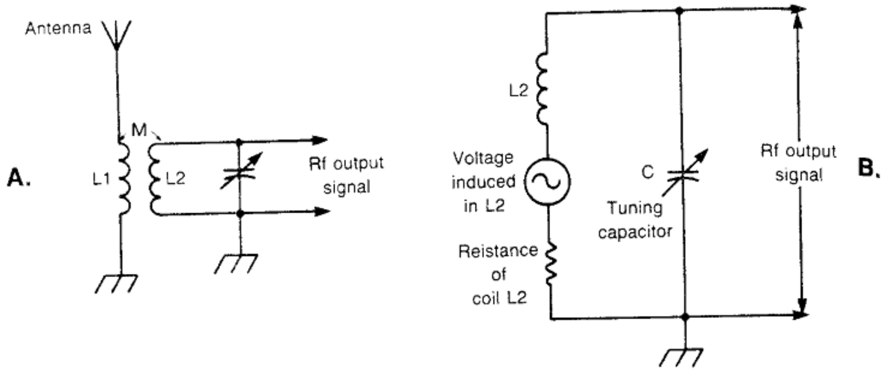

Figure 36.3: Example

Note that the circuit of figure 36.3B is a series arrangement because the wanted signal induced in the antenna drives a current through the coil $L1$. The alternating magnetic flux surrounding $L1$ cuts the other coil so that an rf (radio frequency) voltage is induced in $L2$. This voltage is within the loop containing the coil $L2$ and the capacitor C so that these components and their source voltage are in series.

Pouze cast mathematical derivations...

37 Chap. 80: Q—selectivity—bandwidth

When a series LCR circuit is at resonance, the current has its maximum possible value for a given source voltage. Across the inductor and the capacitor are then developed equal but 180° out-of-phase voltages. These voltages may each be many times greater than the source voltage. The number of times greater is called the voltage magnification factor, which is referred to as Q. The Q factor is just a number and has no units. However, it is a measure of the inductor's merit in the sense that a "good" coil will have a high value of inductive reactance compared with its resistance.

The main importance of Q is its indication of a series tuned circuit's selectivity; this is defined as its ability to distinguish between the signal frequency to which it is resonant and other un-wanted signals on nearby frequencies. It therefore follows that the higher the selectivity, the greater is the freedom from adjacent channel interference. The degree of the selectivity is related to the sharpness of the current response curve (the sharper the curve, the greater the selectivity) and may be measured by the frequency separation between two specific points on the curve (figure 37.1). The points arbitrarily chosen are those for which the true power in the circuit is one half of the maximum true power, which occurs when the circuit is resonant. These positions on the response curve are also referred to as the 3 decibel (dB) points because a loss of 3 dB is equivalent to a power ratio of one half. The frequency separation between these points is called the bandwidth (or bandpass) of the tuned circuit.

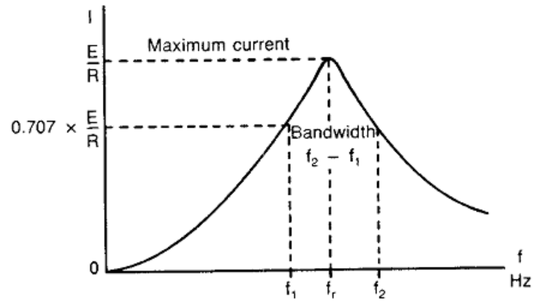

Figure 37.1: Example

At the 3 dB points, the rms circuit current will be $\frac{1}{\sqrt{2}}$ or 0.707 times the maximum rms value of the current at resonance (do not confuse this result with the relationship between the rms and the peak values of a sinewave alternating current). In addition, the overall reactance at the 3 dB points is equal to the circuit's resistance so that the phase angle is 45° and the power factor is $cos45° = 0.707$.

The narrower the bandwidth and the higher the resonant frequency, the sharper is the response curve and the greater is the selectivity. The mathematical derivations will show that the Q value is a direct measure of the selectivity.

38 Chap. 81: Parallel resonant LCR circuit

From the definition of resonance, the supply or source current, i_S must be in phase with the source voltage, e (figure 38.1A). This will occur if the values of the inductor and capacitor currents are equal; only then will the phasor sum, i_X, of their currents be zero (remember that i_L and i_C are 180° out of phase). Then the resistor current (which is independent of the frequency) will be the same as the supply current whose value will be at its minimum level of $\frac{E}{R}$ amperes. It follows that the total impedance, Z_T, of the complete circuit will be entirely resistive and have its maximum value of R ohms.

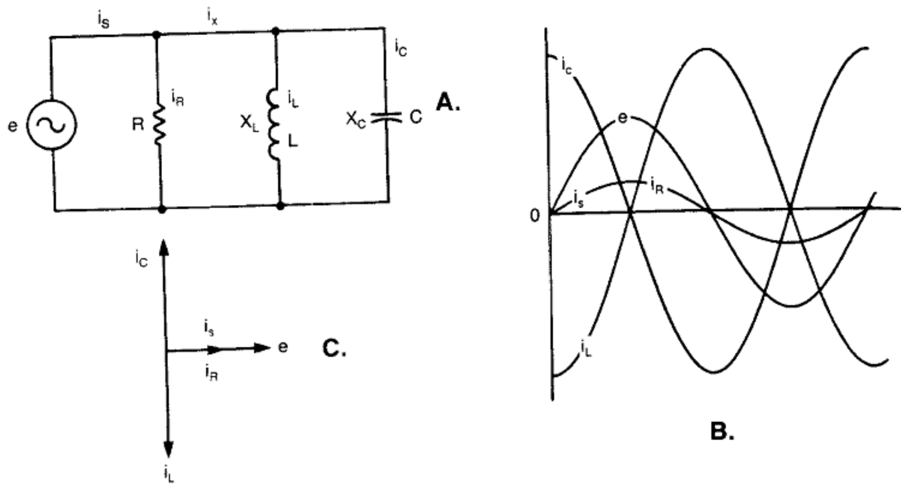

Figure 38.1: Example

At resonance the inductive and capacitive reactances are equal and the parallel LC combination behaves theoretically as an open circuit; the total impedance of the circuit is then equal to the resistance. At frequencies other than the resonant frequency, the LC combination behaves as a certain value of reactance, which when placed in parallel with the resistance, produces a total impedance that is less than the value of the resistance. Combining these two statements, it means that the total impedance can never exceed the value of the resistance.

At frequencies below the resonant frequency, the capacitive reactance is greater than the inductive reactance, the capacitor current is less than the inductor current and the circuit behaves inductively. At frequencies above the resonant frequency, the inductive reactance is greater than the capacitive reactance, the inductor current is less than the capacitor current and the circuit behaves capacitively. These results are the reverse of those for the series LCR circuit, which behaved capacitivelv for frequencies below/rand inductively for frequencies above f. The current and the impedance response curves are illustrated in figures 38.2A and B.

In the resonant phasor diagram of figure 38.1C, the currents I_L and I_C are equal in magnitude and each may be many times greater than the supply current. The number of times greater is equal to the circuit's Q factor which is a direct measure of the selectivity (chapter 37).

38.1 Mathematical derivations

At resonance:

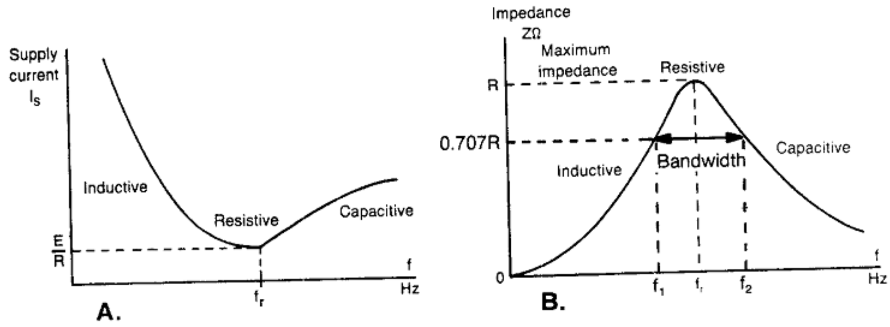

Figure 38.2: Example

- Phase angle of the entire circuit, $\Phi = 0°$.
- Inductor current, $I_L = \frac{E}{X_L}$ is equal in value to the capacitor current $I_C = \frac{E}{X_C}$ amperes.
- Supply current I_S is equal to the resistor current $I_R = \frac{E}{R}$ amperes, which is its minimum value.
- The impedance Z_T of the circuit is at its maximum level and equal to the value of the resistance R ohms.
- Inductive reactance X_L = capacitance reactance X_C

Therefore:

$$2\pi f L = \frac{1}{2\pi f C}$$

This yields resonant frequency:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \tag{38.1}$$

Then

$$L = \frac{1}{4\pi^2 f_r^2 C} = \frac{0.0253}{f_r^2 C} \text{ H} \tag{38.2}$$

and

$$C = \frac{1}{4\pi^2 f_r L} = \frac{0.0253}{f_r^2 C} \text{ F} \tag{38.3}$$

These last three equations are the same as the corresponding expressions for series resonance.

38.1.1 Current magnification

At resonance, the currents I_L and I_C are equal in magnitude and each is Q times the supply current I_S. Then

$$I_L = \frac{E}{X_L} I_C = \frac{E}{X_C} \qquad I_S = I_R = \frac{E}{R} \text{ amperes}$$

and

$$Q = \frac{I_L}{I_S} = \frac{\frac{E}{X_L}}{\frac{E}{R}} = \frac{R}{X_L} = \frac{R}{X_C} \tag{38.4}$$

39 Chap. 86: The power transformer—transformer efficiency

The purpose of a power transformer is to increase or decrease the value of the ac line or supply voltage without altering the frequency. This operation is achieved with a high level of efficiency.

A power transformer consists of a primary coil and a secondary coil whose number of turns are respectively N_p and N_s (figures 39.1A and B). The alternating source voltage, E_p, is applied across the primary coil while the load is connected across the secondary coil. These two coils may each consist of thousands of turns, which are wound on a common soft-iron core so that the leakage flux is reduced to a low value.

In the case of the ideal transformer, the leakage flux is zero so the mutual inductance, $M = \sqrt{L_p L_s}$, and the coupling factor, k, is unity. When an alternating current, I_p, flows in the primary coil, it creates a magnetic flux which links with the secondary coil and induces the secondary voltage, With zero flux leakage, there are the same volts per turn associated with both the primary and secondary coils. It follows that if the number of secondary turns exceeds the number of primary turns, the secondary voltage is greater than the primary voltage and we have a so-called "step-up" transformer. Similarly, if the number of secondary turns is less than the number of primary turns, the secondary voltage is smaller than the primary voltage and the transformer is of the "step-down" type. The terms "step-up" and "step-down" normally refer to the voltage and not to the current. From the phase point of view, the primary and secondary voltages are 180° out of phase provided the two coils are wound in the same sense.

An ideal power transformer has zero power losses and therefore the power input to the primary circuit equals the power output from the secondary circuit. It follows that a step-up of the voltage level from the primary circuit to the secondary circuit must be accompanied by a corresponding reduction in the current levels.

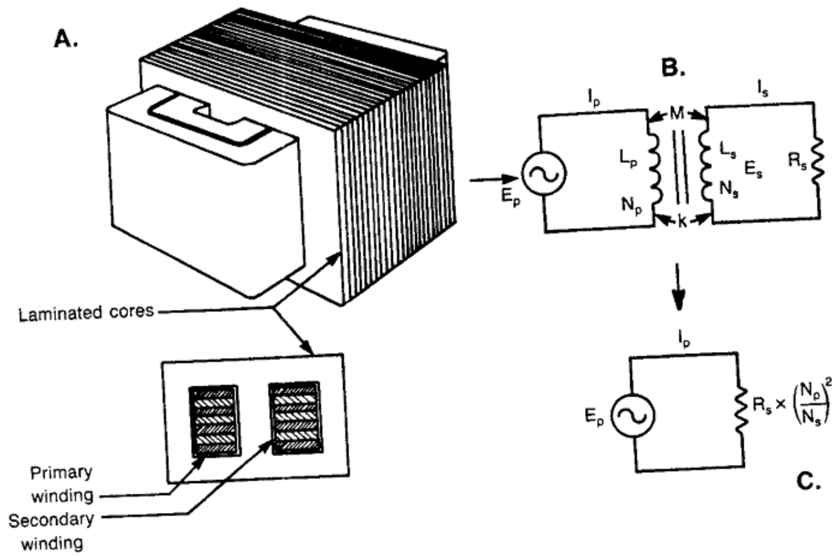

Figure 39.1: Example

39.1 Practical power transformer

The practical power transformer suffers from the following losses:

1. The copper loss which is the power dissipated in the resistances of the primary and secondary windings.
2. The iron loss, which is dissipated in the core. This lost energy is subdivided into:
   (a) the eddy-current loss. This is caused by the moving magnetic flux, which cuts the core and induces circulating currents within the metal. The loss is reduced by laminating the core into thin slices with each slice insulated from its neighbor.
   (b) the hysteresis loss, which is caused by the rapid magnetizing, demagnetizing, and remagnetizing of the core during the cycle of the primary current.

For practical power transformers, the total losses are less than 10% of the primary power. When a load resistance is connected across the secondary coil, the secondary current creates an alternating magnetic flux which, by Lenz's Law, opposes and partially cancels the primary flux. As a result the primary current increases and an effective resistance of value $\frac{E_p}{I_p}$ ohms is presented to the primary source; this ohmic value is called the resistance reflected from the secondary circuit into the primary circuit owing to the presence of the secondary load.

Note that if a steady direct current flows in the primary coil, the leakage flux will be constant in magnitude and direction so that the voltage induced in the secondary coil is zero. However, if the steady dc voltage applied to the primary coil is "chopped" to produce a square wave, the transformer responds to this type of input and some form of alternating voltage (not however, a simple sine wave) will be induced in the secondary coil; the secondary voltage is then rectified to provide a final output dc voltage which is larger than the dc input voltage.

39.2 Mathematical derivations for ideal transformer

Turns ratio:

$$\frac{N_p}{N_s} = \frac{E_p}{E_s} \tag{39.1}$$

Because an ideal transformer has no loses:

$$\text{primary power } P_p = \text{secondary power } P_s \tag{39.2}$$
$$E_p \cdot I_p = E_s \cdot I_s \text{ watts}$$

Combining equations 39.1 and eq22:

$$\frac{E_p}{E_s} = \frac{I_s}{I_p} = \frac{N_p}{N_s} \tag{39.3}$$

Primary voltage:

$$E_p = \frac{E_s I_s}{I_p} = \frac{E_s N_p}{N_s} \text{ volts} \tag{39.4}$$

40 Chap. 87: Complex algebra— operator j— rectangular/polar conversions

In previous chapters an alternating voltage or current has been represented either by a sine wave or a phasor or a trigonometrical expression. While these representations are adequate for simple series and parallel arrangements, they are too cumbersome for the analysis of more complicated circuits such as series-parallel combinations and those circuits that require the use of the network theorems. What is clearly needed is a form of algebra that can be applied directly to the solution of general ac circuits. Such an algebra must be capable of taking into account the circuit's phase relationships by distinguishing between the resistive and reactive elements. This is achieved in complex algebra by the introduction of the operator j. See figure 40.1.

The definition of the operator j is as follows: A phasor when multiplied by the operator j is rotated through 90° or $\frac{\pi}{2}$ radians in the positive or counterclockwise direction but the magnitude of the phasor is unchanged. In a similar way, a phasor when multiplied by the operator $-j$, is rotated through 90° in the clockwise direction.

Consider a simple case in which a resistance of 3Ω is connected in series with an inductive reactance of 4Ω. The corresponding phasor diagram is shown in figure 40.1A. Because the inductive reactance phasor is pointing vertically upwards, it can be said to lie along the axis (assuming the reference line to be horizontal). The phasor equation for the series combination is therefore:

$$\text{Total impedance phasor}\, z = 3 + j4\Omega$$

This expression for the impedance phasor is known as rectangular notation because the 3Ω and the $+j4\Omega$ phasors are perpendicular. Alternatively, we can say that the magnitude of the impedance phasor is $\sqrt{3^2+4^2} = 5\Omega$ while its phase angle is $inv\cos(\frac{3}{5}) = +53.1°$. These results may be combined by stating that the impedance phasor is $5 + 53.1°$(a magnitude of 5Ω with a phase angle of +53.1°): this expression for the impedance is referred to as polar notation.

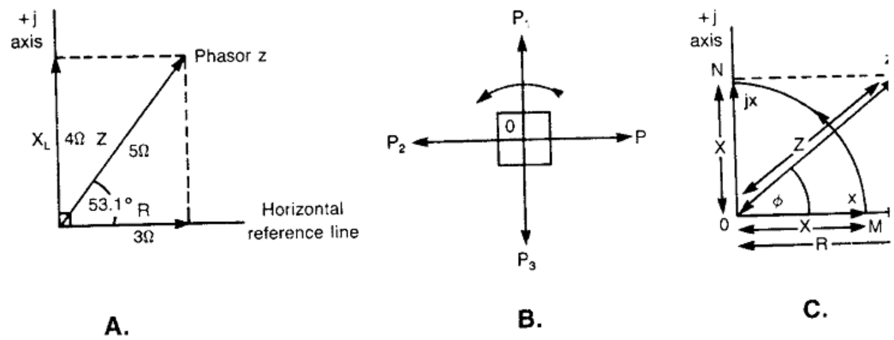

Figure 40.1: Example

41 Chap. 90: Analysis of a series-parallel circuit with the aid of the j operator

In terms of impedance phasors the formula for the total impedance is the same as we earlier derived for comparable resistor networks. In the circuit of figure 41.1B, we would start by combining the parallel z_1,z_2 phasors by using the "product-over-sum" formula. To the $z_1 \| z_2$ phasor we would then add the series phasor and the result is the total impedance phasor, z_T. These operations are performed with the rules of complex algebra.

After obtaining the total impedance phasor, the source voltage, e, is divided by z_T to obtain the total current, i_T. Subsequently, we can calculate the individual voltages across the components, the branch currents and the powers associated with each current.

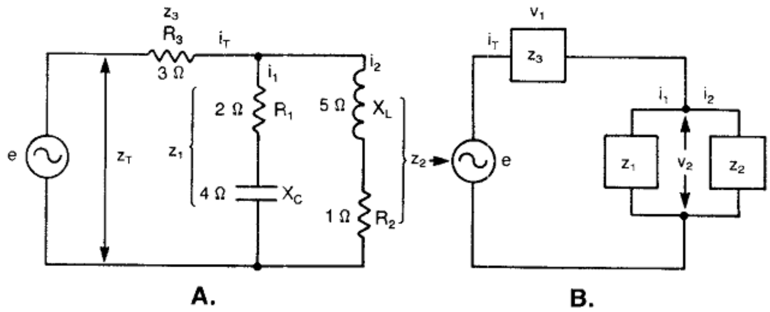

Figure 41.1: Example

41.1 Mathematical derivations

In figures 41.1 A and B:

$$\text{Phasor } z_1 \| z_2 = \frac{z_1 \cdot z_2}{z_1 + z_2} \tag{41.1}$$

Total impedance phasor:

$$z_t = z_3 + z_1 \| z_2 = z_3 + \frac{z_1 \cdot z_2}{z_1 + z_2} \tag{41.2}$$

Total current phasor:

$$i_T = \frac{e}{z_T} \tag{41.3}$$

Voltage phasor:

$$v_1 = i_T \cdot z_3 \tag{41.4}$$

42 Chap. 91: Analysis of a parallel branch circuit with the aid of the j operator

In terms of the impedance phasors the formulate for the total impedance is the same as we earlier derived for comparable resistor networks. In the circuit of figure 42.1B we would use the reciprocal formula to obtain the total impedance phasor, z_T. The calculation of z_T is obtained by using the rules of complex algebra.

The branch currents i_1, i_2 etc., are calculated by dividing the source voltage, e, by each of the branch impedances. The products $e \cdot i_1$, $e \cdot i_2$ etc. will then provide the individual powers associated with the branches.

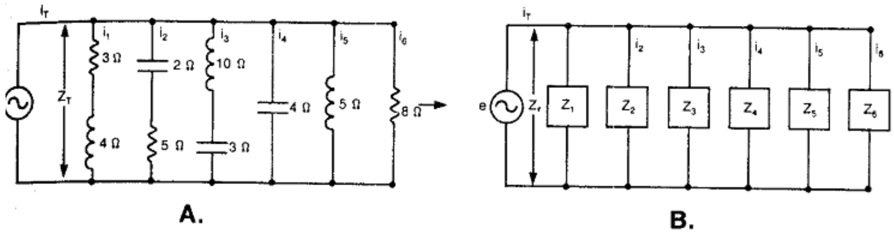

Figure 42.1: Example

42.1 Mathematical derivations in figures 42.1A and B

$$\frac{1}{z_T} = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \frac{1}{z_4} + \frac{1}{z_5} + \frac{1}{z_6} \tag{42.1}$$

Total impedance phasor:

$$z_T = \frac{1}{\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \frac{1}{z_4} + \frac{1}{z_5} + \frac{1}{z_6}} \tag{42.2}$$

Total current phasor:

$$i_T = \frac{e}{z_T} \tag{42.3}$$

Branch current phasors:

$$i_1 = \frac{e}{z_1} i_2 = \frac{e}{z_2} \qquad \text{etc.} \tag{42.4}$$

42.2 Example

In figure 42.1A, $z_1 = 3 + j4$, $z_2 = 5 - j2$, $z_3 = j10 - j3$, $z_4 = 0 - j4$, $z_5 = 0 + j5$, $z_6 = 8 + j0$, $e = 825°$ V. Find the values of the phasors z_T and i_T.

42.2.1 Solution

The total impedance phasor z_T is given by:

$$
\begin{aligned}
\frac{1}{z_T} &= \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \frac{1}{z_4} + \frac{1}{z_5} + \frac{1}{z_6} \qquad (42.5)\\
\frac{1}{z_T} &= \frac{1}{3+j4} + \frac{1}{5-j2} + \frac{1}{j10-j3} + \frac{1}{0-j4} + \frac{1}{0+j5} + \frac{1}{8+j0}\\
&= \frac{3-j4}{3^2+4^2} + \frac{5-j2}{5^2+2^2} + \frac{1}{j7} + \frac{1}{-j4} + \frac{1}{j5} + \frac{1}{8}\\
&= \frac{3-j4}{25} + \frac{5-j2}{29} - j0.142 + j0.25 - j0.2 + 0.125\\
&= 0.12 - j0.16 + 0.172 + j0.069 - j0.142 + j0.25 - j0.2 + 0.125\\
&= 0.417 - j0.183\\
&= 0.455 - 23.7^\circ
\end{aligned}
$$

Total impedance phasor:

$$z_T = \frac{1}{0.455 - 23.7^\circ} = 2.20\Omega 23.7^\circ \qquad (42.6)$$

Total current:

$$
\begin{aligned}
i_T &= \frac{e}{z_T} \qquad (42.7)\\
&= \frac{825^\circ \text{ V}}{2.20 + 23.7^\circ}\\
&= 3.64[(+25^\circ) - (+23.7^\circ)] \text{ A}\\
&= 3.641.3^\circ \text{ A}
\end{aligned}
$$

43 Chap. 100: Waveform analysis

Any single-valued, finite and continuous function $f(t)$ having a period of $\frac{2\pi}{\omega}$ seconds, may be expressed in the following form using **Fourier's theorem**.

$$f(t) = a_0 + A_1 \sin(\omega t + \Phi_1) + A_2 \sin(2\omega t + \Phi_2) + A_3 \sin(3\omega t + \Phi_3) + \dots \quad (43.1)$$

where:

$$\begin{aligned} \omega &= 2\pi f \text{ radians per second} \\ t &= \text{time in seconds} \end{aligned}$$

Because:

$$\begin{aligned} Asin(\omega t + \Phi) &= A \sin \omega t cos \Phi + A \cos \omega t \sin \Phi \\ &= a \cos \omega t + b \sin \omega t \end{aligned}$$

where:

$$\begin{aligned} a &= A \sin \Phi \\ b &= A \cos \Phi \end{aligned}$$

The Fourier expansion may be expressed as:

$$\begin{aligned} f(t) = a_0 &+ a_1 \cos \omega t + a_2 \cos 2\omega t + a_3 \cos 3\omega t + \dots \\ &+ b_1 \sin \omega t + b_2 \sin 2\omega t + b_3 \sin 3\omega t + \dots \end{aligned} \quad (43.2)$$

This means that the waveform may be regarded as composed of a mean level (a_0) together with fundamental sine and cosine waves as well as their harmonics.

In order to use the expansion to analyse a complex waveform, it is necessary to determine the values of the coefficients a_0, a_1,a_2, ..., b_1,b_2, ... This is done by using integral calculus which is outlined in the mathematical derivations. However, the following is the quoted result for the symmetrical square wave (figure 43.1A) which is frequently encountered in communications. For such a square wave whose mean level is zero, the expansion is:

$$f(t) = \frac{2D}{\pi}[\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \dots + \frac{1}{n} \sin \mathrm{n}\omega t + \dots] \quad (43.3)$$

Due to the symmetry of the square wave the expansion contains neither cosine terms nor even harmonic sine terms.

Notice that the amplitude of the nth harmonic is $\frac{1}{n}$ so that the amplitudes of the higher harmonics only decrease slowly. It follows that the symmetrical square wave contains a large number of strong harmonics.

The expansion of equation 43.3 may be verified by adding the fundamental sine wave component and its odd harmonics. This is called waveform synthesis which is illustrated in figure

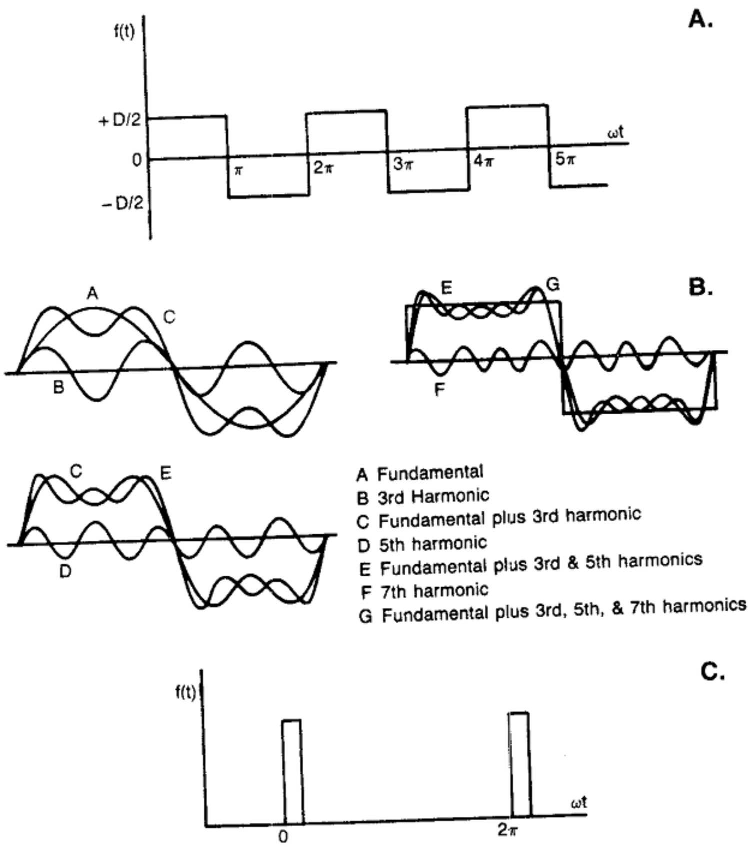

Figure 43.1: Example

43.1B. However, waveform "G" is far from being a symmetrical square and many more odd harmonics would be necessary before achieving a good approximation to the required waveform.

If the square waveform is made very asymmetrical, its appearance is that of a repeating pulse of short duration (figure 43.1C); as an example this would be the modulation waveform in a pulsed radar set. Both odd and even harmonics then appear in the expansion but their amplitudes decrease very slowly. This waveform is therefore extremely rich in harmonics and to achieve a good approximation in its synthesis, we might well have to include harmonics higher than the thousandth.

43.1 Mathematical derivations

Figure 43.2 shows a square waveform which is a single-valued repeating function of time with a period of seconds; consequently, it may be analyzed by Fourier's Theorem. From $\omega t = 0$ to $\omega t = 2\pi$, the equation of the function is $f(t) = D$. From $\omega t = 0$ to $\omega t = 2\pi$, the equation of the function is $f(t) = 0$.

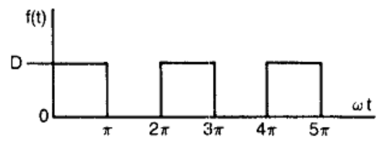

Figure 43.2: Example

Then:

$$
\begin{aligned}
a_0 &= \frac{1}{2\pi}\int_0^{2\pi} f(t)\,d(\omega t) \\
&= \frac{1}{2\pi}\int_0^{\pi} f(t)d(\omega t) + \frac{1}{2\pi}\int_{\pi}^{2\pi} f(t)d(\omega t) \\
&= \frac{1}{2\pi}\cdot D\cdot\pi + 0 = \frac{D}{2}
\end{aligned} \tag{43.4}
$$

$$
\begin{aligned}
a_n &= \frac{1}{\pi}\int_0^{2\pi} f(t)\cos n\omega t d(\omega t) \\
&= \frac{1}{\pi}[\int_0^{\pi} D\cos n\omega t d(\omega t) + \int_{\pi}^{2\pi} 0\cdot\cos n\omega t d(\omega t)]
\end{aligned} \tag{43.5}
$$

44 Chap. 122: Positive or regenerative feedback

Positive feedback may be used to increase the gain of an amplifier circuit. This is illustrated in figure 44.1A where V_i is the input signal from the preceding stage. Such a signal would be applied between the base of a transistor (or the control grid of a tube) and ground. The input signal, V_n, appears between collector (or plate) and ground and a fraction, β (beta), of this output signal is then fed back to the input circuit so that this feedback voltage, $+\beta V_0$, is in phase with V_i. β is called the feedback factor, which may either be expressed as a decimal fraction or as a percentage.

In order for the feedback to be positive, there must be a total of 360° phase shift (equivalent to zero phase shift) around the feedback loop base $\rightarrow$ collector $\rightarrow$ base (or grid $\rightarrow$ plate $\rightarrow$ grid). The total signal voltage when applied between base and emitter (or control grid and cathode) is the sum of the input signal, V_i, and the positive feedback voltage, $+\beta V_0$. The voltage gain (open loop gain) of the active device is A and the mathematical derivations will show that $A^{'} = \frac{A}{1-A\beta}$ where $A^{'}$ is the overall voltage gain with the positive feedback present (closed loop gain). There are then three possible conditions in the circuit:

1. If A and β are chosen so that the value of $A\beta$ is less than 1, then $A^{'}$ is greater than A and the amplifier's gain has been increased as the result of the positive feedback. Such is the case with the so-called regenerative amplifier.

2. If $A\beta = 1$ (for example $A = 10$ and $\beta = 0.1$ or 10%), $A^{'}$ is theoretically infinite. This means that the circuit can provide a continuous output without any input signal from the previous stage. This is the condition in a stable oscillator.

3. If $A\beta > 1$, the oscillator is unstable. The output, V_0, then increases which tends to reduce the value of A until the equilibrium condition of $A\beta = 1$ is reached.

The condition for oscillation is therefore $A\beta = l0°$; this is sometimes referred to as the *Barkhausen* or *Nyquist* criterion. The inclusion of "0°" in the polar value of $A\beta$ means that the resultant phase shift around the loop is zero degrees and, consequently, the feedback is positive. By contrast an angle of 180° would indicate that the feedback is negative.

Figure 44.1B shows the principle of positive feedback in an oscillator circuit. If we assume that the input signal is 1 V rms and the voltage gain of the active device is 10, the output signal is 10 V rms; if the feedback factor is $\frac{1}{10}$ or 10%, the 10 V output signal will be responsible for creating the 1 V input signal (this does not mean that there is only 9 V left of the output signal). This argument sounds rather like the chicken and the egg so the question arises "How does the circuit get started in the first place?" The answer is that all active devices are inherently noisy.

Because the noise is spread throughout the frequency spectrum, it will contain a component at the frequency of oscillation. This component will trigger the positive feedback network so that the oscillation will increase until the equilibrium condition of $A\beta = 10°$ is reached.

44.1 Mathematical derivations

For a regenerative amplifier,total input signal between base and emitter:

$$= V_i + \beta V_0 \,\text{volts} \tag{44.1}$$

Output signal:

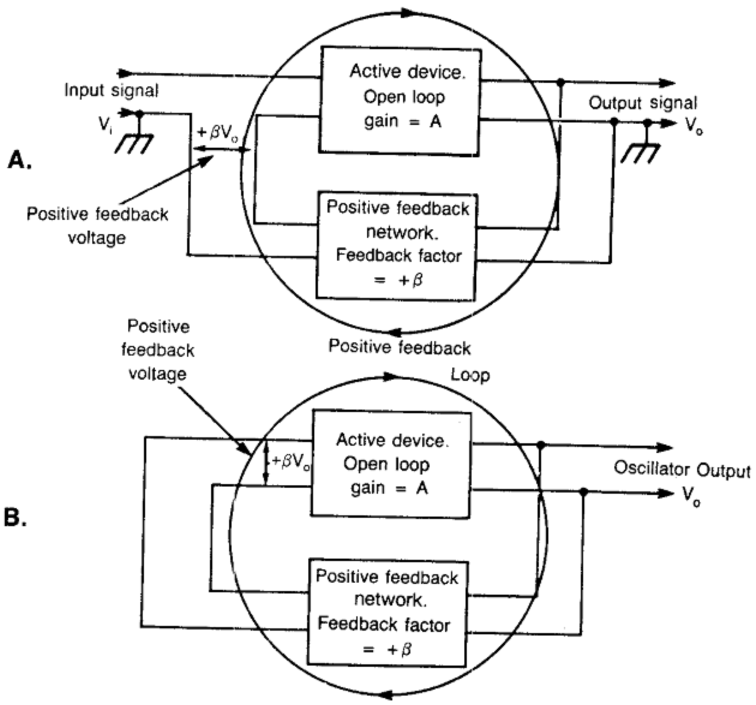

Figure 44.1: Example

$$V_0 = A \cdot (V_i + \beta V_0)\,\text{volts} \tag{44.2}$$

This yields

$$A' = \frac{V_0}{V_i} = \frac{A}{1 - A\beta} \tag{44.3}$$

where A' is the overall voltage gain with the positive feedback present (closed loop gain). For a stable oscillator. For a stable oscillator:

$$A\beta = 10^\circ \tag{44.4}$$

Positive feedback factor $\beta = \frac{1}{A}$, positive feedback percentage $= \beta \cdot 100\%$

44.2 Example

An amplifier provides an open loop gain of 15 and has a positive feedback factor of 3%. What is the value of the closed loop gain?

44.2.1 Solution

Closed loop gain

$$
\begin{aligned}
A' &= \frac{A}{1 - A\beta} && (44.5)\\
&= \frac{15}{1 - \frac{15 \cdot 3}{100}}\\
&= \frac{15}{0.55}\\
&= 27.3
\end{aligned}
$$

44.3 Example

An amplifier has an open loop gain of 50. What percentage of positive feedback is required to sustain a stable oscillation?

44.3.1 Solution

Positive feedback factor

$$
\begin{aligned}
B &= \frac{1}{A} && (44.6)\\
&= \frac{1}{50}
\end{aligned}
$$

Positive feedback percentage:

$$
\begin{aligned}
&= \beta \cdot 100 && (44.7)\\
&= \frac{1}{50} \cdot 100\\
&= 2 && (44.8)
\end{aligned}
$$

45 Chap. 134: Negative feedback

the introduction of negative feedback in an audio amplifier requires that a fraction, β of the output signal is fed back in opposition to the input signal. In contrast with the disadvantage of reducing the amplifier gain, negative or degenerative feedback provides the following features:

1. Stabilization of the amplifier voltage gain against changes in the parameters of the active device such as a transistor or tube.
2. Reduction in the amplitude distortion caused by nonlinearity in the characteristics of the active device.
3. Reduction in frequency and phase distortion produced by the junction and stray capacitances.
4. Reduction in noise.
5. Changes in the amplifier's input and output impedances.

Note that the second, third, and fourth advantages refer to the distortion and the noise created within the amplifier itself. Negative feedback has no effect on the noise and the distortion that are fed in from the previous stage.

45.1 Mathematical derivations

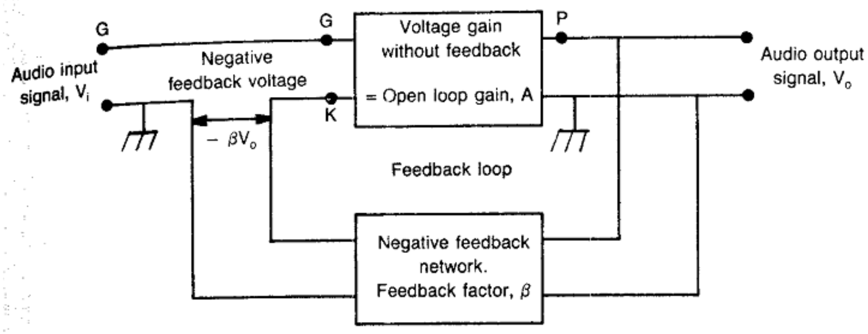

Figure 45.1: Example

In the block diagram of figure 45.1, the signal applied between the grid and the cathode is the sum of the input audio signal, V_i between the control grid and ground, and the negative feedback voltage, $-\beta V_0$. Therefore,

$$\text{Signal between the grid and the cathode } = V_i - \beta V_0 \,\text{volts} \tag{45.1}$$

where V_0 is output audio signal.
Output audio signal

$$V_0 = A \cdot \text{signal between G and K} \tag{45.2}$$
$$= A(V_i - \beta V_0)\,\text{volts} \tag{45.3}$$

where A is open loop gain.
Amplifier gain negative feedback:

$$A' = \frac{V_0}{V_i} = \frac{A}{1 + A\beta} \tag{45.4}$$

if $A\beta$ is appreciably greater than 1

$$A' \to \frac{A}{A\beta} = \frac{1}{\beta} \tag{45.5}$$

which is independent of A; consequently the value of A' will be little affected by any changes in the parameters of the active device. Because:

$$A' = \frac{A}{1 + A\beta'} \tag{45.6}$$

$$A = \frac{A'}{1 - A'\beta}$$

$$\beta = \frac{1}{A'} - \frac{1}{A} = \frac{A - A'}{AA'}$$

The feedback factor, β, may either be expressed as a fraction or as a percentage. The two basic negative feedback circuits are shown in figures 45.2A and B. Figure 45.2A represents voltage negative feedback in which the audio signal voltage, V_0, is divided between $R1$ and $R2$ (C is only a dc blocking capacitor). The voltage $-\beta V_0$ which is developed across $R1$, is applied as degenerative feedback to the input signal, V_i.

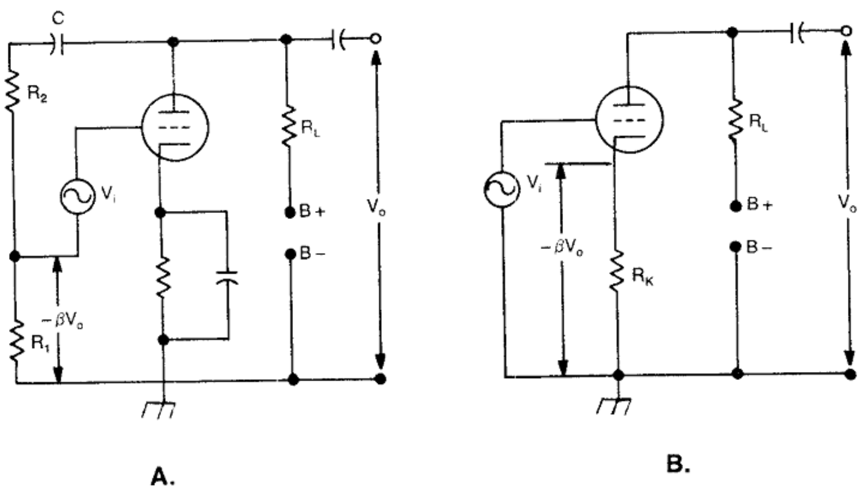

Figure 45.2: Example

Feedback factor:

$$\beta = \frac{R_1}{R_1 + R_2} \tag{45.7}$$

The voltage gain of the amplifier with feedback is:

$$A^{'} = \frac{A}{1 + A\beta} \tag{45.8}$$

Open loop gain:

$$A = \frac{\mu R_L}{r_p + R_L}$$

This expression for the voltage gain without feedback, A, applies when the value $Rl + R2$ is sufficiently large so that the equivalent value of the plate load resistance is not appreciably affected by the shunting action of the two resistors.

Current feedback, as shown in Fig. 134-2B, uses the signal component of the plate current to develop the degenerative feedback voltage across the cathode resistor, EK (3 is then equal to rk!Rl and the voltage gain of the amplifier with feedback,

$$A^{'} = \frac{A}{1 + A\beta}$$

where

$$A = \frac{\mu R_L}{r_p + R_K + R_L} \tag{45.9}$$

The last two equations yield:

$$A^{'} = \frac{\mu R_L}{r_p + R_L + R_K(1 + \mu)} \tag{45.10}$$

45.2 Example

The voltage gain of an audio amplifier without negative feedback is 35. If 20% degenerative feedback is introduced into the circuit, calculate the value of the amplifier gain with feedback.

45.2.1 Solution

Because $A = 35$ and $\beta = \frac{20}{100} = 0.2$, the gain with feedback is:

$$\begin{aligned} A^{'} &= \frac{A}{1 + A\beta} \\ &= \frac{35}{1 + (35 \cdot 0.2)} \\ &= \frac{35}{1 + 7} = \frac{35}{8} \\ &= 4.4 \,\text{rounded off} \end{aligned}$$

45.3 Example

The voltage gain of an audio amplifier with degenerative feedback is 8. If the negative feedback factor is $\frac{1}{10}$, what is the amplifier gain without feedback?

45.3.1 Solution

Because $A^{'} = 8$ and $\beta = \frac{1}{10}$, the gain without feedback is:

$$\begin{aligned} A &= \frac{A'}{1 - A'\beta} \\ &= \frac{8}{1 - (8 \cdot 0.1)} = \frac{8}{0.2} \\ &= 40 \end{aligned} \tag{45.11}$$