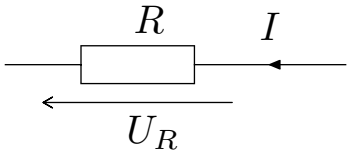
Elektronika pro informační technologie
Teorie obvodů

Sbírka příkladů

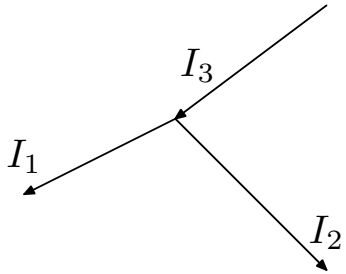
I. Ustálený stejnosměrný stav

1. Ohmův zákon



$$U_R = R \cdot I$$

2. I. K.z. $\Sigma I = 0$ (proudový)
   (Proudy tekoucí z uzlu bereme s kladným znaménkem, proudy tekoucí do uzlu se záporným znaménkem.)

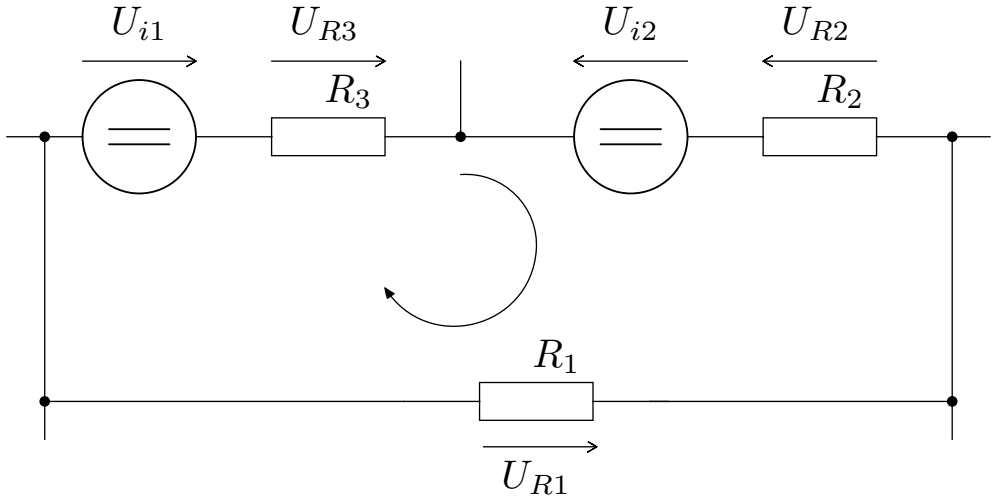


$$I_1 + I_2 - I_3 = 0$$

Pozn. Na přednášce jsme si uváděli jinou konvenci - proudy tekoucí z uzlu bereme se záporným znaménkem, proudy tekoucí do uzlu s kladným znaménkem (výše uvedenou rovnici přenásobíme -1).
$-I_1 - I_2 + I_3 = 0$

Obě rovnice splňují I. K.z. $\Sigma I = 0$

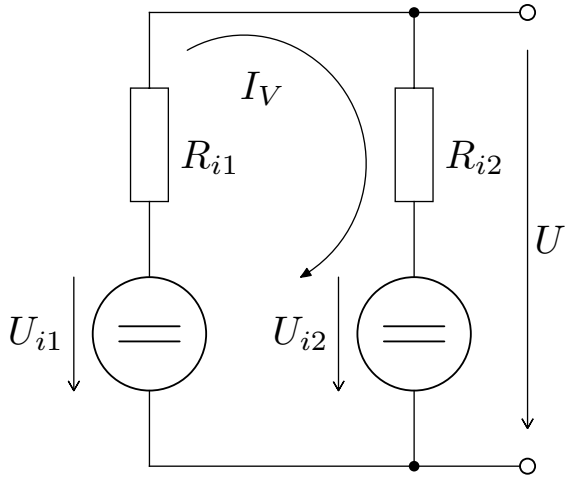
3. II. K.z. $\Sigma U = 0$ (napěťový)
   Napětí (úbytky na rezistorech, napětí zdrojů), jejichž čítající šipka má směr, souhlasící se směrem oběhu kolem smyčky, bereme s kladným znaménkem, ostatní napětí se záporným znaménkem.



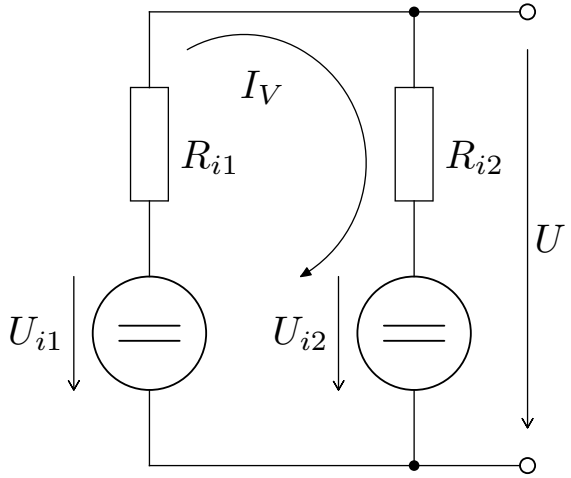
$$U_{i1} + U_{R3} - U_{i2} - U_{R2} - U_{R1} = 0$$

1.1 Základní zákony elektrických obvodů - jejich aplikace

Př.1.1: Určete proud I_V dvou paralelně řazených např. chemických zdrojů el. energie (nový a starší) $R_{i1} = 0,8\Omega, R_{i2} = 1,2\Omega, U_{i1} = 1,6V, U_{i2} = 1,45V$



$$R_{i1}I_V + R_{i2}I_V + U_{i2} - U_{i1} = 0$$



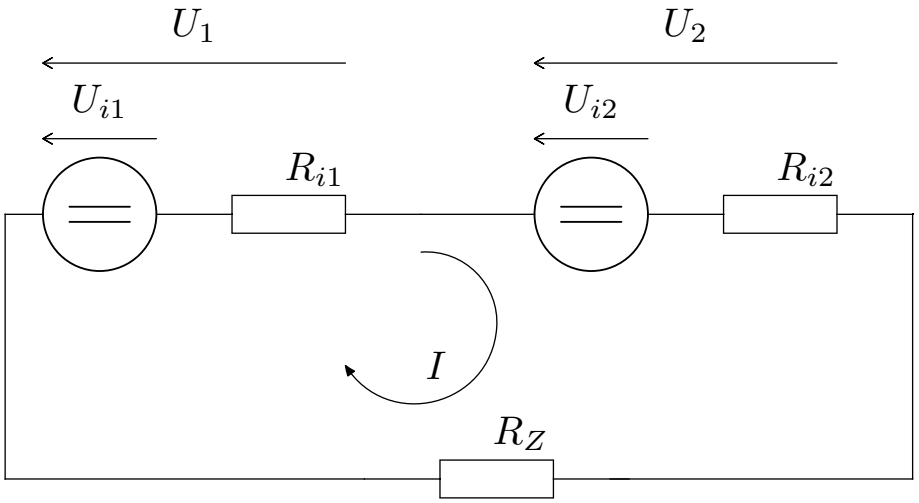
$$I_V = \frac{U_{i1} - U_{i2}}{R_{i1} + R_{i2}} = \frac{1,6 - 1,45}{0,8 + 1,2} = 0,075A \qquad [1]$$

[1] Paralelně řazené články jsou naprázdno a přesto uvnitř baterie teče proud. Proto v tomto zapojení nelze spojovat nové a staré články.

Př.1.2: Určete svorkové napětí zdrojů U_1 a U_2. Srovnejte napětí naprázdno a svorkové. Vnitřní odpor $R_{i1} = 0,8\Omega, R_{i2} = 4\Omega$, vnitřní napětí $U_{i1} = 1,6V, U_{i2} = 1,2V$.

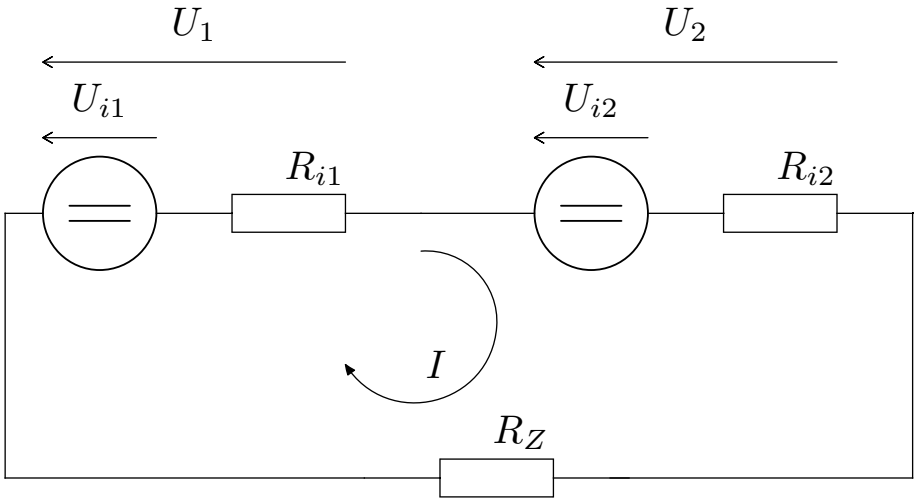
a) $R_Z = 5\Omega$

b) $R_Z = 3\Omega$



$$R_Z I - U_{i1} + R_{i1} I - U_{i2} + R_{i2} I = 0$$

a) $R_Z = 5\Omega$

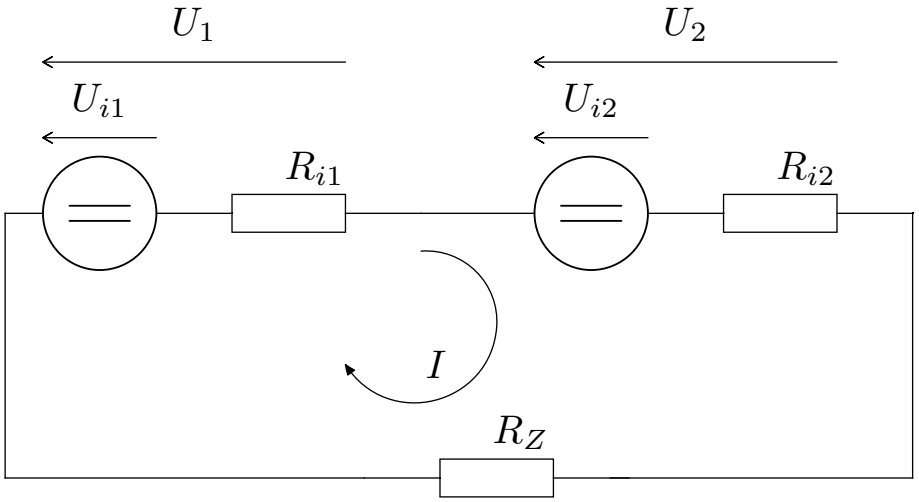


$$I_a = \frac{U_{i1} + U_{i2}}{R_{i1} + R_{i2} + R_Z} = \frac{1,6 + 1,2}{0,8 + 4 + 5} = 0,2857A$$

$$U_{1a} = U_{i1} - R_{i1} I_a = 1,6 - 0,8 \cdot 0,2857 = 1,371V$$

$$U_{2a} = U_{i2} - R_{i2} I_a = 1,2 - 4 \cdot 0,2857 = 0,05714V$$

b) $R_Z = 3\Omega$

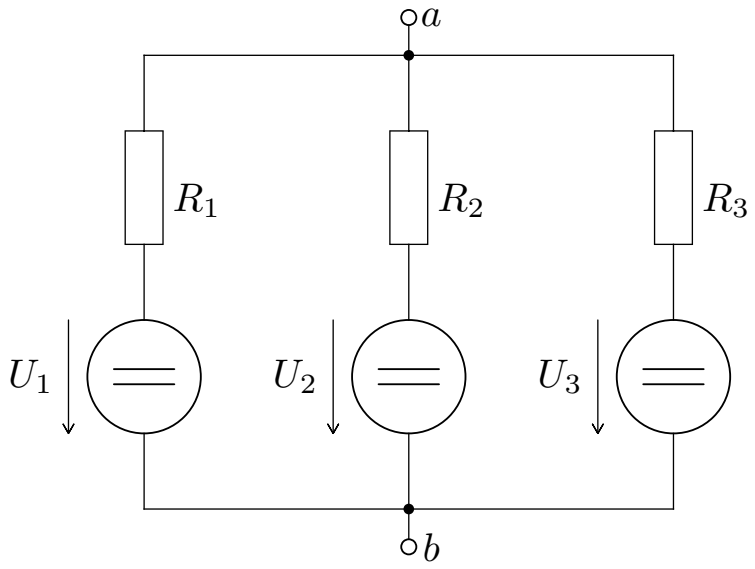


$$I_b = \frac{U_{i1} + U_{i2}}{R_{i1} + R_{i2} + R_Z} = \frac{1,6 + 1,2}{0,8 + 4 + 3} = 0,359A$$

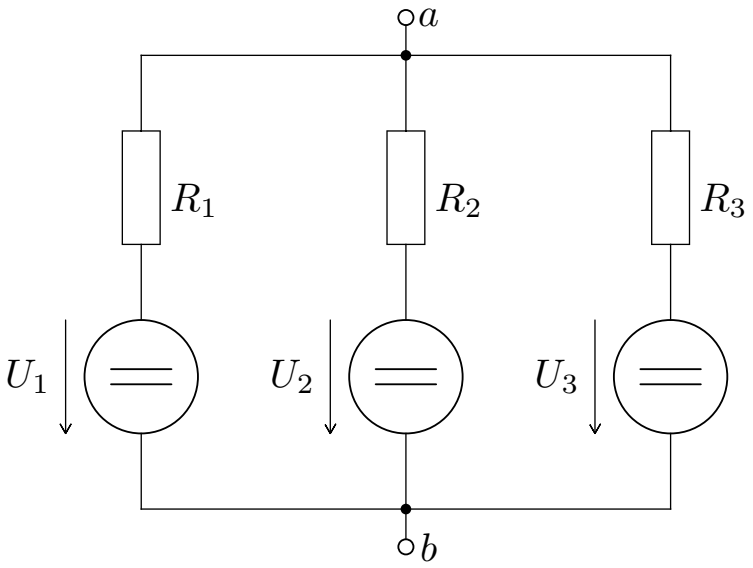
$$U_{1b} = U_{i1} - R_{i1} I_b = 1,6 - 0,8 \cdot 0,359 = 1,313V$$

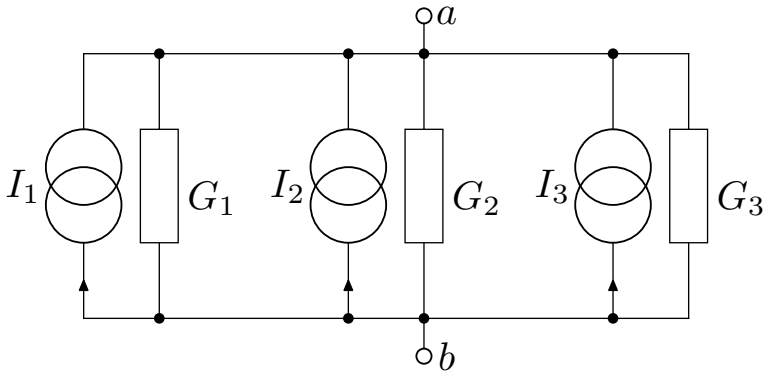
$$U_{2b} = U_{i2} - R_{i2} I_b = 1,2 - 4 \cdot 0,359 = -0,2359V$$

Př.1.3: Určete R_3 tak, aby $U_{ab} = 20V$. $R_1 = 5\Omega, R_2 = 10\Omega, U_1 = 10V, U_2 = 20V$, $U_3 = 30V$



1) Zdroje napětí v serii s odporem převedeme na ekvivalentní proudové zdroje.

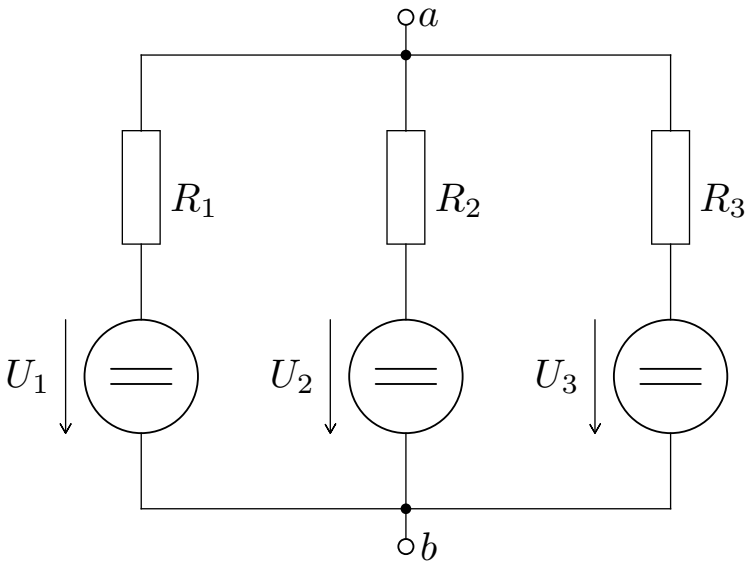


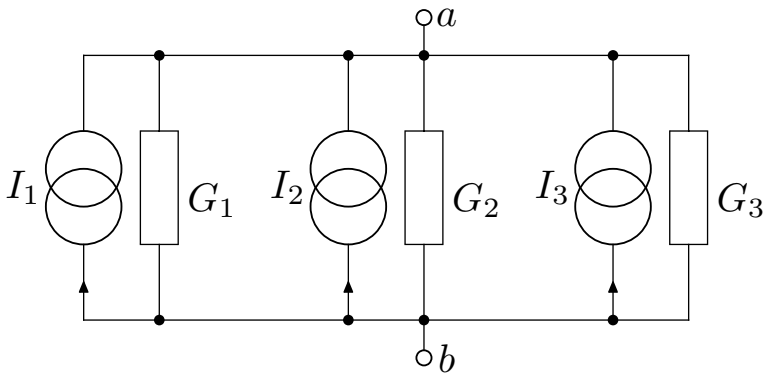


$$I_1 = \frac{U_1}{R_1} = \frac{10}{5} = 2A$$

$$I_2 = \frac{U_2}{R_2} = \frac{20}{10} = 2A$$

$$I_3 = \frac{U_3}{R_3}$$



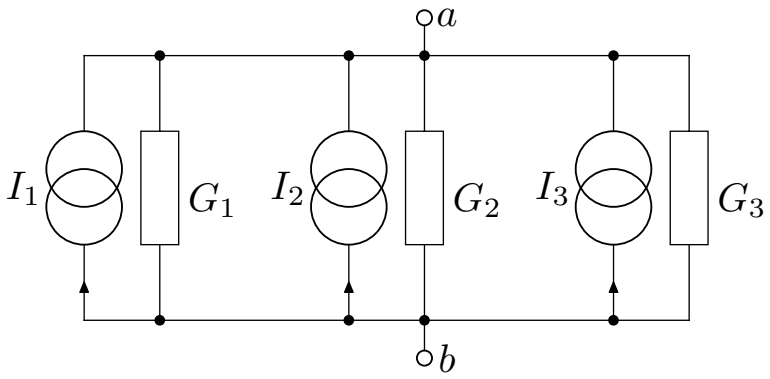


$$G_1 = \frac{1}{R_1} = \frac{1}{5} = 0,2S$$

$$G_2 = \frac{1}{R_2} = \frac{1}{10} = 0,1S$$

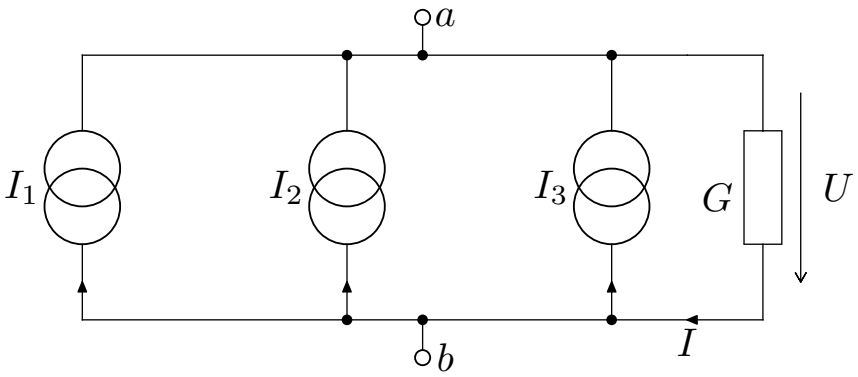
$$G_3 = \frac{1}{R_3}$$

2) Celková vodivost:



$$G = G_1 + G_2 + G_3 = 0,2 + 0,1 + G_3 = 0,3 + G_3$$

3) Rovnice podle I. KZ



$$I_1 + I_2 + I_3 - I = 0$$

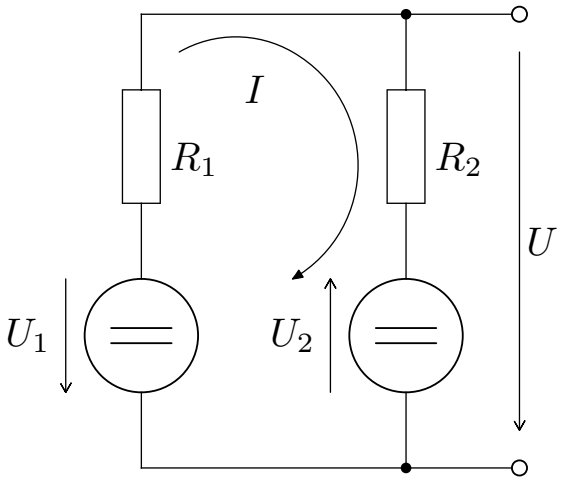
$$2 + 2 + 30G_3 - 20(0,2 + 0,1 + G_3) = 0$$

$$10G_3 = 2$$

$$G_3 = 0,2S$$

$$R_3 = 5\Omega$$

Př.1.4: Určete napětí U. $R_1 = 9\Omega, R_2 = 15\Omega, U_1 = 70V, U_2 = 50V$



$$R_1 I + R_2 I - U_2 - U_1 = 0$$

$$9I + 15I - 50 - 70 = 0$$

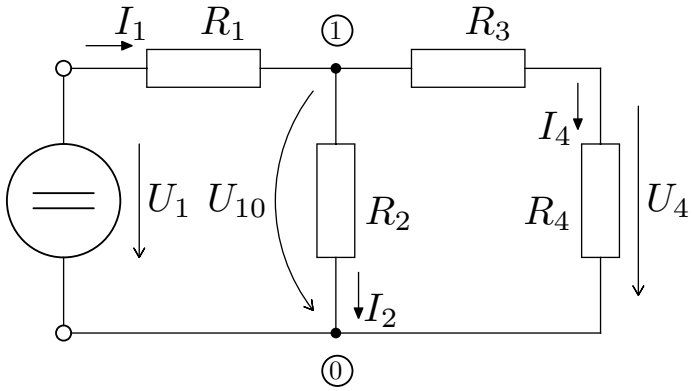
$$24I = 120$$

$$I = 5A$$

$$U = R_2 I - U_2 = 15 \cdot 5 - 50 = 25V$$

1.2 Metody pro speciální případy

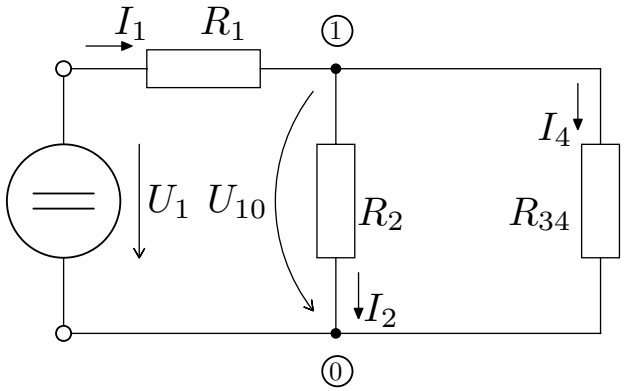
Př.2.1: Určete U_4 a) metodou zjednodušování, b) metodou úměrných veličin.
$U_1 = 10V, R_1 = 20\Omega, R_2 = 100\Omega, R_3 = R_4 = 50\Omega$



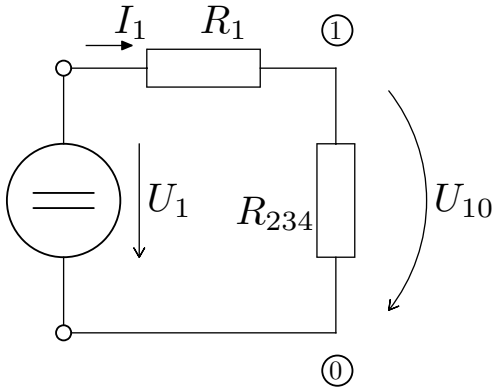
[2]

[2]Očíslujeme uzly: ⓪, ①

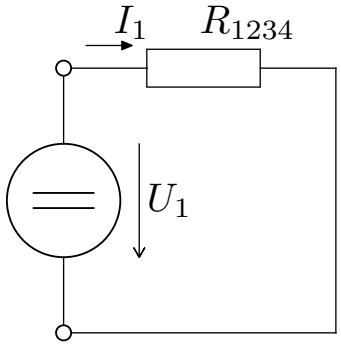
a) metoda zjednodušování



$$R_{34} = R_3 + R_4 = 50 + 50 = 100\Omega$$

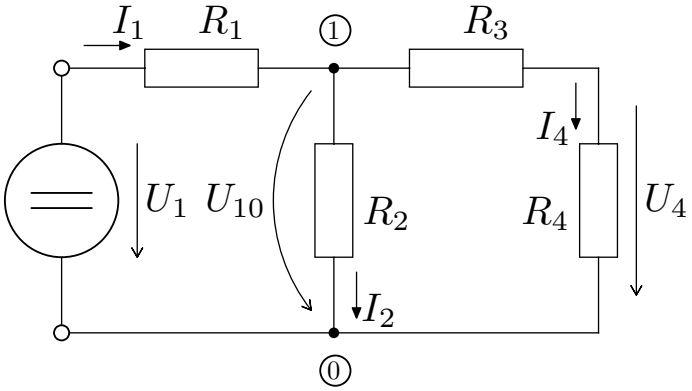


$$R_{234} = \frac{R_2 \cdot R_{34}}{R_2 + R_{34}} = \frac{100 \cdot 100}{100 + 100} = 50\Omega$$



$$R_{1234} = R_1 + R_{234} = 20 + 50 = 70\Omega$$

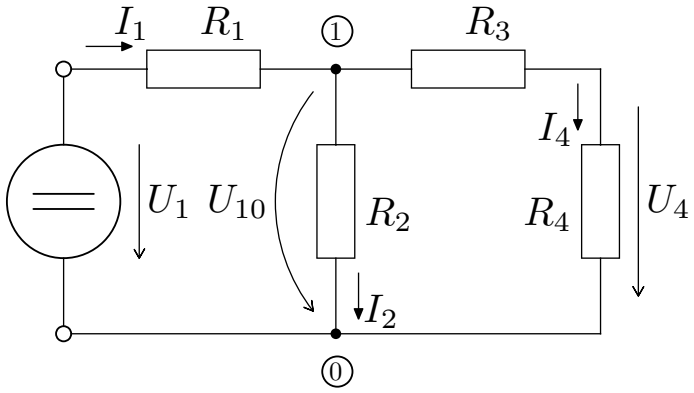
$$I_1 = \frac{U_1}{R_{1234}} = \frac{10}{70} = 0,1429A$$



$$U_{10} = U_1 - R_1 I_1 = 10 - 20 \cdot 0,1429 = 7,143V$$

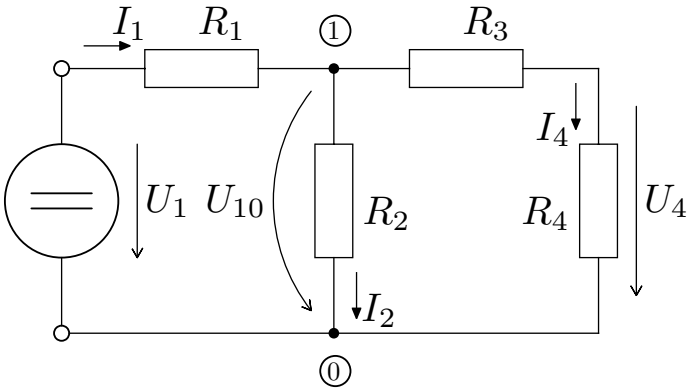
$$U_4 = R_4 \frac{U_{10}}{R_3 + R_4} = 50 \cdot \frac{7,143}{50 + 50} = 3,571V$$

b) metoda úměrných veličin



Volíme $U_4^{\bullet} = 50V$

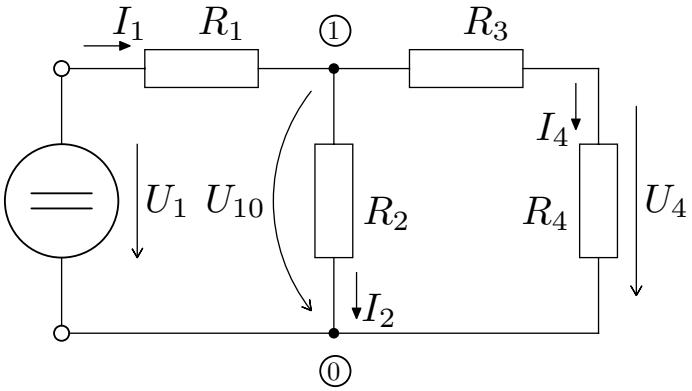
$$I_4^{\bullet} = \frac{U_4^{\bullet}}{R_4} = \frac{50}{50} = 1A$$



$$U_{10}^{\bullet} = (R_3 + R_4) I_4^{\bullet} = (50 + 50) \cdot 1 = 100V$$

$$I_2^{\bullet} = \frac{U_{10}^{\bullet}}{R_2} = \frac{100}{100} = 1A$$

$$I_1^{\bullet} = I_2^{\bullet} + I_4^{\bullet} = 1 + 1 = 2A$$



$$U_{1}^{\bullet} = U_{10}^{\bullet} + R_1 I_{1}^{\bullet} = 100 + 20 \cdot 2 = 140V$$

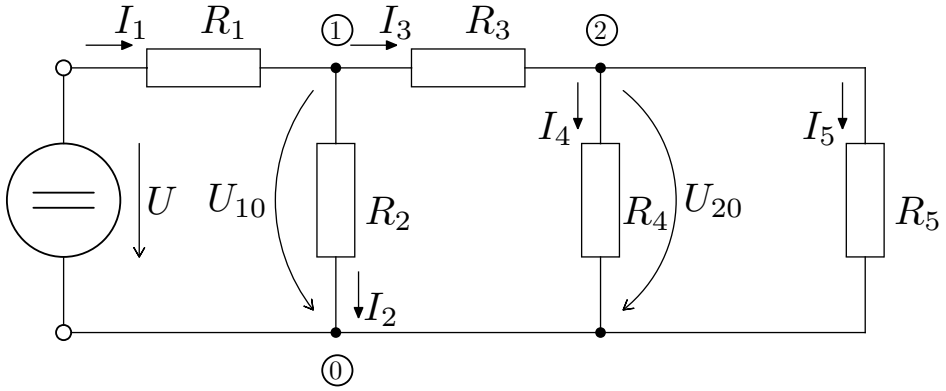
$$k = \frac{U_1}{U_{1}^{\bullet}} = \frac{10}{140} = 0,07143$$

$$U_4 = kU_{4}^{\bullet} = 0,07143 \cdot 50 = 3,571V$$

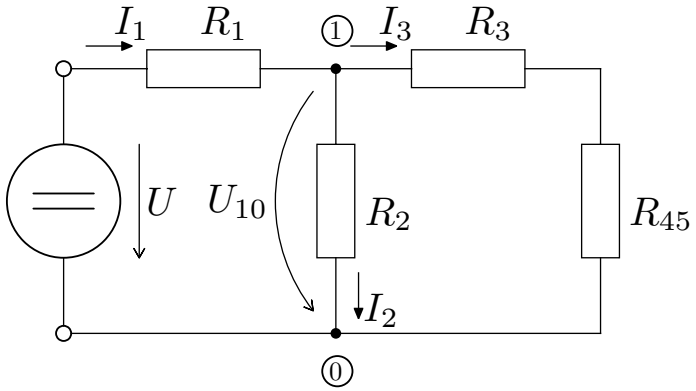
Př.2.2: Určete proudy obvodu: $U = 10V, R_1 = R_3 = 3k\Omega, R_2 = 13k\Omega, R_4 = 2k\Omega$ $R_5 = 4k\Omega$

a) metodou zjednodušování

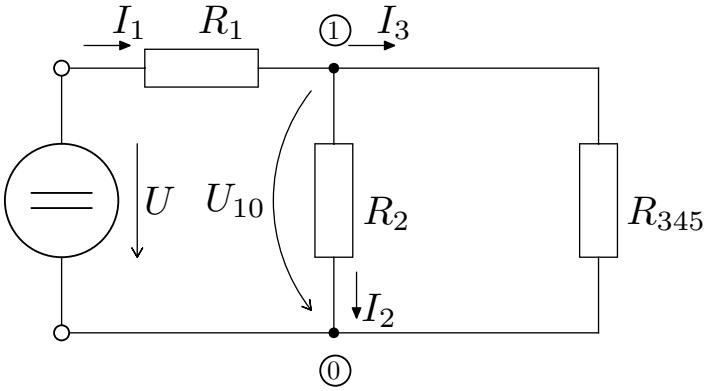
b) metodou úměrných veličin.



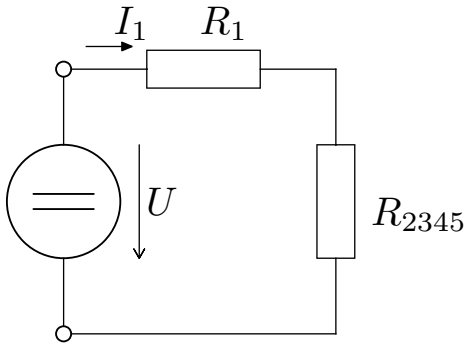
a) metoda zjednodušování



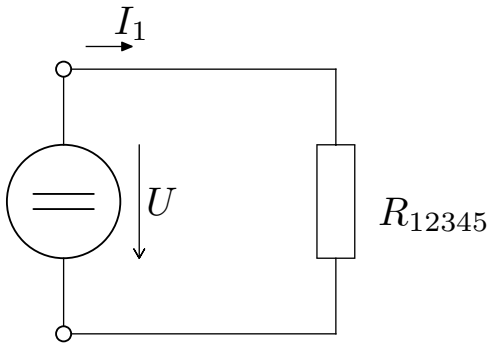
$$R_{45} = \frac{R_4 \cdot R_5}{R_4 + R_5} = \frac{2000 \cdot 4000}{2000 + 4000} = 1,3333k\Omega$$



$$R_{345} = R_3 + R_{45} = 3000 + 1333,3 = 4,333k\Omega$$

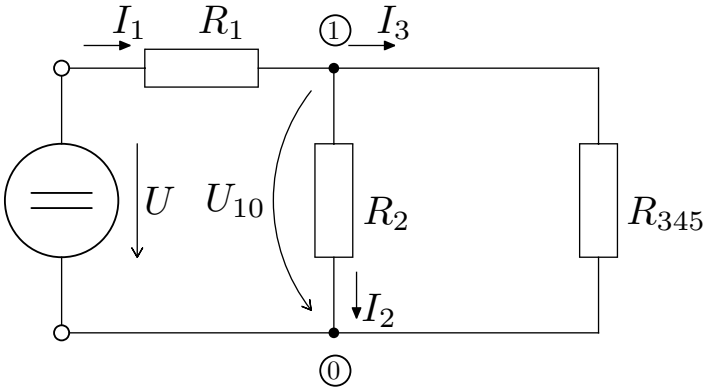


$$R_{2345} = \frac{R_2 \cdot R_{345}}{R_2 + R_{345}} = \frac{13000 \cdot 4333}{13000 + 4333} = 3,25k\Omega$$



$$R_{12345} = R_1 + R_{2345} = 3000 + 3250 = 6,25k\Omega$$

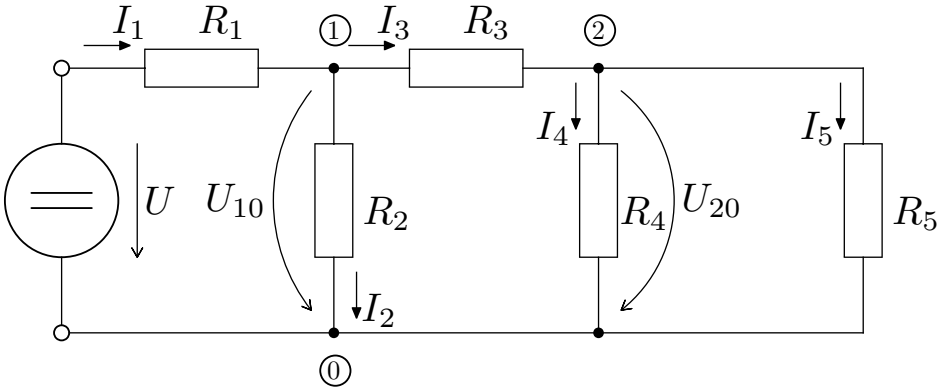
$$I_1 = \frac{U}{R_{12345}} = \frac{10}{6250} = 1,6mA$$



$$U_{10} = U - R_1 I_1 = 10 - 3000 \cdot 0,0016 = 5,2V$$

$$I_2 = \frac{U_{10}}{R_2} = \frac{5,2}{13000} = 0,4mA$$

$$-I_1 + I_2 + I_3 = 0 \Rightarrow I_3 = I_1 - I_2 = 1,6 - 0,4 = 1,2mA$$

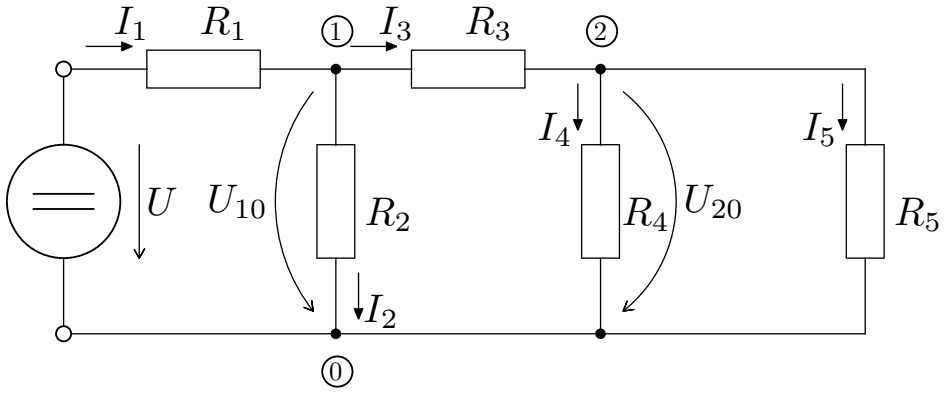


$$U_{20} = U_{10} - R_3 I_3 = 5,2 - 3000 \cdot 0,0012 = 1,6V$$

$$I_4 = \frac{U_{20}}{R_4} = \frac{1,6}{2000} = 0,8mA$$

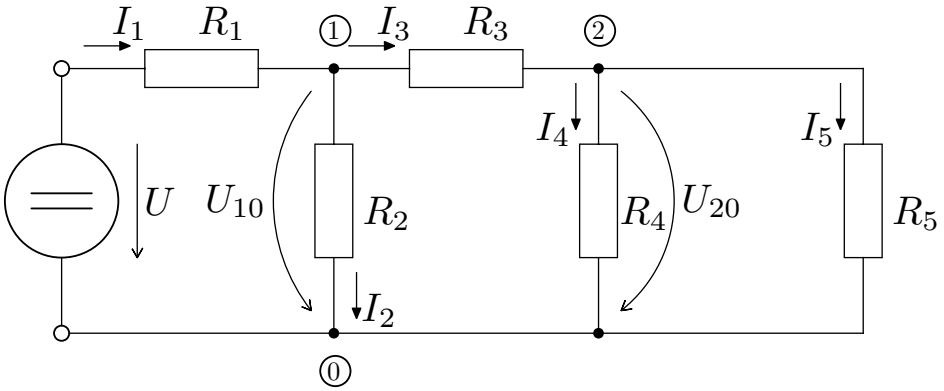
$$I_5 = \frac{U_{20}}{R_5} = \frac{1,6}{4000} = 0,4mA$$

b) metoda úměrných veličin



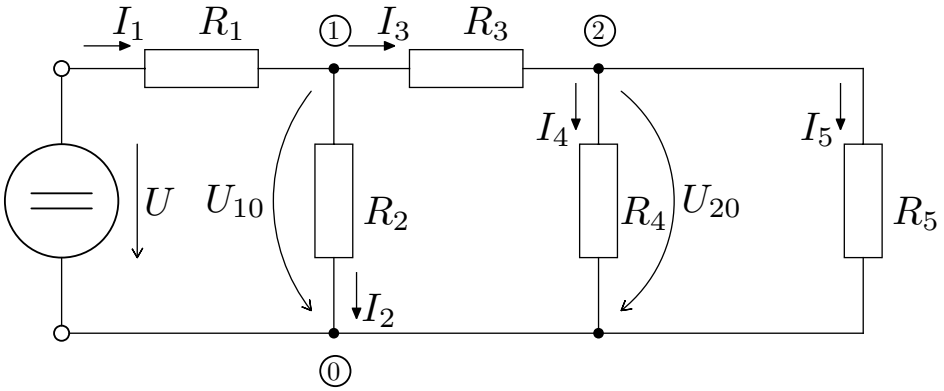
Volíme $I_5^{\bullet} = 1mA$

$$U_{20}^{\bullet} = R_5 I_5^{\bullet} = 4000 \cdot 0,001 = 4V$$



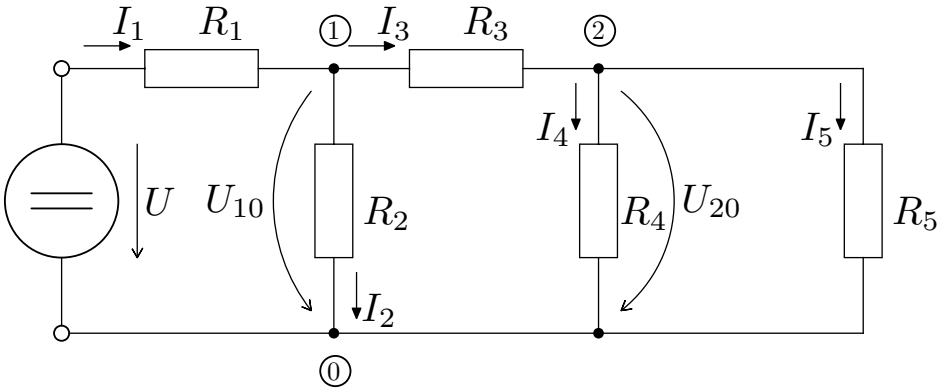
$$I_4^{\bullet} = \frac{U_{20}^{\bullet}}{R_4} = \frac{4}{2000} = 2mA$$

$$-I_3^{\bullet} + I_4^{\bullet} + I_5^{\bullet} = 0 \Rightarrow I_3^{\bullet} = I_4^{\bullet} + I_5^{\bullet} = 0,002 + 0,001 = 3mA$$



$$U_{\overset{\bullet}{10}} = U_{\overset{\bullet}{20}} + R_3 I_{\overset{\bullet}{3}} = 4 + 3000 \cdot 0,003 = 13V$$

$$I_{\overset{\bullet}{2}} = \frac{U_{\overset{\bullet}{10}}}{R_2} = \frac{13}{13000} = 1mA$$



$$-I_1^{\bullet} + I_2^{\bullet} + I_3^{\bullet} = 0 \Rightarrow I_1^{\bullet} = I_2^{\bullet} + I_3^{\bullet} = 0,001 + 0,003 = 4mA$$

$$U_{\bullet} = U_{10}^{\bullet} + R_1 I_1^{\bullet} = 13 + 3000 \cdot 0,004 = 25V$$

$$k = \frac{U}{U_{\bullet}} = \frac{10}{25} = 0,4$$

$$I_1 = kI_{\bullet 1} = 0,4 \cdot 0,004 = 1,6mA$$

$$I_2 = kI_{\bullet 2} = 0,4 \cdot 0,001 = 0,4mA$$

$$I_3 = kI_{\bullet 3} = 0,4 \cdot 0,003 = 1,2mA$$

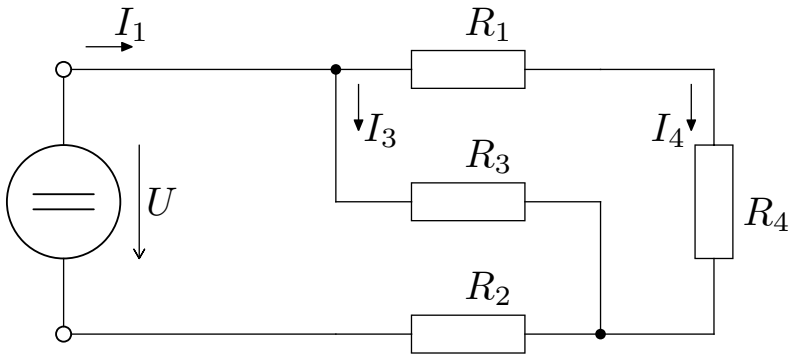
$$I_4 = kI_{\bullet 4} = 0,4 \cdot 0,002 = 0,8mA$$

$$I_5 = kI_{\bullet 5} = 0,4 \cdot 0,001 = 0,4mA$$

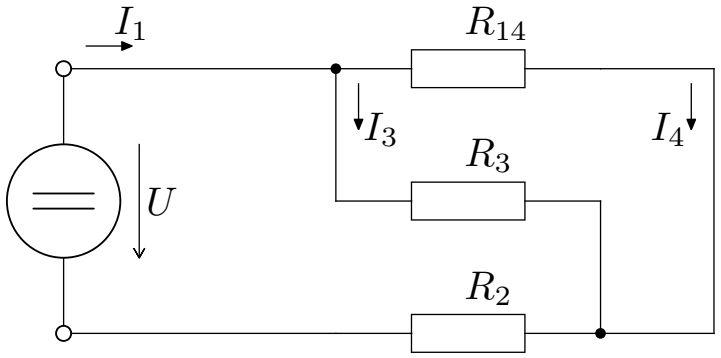
Př.2.3: Určete proudy obvodu: $U = 10V, R_1 = R_3 = 3k\Omega, R_2 = 13k\Omega, R_4 = 2k\Omega$

a) metodou zjednodušování

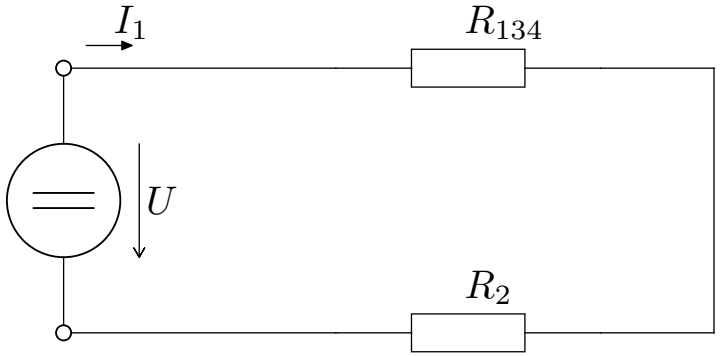
b) metodou úměrných veličin.



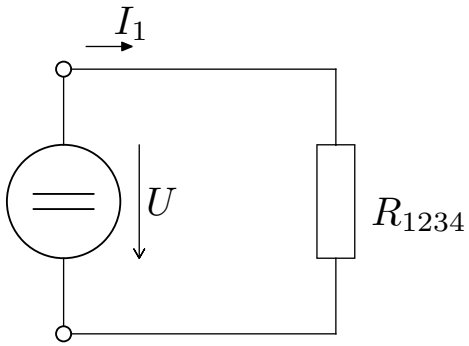
a) metoda zjednodušování



$$R_{14} = R_1 + R_4 = 3000 + 2000 = 5k\Omega$$

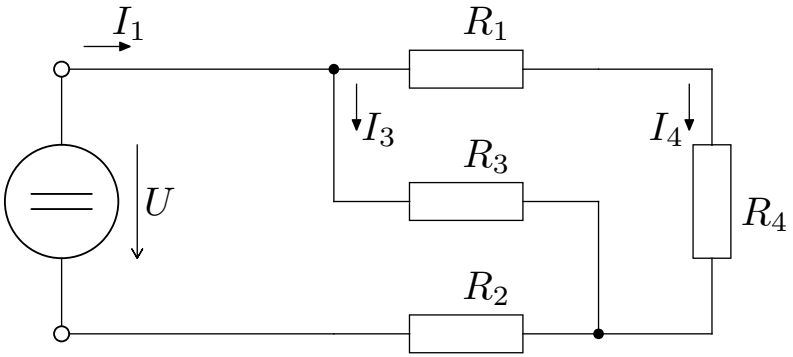


$$R_{134} = \frac{R_3 \cdot R_{14}}{R_3 + R_{14}} = \frac{3000 \cdot 5000}{3000 + 5000} = 1,875k\Omega$$



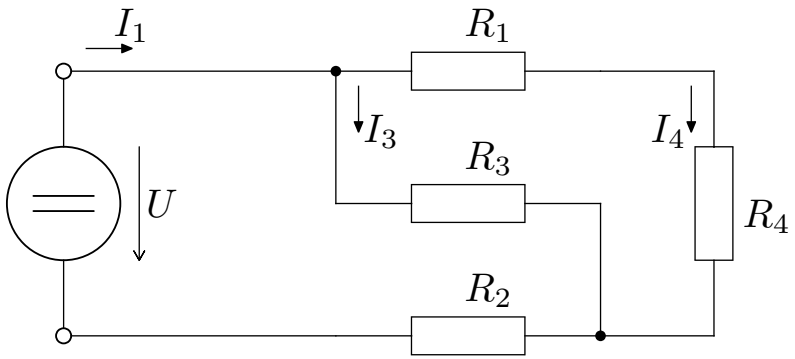
$$R_{1234} = R_{134} + R_2 = 1875 + 13000 = 14,875k\Omega$$

$$I_1 = \frac{U}{R_{1234}} = \frac{10}{14875} = 0,6723mA$$



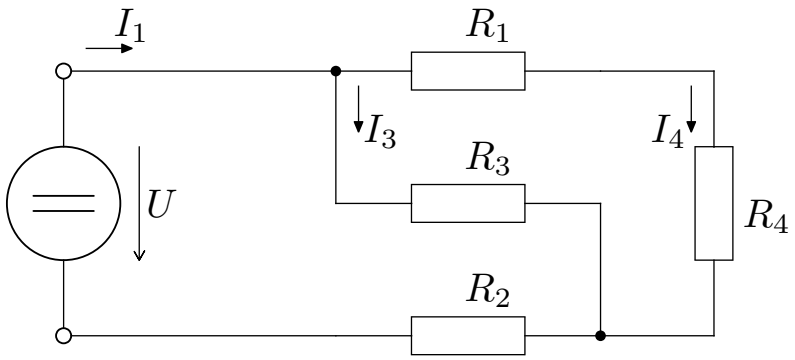
$$U_{R3} = U - R_2 I_1 = 10 - 13000 \cdot 0,6723 \cdot 10^{-3} = 1,26V$$

$$I_3 = \frac{U_{R3}}{R_3} = \frac{1,26}{3000} = 0,42mA$$



$$I_1 - I_3 - I_4 = 0 \Rightarrow I_4 = I_1 - I_3 = 0,6723 \cdot 10^{-3} - 0,42 \cdot 10^{-3} = 0,252mA$$

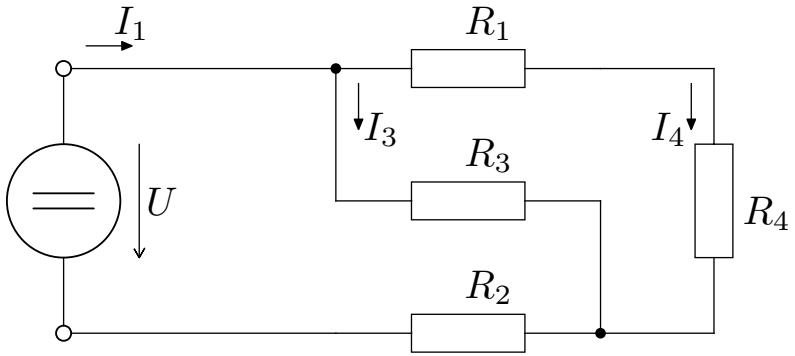
b) metoda úměrných veličin



Volíme $I_4^{\bullet} = 1mA$ [3]

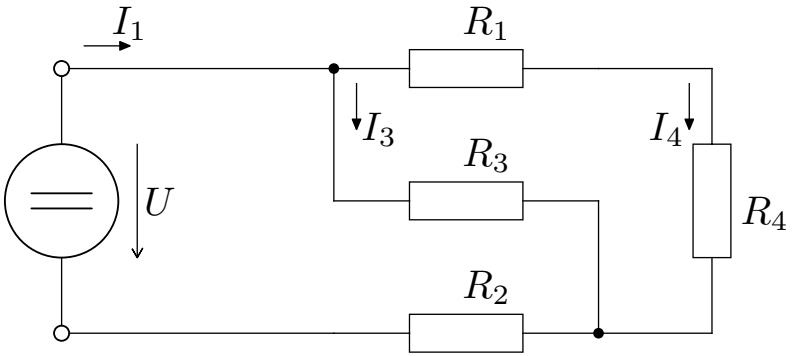
$$U_{R3}^{\bullet} = (R_1 + R_4)I_4^{\bullet} = (3000 + 2000) \cdot 0,001 = 5V$$

[3] Jestliže provedeme typ tohoto proudu s velkou chybou, není to na závadu, koeficienty budou úměrně větší.



$$\overset{\bullet}{I}_3 = \frac{\overset{\bullet}{U}_{R3}}{R_3} = \frac{5}{3000} = 1,67mA$$

$$\overset{\bullet}{I}_1 - \overset{\bullet}{I}_3 - \overset{\bullet}{I}_4 = 0 \Rightarrow \overset{\bullet}{I}_1 = \overset{\bullet}{I}_3 + \overset{\bullet}{I}_4 = 0,00167 + 0,001 = 2,67mA$$



$$U_{\bullet} = U_{\underset{R3}{\bullet}} + R_2 I_{\underset{1}{\bullet}} = 5 + 13000 \cdot 0,00267 = 39,71V$$

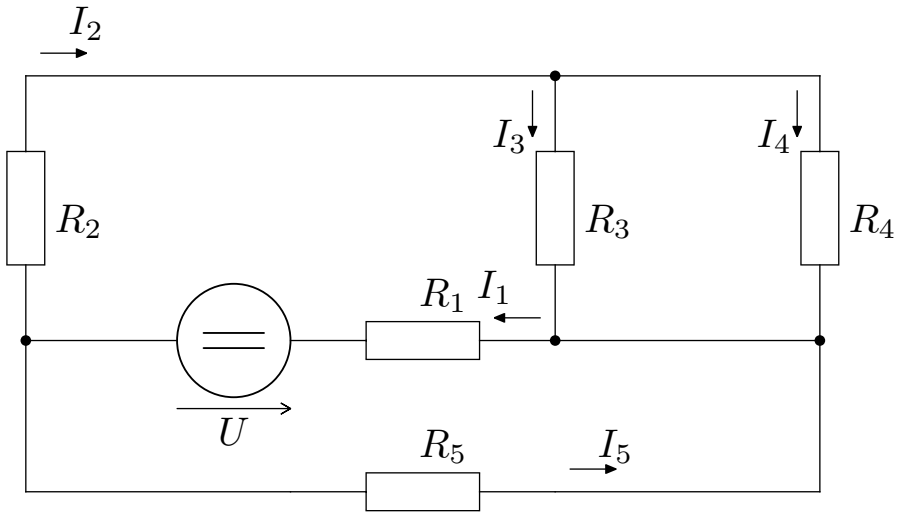
$$k = \frac{U}{U_{\bullet}} = \frac{10}{39,71} \doteq 0,25$$

$$I_1 = kI_{\bullet_1} = 0,25 \cdot 0,00266 = 0,6723mA$$

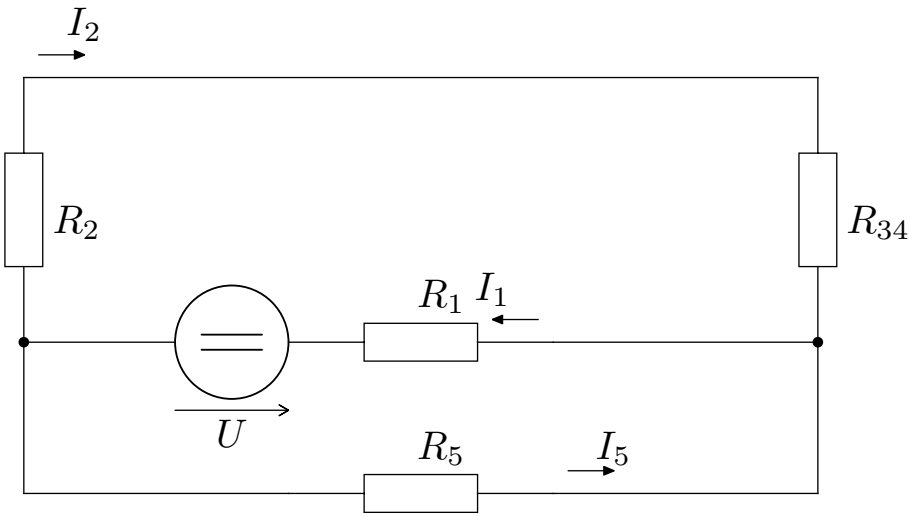
$$I_3 = kI_{\bullet_3} = 0,25 \cdot 0,00166 = 0,42mA$$

$$I_4 = kI_{\bullet_4} = 0,25 \cdot 0,001 = 0,252mA$$

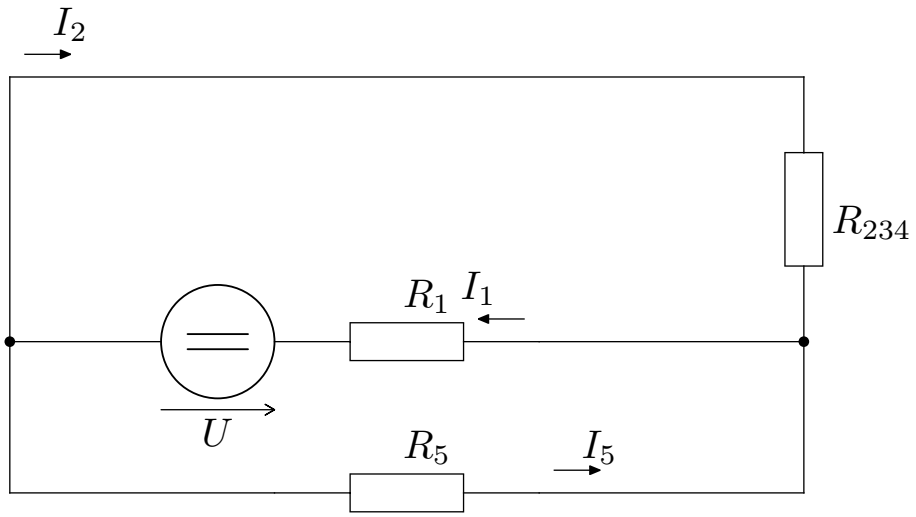
Př.2.4: Metodou zjednodušování a úměrných veličin určete všechny proudy obvodu. $U = 48V, R_1 = 2\Omega, R_2 = 30\Omega, R_3 = 40\Omega, R_4 = 10\Omega, R_5 = 20\Omega$



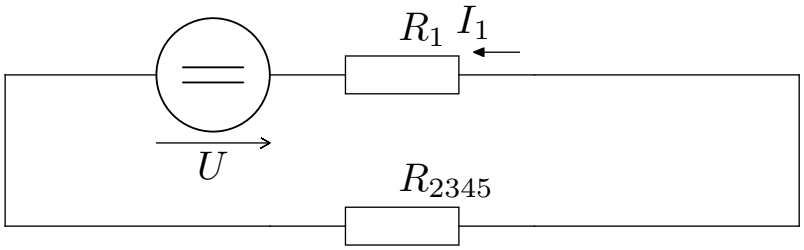
a) metoda zjednodušování



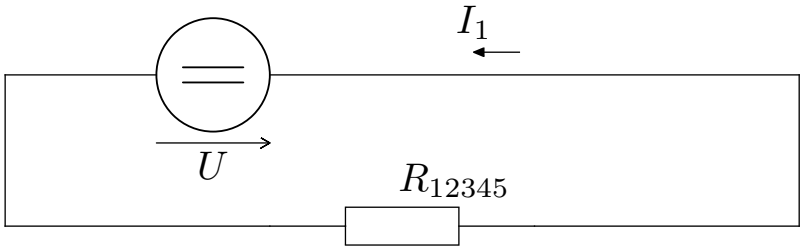
$$R_{34} = \frac{R_3 \cdot R_4}{R_3 + R_4} = \frac{40 \cdot 10}{40 + 10} = 8\Omega$$



$$R_{234} = R_2 + R_{34} = 30 + 8 = 38\Omega$$

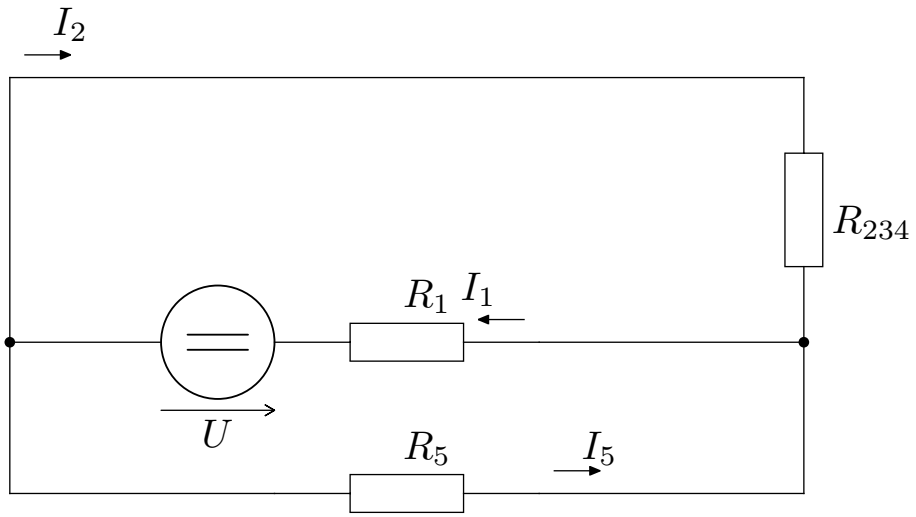


$$R_{2345} = \frac{R_{234} \cdot R_5}{R_{234} + R_5} = \frac{38 \cdot 20}{38 + 20} = 13,1\Omega$$



$$R_{12345} = R_1 + R_{2345} = 2 + 13,1 = 15,1\Omega$$

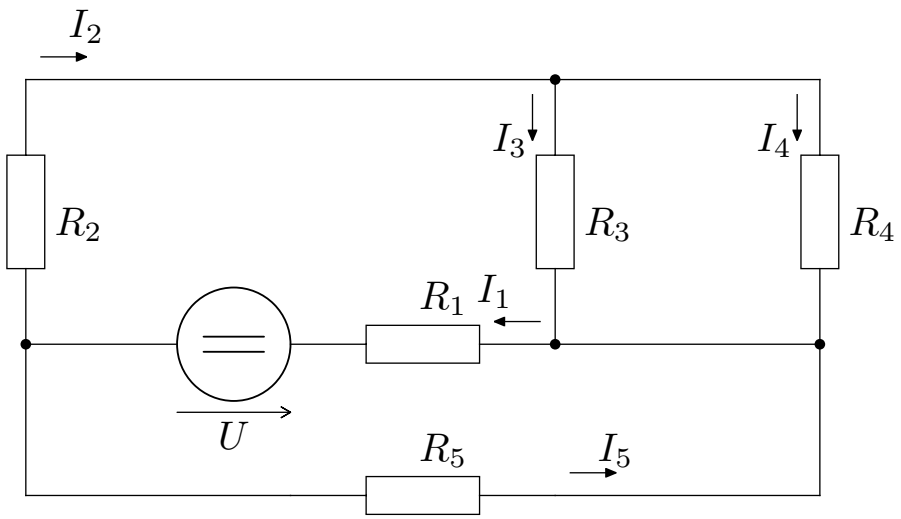
$$I_1 = \frac{U}{R_{12345}} = \frac{48}{15,1} = 3,178A$$



$$U_{R5} = U - R_1 I_1 = 48 - 2 \cdot 3,178 = 41,64V$$

$$I_5 = \frac{U_{R5}}{R_5} = \frac{41,46}{20} = 2,082A$$

$$I_1 - I_2 - I_5 = 0 \Rightarrow I_2 = I_1 - I_5 = 3,178 - 2,082 = 1,096A$$

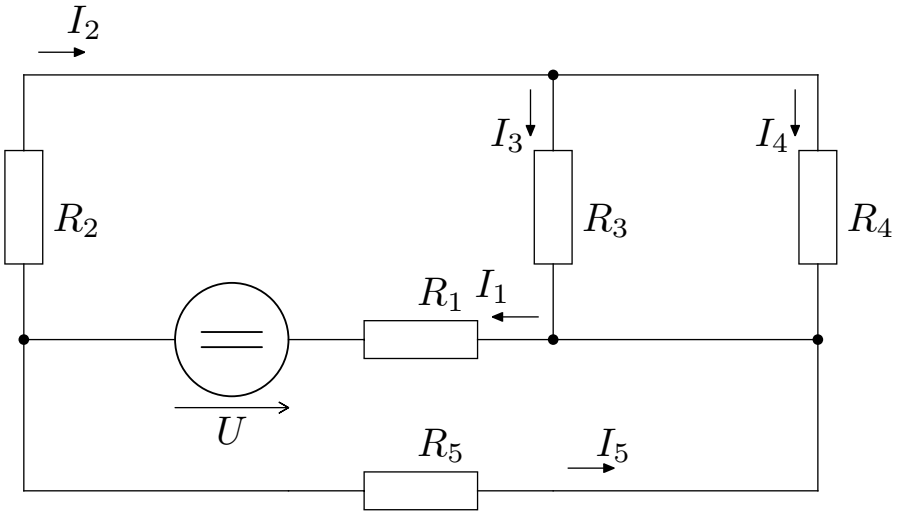


$$U_{R3} = U - R_1 I_1 - R_2 I_2 = 48 - 2 \cdot 3,178 - 30 \cdot 1,096 = 8,764V$$

$$I_3 = \frac{U_{R3}}{R_3} = \frac{8,764}{40} = 0,219A$$

$$I_4 = \frac{U_{R3}}{R_4} = \frac{8,764}{10} = 0,877A$$

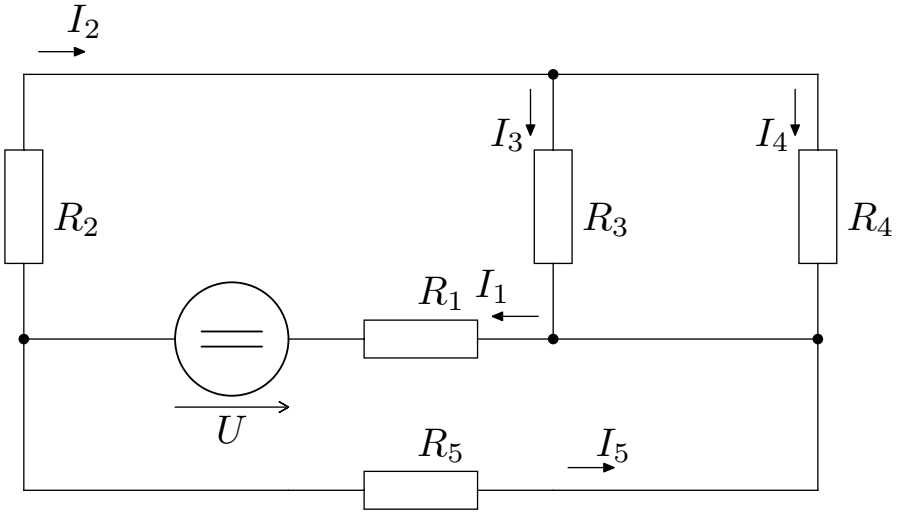
b) metoda úměrných veličin



Volíme $I_4^{\bullet} = 1A$

$$U_{R4}^{\bullet} = R_4 I_4 = 10 \cdot 1 = 10V$$

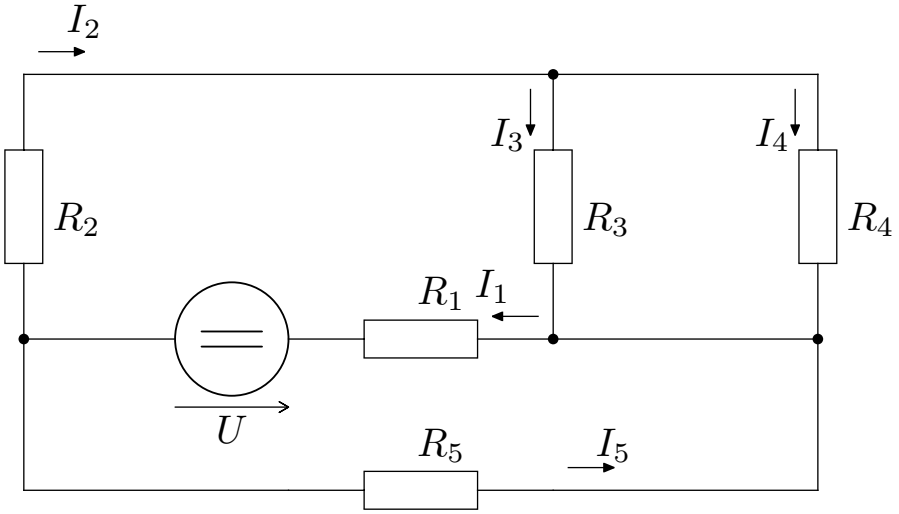
$$I_3^{\bullet} = \frac{U_{R4}^{\bullet}}{R_4} = \frac{10}{40} = 0,25A$$



$$I_{\bullet 2} - I_{\bullet 3} - I_{\bullet 4} = 0 \Rightarrow I_{\bullet 2} = I_{\bullet 3} + I_{\bullet 4} = 0,25 + 1 = 1,25A$$

$$U_{\bullet R5} = U_{\bullet R4} + R_2 I_{\bullet 2} = 10 + 30 \cdot 1,25 = 47,5V$$

$$I_{\bullet 5} = \frac{U_{\bullet R5}}{R_5} = \frac{47,5}{20} = 2,375A$$



$$I_1^\bullet - I_2^\bullet - I_5^\bullet = 0 \Rightarrow I_1^\bullet = I_2^\bullet + I_5^\bullet = 1,25 + 2,375 = 3,625A$$

$$U^\bullet = R_1 I_1^\bullet + R_5 I_5^\bullet = 2 \cdot 3,625 + 20 \cdot 2,375 = 54,75V$$

$$k = \frac{U}{U^\bullet} = \frac{10}{54,75} = 0,8767$$

$$I_1 = kI_{\overset{\bullet}{1}} = 0,8767 \cdot 3,625 = 3,178A$$

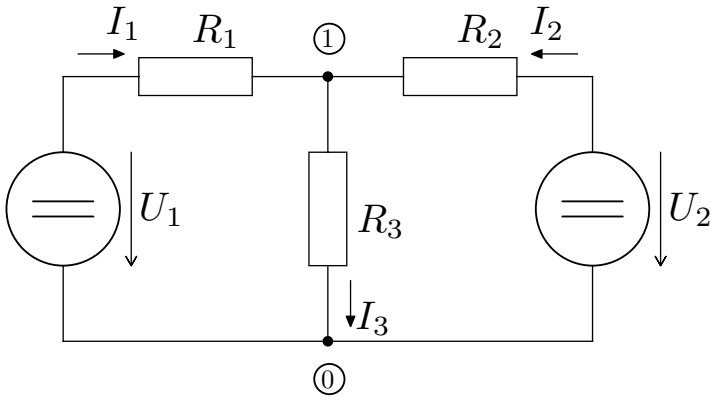
$$I_2 = kI_{\overset{\bullet}{2}} = 0,8767 \cdot 1,25 = 1,096A$$

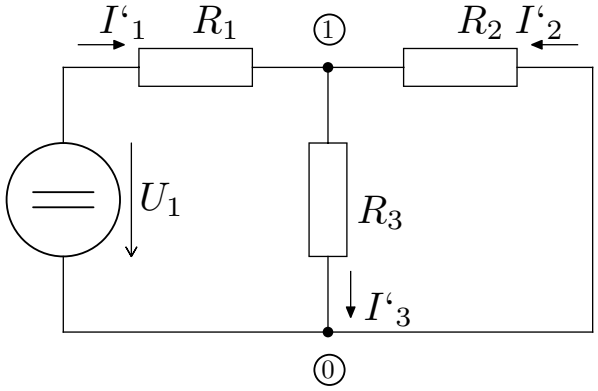
$$I_3 = kI_{\overset{\bullet}{3}} = 0,8767 \cdot 0,25 = 0,219A$$

$$I_4 = kI_{\overset{\bullet}{4}} = 0,8767 \cdot 1 = 0,8767A$$

$$I_5 = kI_{\overset{\bullet}{5}} = 0,8767 \cdot 2,375 = 2,082A$$

Př.2.5: Proudy řešte superpozicí. $U_1 = 6V, U_2 = 18V, R_1 = 10\Omega, R_2 = 20\Omega, R_3 = 15\Omega$





$$R_{23} = \frac{R_2 \cdot R_3}{R_2 + R_3} = \frac{20 \cdot 15}{20 + 15} = 8,571\Omega$$

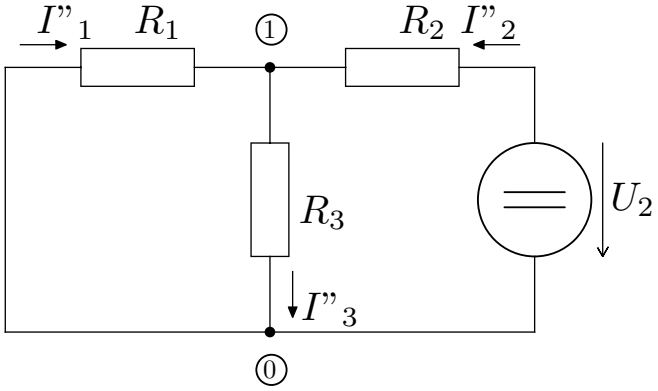
$$R_{123} = R_1 + R_{23} = 10 + 8,571 = 18,571\Omega$$

$$I'_1 = \frac{U_1}{R_{123}} = \frac{6}{18,571} = 0,3231A$$

$$U'_{01} = U_1 - R_1 I'_1 = 6 - 10 \cdot 0,3231 = 2,769V$$

$$I'_3 = \frac{U'_{10}}{R_3} = \frac{2,769}{15} = 0,1846A$$

$$I'_2 = -\frac{U'_{10}}{R_2} = -\frac{2,769}{20} = -0,1385A$$



$$R_{13} = \frac{R_1 \cdot R_3}{R_1 + R_3} = \frac{10 \cdot 15}{10 + 15} = 6\Omega$$

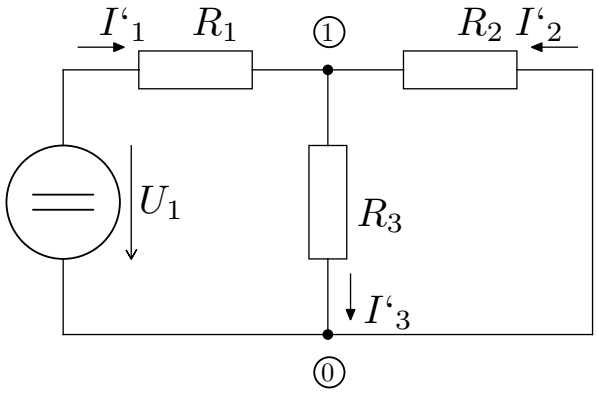
$$R_{123} = R_2 + R_{13} = 20 + 6 = 26\Omega$$

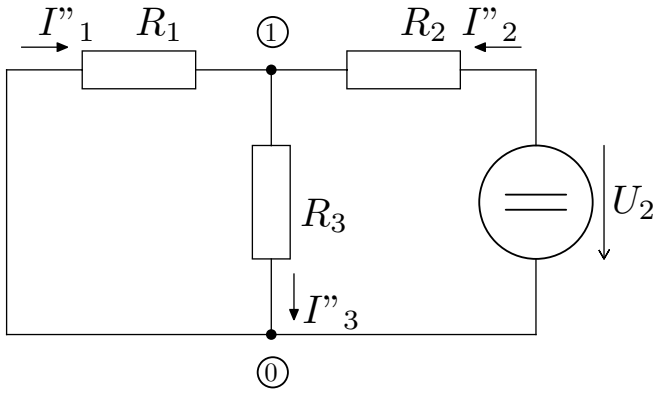
$$I\text{“}_2 = \frac{U_2}{R_{123}} = \frac{18}{26} = 0,6923A$$

$$U\text{“}_{01} = U_2 - R_2 I\text{“}_2 = 18 - 20 \cdot 0,6923 = 4,1538V$$

$$I\text{“}_3 = \frac{U\text{“}_{10}}{R_3} = \frac{4,1538}{15} = 0,2769A$$

$$I\text{“}_1 = -\frac{U\text{“}_{10}}{R_1} = -\frac{4,1538}{10} = -0,4153A$$





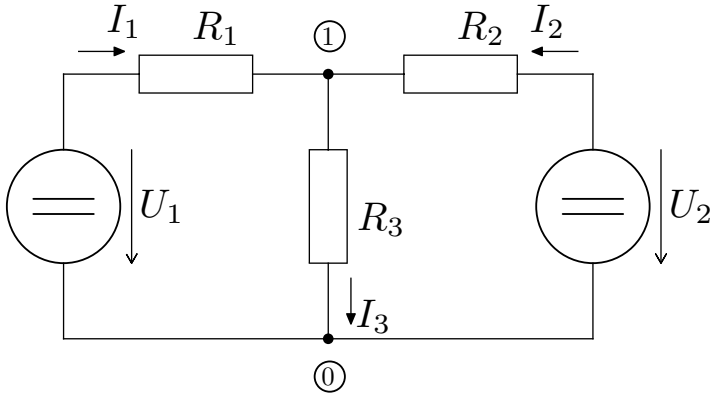
$$I_1 = I‘_1 + I“_1 = 0,3231 - 0,4153 = -0,0923A$$

$$I_2 = I‘_2 + I“_2 = -0,1385 + 0,6923 = 0,5538A$$

$$I_3 = I‘_3 + I“_3 = 0,1846 + 0,2769 = 0,4615A$$

1.3 Univerzální metody

Př.3.1: Obvod řešte aplikací Kirchhoffových zákonů. $U_1 = 6V, U_2 = 18V, R_1 = 10\Omega$, $R_2 = 20\Omega, R_3 = 15\Omega$



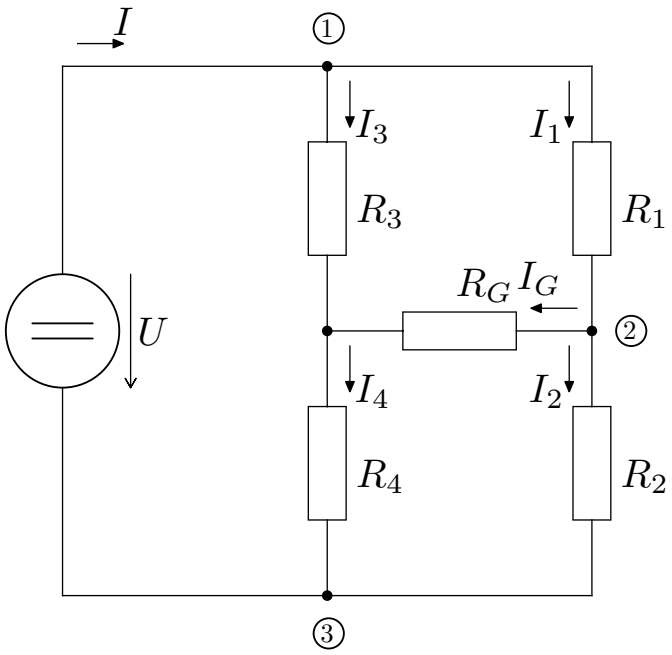
$$-I_1 - I_2 + I_3 = 0$$
$$R_1 I_1 + R_3 I_3 - U_1 = 0$$
$$R_2 I_2 + R_3 I_3 - U_2 = 0$$

$$\begin{pmatrix} -1 & -1 & 1 \\ R_1 & 0 & R_3 \\ 0 & R_2 & R_3 \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ U_1 \\ U_2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 & 1 \\ 10 & 0 & 15 \\ 0 & 20 & 15 \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 18 \end{pmatrix}$$

$$I_1 = -0,09231A$$
$$I_2 = 0,5538A$$
$$I_3 = 0,4615A$$

Př.3.2: Obvod popište pomocí K.z.



Nezávislé smyčky s = 3,
nezávislé uzly n = 3.

$$I.K.z.: \quad \begin{aligned} I - I_1 - I_3 &= 0 \\ I_1 - I_2 - I_G &= 0 \\ -I + I_2 + I_4 &= 0 \end{aligned}$$

$$II.K.z.: \quad \begin{aligned} R_1 I_1 + R_G I_G - R_3 I_3 &= 0 \\ R_2 I_2 - R_4 I_4 - R_G I_G &= 0 \\ R_3 I_3 + R_4 I_4 - U &= 0 \end{aligned}$$

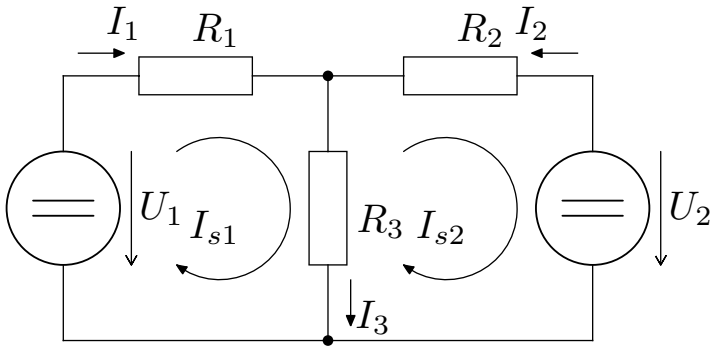
[4]

$$\begin{pmatrix} -1 & 0 & -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ R_1 & 0 & -R_3 & 0 & R_G & 0 \\ 0 & R_2 & 0 & -R_4 & -R_G & 0 \\ 0 & 0 & R_3 & R_4 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_G \\ I \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ U \end{pmatrix}$$

[4]Popis obvodu podle K.z. vede zřejmě na velké množství rovnic, proto často používáme MSP nebo MUN.

Př.3.3: Řešte metodou smyčkových proudů. $U_1 = 6V, U_2 = 18V, R_1 = 10\Omega$, $R_2 = 20\Omega, R_3 = 15\Omega$

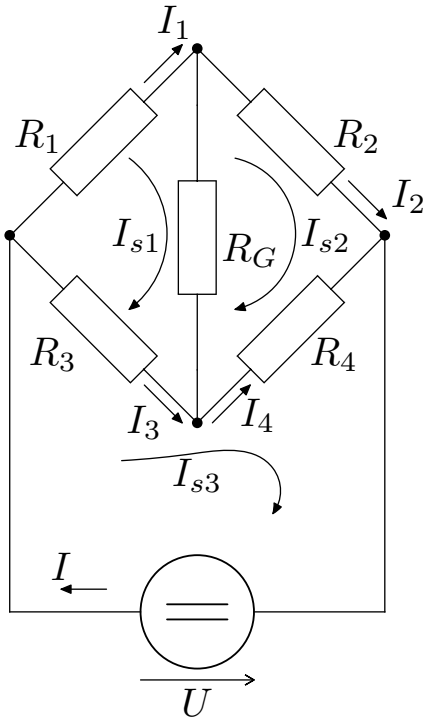
II.K.z.[5]:



$$R_1 I_{s1} + R_3(I_{s1} - I_{s2}) - U_1 = 0$$
$$R_2 I_{s2} + R_3(I_{s2} - I_{s1}) + U_2 = 0$$
$$\begin{pmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \end{pmatrix} = \begin{pmatrix} U_1 \\ -U_2 \end{pmatrix}$$
$$\begin{pmatrix} 25 & -15 \\ -15 & 35 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \end{pmatrix} = \begin{pmatrix} 6 \\ -18 \end{pmatrix}$$
$$I_{s1} = -0,09231A \quad I_{s2} = -0,5538A$$
$$I_1 = I_{s1} = -0,09231A$$
$$I_2 = -I_{s2} = 0,5538A$$
$$I_3 = I_{s1} - I_{s2} = -0,09231 + 0,5538 = 0,4615A$$

[5]Soustavu rovnic pro MSP lze zapsat přímo v maticovém tvaru: $\underline{R} \cdot \underline{I_s} = \underline{U}$. Prvky hlavní diagonály odporové matice jsou dány součtem rezistorů příslušné smyčky. Při souhlasném smyslu smyčkových proudů jsou pak ostatní prvky určené záporně vzatou hodnotou rezistorů společných větví.

Př.3.4: MSP určete proudy obvodu. $U = 2V, R_1 = R_3 = 20\Omega, R_2 = 40\Omega, R_4 = 10\Omega, R_G = 25\Omega$



$$\begin{pmatrix} R_1+R_G+R_3 & -R_G & -R_3 \\ -R_G & R_2+R_4+R_G & -R_4 \\ -R_3 & -R_4 & R_3+R_4 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ U \end{pmatrix}$$

$$\begin{pmatrix} 65 & -25 & -20 \\ -25 & 75 & -10 \\ -20 & -10 & 30 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

$$I_{s1} = 0,04321A \quad I_{s2} = 0,0284A \quad I_{s3} = 0,1049A$$

$$I_1 = I_{s1} = 0,04321A$$

$$I_2 = I_{s2} = 0,0284A$$

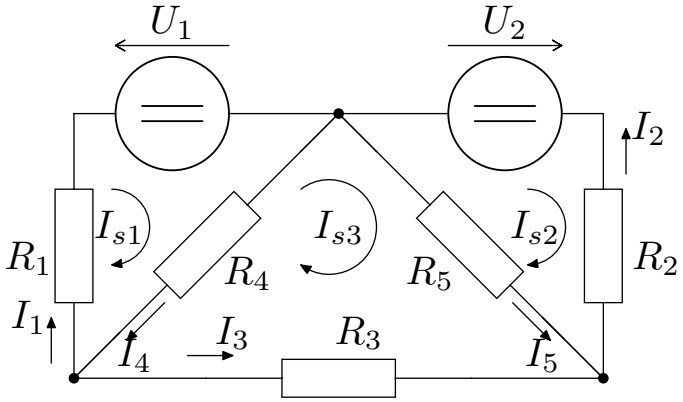
$$I_3 = I_{s3} - I_{s1} = 0,1049 - 0,04321 = 0,06169A$$

$$I_4 = I_{s3} - I_{s2} = 0,1049 - 0,0284 = 0,0765A$$

$$I_G = I_{s1} - I_{s2} = 0,04321 - 0,0284 = 0,01481A$$

$$I = I_{s3} = 0,1049A$$

Př.3.5: MSP určete proudy obvodu. $U_1 = 5V, U_2 = 7V, R_1 = 7,5\Omega, R_2 = 2,5\Omega,$ $R_3 = 5\Omega, R_4 = 2\Omega, R_5 = 25\Omega$



$$\begin{pmatrix} R_1+R_4 & 0 & -R_4 \\ 0 & R_2+R_5 & -R_5 \\ -R_4 & -R_5 & R_3+R_4+R_5 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} U_1 \\ -U_2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 9,5 & 0 & -2 \\ 0 & 27,5 & -25 \\ -2 & -25 & 32 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} 5 \\ -7 \\ 0 \end{pmatrix}$$

$$I_{s1} = 0,4A \qquad I_{s2} = -0,8A \qquad I_{s3} = -0,6A$$

$$I_1 = I_{s1} = 0,4A$$

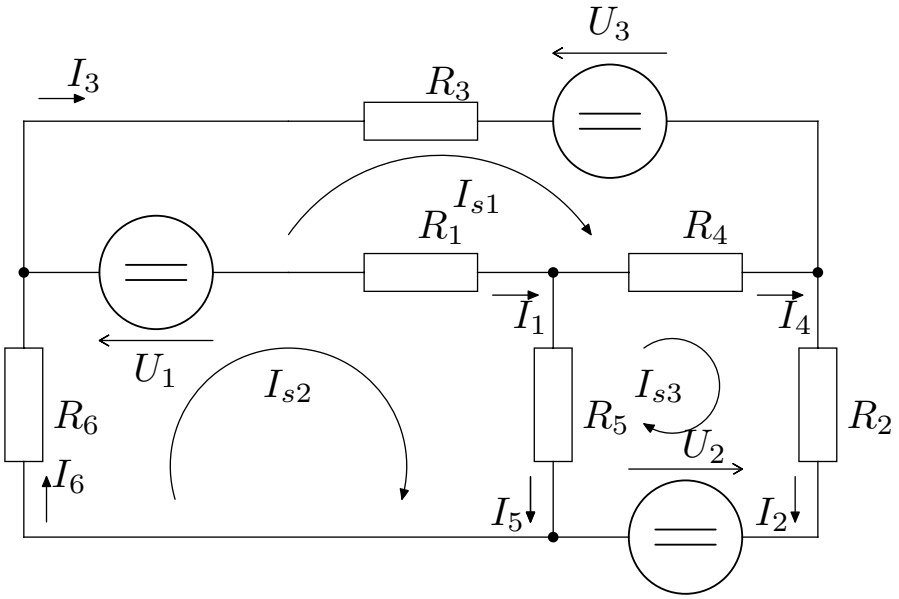
$$I_2 = -I_{s3} = 0,6A$$

$$I_3 = -I_{s2} = 0,8A$$

$$I_4 = I_{s1} - I_{s3} = 0,4 + 0,6 = 1A$$

$$I_5 = I_{s3} - I_{s2} = -0,6 + 0,8 = 0,2A$$

Př.3.6: MSP určete proudy obvodu. $U_1 = 110V, U_2 = 15V, U_3 = 90V, R_1 = 500\Omega$, $R_2 = 300\Omega, R_3 = 500\Omega, R_4 = 1000\Omega, R_5 = 200\Omega, R_6 = 700\Omega$



$$\begin{pmatrix} R_1+R_3+R_4 & -R_1 & -R_4 \\ -R_1 & R_1+R_5+R_6 & -R_5 \\ -R_4 & -R_5 & R_2+R_4+R_5 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} U_3-U_1 \\ U_1 \\ U_2 \end{pmatrix}$$

$$\begin{pmatrix} 2000 & -500 & -1000 \\ -500 & 1400 & -200 \\ -1000 & -200 & 1500 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} -20 \\ 110 \\ 15 \end{pmatrix}$$

$$I_{s1} = 0,04A \qquad I_{s2} = 0,1A \qquad I_{s3} = 0,05A$$

$$I_1 = I_{s2} + I_{s1} = 0,1 - 0,04 = 0,06A$$

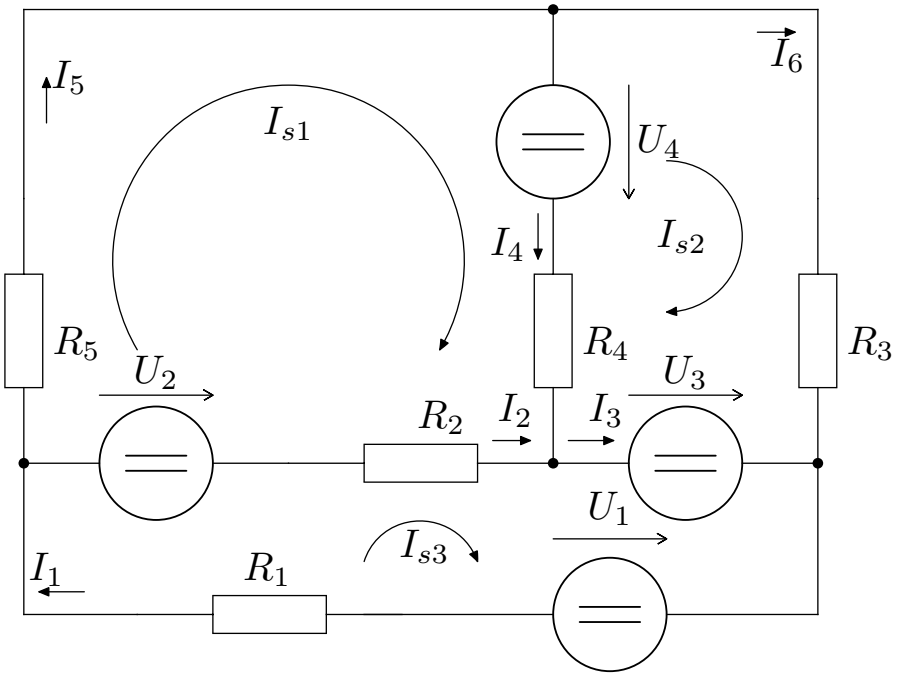
$$I_2 = I_{s3} = 0,05A \qquad I_3 = I_{s1} = 0,04A$$

$$I_4 = I_{s3} - I_{s1} = 0,05 - 0,04 = 0,01A$$

$$I_5 = I_{s2} - I_{s3} = 0,1 - 0,05 = 0,05A$$

$$I_6 = I_{s2} = 0,1A$$

Př.3.7: MSP určete proudy obvodu. $U_1 = 100V, U_2 = 30V, U_3 = 10V, U_4 = 6V$, $R_1 = 10\Omega, R_2 = 10\Omega, R_3 = 15\Omega, R_4 = 6\Omega, R_5 = 5\Omega$



$$\begin{pmatrix} R_2+R_4+R_5 & -R_4 & -R_2 \\ -R_4 & R_3+R_4 & 0 \\ -R_2 & 0 & R_1+R_2 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} U_2-U_4 \\ U_3-U_4 \\ U_1-U_2-U_3 \end{pmatrix}$$

$$\begin{pmatrix} 21 & -6 & -10 \\ -6 & 21 & 0 \\ -10 & 0 & 20 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} 24 \\ 16 \\ 60 \end{pmatrix}$$

$$I_{s1} = 4,1A \qquad I_{s2} = 1,933A \qquad I_{s3} = 5,05A$$

$$I_1 = I_{s3} = 5,05A$$

$$I_2 = I_{s3} - I_{s1} = 0,95A$$

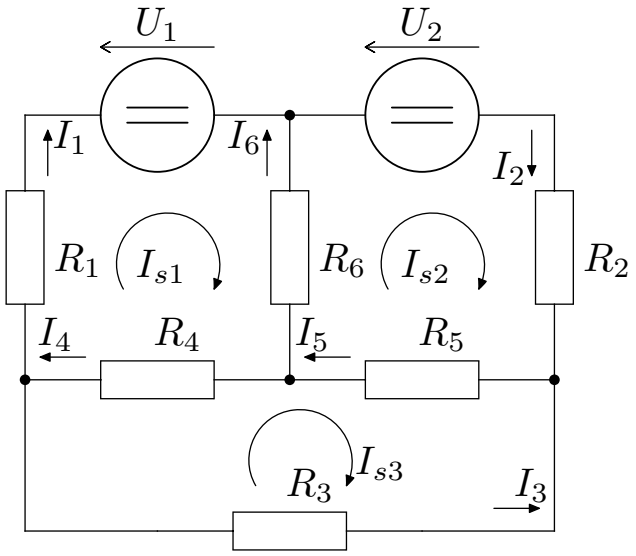
$$I_3 = I_{s3} - I_{s2} = 3,1167A$$

$$I_4 = I_{s1} - I_{s2} = 2,167A$$

$$I_5 = I_{s2} = 4,1A$$

$$I_6 = I_{s2} = 1,933A$$

Př.3.8: MSP určete proudy obvodu a také výkony dodávané zdroji a spotřebované rezistory. $U_1 = 8V, U_2 = 8V, R_1 = 22\Omega, R_2 = 5\Omega, R_3 = 16\Omega, R_4 = 15\Omega, R_5 = 9\Omega,$ $R_6 = 14\Omega$



$$\begin{pmatrix} R_1+R_4+R_6 & -R_6 & -R_4 \\ -R_6 & R_2+R_5+R_6 & -R_5 \\ -R_4 & -R_5 & R_3+R_4+R_5 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} U_1 \\ U_2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 46 & -14 & -15 \\ -14 & 28 & -9 \\ -15 & -9 & 40 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \\ 0 \end{pmatrix}$$

$$I_{s1} = 3,9556A \qquad I_{s2} = 0,57259A \qquad I_{s3} = 0,27716A$$

$$\begin{aligned} I_1 &= I_{s3} = 0,39556A \\ I_2 &= I_{s2} = 0,57259A \\ I_3 &= -I_{s3} = -0,27716A \\ I_4 &= I_{s1} - I_{s3} = 0,11839A \\ I_5 &= I_{s2} - I_{s3} = 0,2954A \\ I_6 &= I_{s2} - I_{s1} = 0,177A \end{aligned}$$

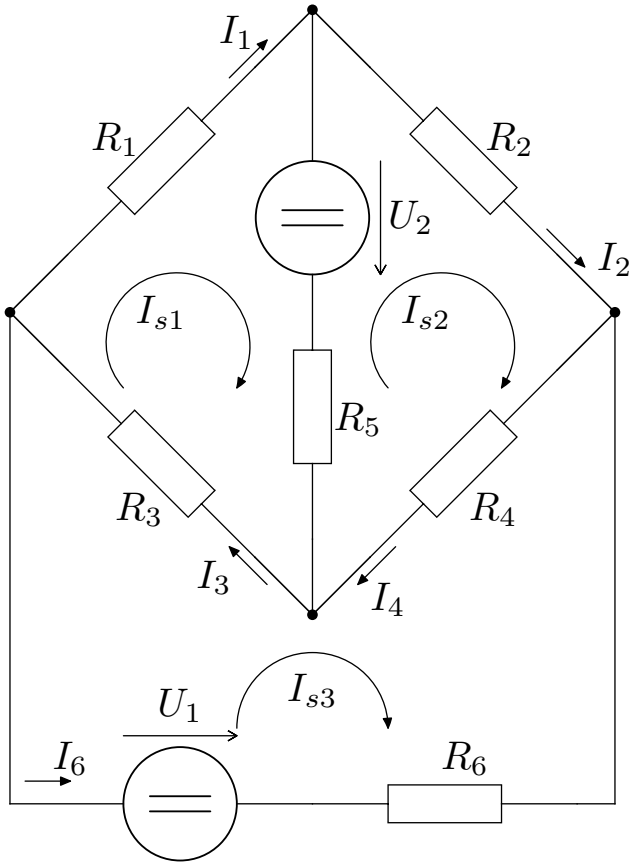
$$P_{R1} = R_1 I_1^2 = 3,443W$$
$$P_{R2} = R_2 I_2^2 = 1,639W$$
$$P_{R3} = R_3 I_3^2 = 1,229W$$
$$P_{R4} = R_4 I_4^2 = 0,2103W$$
$$P_{R5} = R_5 I_5^2 = 0,7854W$$
$$P_{R6} = R_6 I_6^2 = 0,4386W$$

$$P = \sum_{i=1}^{6} P_{Ri} = 7,745W$$

$$P_1 = U_1 I_1 = 3,165VA$$
$$P_2 = U_2 I_2 = 4.581VA$$

$$P = \sum_{i=1}^{2} P_i = 7,745W$$

Př.3.9: MSP určete proudy obvodu. $U_1 = 35V, U_2 = 11V, R_1 = 5k\Omega, R_2 = 3k\Omega,$ $R_3 = 2k\Omega, R_4 = 5k\Omega, R_5 = 1k\Omega, R_6 = 0,5k\Omega$



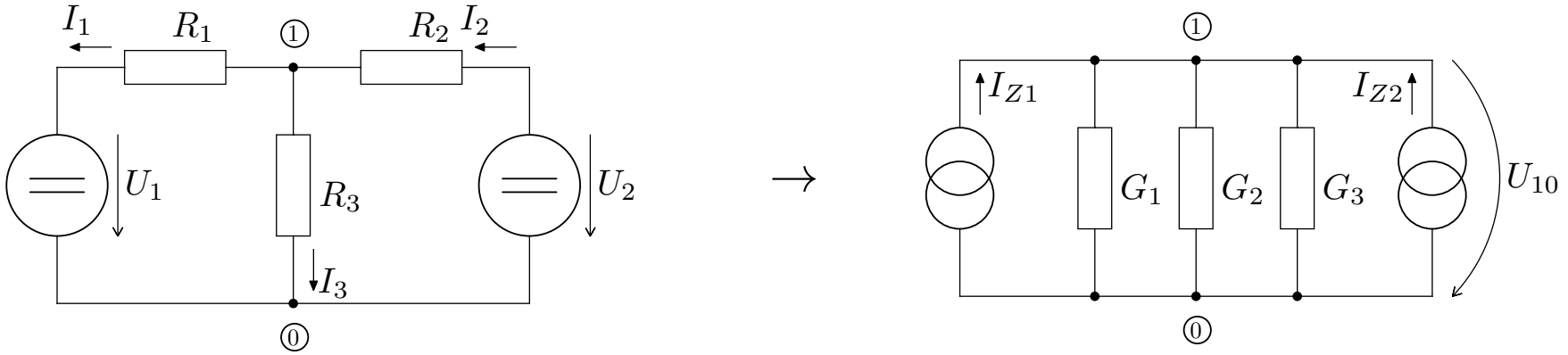
$$\begin{pmatrix} R_1+R_3+R_5 & -R_5 & -R_3 \\ -R_5 & R_2+R_4+R_5 & -R_4 \\ -R_3 & -R_4 & R_3+R_4+R_6 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} -U_2 \\ U_2 \\ U_1 \end{pmatrix}$$

$$\begin{pmatrix} 8000 & -1000 & -2000 \\ -1000 & 9000 & -5000 \\ -2000 & -5000 & 7500 \end{pmatrix} \cdot \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \begin{pmatrix} -11 \\ 11 \\ 35 \end{pmatrix}$$

$$I_{s1} = 1,917mA \qquad I_{s2} = 6,848mA \qquad I_{s3} = 9,743mA$$

$$\begin{aligned} I_1 &= I_{s1} = 1,917mA \\ I_2 &= I_{s2} = 6,848mA \\ I_3 &= I_{s1} - I_{s3} = -7,826mA \\ I_4 &= I_{s2} - I_{s3} = -2,895mA \\ I_5 &= I_{s2} - I_{s1} = 4,93132116mA \\ I_6 &= I_{s3} = 9,743mA \end{aligned}$$

Př.4.1: Obvod řešte MUN. $U_1 = 6V, U_2 = 18V, R_1 = 10\Omega, R_2 = 20\Omega, R_3 = 15\Omega$



$$G_1 = 0,1S; \quad G_2 = 0,05S; \quad G_3 = 0,06667S$$

$$I_{Z1} = \frac{U_1}{R_1} = \frac{6}{10} = 0,6A \qquad I_{Z2} = \frac{U_2}{R_2} = \frac{18}{20} = 0,6A$$

$$I.K.z.: \qquad -I_{Z1} - I_{Z2} + G_1U_{10} + G_2U_{10} + G_3U_{10} = 0$$

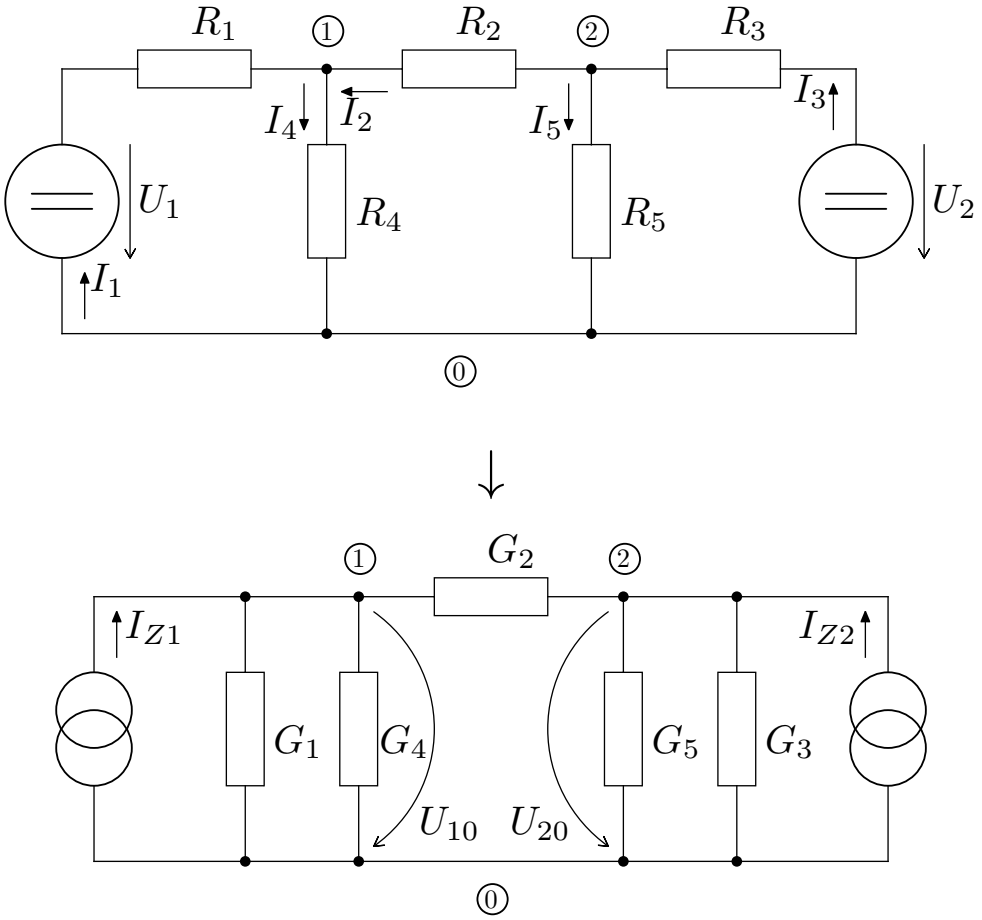
$$U_{10} = \frac{I_{Z1} + I_{Z2}}{G_1 + G_2 + G_3} = \frac{0,6 + 0,9}{0,1 + 0,05 + 0,06667} = 6,923V$$

$$I_1 = I_{Z1} - G_1 U_{10} = 0,6 - 0,1 \cdot 6,923 = -0,0923A$$

$$I_2 = I_{Z2} - G_2 U_{10} = 0,9 - 0,05 \cdot 6,923 = 0,5539A$$

$$I_3 = G_3 U_{10} = 0,06667 \cdot 6,923 = 0,4616A$$

Př.4.2: Určete proudy větví obvodu z obrázku pomocí MUN. $U_1 = 5V, U_2 = 10V$, $R_1 = 2k\Omega, R_2 = 2k\Omega, R_3 = 5k\Omega, R_4 = 3k\Omega, R_5 = 1k\Omega$



$$I_{Z1} = \frac{U_1}{R_1} = \frac{5}{2000} = 2,5mA$$
$$I_{Z2} = \frac{U_2}{R_2} = \frac{10}{2000} = 0,6A$$
$$-I_{Z1} + (G_1 + G_4)U_{10} + G_2(U_{10} - U_{20}) = 0$$
$$-I_{Z2} + (G_3 + G_5)U_{20} + G_2(U_{20} - U_{10}) = 0$$
$$(G_1 + G_4 + G_2)U_{10} - G_2U_{20} = I_{Z1}$$
$$-G_2U_{10} + (G_3 + G_5 + G_2)U_{20} = I_{Z2}$$

$$\begin{pmatrix} G_1 + G_2 + G_4 & -G_2 \\ -G_2 & G_2 + G_3 + G_5 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} I_{Z1} \\ I_{Z2} \end{pmatrix}$$
$$\begin{pmatrix} 1,\overline{3} \cdot 10^{-3} & -5 \cdot 10^{-4} \\ -5 \cdot 10^{-4} & 1,7 \cdot 10^{-3} \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} 2,5 \cdot 10^{-3} \\ 2 \cdot 10^{-3} \end{pmatrix}$$

$$U_{10} = 2,6033V \qquad U_{20} = 1,9421V$$

$$I_1 = G_1(U_1 - U_{10}) = 5 \cdot 10^{-4} \cdot (5 - 2,6033) = 1,1983mA$$

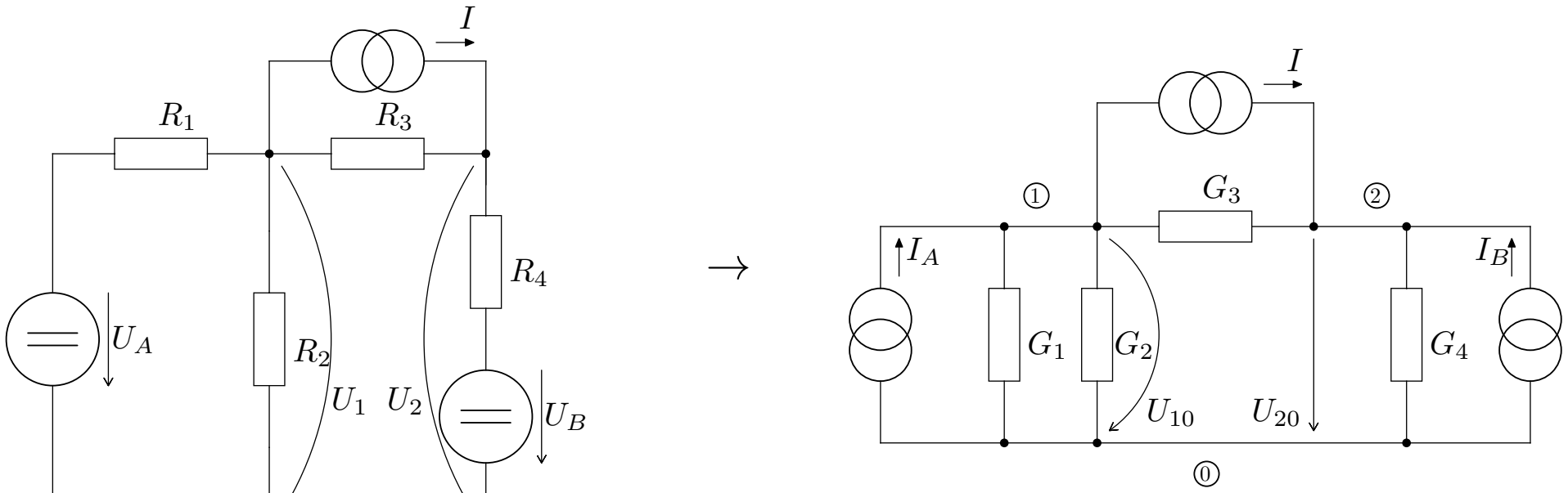
$$I_2 = G_2(U_{20} - U_{10}) = 5 \cdot 10^{-4} \cdot (1,9421 - 2,6033) = -3,306mA$$

$$I_3 = G_3(U_2 - U_{20}) = 2 \cdot 10^{-4} \cdot (10 - 1,9421) = 1,6116mA$$

$$I_4 = G_4U_{10} = 0,\overline{3} \cdot 10^{-3} \cdot 2,6033 = 0,8678mA$$

$$I_5 = G_5U_{20} = 10^{-3} \cdot 1,9421 = 1,9421mA$$

Př.4.3: Určete napětí U_1 a U_2 pomocí MUN. $U_A = 20V, U_B = 15V, I = 1,5A$, $R_1 = R_2 = 20\Omega, R_3 = R_4 = 40\Omega$



$$I_A = \frac{U_A}{R_1} = \frac{20}{20} = 1A \qquad I_B = \frac{U_B}{R_4} = \frac{15}{40} = 0,375A$$

Rovnice je možné psát přímo v maticovém tvaru[6]: $\underline{G} \cdot \underline{U} = \underline{I}$.

[6]Na hlavní diagonále matice jsou vlastní vodivosti uzlů, tedy suma vodivostí k uzlu připojených. Mimo hlavní diagonálu je záporně vzatý součet vodivostí větví spojujících příslušné uzly.

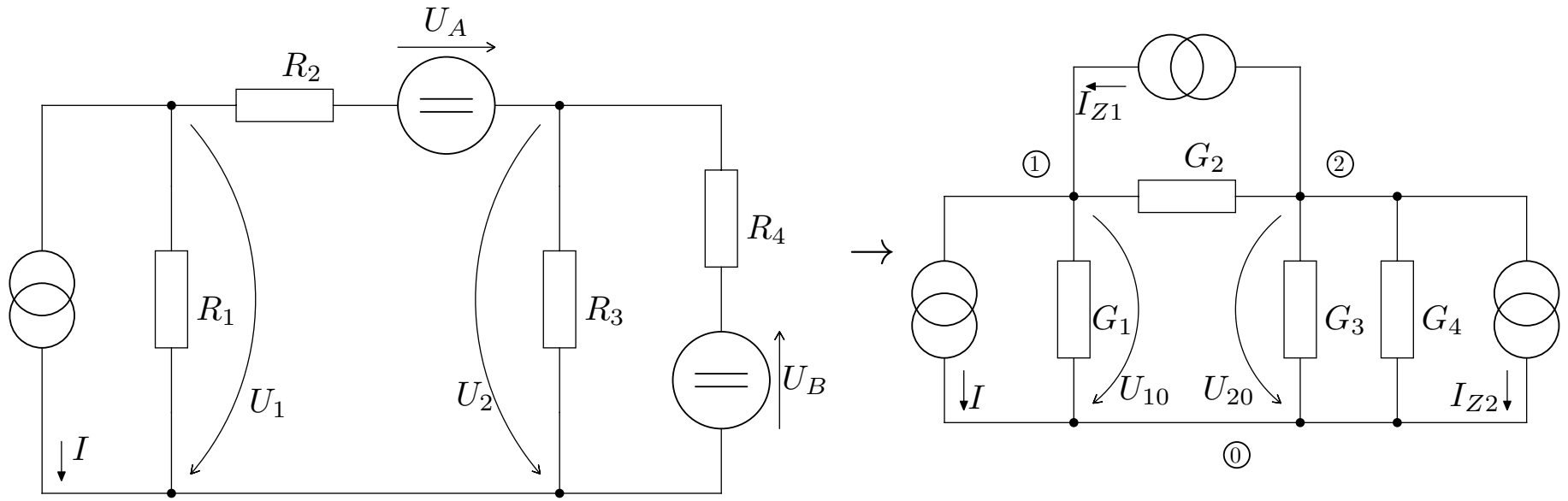
$$\begin{pmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3 & G_3+G_4 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} -I_A - I \\ I + I_B \end{pmatrix}$$

$$\begin{pmatrix} 0,125 & -0,025 \\ -0,025 & 0,05 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} -2,5 \\ 1,875 \end{pmatrix}$$

$$U_{10} = -13,\overline{8}V$$

$$U_{20} = 30,\overline{5}V$$

Př.4.4: Určete napětí U_1 a U_2 pomocí MUN. $U_A = 20V, U_B = 15V, I = 1,5A$, $R_1 = R_2 = 20\Omega, R_3 = R_4 = 40\Omega$



$$I_{Z1} = \frac{U_A}{R_2} = \frac{20}{20} = 1A$$

$$I_{Z2} = \frac{U_B}{R_4} = \frac{15}{40} = 0,375A$$

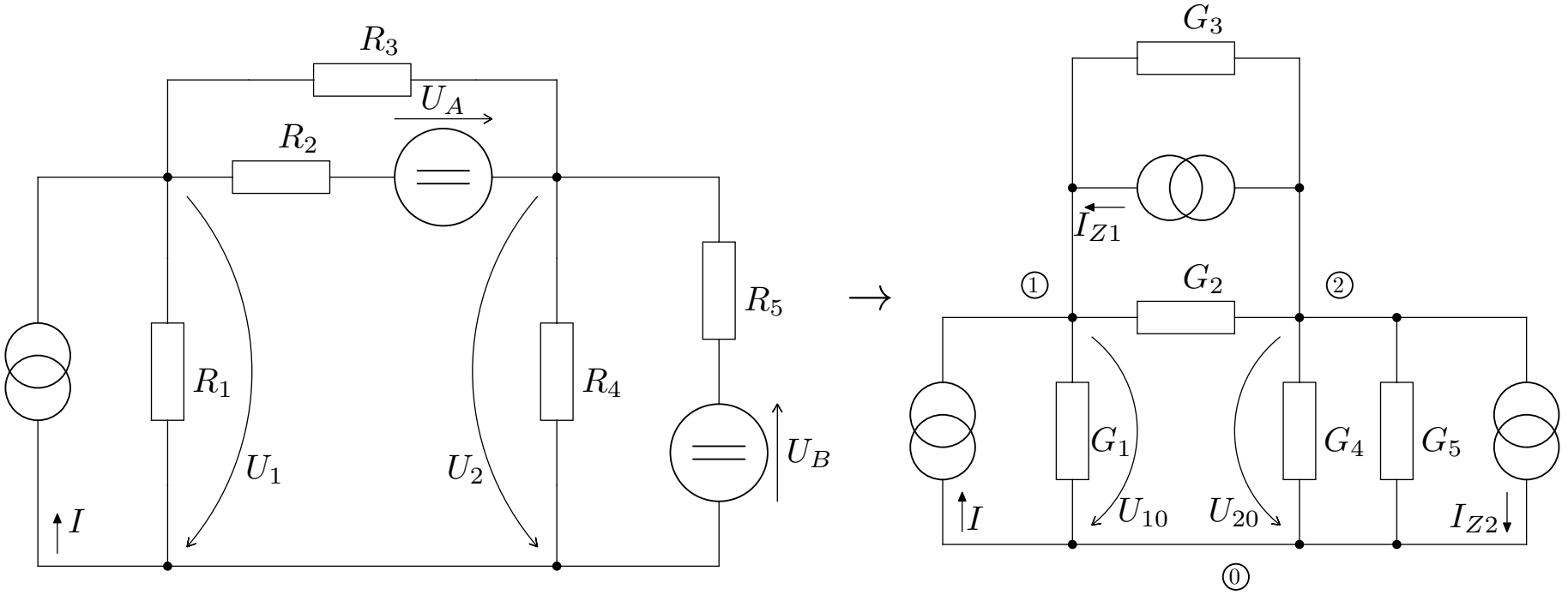
$$\begin{pmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 + G_4 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} I_{Z1} - I \\ -I_{Z1} - I_{Z2} \end{pmatrix}$$

$$\begin{pmatrix} 0,1 & -0,05 \\ -0,05 & 0,1 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} -0,5 \\ -1,375 \end{pmatrix}$$

$$U_{10} = -15,8\overline{3}V$$

$$U_{20} = -21,\overline{6}V$$

Př.4.5: Určete napětí U_1 a U_2 pomocí MUN. $U_A = 5V, U_B = 10V, I = 2mA$, $R_1 = R_2 = 2,2k\Omega, R_3 = 5,6k\Omega, R_4 = 3,3k\Omega, R_5 = 1k\Omega$



$$I_{Z1} = \frac{U_A}{R_2} = \frac{5}{2200} = 2,\overline{27}mA$$

$$I_{Z2} = \frac{U_B}{R_5} = \frac{10}{1000} = 10mA$$

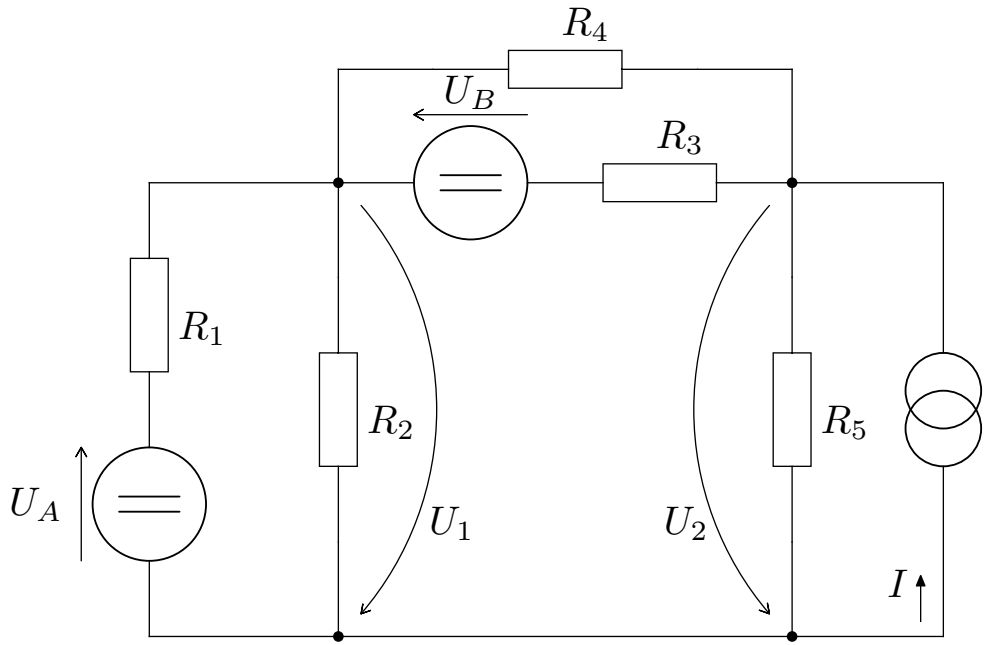
$$\begin{pmatrix} G_1+G_2+G_3 & -G_2-G_3 \\ -G_2-G_3 & G_2+G_3+G_4+G_5 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} I+I_{Z1} \\ -I_{Z1}-I_{Z2} \end{pmatrix}$$

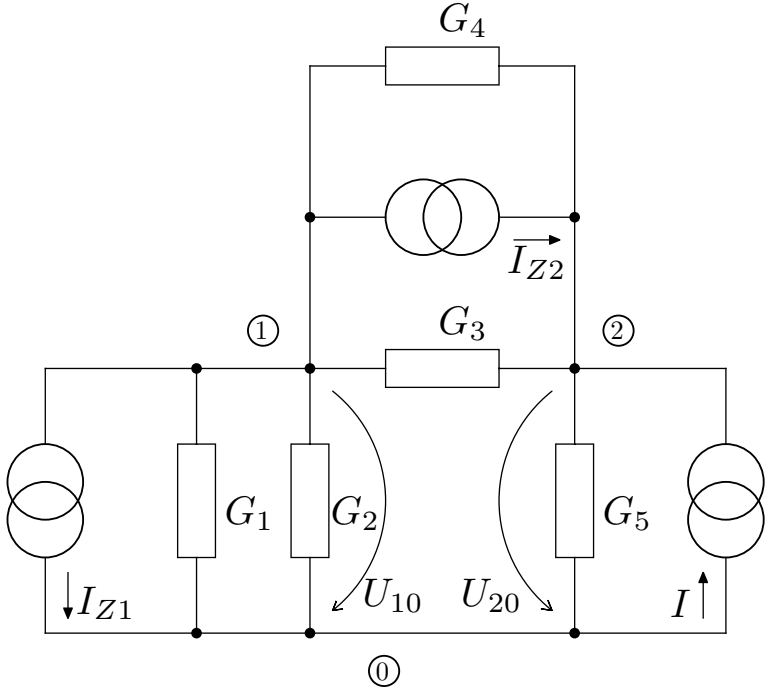
$$\begin{pmatrix} 1,0877 \cdot 10^{-3} & -6,3312 \cdot 10^{-3} \\ -6,3312 \cdot 10^{-3} & 1,9361 \cdot 10^{-3} \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} 4,\overline{27} \cdot 10^{-3} \\ -0,012\overline{27} \end{pmatrix}$$

$$U_{10} = 0,2947V$$

$$U_{20} = -6,2423V$$

Př.4.6: Určete napětí U_1 a U_2 pomocí MUN. $U_A = 15V, U_B = 10V, I = 2,5mA$, $R_1 = R_2 = 2,2k\Omega, R_3 = 5,6k\Omega, R_4 = 2,7k\Omega, R_5 = 1k\Omega$





$$I_{Z1} = \frac{U_A}{R_2} = \frac{15}{2200} = 6,\overline{81}mA$$

$$I_{Z2} = \frac{U_B}{R_5} = \frac{10}{1000} = 10mA$$

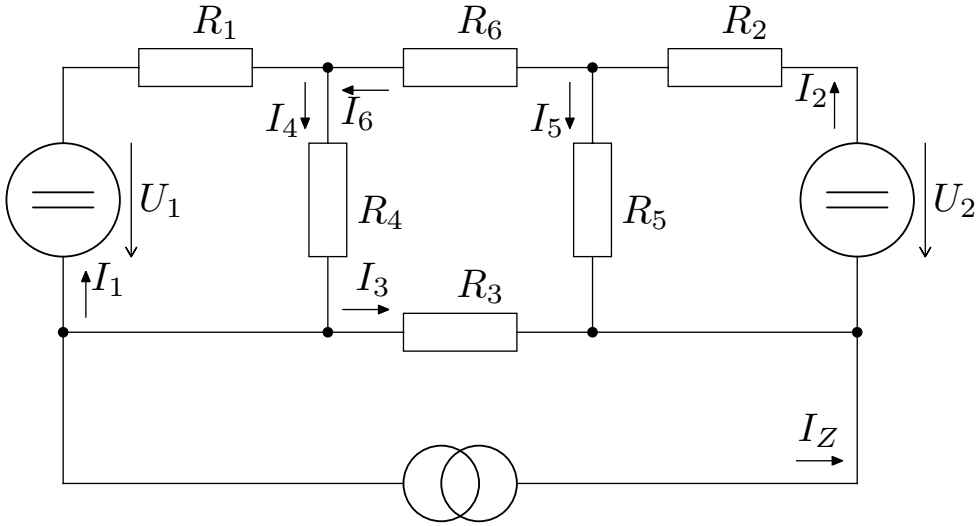
$$\begin{pmatrix} G_1+G_2+G_3+G_4 & -G_3-G_4 \\ -G_3-G_4 & G_3+G_4+G_5 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} -I_{Z1}-I_{Z2} \\ I+I_{Z2} \end{pmatrix}$$

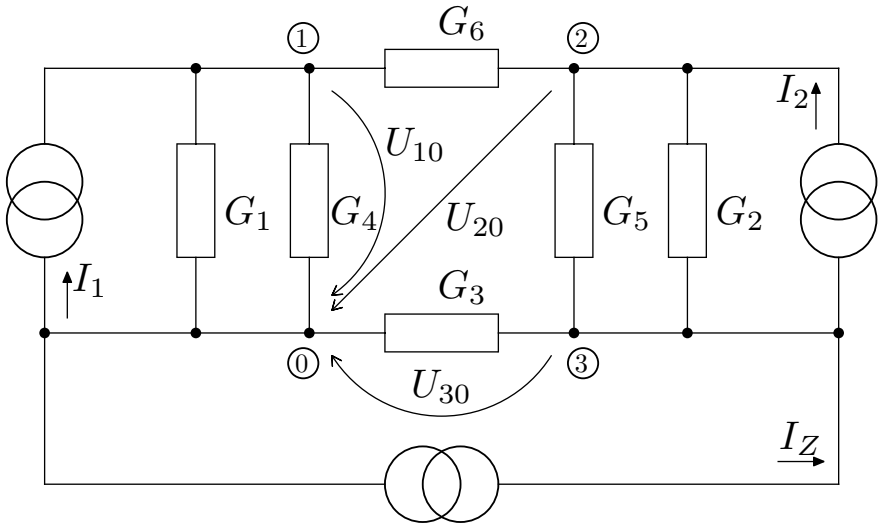
$$\begin{pmatrix} 1,4580 \cdot 10^{-3} & -5,4894 \cdot 10^{-3} \\ -5,4894 \cdot 10^{-3} & 1,5489 \cdot 10^{-3} \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \end{pmatrix} = \begin{pmatrix} -8,6039 \cdot 10^{-3} \\ -4,2857 \cdot 10^{-3} \end{pmatrix}$$

$$U_{10} = -5,5057V$$

$$U_{20} = -0,7796V$$

Př.4.7: Proudy obvodu určete pomocí MUN. $U_1 = 12V, U_2 = 16V, I_Z = 3mA$, $R_1 = 1k\Omega, R_2 = 2k\Omega, R_3 = 1k\Omega, R_4 = 5k\Omega, R_5 = 4k\Omega, R_6 = 2k\Omega$





$$I_{Z1} = \frac{U_1}{R_1} = \frac{12}{1000} = 12mA$$
$$I_{Z2} = \frac{U_2}{R_2} = \frac{16}{2000} = 8mA$$

$$\begin{pmatrix} G_1 + G_4 + G_6 & -G_6 & 0 \\ -G_6 & G_2 + G_5 + G_6 & -G_2 - G_5 \\ 0 & -G_2 - G_5 & G_2 + G_3 + G_5 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ U_{30} \end{pmatrix} = \begin{pmatrix} I_1 \\ I_2 \\ I_Z + I_2 \end{pmatrix}$$

$$\begin{pmatrix} 1,7 \cdot 10^{-3} & -5 \cdot 10^{-4} & 0 \\ -5 \cdot 10^{-4} & 1,25 \cdot 10^{-3} & -7,5 \cdot 10^{-4} \\ 0 & -7,5 \cdot 10^{-4} & 1,75 \cdot 10^{-3} \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ U_{30} \end{pmatrix} = \begin{pmatrix} 12 \cdot 10^{-3} \\ 8 \cdot 10^{-3} \\ -5 \cdot 10^{-3} \end{pmatrix}$$

$$U_{10} = 7,6953V$$
$$U_{20} = 2,1641V$$
$$U_{20} = -1,9297V$$

$$I_4 = \frac{U_{10}}{R_4} = \frac{7,6953}{5000} = 1,5391mA$$

$$I_5 = \frac{U_{20}}{R_5} = \frac{2,1641}{4000} = 5,4102mA$$

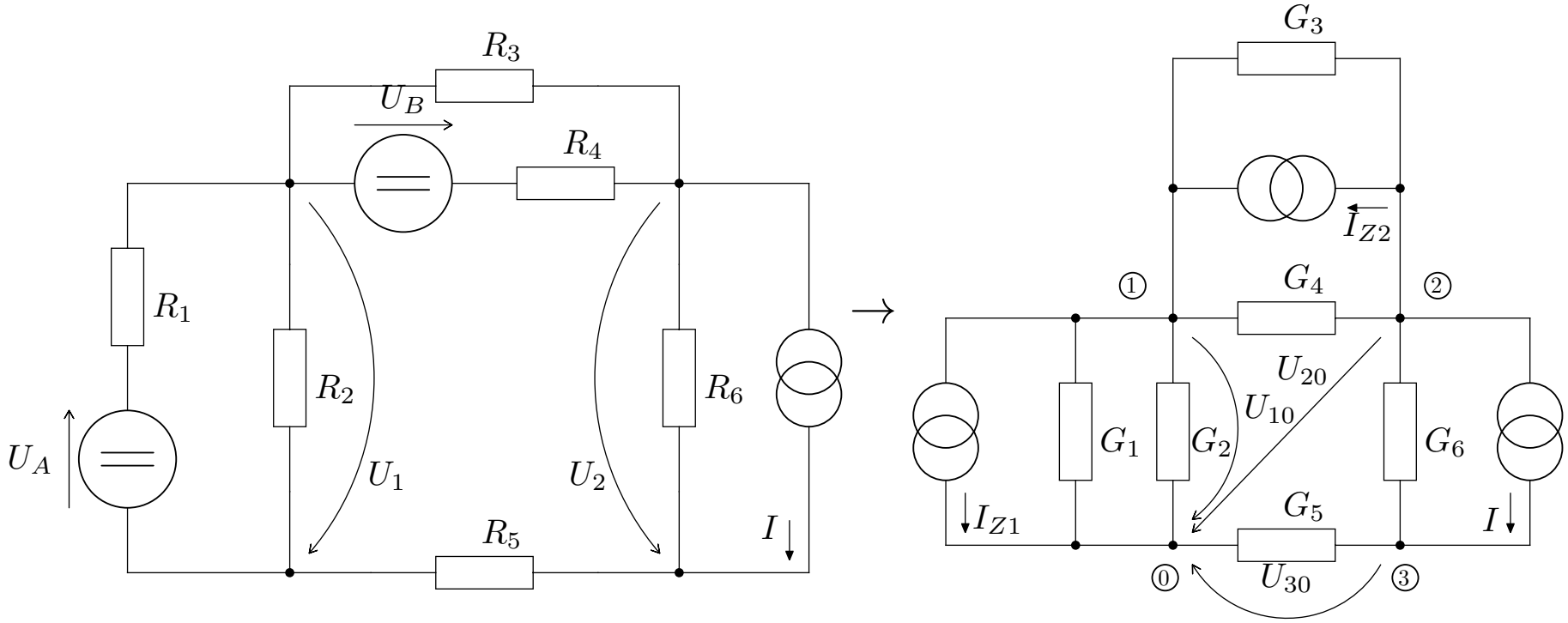
$$I_3 = \frac{U_{30}}{R_3} = \frac{-1,9297}{1000} = -1,9297mA$$

$$\begin{aligned} I_2 &= I_5 + I_3 + I_Z = \\ &= 5,4102 \cdot 10^{-3} - 1,9297 \cdot 10^{-3} + 3 \cdot 10^{-3} = 6,4805mA \end{aligned}$$

$$\begin{aligned} I_1 &= I_4 - I_3 - I_Z = \\ &= 1,5391 \cdot 10^{-3} + 1,9297 \cdot 10^{-3} - 3 \cdot 10^{-3} = 0,4688mA \end{aligned}$$

$$I_6 = I_2 - I_5 = 6,4805 \cdot 10^{-3} - 5,4102 \cdot 10^{-3} = 1,0703mA$$

Př.4.8: Určete napětí U_1 a U_2 pomocí MUN. $U_A = 15V, U_B = 10V, I = 2,5mA$, $R_1 = R_2 = 2,2k\Omega, R_3 = 5,6k\Omega, R_4 = 2,7k\Omega, R_5 = 1k\Omega, R_6 = 4,7k\Omega$



$$I_{Z1} = \frac{U_A}{R_1} = \frac{5}{2200} = 2,\overline{27}mA$$

$$I_{Z2} = \frac{U_B}{R_4} = \frac{10}{2700} = 3,\overline{703}mA$$

$$\begin{pmatrix} G_1+G_2+G_3+G_4 & -G_3-G_4 & 0 \\ -G_3-G_4 & G_3+G_4+G_6 & -G_6 \\ 0 & -G_6 & G_5+G_6 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ U_{30} \end{pmatrix} = \begin{pmatrix} I_{Z2}-I_{Z1} \\ -I-I_{Z2} \\ I \end{pmatrix}$$

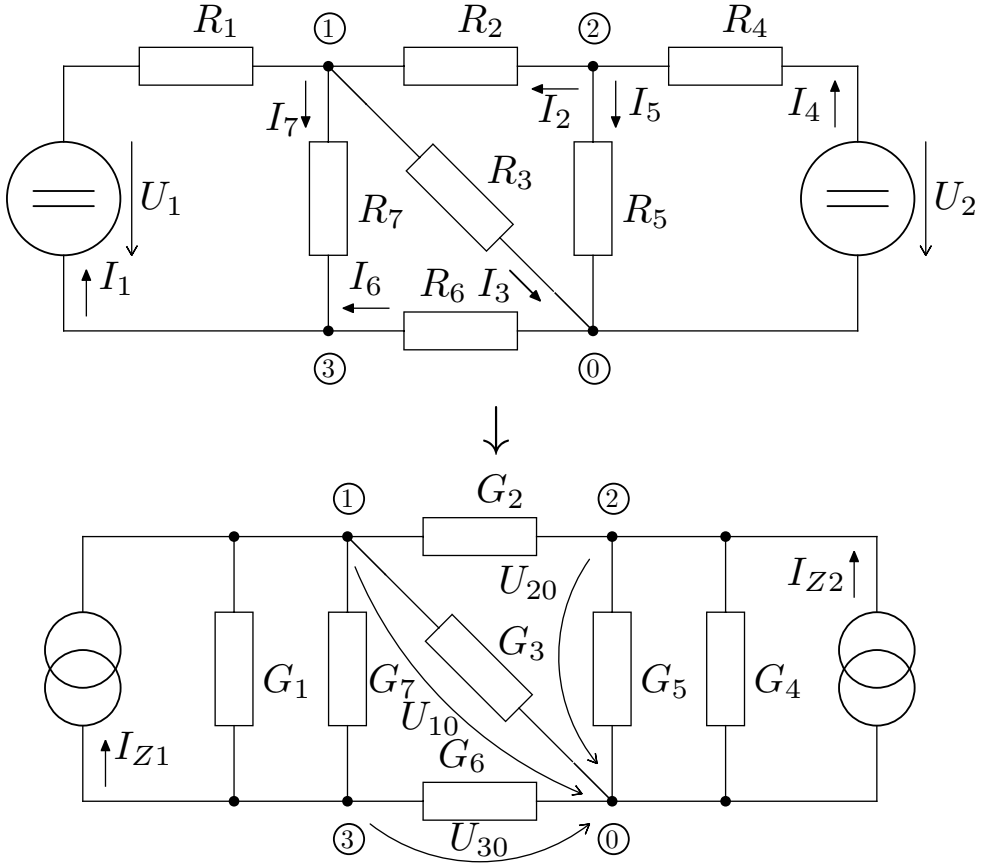
$$\begin{pmatrix} 1,4580 \cdot 10^{-3} & -5,4894 \cdot 10^{-4} & 0 \\ -5,4894 \cdot 10^{-4} & 7,6171 \cdot 10^{-4} & -2,1277 \cdot 10^{-4} \\ 0 & -2,1277 \cdot 10^{-4} & 1,2128 \cdot 10^{-3} \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ U_{30} \end{pmatrix} = \begin{pmatrix} 1,4310 \cdot 10^{-3} \\ -5,7037 \cdot 10^{-3} \\ 2 \cdot 10^{-3} \end{pmatrix}$$

$$U_{10} = -2,5195V$$

$$U_{20} = -9,2988V$$

$$U_{20} = 0,01775V$$

Př.4.9: Určete napětí U_1 a U_2 pomocí MUN. $U_1 = 5V, U_2 = 10V, R_1 = 2k\Omega$, $R_2 = 2k\Omega, R_3 = 5k\Omega, R_4 = 3k\Omega, R_5 = 1k\Omega, R_6 = 4k\Omega, R_7 = 10k\Omega$



$$I_{Z1} = \frac{U_1}{R_1} = \frac{5}{2000} = 2,5mA$$
$$I_{Z2} = \frac{U_2}{R_4} = \frac{10}{3000} = 3,\overline{3}mA$$

$$\begin{pmatrix} G_1+G_2+G_3+G_7 & -G_2 & -G_1-G_7 \\ -G_2 & G_2+G_4+G_5 & 0 \\ -G_1-G_7 & 0 & G_1+G_6+G_7 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ U_{30} \end{pmatrix} = \begin{pmatrix} I_{Z1} \\ I_{Z2} \\ -I_{Z1} \end{pmatrix}$$

$$\begin{pmatrix} 1,3\cdot 10^{-3} & -5\cdot 10^{-4} & -6\cdot 10^{-4} \\ -5\cdot 10^{-4} & 1,8\overline{3}\cdot 10^{-3} & 0 \\ -6\cdot 10^{-4} & 0 & 8,5\cdot 10^{-4} \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ U_{30} \end{pmatrix} = \begin{pmatrix} 2,5\cdot 10^{-3} \\ 3,\overline{3}\cdot 10^{-3} \\ -2,5\cdot 10^{-3} \end{pmatrix}$$

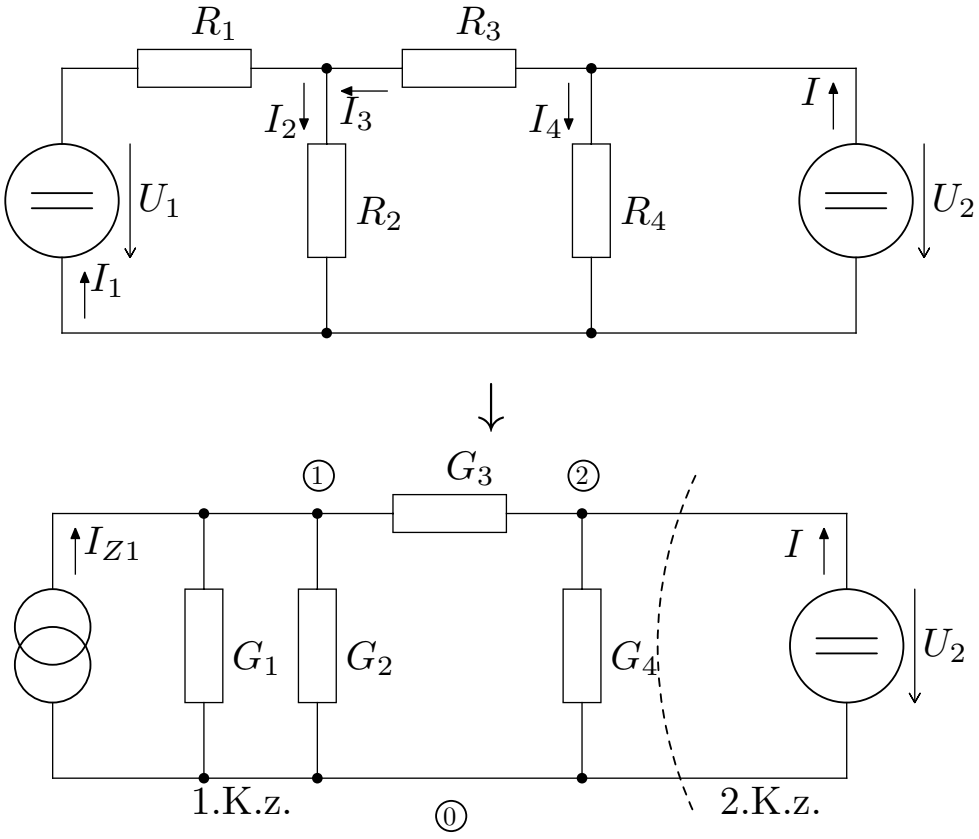
$$U_{10} = -2,5195V$$

$$U_{20} = -9,2988V$$

$$U_{20} = 0,01775V$$

$$
\begin{aligned}
I_3 &= \frac{U_{10}}{R_3} = \frac{2,2218}{5000} = 0,4444mA \\
I_5 &= \frac{U_{20}}{R_5} = \frac{2,4241}{1000} = 2,4241mA \\
I_6 &= -\frac{U_{30}}{R_6} = -\frac{-1,3728}{4000} = 0,3432mA \\
I_4 &= I_5 + I_3 - I_6 = \\
&= 2,4241 \cdot 10^{-3} + 0,4444 \cdot 10^{-3} - 0,3432 \cdot 10^{-3} = \\
&= 2,5253mA \\
I_2 &= I_4 - I_5 = 2,5253 \cdot 10^{-3} - 2,4241 \cdot 10^{-3} = 0,1012mA \\
I_7 &= \frac{U_{10} - U_{20}}{R_5} = \frac{-0,2023}{5000} = 0,3595mA \\
I_1 &= I_6 + I_7 = 0,3432 \cdot 10^{-3} + 0,3595 \cdot 10^{-3} = 0,7027mA
\end{aligned}
$$

Př.5.1: Určete proudy obvodu pomocí MMUN[7]. $U_1 = 5V, U_2 = 10V$, $R_1 = R_3 = 5\Omega, R_2 = R_4 = 10\Omega$



[7]MMUN lze použít i při reálných zdrojích napětí, jestliže to má význam pro řešení problémů (tj. když se zajímáme o proud tekoucí zdrojem).

$$I_{Z1} = \frac{U_1}{R_1} = \frac{5}{5} = 1A$$

$$\begin{pmatrix} G_1 + G_2 + G_3 & -G_3 & 0 \\ -G_3 & G_3 + G_4 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ I \end{pmatrix} = \begin{pmatrix} I_{Z1} \\ 0 \\ U_2 \end{pmatrix}$$

$$\begin{pmatrix} 0,5 & -1,5 & 0 \\ -1,5 & 0,3 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ I \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 10 \end{pmatrix}$$

$$U_{10} = 6V$$

$$U_{20} = 10V$$

$$I = 1,8A$$

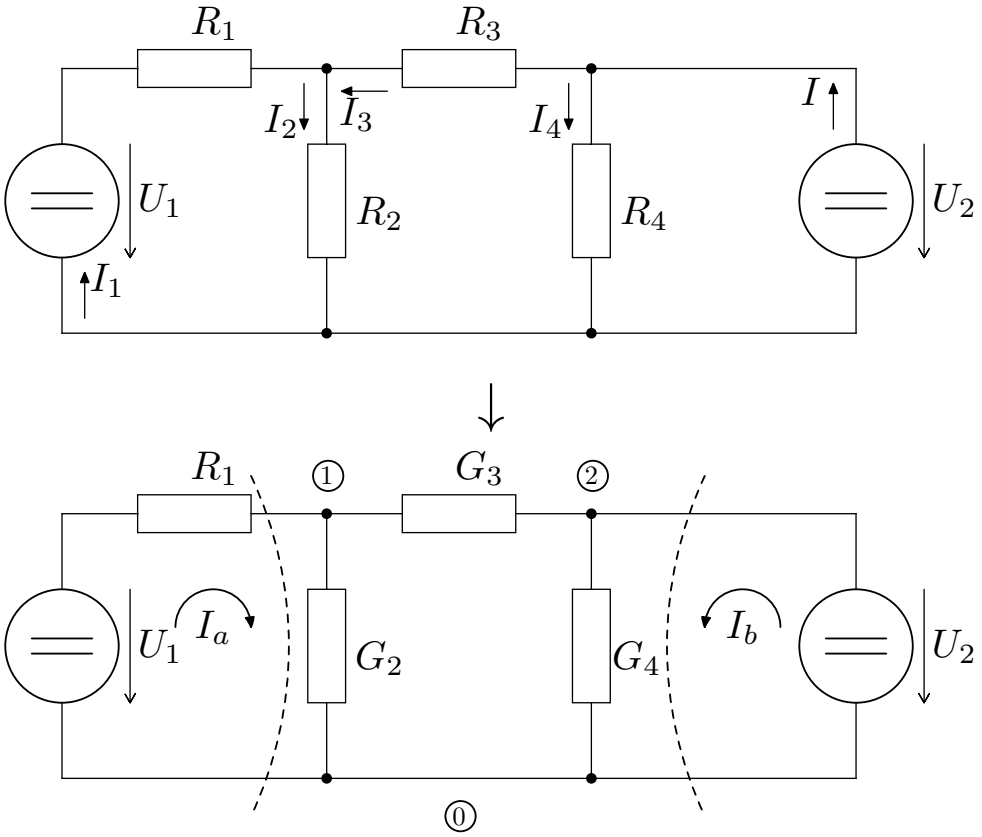
$$I_1 = I_{Z1} - \frac{U_{10}}{R1} = 1 - \frac{6}{5} = -0,2A$$

$$I_2 = \frac{U_{10}}{R2} = \frac{6}{10} = 0,6A$$

$$I_3 = \frac{U_{20} - U_{10}}{R3} = \frac{10 - 6}{5} = 0,8A$$

$$I_4 = \frac{U_{20}}{R4} = \frac{10}{10} = 1A$$

Př.5.2: Předešlý příklad řešte modifikací obou zdrojů.



Podle II. K.z. napíšeme 2 rovnice:

$$U_{10} - U_1 + R_1 I_a = 0$$
$$\underline{U_{10} + R_1 I_a = U_1}$$
$$U_{20} - U_2 = 0$$
$$\underline{U_{20} = U_2}$$

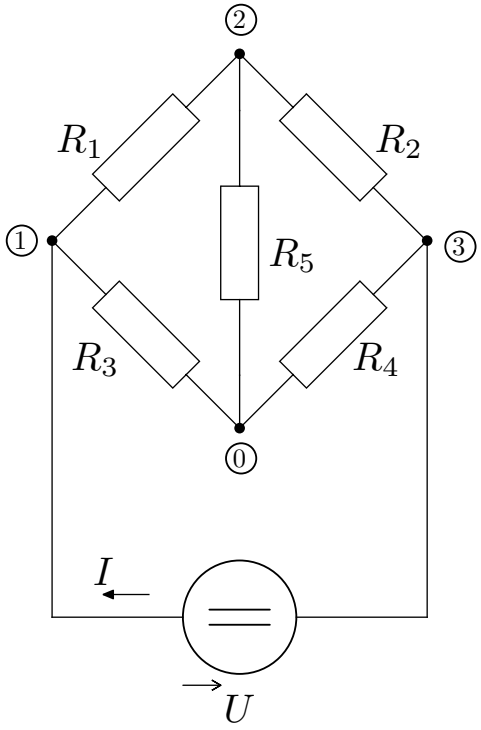
$$\begin{pmatrix} G_2+G_3 & -G_3 & -1 & 0 \\ -G_3 & G_3+G_4 & 0 & -1 \\ 1 & 0 & R_1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ I_a \\ I_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ U_1 \\ U_2 \end{pmatrix}$$

$$\begin{pmatrix} 0,3 & -1,5 & -1 & 0 \\ -1,5 & 0,3 & 0 & -1 \\ 1 & 0 & 5 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ I_a \\ I_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \\ 10 \end{pmatrix}$$

$$U_{10} = 6V$$
$$U_{20} = 10V$$

$$I_a = -0,2A$$
$$I_b = 1,8A$$

Př.5.3: Vypočítejte uzlová napětí a proud I v uvedeném obvodu pomocí MMUN[8].$U = 2V, R_1 = R_3 = 20\Omega, R_2 = 40\Omega, R_4 = 10\Omega, R_5 = 25\Omega$



Podle II. K.z.:

$$U_{10} - U_{30} - U = 0$$
$$\underline{U_{10} - U_{30} = U}$$

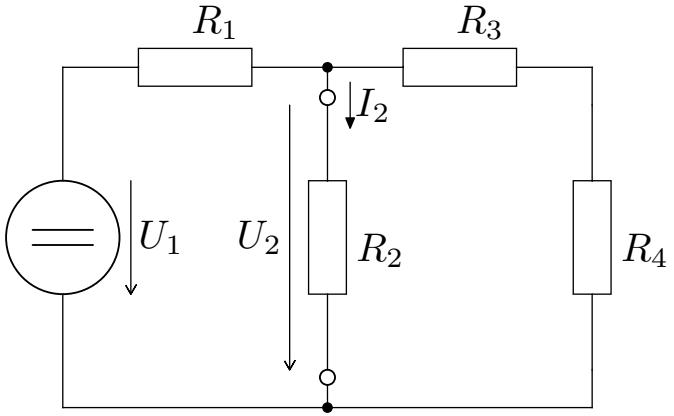
[8]Je zřejmé, že MMUN vede na větší počet rovnic, což není na závadu při počítačovém zpracování.

$$\begin{pmatrix} G_1+G_3 & -G_1 & 0 & -1 \\ -G_1 & G_1+G_2+G_5 & -G_2 & 0 \\ 1 & 0 & R_1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ U_{30} \\ I_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ U \end{pmatrix}$$

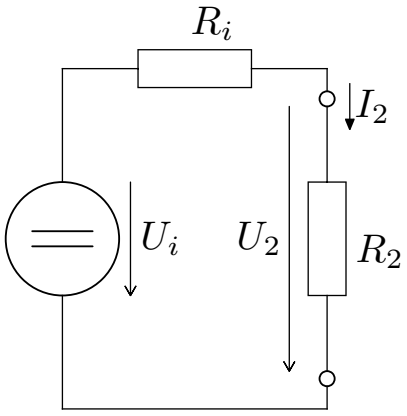
$$\begin{pmatrix} 0,1 & -0,05 & 0 & -1 \\ -0,05 & 0,115 & -0,025 & 0 \\ 0 & -0,025 & 0,125 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} U_{10} \\ U_{20} \\ I_a \\ I_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 2 \end{pmatrix}$$

$$U_{10} = 1,1235V$$
$$U_{20} = 0,3704V$$
$$U_{30} = -0,7654V$$
$$I_b = 0,1049A$$

Př.5.4: V obvodu na obrázku vypočítejte pomocí Theveninovy věty napětí, proud a výkon rezistoru R_2, je-li $U = 20V, R_1 = R_3 = 10\Omega, R_2 = 20\Omega, R_4 = 40\Omega$.



$\rightarrow$

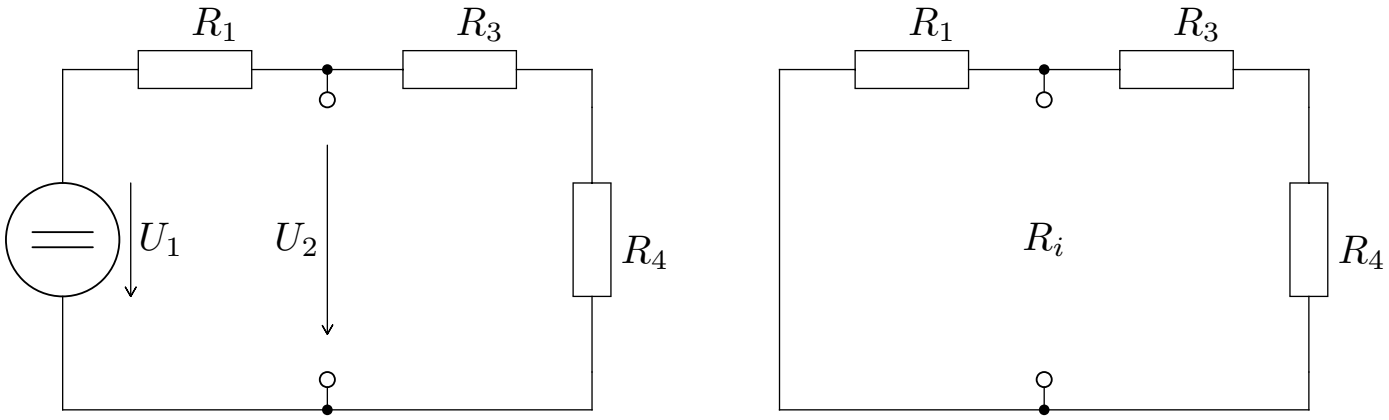


$$I_2 = \frac{U_i}{R_i + R_2}$$

$$U_2 = I_2 R_2$$

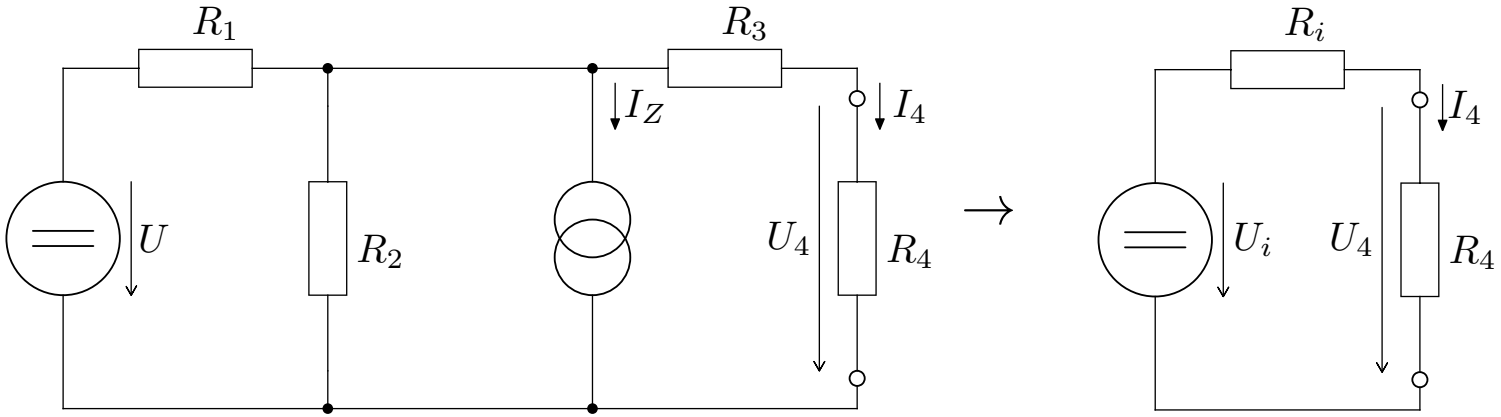
$$P_2 = U_2 I_2$$

Řešení:



$$
\begin{aligned}
U_i &= U\frac{R_3+R_4}{R_1+R_3+R_4} = 20\cdot\frac{10+40}{10+10+40} = 16,\overline{6}V \\
R_i &= \frac{R_1(R_3+R_4)}{R_1+R_3+R_4} = \frac{10\cdot(10+40)}{10+10+40} = 8,\overline{3}\Omega \\
I_2 &= \frac{U_i}{R_i+R_2} = \frac{16,\overline{6}}{8,\overline{3}+20} = 0,5882A \\
U_2 &= I_2R_2 = 0,5882\cdot 20 = 11,7648V \\
P_2 &= U_2I_2 = 11,7648\cdot 0,5882 = 6,9200W
\end{aligned}
$$

Př.5.5: V obvodu na obrázku vypočítejte pomocí Theveninovy věty napětí, proud a výkon rezistoru R_4. $U = 10V, I_Z = 2A, R_1 = R_3 = 10\Omega, R_2 = 20\Omega, R_4 = 40\Omega$.

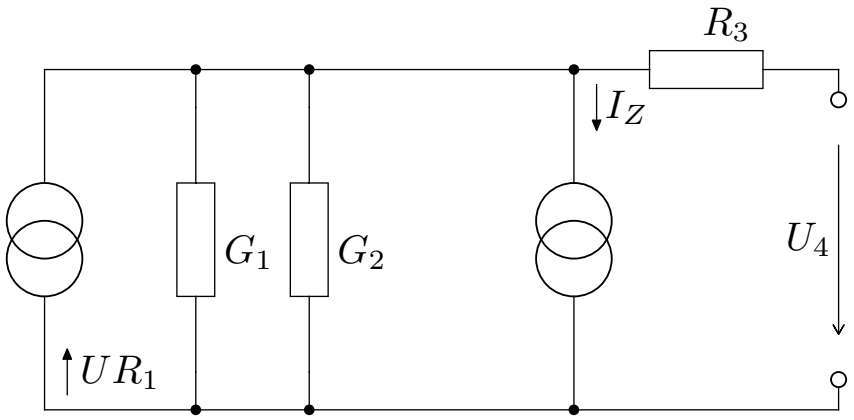


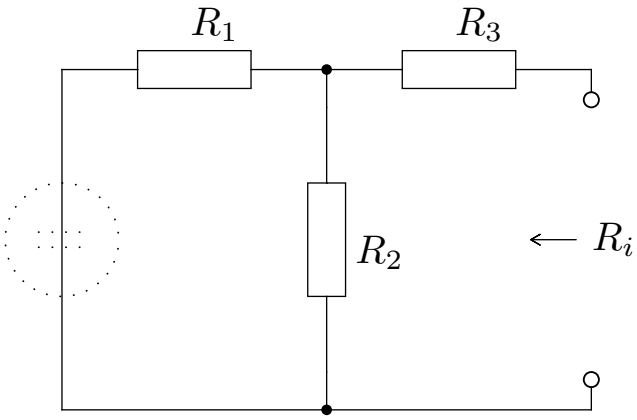
$$I_4 = \frac{U_i}{R_1 + R_4}$$

$$U_4 = I_4 R_4$$

$$P_4 = U_4 I_4$$

Řešení:





$$(G_1 + G_2)U_i = \frac{U}{R_1} \Rightarrow U_i = \frac{UG_1 - I_Z}{G_1 + G_2} = -6,\overline{6}V$$

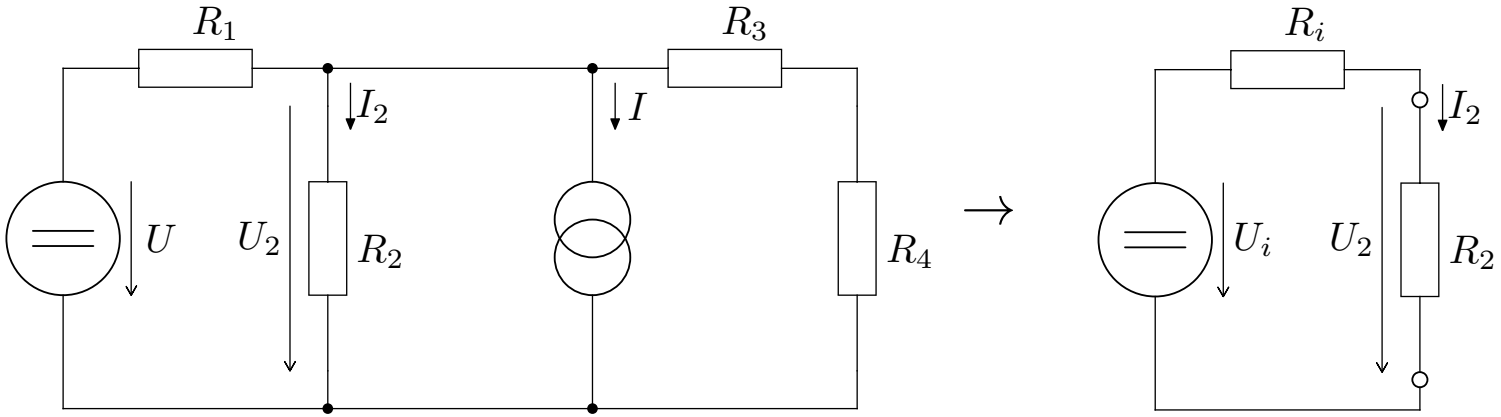
$$R_i = R_3 + \frac{R_1 R_2}{R_1 + R_2} = 16,\overline{6}\Omega$$

$$I_4 = \frac{U_i}{R_1 + R_4} = -0,11765A$$

$$U_4 = I_4 R_4 = -4,7059V$$

$$P_4 = U_4 I_4 = 0,5536W$$

Př.5.6: V obvodu na obrázku vypočítejte pomocí Theveninovy věty napětí, proud a výkon rezistoru R_2. $U = 10V, I_Z = 2A, R_1 = R_3 = 10\Omega, R_2 = 20\Omega, R_4 = 40\Omega$.



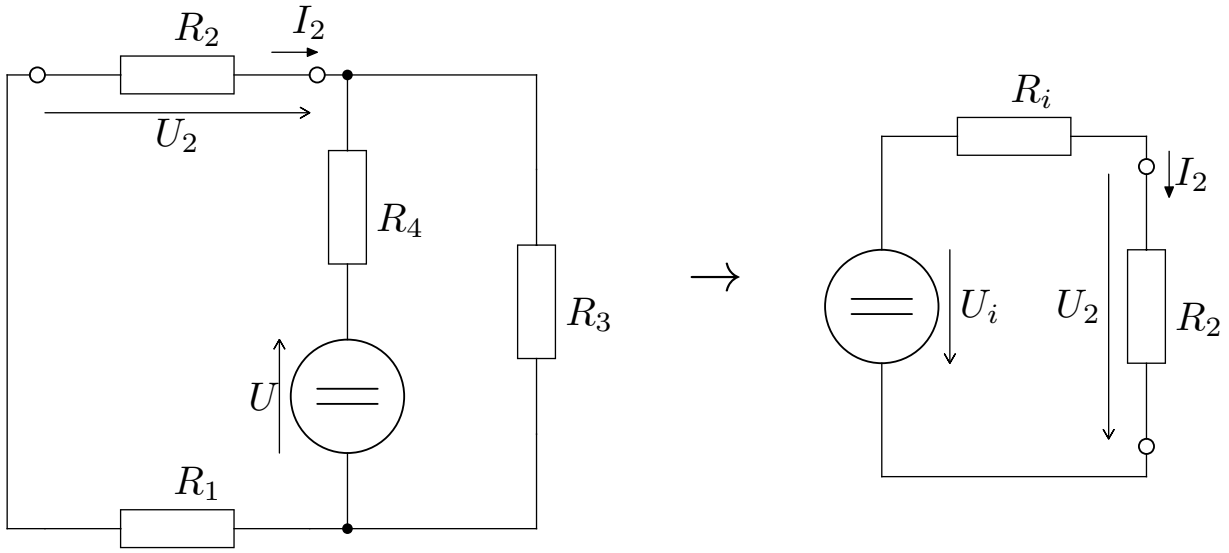
$$I_2 = \frac{U_i}{R_i + R_2}$$

$$U_2 = I_2 R_2$$

$$P_2 = U_2 I_2$$

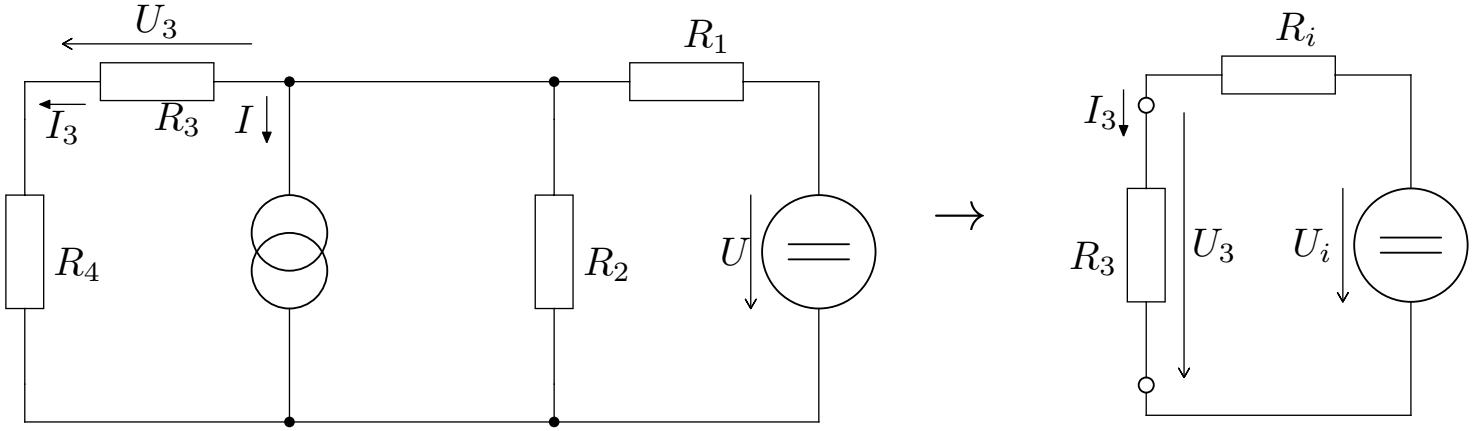
$$
\begin{aligned}
(G_1 + \frac{G_3 G_4}{G_3 + G_4}) U_i = \frac{U}{R_1} - I_Z \Rightarrow U_i &= \frac{\frac{U}{R_1} - I_Z}{G_1 + \frac{G_3 G_4}{G_3 + G_4}} = -8,\overline{3}V \\
R_i &= \frac{(R_3 + R_4) R_1}{R_1 + R_3 + R_4} = 8,\overline{3}\Omega \\
I_2 &= \frac{U_i}{R_i + R_2} = -0,29412A \\
U_2 &= I_2 R_2 = -5,88235V \\
P_2 &= U_2 I_2 = 1,7301W
\end{aligned}
$$

Př.5.7: V obvodu na obrázku vypočítejte pomocí Theveninovy věty napětí, proud a výkon rezistoru R_2. $U = 20V, R_1 = 15\Omega, R_2 = 20\Omega, R_3 = R_4 = 10\Omega$.



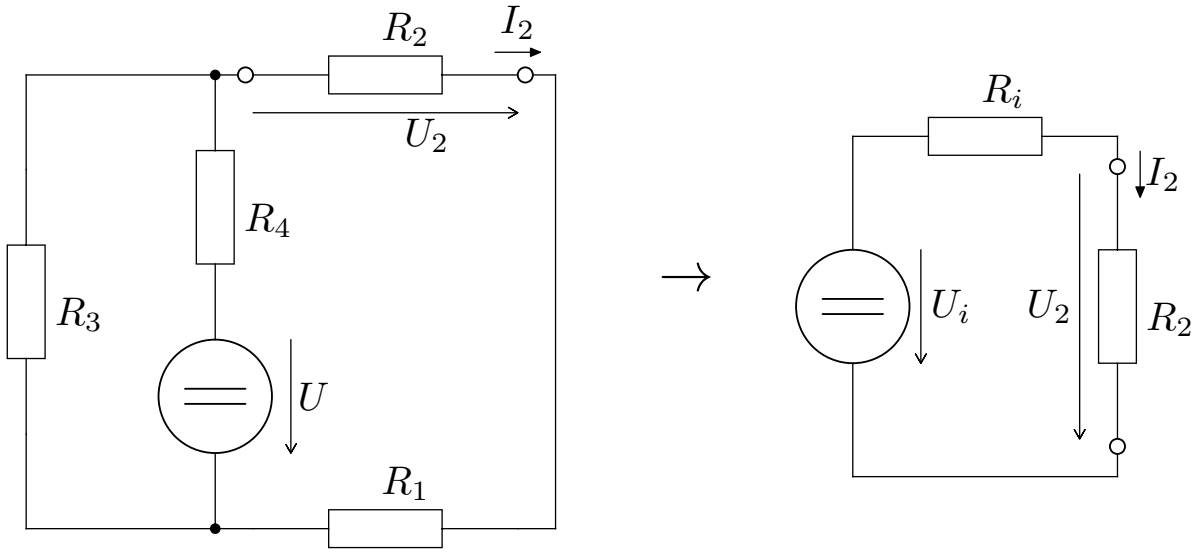
$$
\begin{aligned}
(G_3 + G_4)U_i = \frac{U}{R_4} \Rightarrow U_i &= \frac{\frac{U}{R_4}}{G_3 + G_4} = 10V \\
R_i &= R_1 + \frac{R_3 R_4}{R_3 + R_4} = 20\Omega \\
I_2 &= \frac{U_i}{R_i + R_2} = 0,25A \\
U_2 &= I_2 R_2 = 5V \\
P_2 &= U_2 I_2 = 1,25W
\end{aligned}
$$

Př.5.8: V obvodu na obrázku vypočítejte pomocí Theveninovy věty napětí, proud a výkon rezistoru R_3. $U = 10V, I = 0,5A, R_1 = R_3 = 10\Omega, R_2 = 20\Omega, R_4 = 40\Omega$.



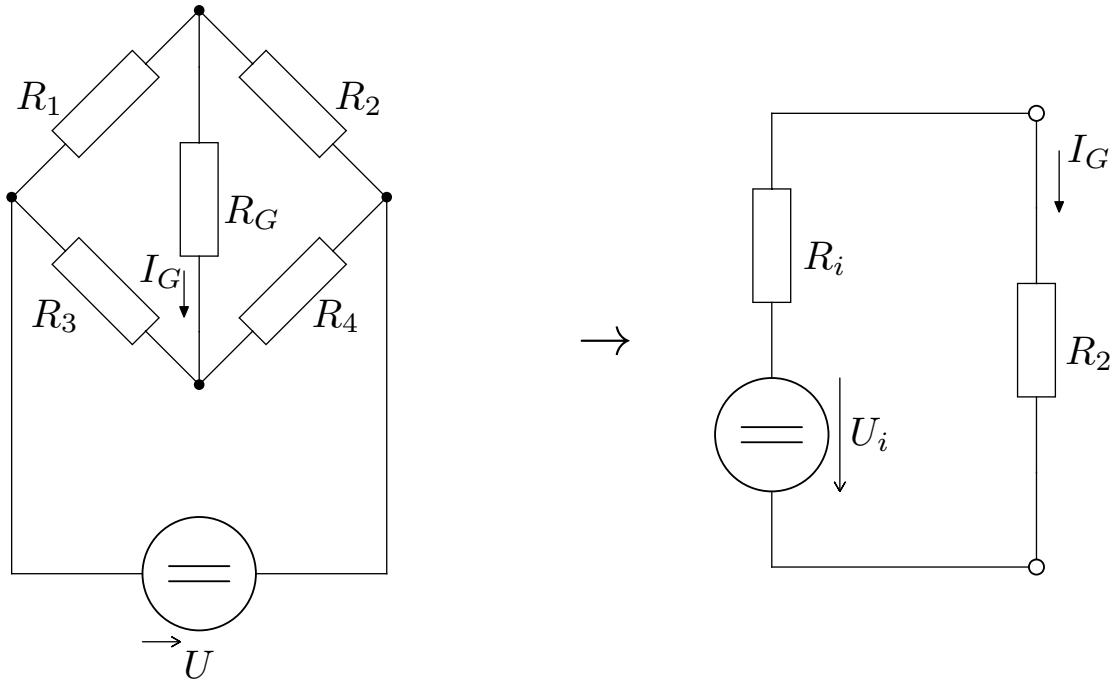
$$
\begin{aligned}
(G_1 + G_2)U_i = \frac{U}{R_1} - I \Rightarrow U_i &= \frac{\frac{U}{R_1} - I}{G_1 + G_2} = 3,\overline{3}V \\
R_i &= R_4 + \frac{R_1 R_2}{R_1 + R_2} = 46,\overline{6}\Omega \\
I_3 &= \frac{U_i}{R_i + R_3} = 0,05882A \\
U_3 &= I_3 R_3 = 0,5882V \\
P_3 &= U_3 I_3 = 0,0346W
\end{aligned}
$$

Př.5.8: V obvodu na obrázku vypočítejte pomocí Theveninovy věty napětí, proud a výkon rezistoru R_2. $U = 30V, R_1 = 15\Omega, R_2 = 20\Omega, R_3 = R_4 = 10\Omega$.



$$
\begin{aligned}
(G_3 + G_4)U_i = \frac{U}{R_4} \Rightarrow U_i &= \frac{\frac{U}{R_4}}{G_3 + G_4} = 15V \\
R_i &= R_1 + \frac{R_3 R_4}{R_3 + R_4} = 20\Omega \\
I_2 &= \frac{U_i}{R_i + R_2} = 0,375A \\
U_2 &= I_2 R_2 = 7,5V \\
P_2 &= U_2 I_2 = 2,8125W
\end{aligned}
$$

Př.6.1: V můstkovém zapojení určete proud I_G pomocí věty o náhradním napěťovém zdroji. $U = 2V, R_1 = 20\Omega, R_2 = 40\Omega, R_3 = 20\Omega, R_4 = 10\Omega, R_G = 25\Omega$.



$$\begin{pmatrix} R_3+R_4 & 0 \\ 0 & R_1+R_2 \end{pmatrix} \cdot \begin{pmatrix} I_a \\ I_b \end{pmatrix} = \begin{pmatrix} -U \\ U \end{pmatrix}$$

$$\begin{pmatrix} 30 & 0 \\ 0 & 60 \end{pmatrix} \cdot \begin{pmatrix} I_a \\ I_b \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$I_a = -0,0\overline{6}A$$

$$I_b = 0,0\overline{3}A$$

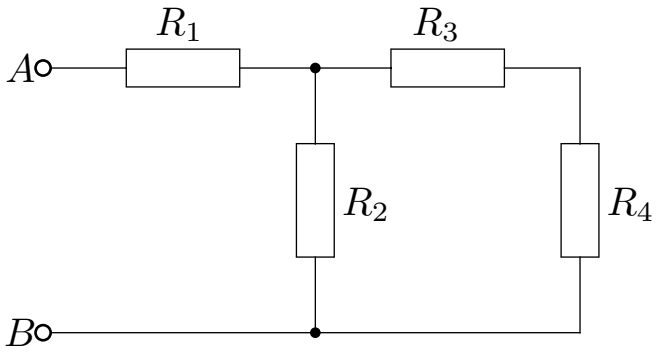
$$U_i = R_4 I_a + R_2 I_b = 0,\overline{6}V$$

$$R_i = \frac{R_1 R_2}{R_1+R_2} + \frac{R_3 R_4}{R_3+R_4} = 20\Omega$$

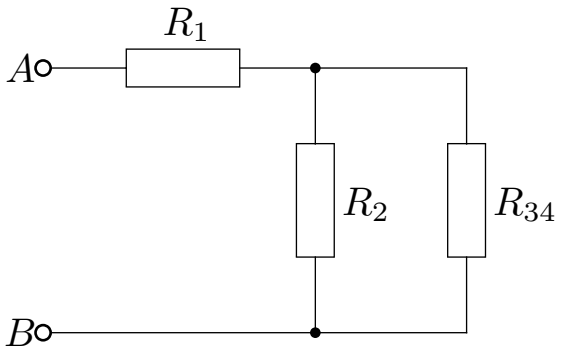
$$I_G = \frac{U_i}{R_i + R_G} = 14,82mA$$

Dodatek - Ustálený stejnosměrný proud

Př.D1: Vypočítejte celkový odpor obvodu. $R_1 = 100\Omega, R_2 = 400\Omega, R_3 = R_4 = 200\Omega$

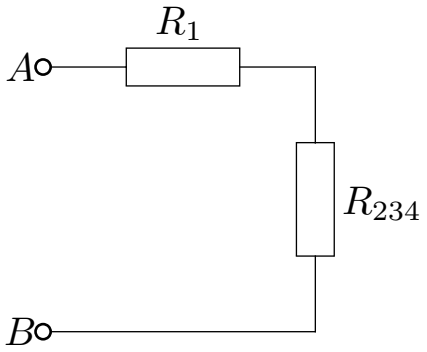


Krok 1.



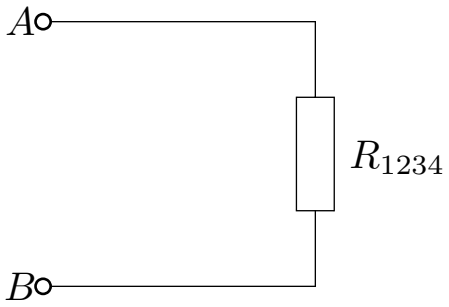
$$R_{34} = R_3 + R_4 = 200\Omega + 200\Omega = 400\Omega$$

Krok 2.



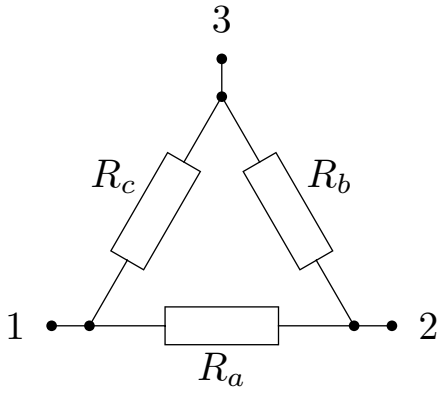
$$R_{234} = \frac{R_2 \cdot R_{34}}{R_2 + R_{34}} = \frac{400\Omega \cdot 400\Omega}{400\Omega + 400\Omega} = 200\Omega$$

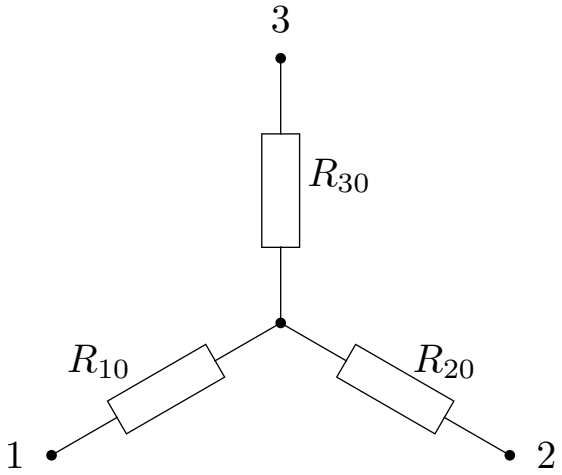
Krok 3.



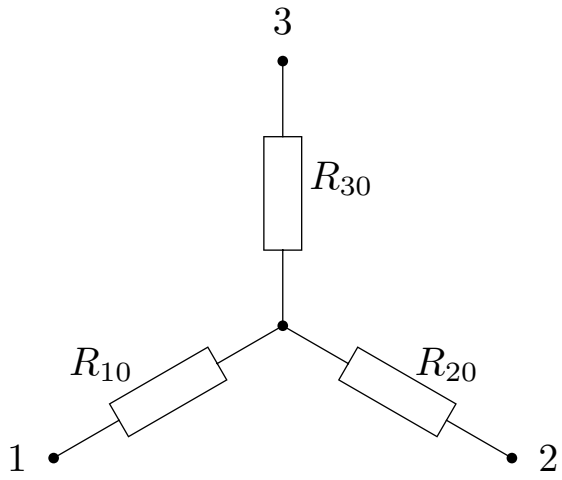
$$R_{1234} = R_1 + R_2 34 = 100\Omega + 200\Omega = 300\Omega$$

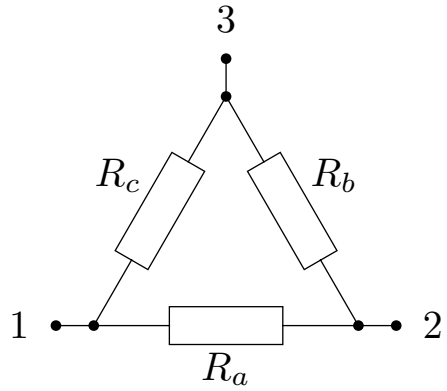
Transfigurace hvězda – trojúhelník a trojúhelník – hvězda





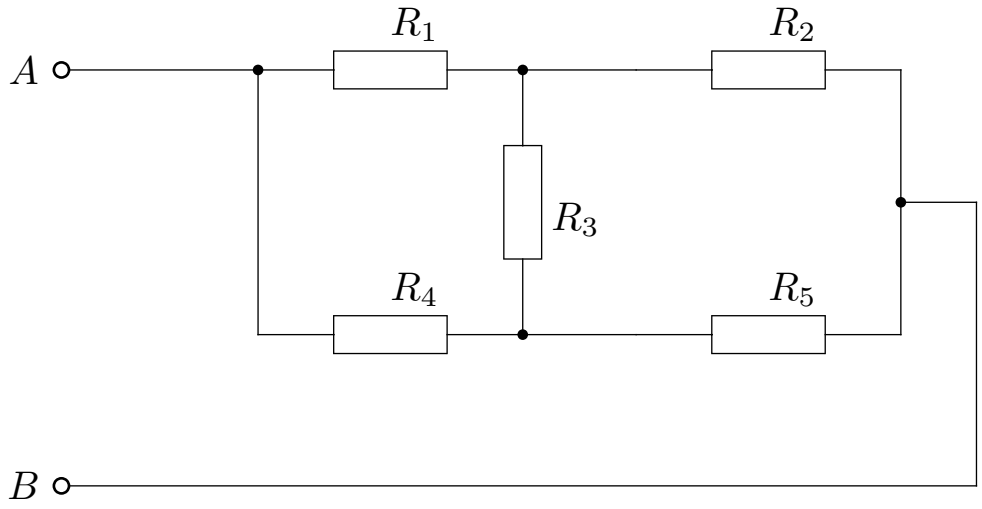
$$
\begin{aligned}
R_{10} &= \frac{R_a \cdot R_c}{R_a + R_b + R_c} \\
R_{20} &= \frac{R_a \cdot R_b}{R_a + R_b + R_c} \\
R_{30} &= \frac{R_b \cdot R_c}{R_a + R_b + R_c}
\end{aligned}
\tag{1}
$$



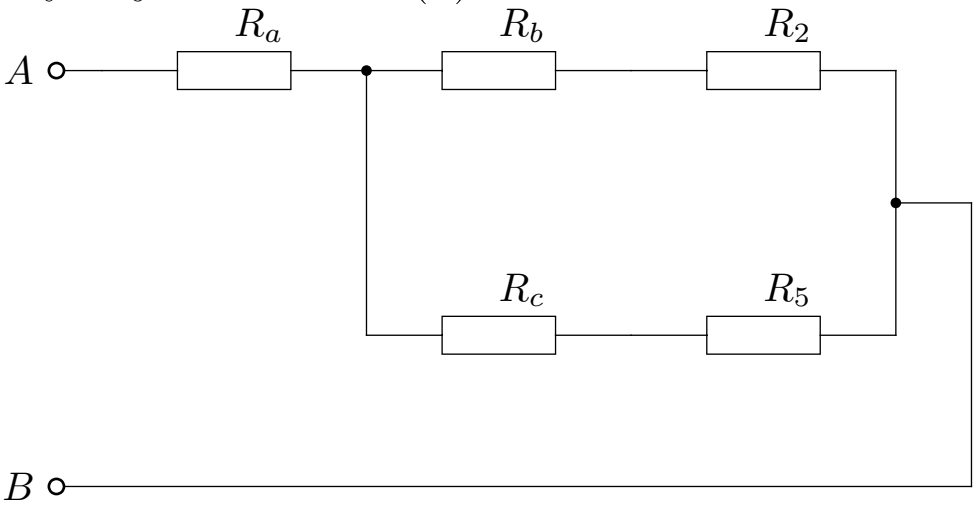


$$
\begin{aligned}
R_a &= R_{10} + R_{20} + R_{10} \cdot \frac{R_{20}}{R_{30}} \\
R_b &= R_{20} + R_{30} + R_{20} \cdot \frac{R_{30}}{R_{10}} \\
R_c &= R_{10} + R_{30} + R_{10} \cdot \frac{R_{30}}{R_{20}}
\end{aligned}
\tag{2}
$$

Př.D2: Vypočítejte celkový odpor obvodu. $R_1 = 100\Omega$, $R_2 = 200\Omega$, $R_3 = 300\Omega$, $R_4 = 400\Omega$, $R_5 = 500\Omega$



Krok 1. Využijeme vzorce (1)[9]



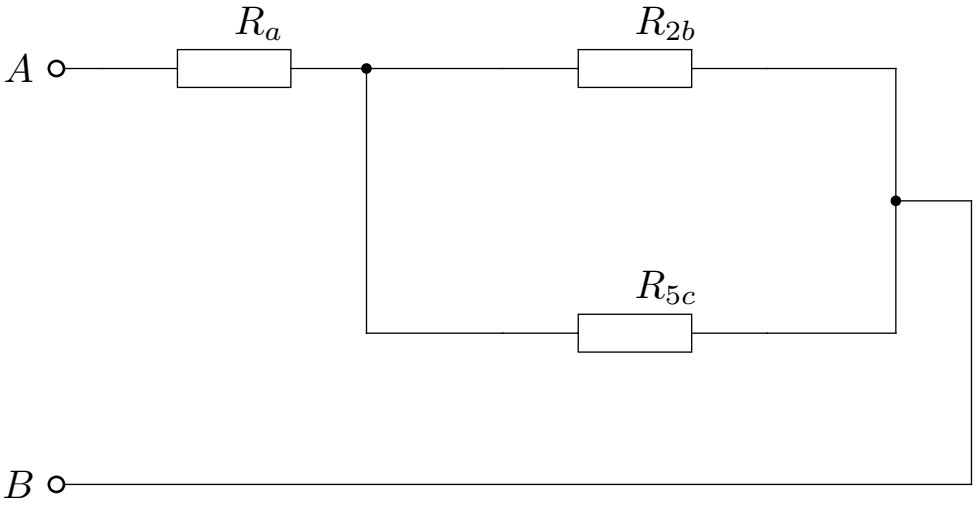
$$R_a = \frac{R_1 \cdot R_4}{R_1 + R_3 + R_4} = \frac{100 \cdot 400}{100 + 300 + 400} = 50\Omega$$

$$R_b = \frac{R_1 \cdot R_3}{R_1 + R_3 + R_4} = \frac{100 \cdot 300}{100 + 300 + 400} = 37,5\Omega$$

$$R_c = \frac{R_3 \cdot R_4}{R_1 + R_3 + R_4} = \frac{300 \cdot 400}{100 + 300 + 400} = 150\Omega$$

[9]Na označení rezistorů nezáleží, je důležité umístění mezi uzly.

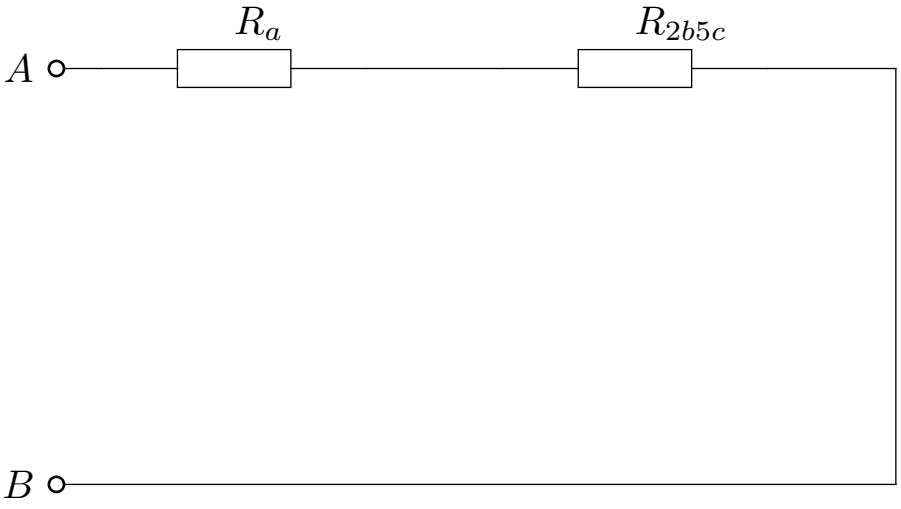
Krok 2.



$$R_{2b} = R_b + R_2 = 37,5 + 200 = 237,5\Omega$$

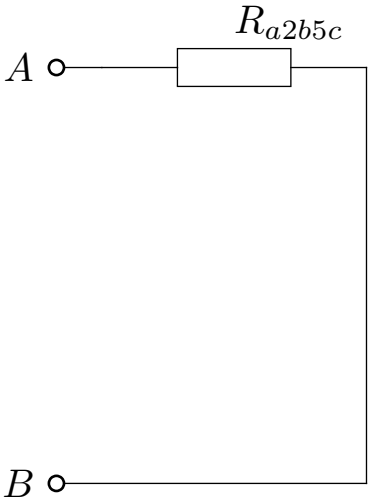
$$R_{5c} = R_c + R_5 = 150 + 500 = 650\Omega$$

Krok 3.



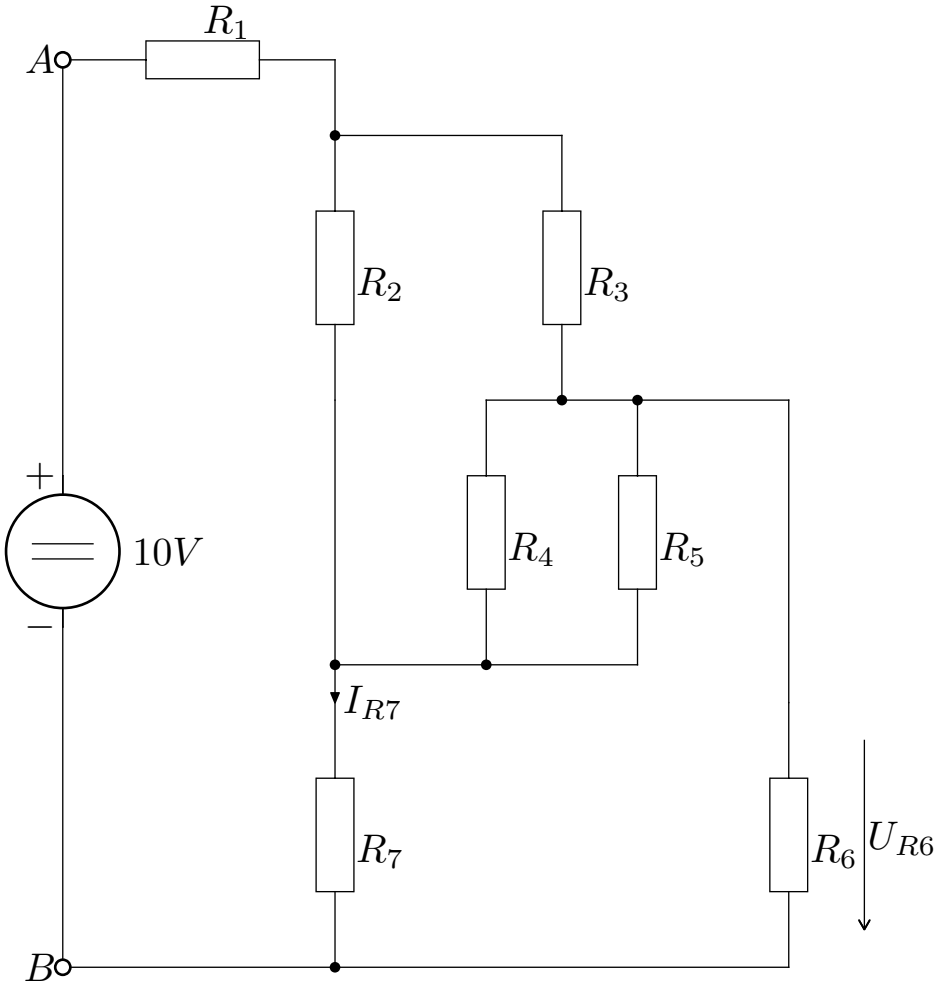
$$R_{2b5c} = \frac{R_{2b} \cdot R_{5c}}{R_{2b} + R_{5c}} = \frac{237,5 \cdot 650}{237,5 + 650} = 173,94\Omega$$

Krok 4.

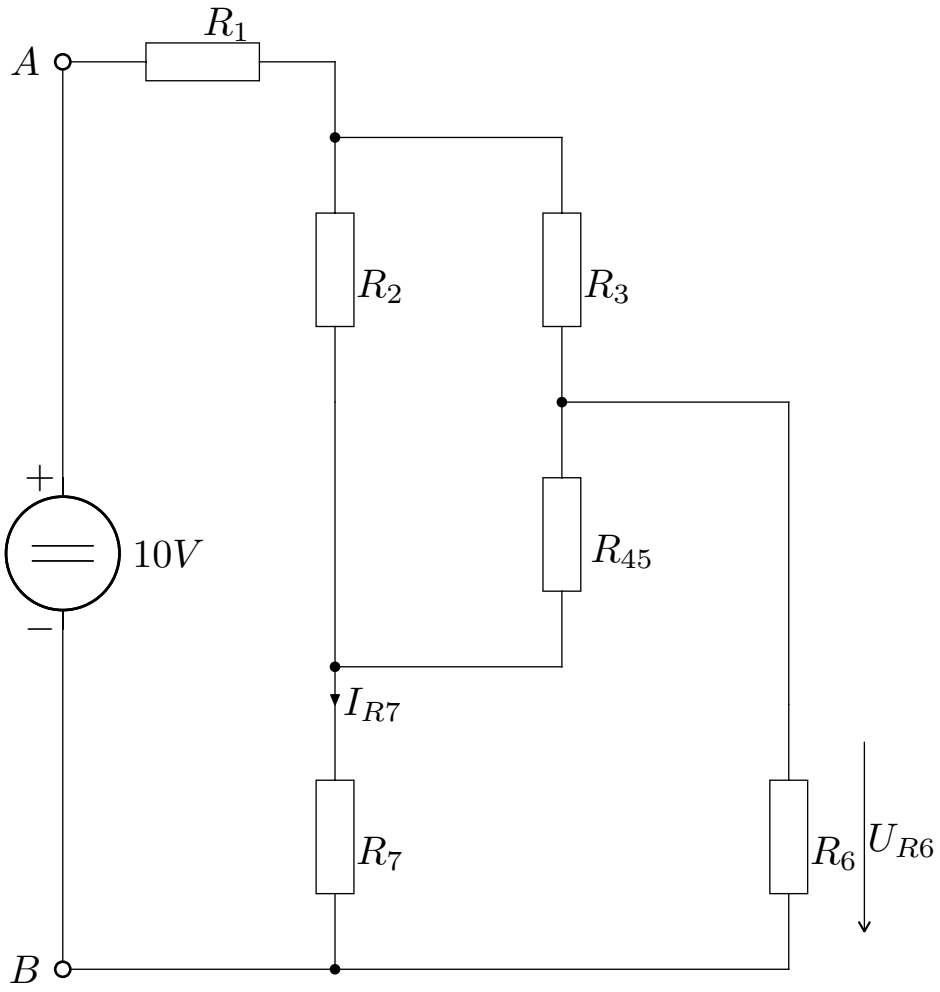


$$R_{a2b5c} = R_a + R_{2b5c} = 50 + 173,94 = 223,94\Omega$$

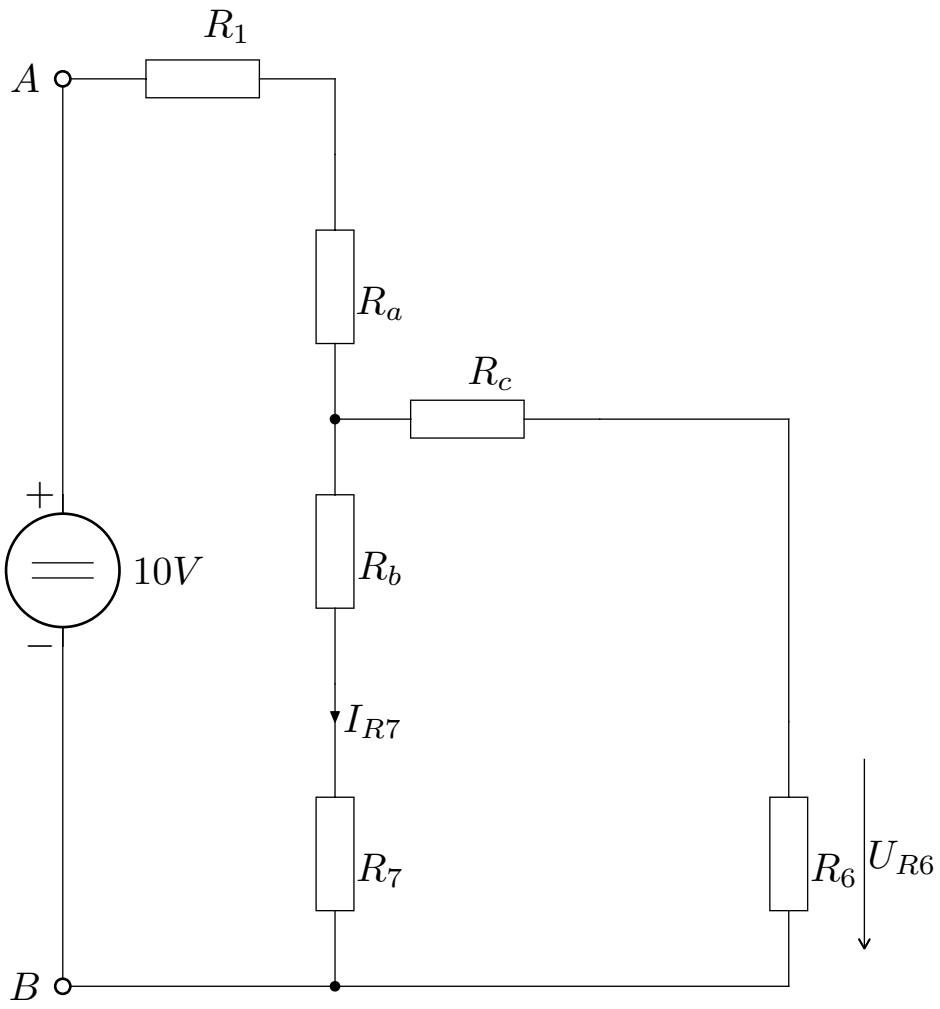
Př.D3: Vypočítejte I_{R7} a U_{R6}. $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = 200\Omega$



Krok 1.



Krok 2. Využijeme vzorce (1)



/ad krok 1.

$$R_{45} = \frac{R_4 \cdot R_5}{R_4 + R_5} = \frac{200 \cdot 200}{200 + 200} = 100\Omega$$

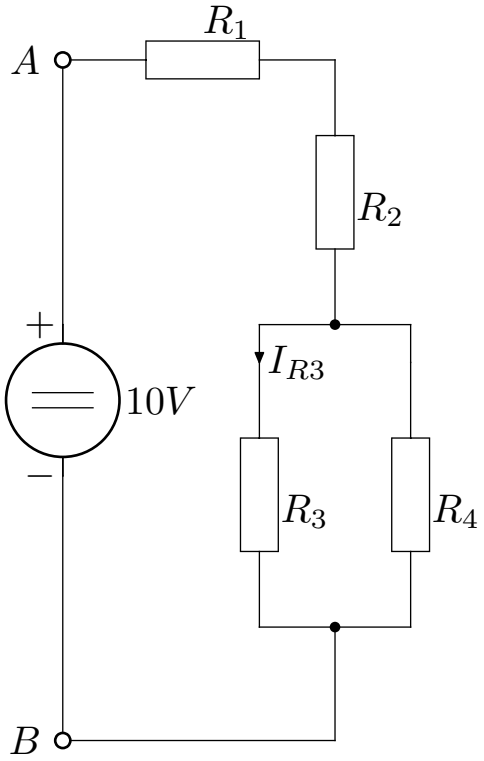
/ad krok 2. Využijeme vzorce (1)

$$R_a = \frac{R_3 \cdot R_2}{R_3 + R_{45} + R_2} = \frac{200 \cdot 200}{200 + 100 + 200} = 80\Omega$$

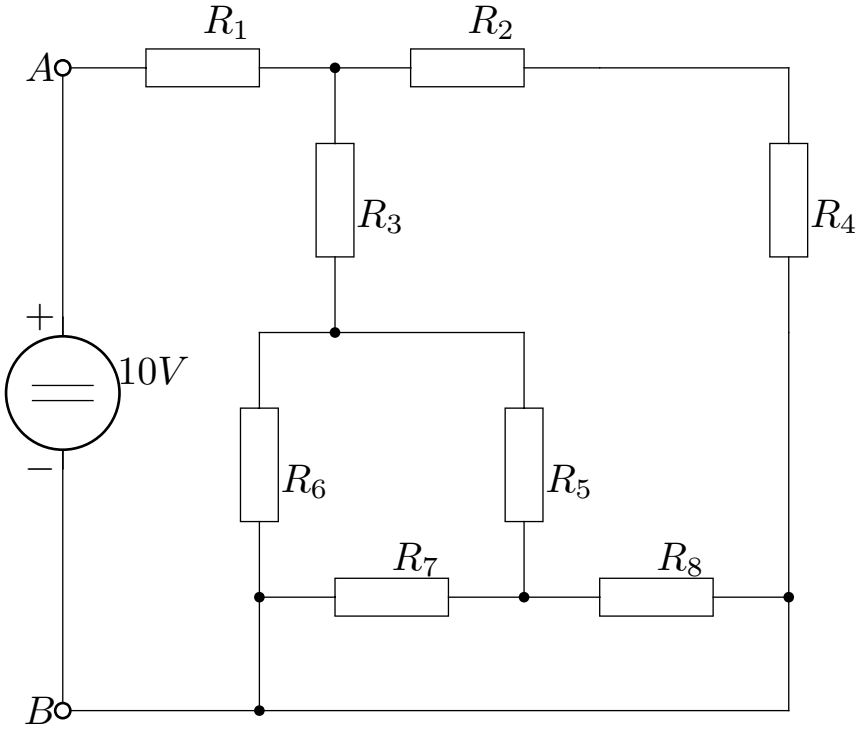
$$R_b = \frac{R_3 \cdot R_{45}}{R_3 + R_{45} + R_2} = \frac{200 \cdot 100}{200 + 100 + 200} = 40\Omega$$

$$R_c = \frac{R_{45} \cdot R_2}{R_3 + R_{45} + R_2} = \frac{100 \cdot 200}{200 + 100 + 200} = 40\Omega$$

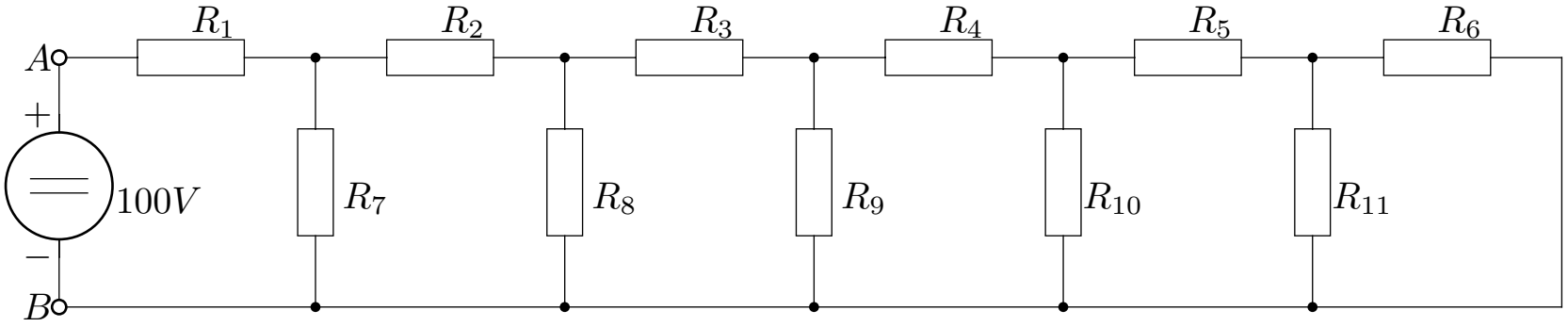
Př.D4: Vypočítejte I_{R3} a U_{R4}. $R_1 = 100\Omega, R_2 = 200\Omega, R_3 = 300\Omega, R_4 = 400\Omega$



Př.D5: Vypočítejte napětí na všech rezistorech, všechny proudy protékající rezistory.
$R_1 = \Omega, R_2 = \Omega, R_3 = \Omega, R_4 = \Omega, R_5 = \Omega, R_6 = \Omega, R_7 = \Omega$



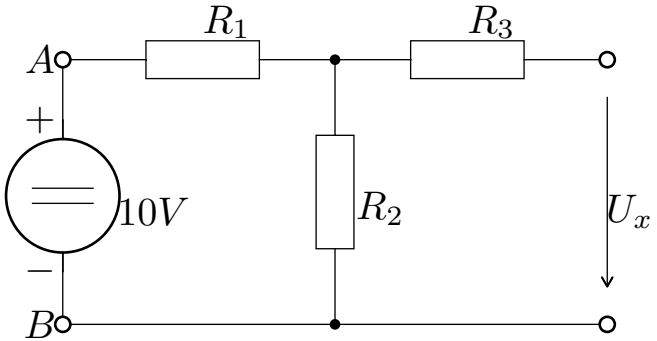
Př.D6: Vypočítejte napětí U_{R7}, U_{R8}, U_{R9}, U_{R10}, U_{R11} a proudy I_{R1}, I_{R2}, I_{R3}, I_{R4}, I_{R5}, I_{R6}, . $R_1 = R_2 = R_3 = R_4 = R_5 = 100\Omega, R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = 200\Omega$



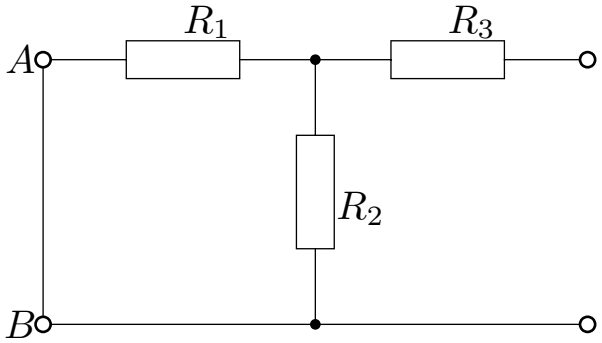
Theveninova věta

Libovolně složitý lineární obvod lze k jeho libovolným dvěma svorkám nahradit obvodem ideálního zdroje napětí U_n v sérii s rezistorem R_n. Napětí U_n bude napětí na těchto svorkách naprázdno. Vnitřní odpor tohoto zdroje vypočítáme jako odpor mezi výstupními svorkami, pokud je zátěž odpojena, zdroje napětí zkratovány a zdroje proudu odpojeny.

Př.D7: Vypočítejte napětí U_x, $R_1 = R_2 = R_3 = 200\Omega$



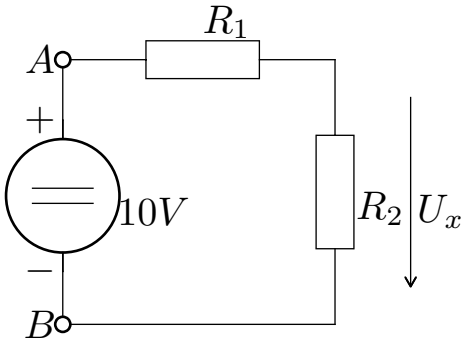
Krok 1. Celkový odpor:



$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{200 \cdot 200}{200 + 200} = 100\Omega$$

$$R_n = R_{123} = R_{12} + R_3 = 100 + 200 = 300\Omega$$

Krok 2. Napětí na svorkách U_x

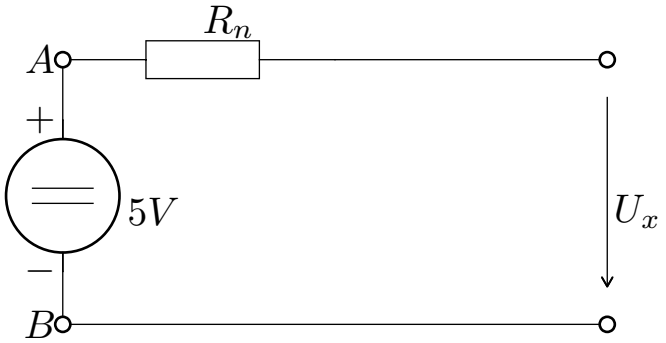


$$R_{12} = R_1 + R_2 = 200 + 200 = 400\Omega$$

$$I = \frac{U}{R_{12}} = \frac{10}{400} = 0,025A$$

$$U_x = R_2 \cdot I = 200 \cdot 0,025 = 5V$$

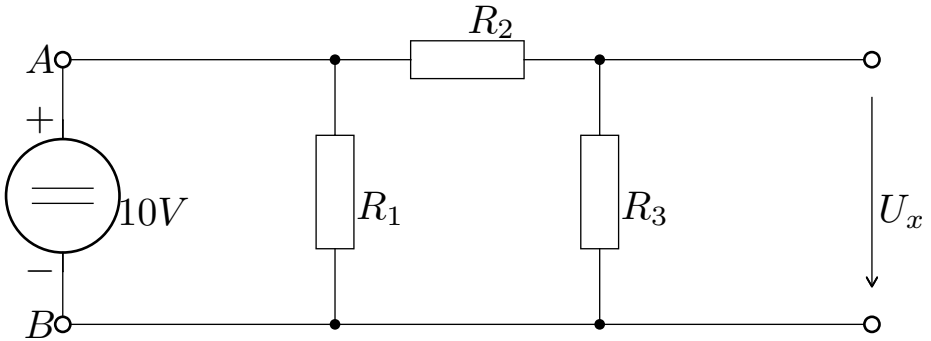
Výsledek:



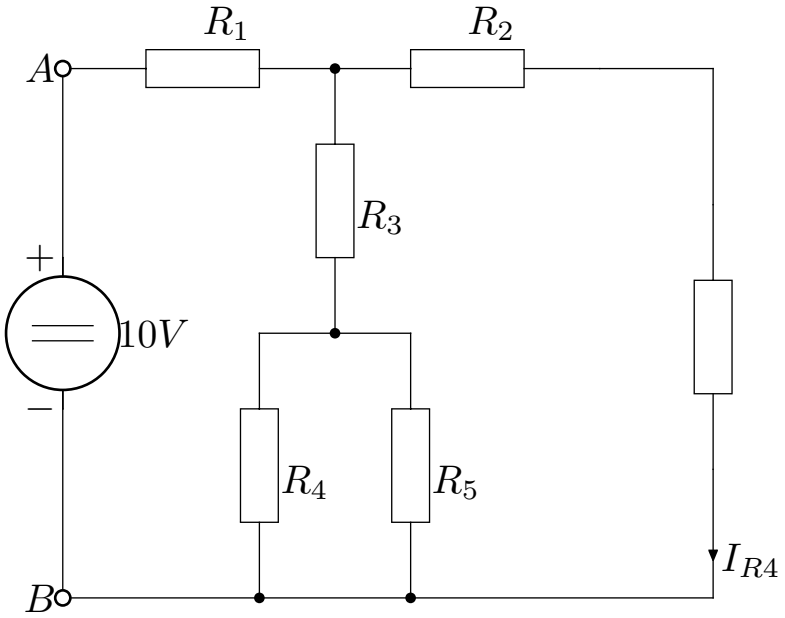
$$R_n = 300\Omega$$

$$U_x = 5V$$

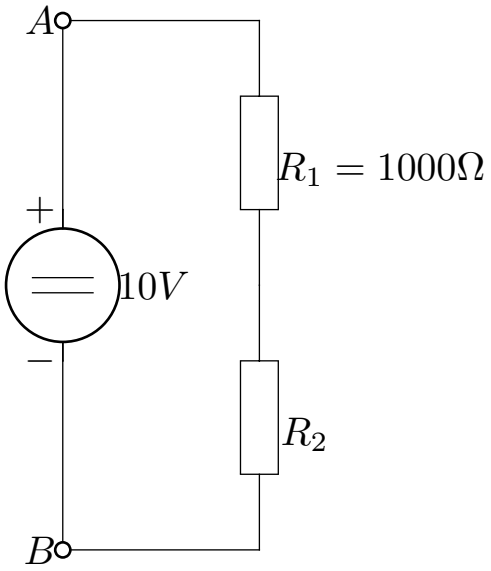
Př.D8: Vypočítejte napětí U_x, $R_1 = R_2 = R_3 = 200\Omega$



Př.D9: Vypočítejte R_1. $R_2 = R_4 = 200\Omega, R_3 = 100\Omega, R_5 = R_6 = 300\Omega, I_{R4} = 0,02A$



Př.D10: Určete maximální hodnoty odporu R_2 tak, aby U_{R2} mělo hodnotu $0,5V$. Ověřte, že pro hodnotu $R_2 > R_{2MIN} U_{R2} > 0,5V$



II. Obvody ustáleného harmonického stavu

2.1 Symbolická metoda řešení

Symbolická metoda - transformace z časové oblasti do komplexní roviny

Komplexor = rotující fázor:

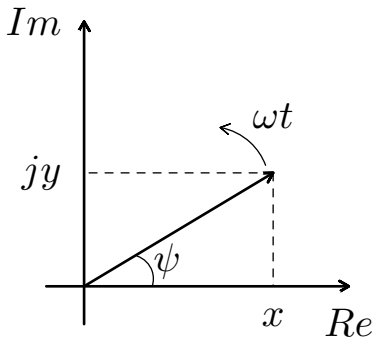
$$\hat{i}(t) = \widehat{I}_m e^{j\omega t} = I_m e^{j\omega} \cdot e^{j\omega t} = I_m \left[cos(\omega t + \psi) + j\ sin(\omega t + \psi)\right]$$

Okamžizá hodnota je rovna imaginární složce komplexoru:

$$i(t) = I_m \left[\hat{i}(t)\right] = I_m sin(\omega t + \psi)$$

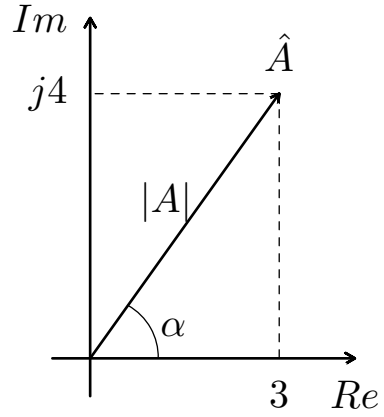
Fázor maximální hodnoty:

$$\widehat{I}_m = I_m e^{j\psi} = I_m \angle\psi = I_m(cos\psi + j\ sin\psi) = x + jy$$



Fázor efektivní hodnoty: $\widehat{I} = \frac{I_m}{\sqrt{2}} \cdot e^{j\psi}$

Př.7.1: Převeďte komplexní číslo ze složkového tvaru do exponenciálního a vektorového tvaru.



a) $\hat{A} = 3 + 4j$

$$|A| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\alpha = arctg\frac{4}{3} = 53,13°$$

$$\hat{A} = 5 \cdot e^{j53,13°} = 5\angle 53,13°$$

b) $\hat{A} = -5 + 4j$

$$|A| = 6,4031 \cdot e^{j141,34^{\cdot}} = 6,4031\angle 141,34°$$

c) $\hat{A} = 2 - 5j$

$$|A| = 5,3852 \cdot e^{-j68,2^{\cdot}} = 5,3852\angle 68,2°$$

Př.7.2: Vyjádřete k zadanému proudu $i(t) = 10 \cdot sin(\omega t + 30°)$: a) efektivní hodnotu, b) fázor ve všech tvarech, c) komplexor.

a)

$$I = \frac{10}{\sqrt{2}} A$$

b)

$$\hat{I}_m = 10 \cdot e^{j\frac{\pi}{6}} = 10\angle 30° = 10 \cdot (cos30° + j\ sin30°) = 10 \cdot \left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = (8,6602 + j5)A$$

c)

$$\hat{i}(t) = 10 \cdot e^{j\frac{\pi}{6}} \cdot e^{j\omega t} = \hat{I}_m e^{j\omega t}$$

$$\hat{I} = \frac{\hat{I}_m}{\sqrt{2}} = (6,124 + j3,536)A$$

Př.7.3: Vypočítejte pomocí symbolické metody součet harmonických proudů (stejného kmitočtu), které jsou dány okamžitými hodnotami:

$i_1(t) = 10 \cdot sin\,(\omega t + 45°)$A a $i_2(t) = 15 \cdot sin\,(\omega t + 70°)$A

$\hat{I}_{1m} = 10 \cdot e^{j45°} = 10 \cdot (cos45° + j\ sin45°) = (7,07 + j\ 7,07)A$

$\hat{I}_{2m} = 15 \cdot e^{j70°} = 15 \cdot (cos70° + j\ sin70°) = (5,13 + j\ 14,09)A$

Součtový proud je: $\hat{I} = \hat{I}_{1m} + \hat{I}_{2m} = (12,2 + j\ 21,16)A = 24,42 \cdot e^{j60,03°}A$

Okamžitá hodnota součtového proudu je:

$$i(t) = Im\left[I_m \cdot e^{j\varphi} \cdot e^{j\omega t}\right] = 24,42 \cdot sin\,(\omega t + 60,03°)\,A$$

Př.7.4: Vypočítejte pomocí symbolické metody rozdíl dvou harmonických napětí (stejného kmitočtu), která jsou dána okamžitými hodnotami:

$u_1(t) = 45 \cdot sin\,(\omega t + 25°)\text{V}$ a $u_2(t) = 15 \cdot sin\,(\omega t + 40°)\text{V}$

$$\hat{U}_{1m} = 45 \cdot e^{j25°} = (40,78 + j\ 19,02)V$$

$$\hat{U}_{2m} = 15 \cdot e^{j-40°} = (11,49 - j\ 9,64)V$$

$$\hat{U} = \hat{U}_{1m} - \hat{U}_{2m} = (29,29 + j\ 28,66)V = 40,98 \cdot e^{j44,37°}A$$

$$u(t) = Im\left[U_m \cdot e^{j\varphi} \cdot e^{j\omega t}\right] = 40,98 \cdot sin\,(\omega t + 44,37°)\,A$$

Př.7.5: Vypočítejte pomocí symbolické metody součet harmonických proudů (stejného kmitočtu), které jsou dány okamžitými hodnotami:

$$i_1(t) = 10 \cdot sin\,(\omega t + 15°)\text{A},\; i_2(t) = 15 \cdot sin\,(\omega t + 50°)\,A, i_3(t) = 6 \cdot sin\,(\omega t - 20°)\text{A}$$

$$\hat{I}_{1m} = 10 \cdot e^{j15°} = (9,66 + j\;2,59)A$$

$$\hat{I}_{2m} = 15 \cdot e^{j50°} = (9,64 + j\;11,49)A$$

$$\hat{I}_{3m} = 6 \cdot e^{j-20°} = (5,64 + j\;-2,05)A$$

$$\hat{I} = \hat{I}_{1m} + \hat{I}_{2m} + \hat{I}_{3m} = (24,94 + j\;12,03)A = 27,69 \cdot e^{j25,75°}A$$

$$i(t) = Im\left[I_m \cdot e^{j\varphi} \cdot e^{j\omega t}\right] = 27,69 \cdot sin\,(\omega t + 25,75°)\,A$$

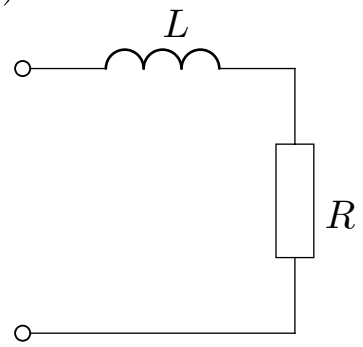
Impedance, admitance, metody pro speciální případy

Př.7.6: Vypočítejte impedanci $\hat{Z}$ reálné cívky při kmitočtu $f = 50Hz$ (Indukčnost cívky $L = 159mH$, odpor $R = 10\Omega$.

$$\begin{aligned}\hat{Z} &= R + j\omega \cdot L = R + j2\pi fL = 10 + j50 = \sqrt{10^2 + 50^2} \cdot e^{jarctg\frac{50}{10}} = \\ &= 50,99e^{j78,69^\circ} = 50,99\angle 78,69^\circ\Omega\end{aligned}$$

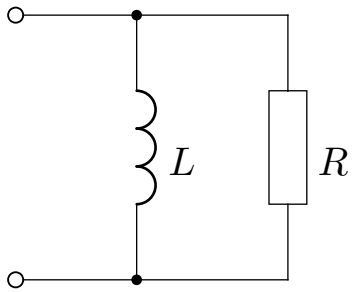
Př.7.7: Pro obvody na obrázcích a) až i) vypočtěte impedanci $\hat{Z}(j\omega)$ a admitanci $\hat{Y}(j\omega)$ při zadaných kmitočtech a pro kmitočty $\omega = 0$ a $\omega \to \infty$.

a)



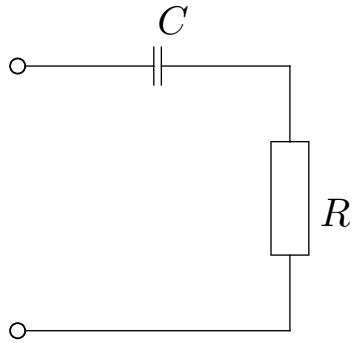
$$\begin{aligned} R &= 20\Omega \\ L &= 1,5mH \\ f &= 100Hz, 1kHz, 10kHz \end{aligned}$$

b)



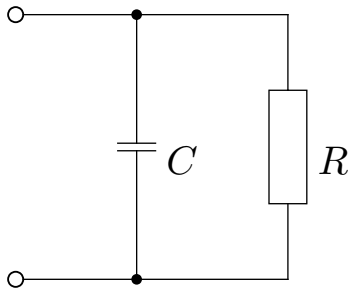
$$\begin{aligned} R &= 20\Omega \\ L &= 1,5mH \\ f &= 100Hz, 1kHz, 10kHz \end{aligned}$$

c)



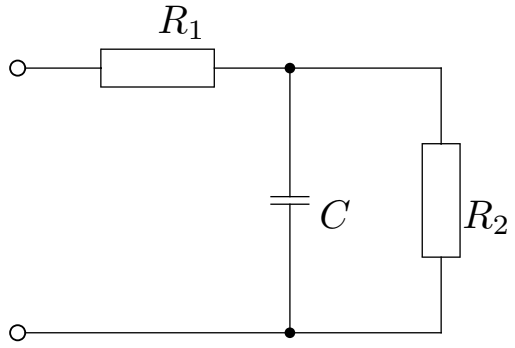
$$
\begin{aligned}
R &= 100\Omega \\
C &= 1\mu F \\
f &= 50Hz, 500Hz, 5kHz
\end{aligned}
$$

d)



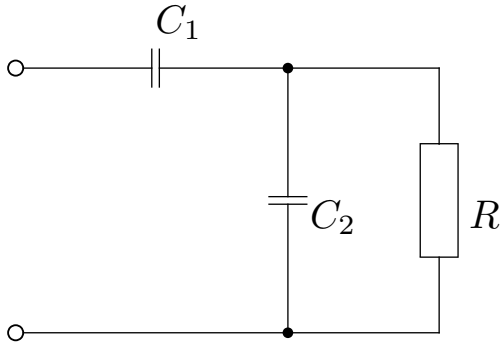
$$
\begin{aligned}
R &= 100\Omega \\
C &= 1\mu F \\
f &= 50Hz, 500Hz, 5kHz
\end{aligned}
$$

e)



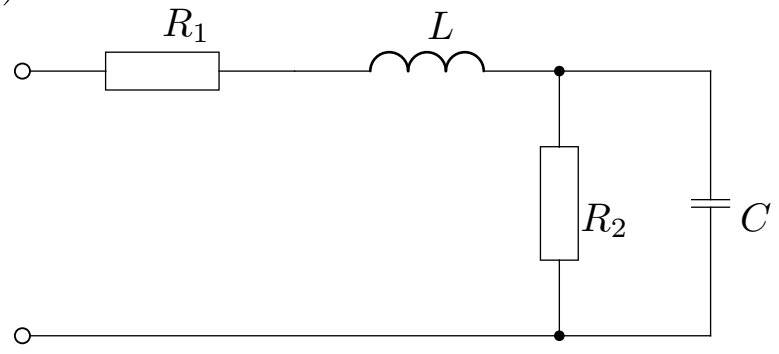
$$
\begin{aligned}
R_1 &= 800\Omega \\
R_2 &= 200\Omega \\
C &= 500pF \\
f &= 10kHz, 30kHz, 100kHz
\end{aligned}
$$

f)



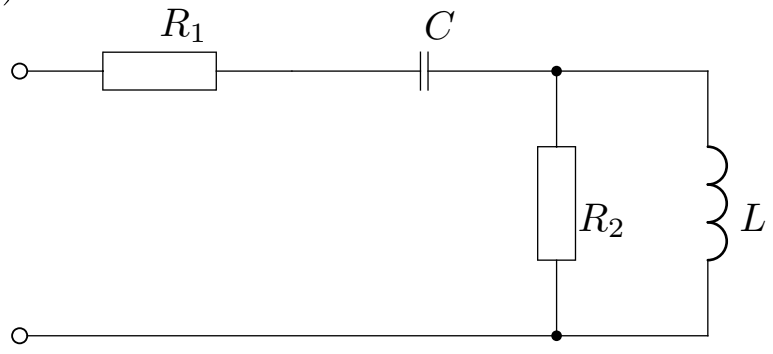
$$
\begin{aligned}
R &= 2k\Omega \\
C_1 &= 20nF \\
C_2 &= 4nF \\
f &= 1kHz, 10kHz, 100kHz
\end{aligned}
$$

g)



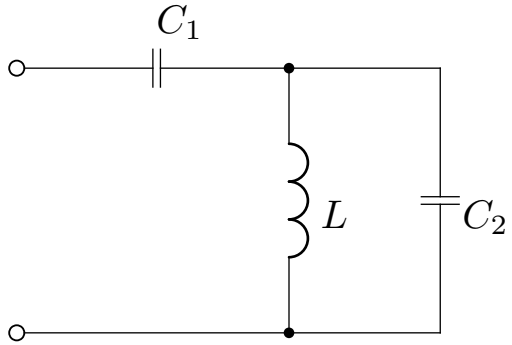
$$
\begin{aligned}
R_1 &= 10\Omega \\
R_2 &= 50\Omega \\
L &= 180\mu H \\
C &= 500pF \\
f &= 400kHz, 500kHz, 600kHz
\end{aligned}
$$

h)



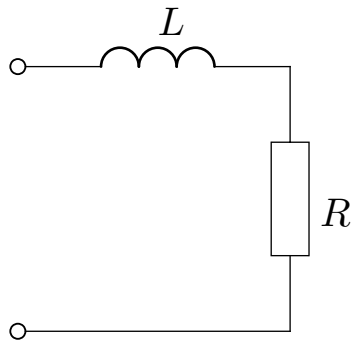
$$
\begin{aligned}
R_1 &= 1\Omega \\
R_2 &= 5\Omega \\
L &= 180\mu H \\
C &= 0,5\mu F \\
f &= 400kHz, 500kHz, 600kHz
\end{aligned}
$$

i)



$$\begin{aligned} C_1 &= C_2 = 0,8\mu F \\ L &= 1,2mH \\ f &= 3kHz, 5kHz, 10kHz \end{aligned}$$

a)

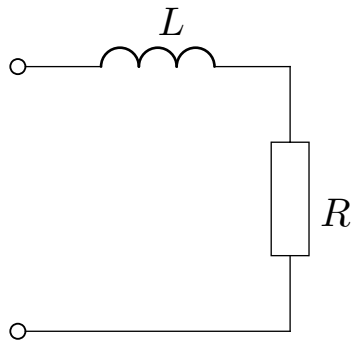


$$\begin{aligned} R &= 20\Omega \\ L &= 1,5mH \\ f &= 100Hz \end{aligned}$$

$$\hat{Z} = R + j\omega \cdot L$$

$$\begin{aligned} \hat{Z} &= (20 + 0,9425j)\Omega = 20,02\angle 2,7°\Omega \\ \hat{Y} &= (0,0499 - 0,0024j)S = 0,0499\angle -2,7°S \end{aligned}$$

a)

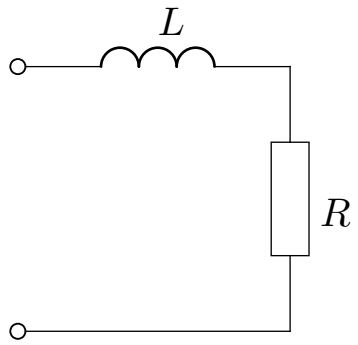


$$\begin{aligned} R &= 20\Omega \\ L &= 1,5mH \\ f &= 1kHz \end{aligned}$$

$$\hat{Z} = R + j\omega \cdot L$$

$$\begin{aligned} \hat{Z} &= (20 + 9,4248j)\Omega = 22,11\angle 25,23°\Omega \\ \hat{Y} &= (0,0409 - 0,0193j)S = 0,04523\angle -25,22°S \end{aligned}$$

a)

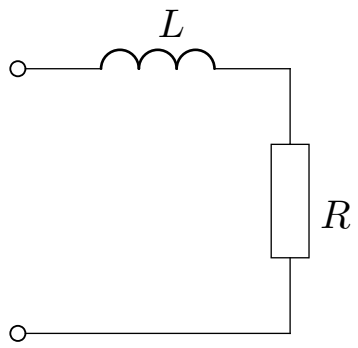


$$\begin{aligned} R &= 20\Omega \\ L &= 1,5mH \\ f &= 10kHz \end{aligned}$$

$$\hat{Z} = R + j\omega \cdot L$$

$$\begin{aligned} \hat{Z} &= (20 + 94,2478j)\Omega = 96,346\angle 78,019°\Omega \\ \hat{Y} &= (0,022 - 0,0102j)S = 0,01038\angle -78,02°S \end{aligned}$$

a)



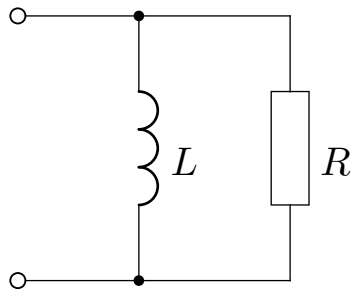
$$\begin{aligned} R &= 20\Omega \\ L &= 1,5mH \end{aligned}$$

$$\hat{Z} = R + j\omega \cdot L$$

$$\begin{aligned} \omega = 0: \qquad \hat{Z} &= 20\angle 0°\Omega \\ \hat{Y} &= 0,05\angle 0°S \end{aligned}$$

$$\begin{aligned} \omega \to \infty: \qquad \hat{Z} &\to \infty \\ \hat{Y} &= 0 \end{aligned}$$

b)

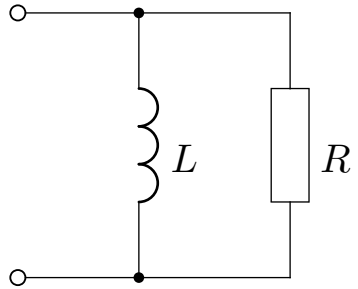


$$\hat{Z} = \frac{R \cdot j\omega \cdot L}{R + j\omega \cdot L}$$

$$\begin{aligned} R &= 20\Omega \\ L &= 1,5mH \\ f &= 100Hz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (0,0443 + 0,9404j)\Omega = 0,9415\angle 87,30°\Omega \\ \hat{Y} &= (0,05 - 1,00610j)S = 1,0622\angle -87,30°S \end{aligned}$$

b)

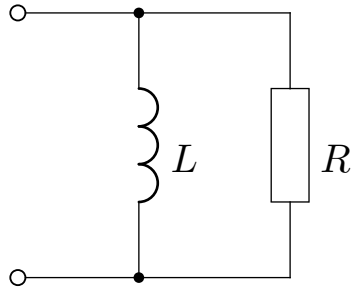


$$\hat{Z} = \frac{R \cdot j\omega \cdot L}{R + j\omega \cdot L}$$

$$\begin{aligned} R &= 20\Omega \\ L &= 1,5mH \\ f &= 1kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (3,6345 + 7,7123j)\Omega = 8,5258\angle 64,77°\Omega \\ \hat{Y} &= (0,05 - 0,1061j)S = 0,1173\angle -64,77°S \end{aligned}$$

b)

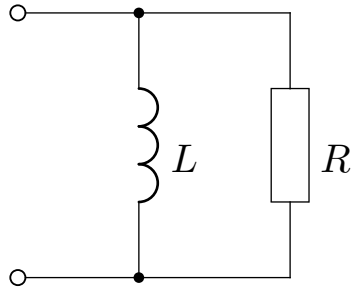


$$\hat{Z} = \frac{R \cdot j\omega \cdot L}{R + j\omega \cdot L}$$

$$\begin{aligned} R &= 20\Omega \\ L &= 1,5mH \\ f &= 10kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (19,1382 + 4,0611j)\Omega = 19,5644\angle 11,98°\Omega \\ \hat{Y} &= (0,05 - 0,0106j)S = 0,0511\angle -11,98°S \end{aligned}$$

b)



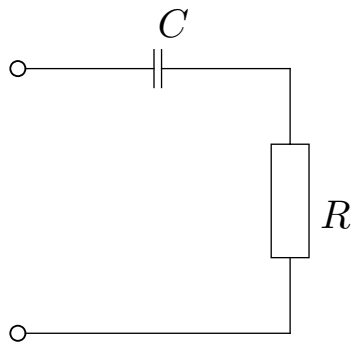
$$\begin{aligned} R &= 20\Omega \\ L &= 1,5mH \end{aligned}$$

$$\hat{Z} = \frac{R \cdot j\omega \cdot L}{R + j\omega \cdot L}$$

$$\begin{aligned} \omega = 0: \qquad \hat{Z} &= 0 \\ \hat{Y} &\to \infty \end{aligned}$$

$$\begin{aligned} \omega \to \infty: \qquad \hat{Z} &= 20\angle 0°\Omega \\ \hat{Y} &= 0,05\angle 0°S \end{aligned}$$

c)

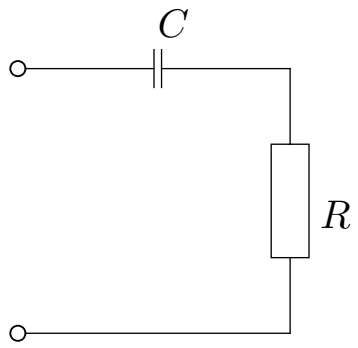


$$\hat{Z} = R + \frac{1}{j\omega \cdot C}$$

$$\begin{aligned} R &= 100\Omega \\ C &= 1\mu F \\ f &= 50Hz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (100 - 3183,098j)\Omega = 3184,669\angle -88,20°\Omega \\ \hat{Y} &= (9,86 + 313,86j) \cdot 10^{-6} S = 314.004\angle 88,20°\mu S \end{aligned}$$

c)

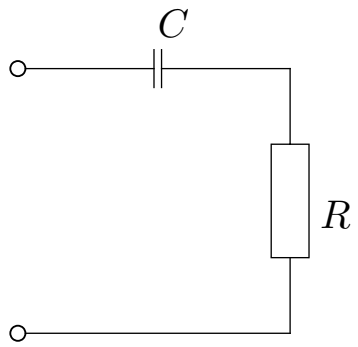


$$\hat{Z} = R + \frac{1}{j\omega \cdot C}$$

$$\begin{aligned} R &= 100\Omega \\ C &= 1\mu F \\ f &= 500Hz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (100 - 318,309j)\Omega = 336,648\angle -72,56°\Omega \\ \hat{Y} &= (0,898 + 2,859j) \cdot 10^{-3}S = 2,997\angle 72,56°mS \end{aligned}$$

c)

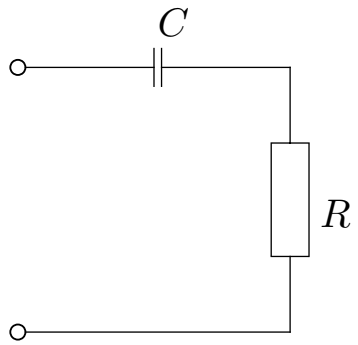


$$\hat{Z} = R + \frac{1}{j\omega \cdot C}$$

$$\begin{aligned} R &= 100\Omega \\ C &= 1\mu F \\ f &= 5kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (100 - 31,831j)\Omega = 109,944\angle -17,66°\Omega \\ \hat{Y} &= (9,080 - 2,890j) \cdot 10^{-3}S = 9,529\angle 17,66°mS \end{aligned}$$

c)



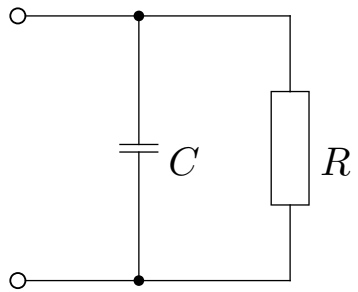
$$\begin{aligned} R &= 100\Omega \\ C &= 1\mu F \end{aligned}$$

$$\hat{Z} = R + \frac{1}{j\omega \cdot C}$$

$$\begin{aligned} \omega = 0: \qquad \hat{Z} &\rightarrow \infty \\ \hat{Y} &= 0 \end{aligned}$$

$$\begin{aligned} \omega \rightarrow \infty: \qquad \hat{Z} &= 100\angle 0^\circ\Omega \\ \hat{Y} &= 0,01\angle 0^\circ S \end{aligned}$$

d)

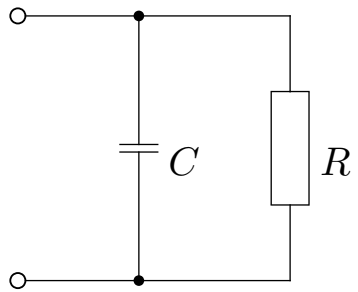


$$\hat{Z} = \frac{R \cdot \frac{1}{j\omega \cdot C}}{R + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R &= 100\Omega \\ C &= 1\mu F \\ f &= 50Hz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (99,901 - 3,1385j)\Omega = 99,9507\angle -1,8°\Omega \\ \hat{Y} &= (0,01 + 0,000314j)S = 0,010005\angle 1,80°S \end{aligned}$$

d)

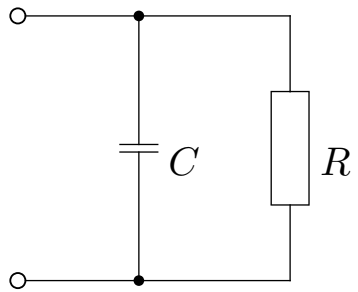


$$\hat{Z} = \frac{R \cdot \frac{1}{j\omega \cdot C}}{R + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R &= 100\Omega \\ C &= 1\mu F \\ f &= 500Hz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (91,0170 - 28,5939j)\Omega = 95,402\angle -17,44°\Omega \\ \hat{Y} &= (0,01 + 0,00314j)S = 0,01048\angle 17,44°S \end{aligned}$$

d)

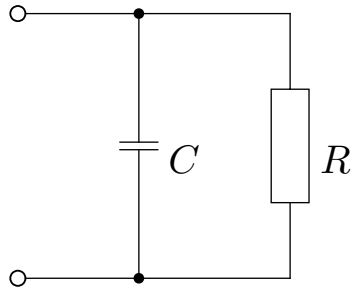


$$\hat{Z} = \frac{R \cdot \frac{1}{j\omega \cdot C}}{R + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R &= 100\Omega \\ C &= 1\mu F \\ f &= 5kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (9,199 - 28,902j)\Omega = 30,331\angle -72,34°\Omega \\ \hat{Y} &= (0,01 + 0,0314j)S = 0,0329\angle 72,34°S \end{aligned}$$

d)



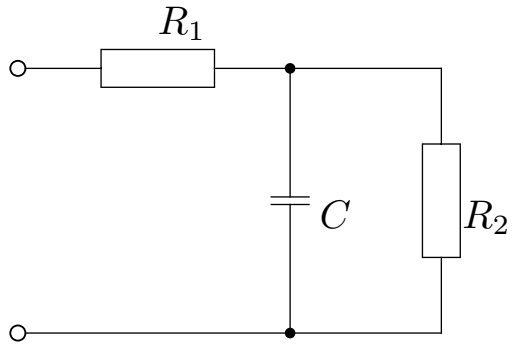
$$\hat{Z} = \frac{R \cdot \frac{1}{j\omega \cdot C}}{R + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R &= 100\Omega \\ C &= 1\mu F \end{aligned}$$

$$\omega = 0: \qquad \begin{aligned} \hat{Z} &= 100\angle 0°\Omega \\ \hat{Y} &= 0,01\angle 0°S \end{aligned}$$

$$\omega \to \infty: \qquad \begin{aligned} \hat{Z} &= 0 \\ \hat{Y} &\to \infty \end{aligned}$$

e)

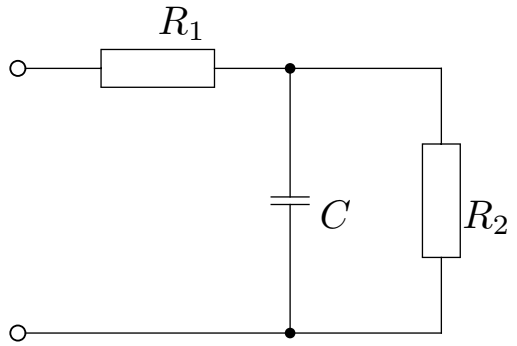


$$\hat{Z} = R_1 + \frac{R_2 \cdot \frac{1}{j\omega \cdot C}}{R_2 + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R_1 &= 800\Omega \\ R_2 &= 200\Omega \\ C &= 5nF \\ f &= 10kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (999,2135 - 12,5170j)\Omega = 999,292\angle -0,72°\Omega \\ \hat{Y} &= (1,00063 + 0,01254j) \cdot 10^{-3}S = 1,000709\angle 0,72°mS \end{aligned}$$

e)

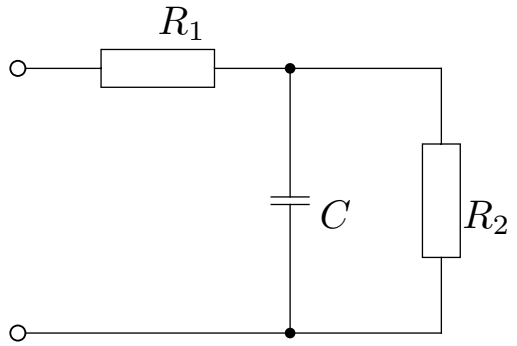


$$\hat{Z} = R_1 + \frac{R_2 \cdot \frac{1}{j\omega \cdot C}}{R_2 + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R_1 &= 800\Omega \\ R_2 &= 200\Omega \\ C &= 5nF \\ f &= 30kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (999,1377 - 36,4056j)\Omega = 993,8047\angle -2,10°\Omega \\ \hat{Y} &= (1,00556 + 0,03686j) \cdot 10^{-3}S = 1,006234\angle 2,10°mS \end{aligned}$$

e)

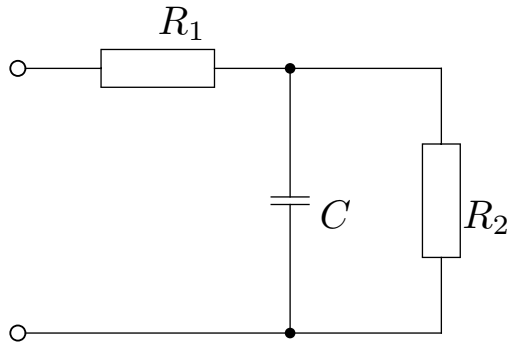


$$\hat{Z} = R_1 + \frac{R_2 \cdot \frac{1}{j\omega \cdot C}}{R_2 + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R_1 &= 800\Omega \\ R_2 &= 200\Omega \\ C &= 5nF \\ f &= 100kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (943,3914 - 90,0954j)\Omega = 947,6837\angle -5,46°\Omega \\ \hat{Y} &= (1,0504 + 0,1003j) \cdot 10^{-3}S = 1,0552\angle 5,46°S \end{aligned}$$

e)



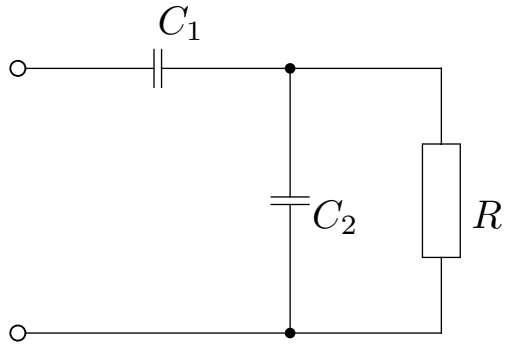
$$\hat{Z} = R_1 + \frac{R_2 \cdot \frac{1}{j\omega \cdot C}}{R_2 + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R_1 &= 800\Omega \\ R_2 &= 200\Omega \\ C &= 5nF \end{aligned}$$

$$\omega = 0: \qquad \begin{aligned} \hat{Z} &= R_1 + R_2 = 1000\angle 0°\Omega \\ \hat{Y} &= 1\angle 0°mS \end{aligned}$$

$$\omega \to \infty: \qquad \begin{aligned} \hat{Z} &= R_1 = 200\angle 0°\Omega \\ \hat{Y} &= 5\angle 0°mS \end{aligned}$$

f)

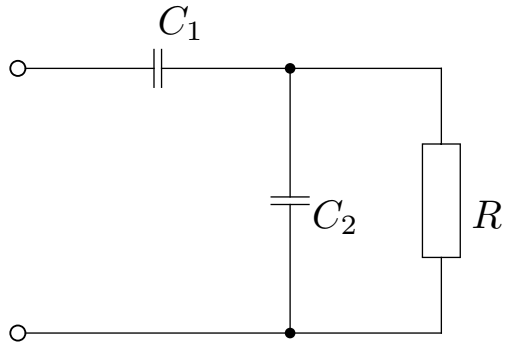


$$\hat{Z} = \frac{1}{j\omega \cdot C_1} + \frac{R \cdot \frac{1}{j\omega \cdot C_2}}{R + \frac{1}{j\omega \cdot C_2}}$$

$$\begin{aligned} R &= 2k\Omega \\ C_1 &= 20nF \\ C_2 &= 4nF \\ f &= 1kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (1994,959 - 8058,0248j)\Omega = 8301,3027\angle -76,09°\Omega \\ \hat{Y} &= (1,701 + 130,224j) \cdot 10^{-6}S = 130,235\angle 89,25°\mu S \end{aligned}$$

f)

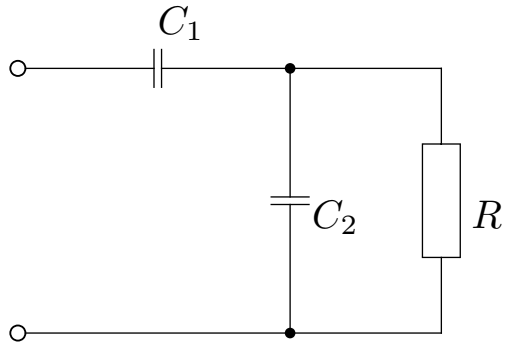


$$\hat{Z} = \frac{1}{j\omega \cdot C_1} + \frac{R \cdot \frac{1}{j\omega \cdot C_2}}{R + \frac{1}{j\omega \cdot C_2}}$$

$$\begin{aligned} R &= 2k\Omega \\ C_1 &= 20nF \\ C_2 &= 4nF \\ f &= 10kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (1596,60 - 1598,313j)\Omega = 2259,145\angle -45,03°\Omega \\ \hat{Y} &= (0,3128 + 0,3132) \cdot 10^{-3}S = 0,4426\angle 45,03°mS \end{aligned}$$

f)

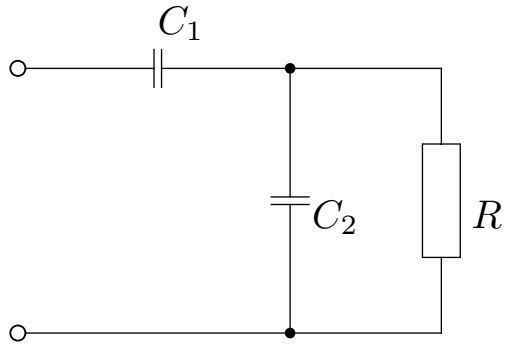


$$\hat{Z} = \frac{1}{j\omega \cdot C_1} + \frac{R \cdot \frac{1}{j\omega \cdot C_2}}{R + \frac{1}{j\omega \cdot C_2}}$$

$$\begin{aligned} R &= 2k\Omega \\ C_1 &= 20nF \\ C_2 &= 4nF \\ f &= 100kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (76,1435 - 462,3166j)\Omega = 468,545\angle -80,65°\Omega \\ \hat{Y} &= (0,34684 + 2,1059j) \cdot 10^{-3}S = 2,1343\angle 80,65°mS \end{aligned}$$

f)



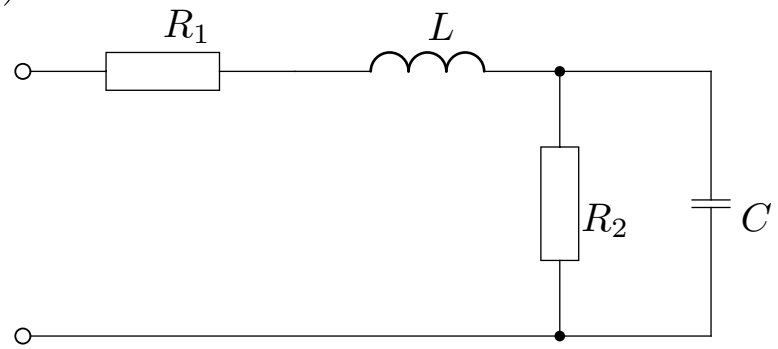
$$\hat{Z} = \frac{1}{j\omega \cdot C_1} + \frac{R \cdot \frac{1}{j\omega \cdot C_2}}{R + \frac{1}{j\omega \cdot C_2}}$$

$$\begin{aligned} R &= 2k\Omega \\ C_1 &= 20nF \\ C_2 &= 4nF \end{aligned}$$

$$\begin{aligned} \omega = 0: \qquad \hat{Z} &\to \infty \\ \hat{Y} &= 0 \end{aligned}$$

$$\begin{aligned} \omega \to \infty: \qquad \hat{Z} &= 0 \\ \hat{Y} &\to \infty \end{aligned}$$

g)

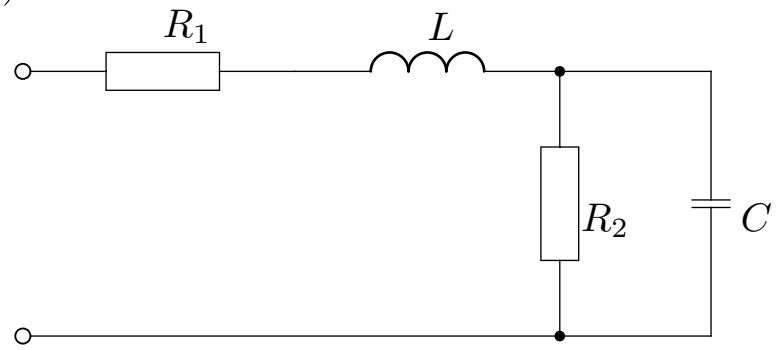


$$\hat{Z} = R_1 + j\omega \cdot L + \frac{R_2 \cdot \frac{1}{j\omega \cdot C}}{R_2 + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R_1 &= 10\Omega \\ R_2 &= 50k\Omega \\ L &= 180\mu H \\ C &= 500pF \\ f &= 400kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (22,66 - 343,18j)\Omega = 343,93\angle -86,22°\Omega \\ \hat{Y} &= (0,1916 + 2,9013j) \cdot 10^{-3}S = 2,9076\angle 86,22°mS \end{aligned}$$

g)

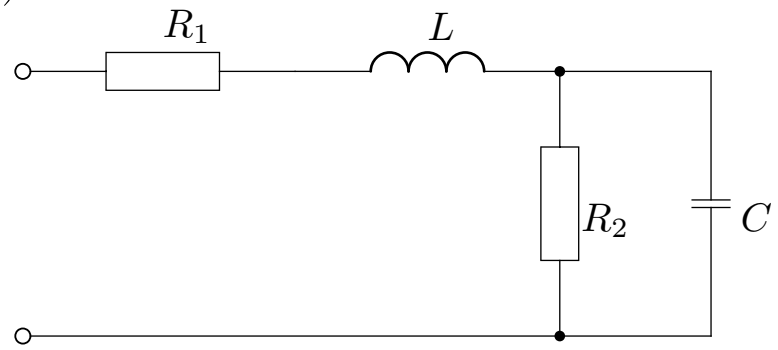


$$\hat{Z} = R_1 + j\omega \cdot L + \frac{R_2 \cdot \frac{1}{j\omega \cdot C}}{R_2 + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R_1 &= 10\Omega \\ R_2 &= 50k\Omega \\ L &= 180\mu H \\ C &= 500pF \\ f &= 500kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (18,104 - 71,028j)\Omega = 73,299\angle -75,70°\Omega \\ \hat{Y} &= (3,3696 + 13,2201j) \cdot 10^{-3}S = 13,6428\angle 75,70°mS \end{aligned}$$

g)

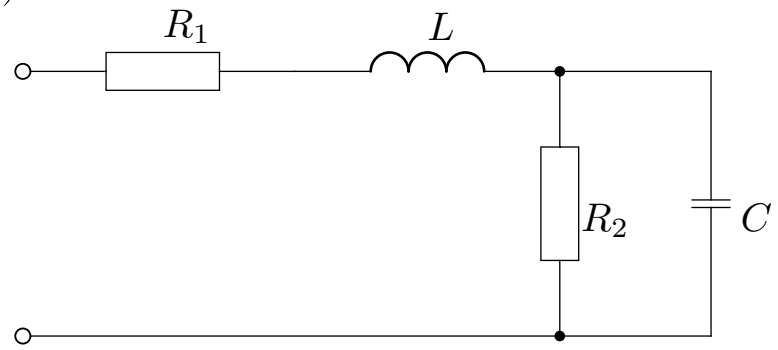


$$\hat{Z} = R_1 + j\omega \cdot L + \frac{R_2 \cdot \frac{1}{j\omega \cdot C}}{R_2 + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R_1 &= 10\Omega \\ R_2 &= 50k\Omega \\ L &= 180\mu H \\ C &= 500pF \\ f &= 600kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (15,6283 + 148,1295j)\Omega = 148,952\angle 83,98°\Omega \\ \hat{Y} &= (0,7044 - 6,6766j) \cdot 10^{-3}S = 6,7136\angle -83,98°mS \end{aligned}$$

g)



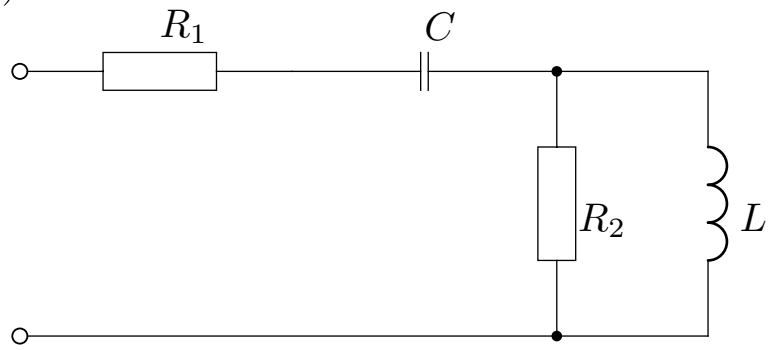
$$\hat{Z} = R_1 + j\omega \cdot L + \frac{R_2 \cdot \frac{1}{j\omega \cdot C}}{R_2 + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} R_1 &= 10\Omega \\ R_2 &= 50k\Omega \\ L &= 180\mu H \\ C &= 500pF \end{aligned}$$

$$\begin{aligned} \omega = 0: \qquad \hat{Z} &= R_1 + R_2 = 50010\angle 0^\circ\Omega \\ \hat{Y} &= 19,996\angle 0^\circ\mu S \end{aligned}$$

$$\begin{aligned} \omega \to \infty: \qquad \hat{Z} &\to \infty \\ \hat{Y} &= 0 \end{aligned}$$

h)

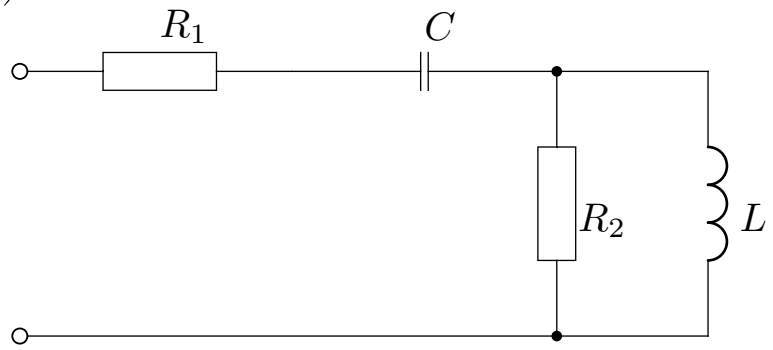


$$\hat{Z} = R_1 + \frac{1}{j\omega \cdot C} + \frac{R_2 \cdot j\omega \cdot L}{R_2 + j\omega \cdot L}$$

$$\begin{aligned} R_1 &= 1\Omega \\ R_2 &= 5\Omega \\ L &= 180\mu H \\ C &= 500pF \\ f &= 400kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (1,0406 + 1,2445j)\Omega = 1,6222\angle 50,10°\Omega \\ \hat{Y} &= (0,3954 - 0,4759j)S = 0,6164\angle -50,10°S \end{aligned}$$

h)

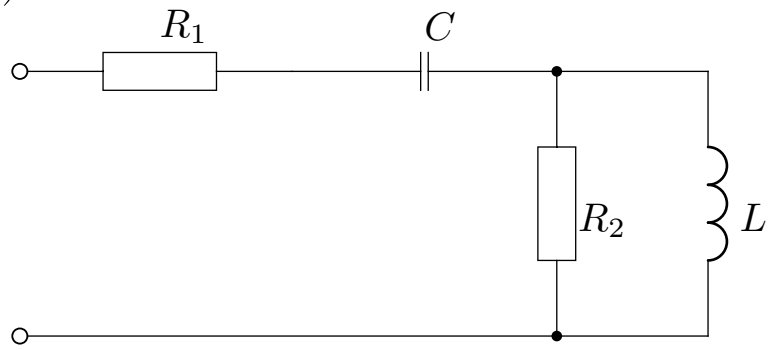


$$\hat{Z} = R_1 + \frac{1}{j\omega \cdot C} + \frac{R_2 \cdot j\omega \cdot L}{R_2 + j\omega \cdot L}$$

$$\begin{aligned} R_1 &= 1\Omega \\ R_2 &= 5\Omega \\ L &= 180\mu H \\ C &= 500pF \\ f &= 500kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (1,0631 - 0,0783j)\Omega = 1,0660\angle -4,21°\Omega \\ \hat{Y} &= (0,9355 + 0,0689j)S = 0,9381\angle 4,21°S \end{aligned}$$

h)

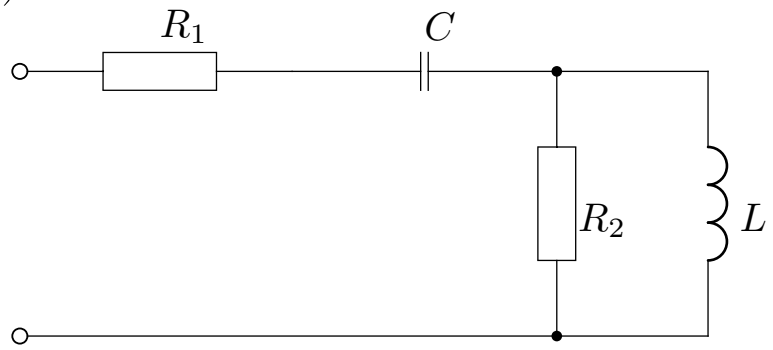


$$\hat{Z} = R_1 + \frac{1}{j\omega \cdot C} + \frac{R_2 \cdot j\omega \cdot L}{R_2 + j\omega \cdot L}$$

$$\begin{aligned} R_1 &= 1\Omega \\ R_2 &= 5\Omega \\ L &= 180\mu H \\ C &= 500pF \\ f &= 600kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= (1,0905 + 0,1358j)\Omega = 1,0989\angle 7,10^\circ\Omega \\ \hat{Y} &= (0,9031 - 0,1125j)S = 0,9100\angle -7,10^\circ S \end{aligned}$$

h)



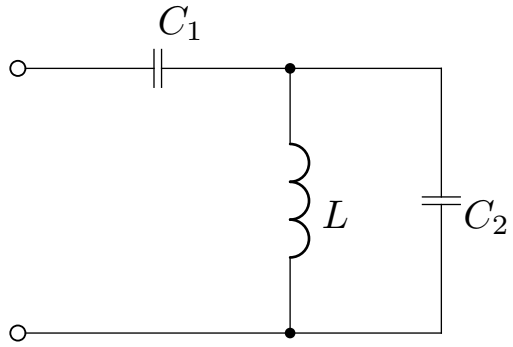
$$\hat{Z} = R_1 + \frac{1}{j\omega \cdot C} + \frac{R_2 \cdot j\omega \cdot L}{R_2 + j\omega \cdot L}$$

$$\begin{aligned} R_1 &= 1\Omega \\ R_2 &= 5\Omega \\ L &= 180\mu H \\ C &= 500pF \end{aligned}$$

$$\omega = 0: \qquad \begin{aligned} \hat{Z} &\rightarrow \infty \\ \hat{Y} &= 0 \end{aligned}$$

$$\omega \rightarrow \infty: \qquad \begin{aligned} \hat{Z} &= R_1 + R_2 = 6\angle 0^\circ\Omega \\ \hat{Y} &= 0,16\angle 0^\circ S \end{aligned}$$

i)

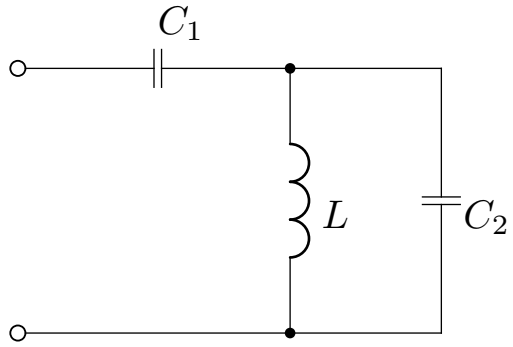


$$\hat{Z} = \frac{1}{j\omega \cdot C_1} + \frac{j\omega \cdot L \cdot \frac{1}{j\omega \cdot C_2}}{j\omega \cdot L + \frac{1}{j\omega \cdot C_2}}$$

$$\begin{aligned} C_1 &= 0,8\mu F \\ C_2 &= 0,8\mu F \\ L &= 1,2mH \\ f &= 3kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= -31,9858j\Omega = 31,9858\angle -90°\Omega \\ \hat{Y} &= 31,2639j \cdot 10^{-3}S = 31,2639\angle 90°mS \end{aligned}$$

i)

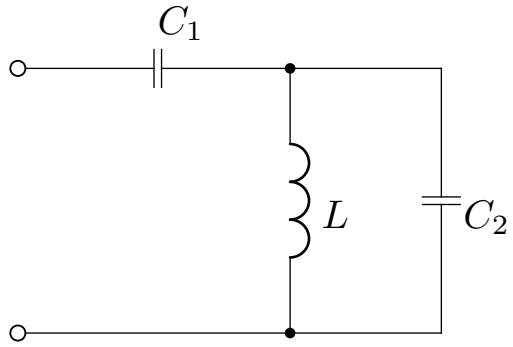


$$\hat{Z} = \frac{1}{j\omega \cdot C_1} + \frac{j\omega \cdot L \cdot \frac{1}{j\omega \cdot C_2}}{j\omega \cdot L + \frac{1}{j\omega \cdot C_2}}$$

$$\begin{aligned} C_1 &= 0,8\mu F \\ C_2 &= 0,8\mu F \\ L &= 1,2mH \\ f &= 5kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= 678,0438j \cdot 10^{-3}\Omega = 678,0438\angle 90°\Omega \\ \hat{Y} &= -1,4748j \cdot 10^{-3}S = 1,4748\angle -90°mS \end{aligned}$$

i)

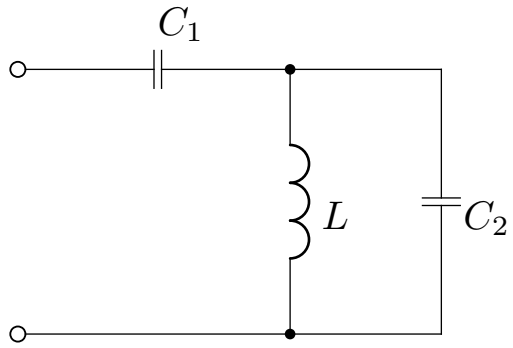


$$\hat{Z} = \frac{1}{j\omega \cdot C_1} + \frac{j\omega \cdot L \cdot \frac{1}{j\omega \cdot C_2}}{j\omega \cdot L + \frac{1}{j\omega \cdot C_2}}$$

$$\begin{aligned} C_1 &= 0,8\mu F \\ C_2 &= 0,8\mu F \\ L &= 1,2mH \\ f &= 10kHz \end{aligned}$$

$$\begin{aligned} \hat{Z} &= -46,9195j\Omega = 46,9195\angle -90°\Omega \\ \hat{Y} &= 21,3131j \cdot 10^{-3}S = 21,3131\angle 90°mS \end{aligned}$$

i)



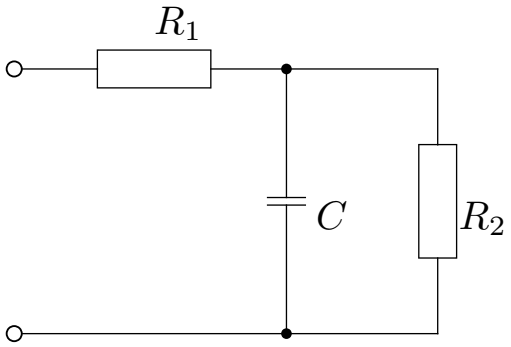
$$\begin{aligned} C_1 &= 0,8\mu F \\ C_2 &= 0,8\mu F \\ L &= 1,2mH \end{aligned}$$

$$\hat{Z} = \frac{1}{j\omega \cdot C_1} + \frac{j\omega \cdot L \cdot \frac{1}{j\omega \cdot C_2}}{j\omega \cdot L + \frac{1}{j\omega \cdot C_2}}$$

$$\begin{aligned} \omega = 0: \qquad \hat{Z} &\to \infty \\ \hat{Y} &= 0 \end{aligned}$$

$$\begin{aligned} \omega \to \infty: \qquad \hat{Z} &= 0 \\ \hat{Y} &\to \infty \end{aligned}$$

Př.7.8: Vypočtěte celkovou impedanci a admitanci obvodu při kmitočtu $f = 30HZ$.

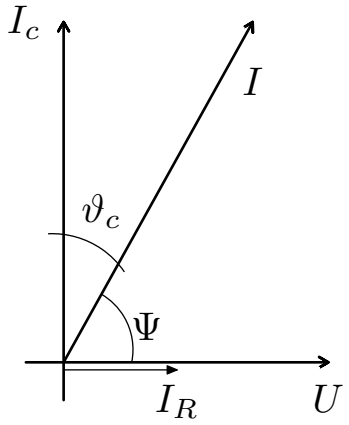


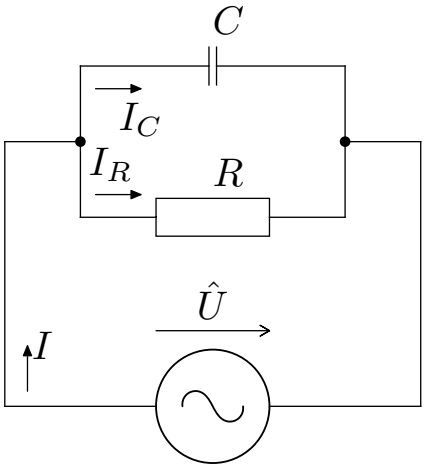
$$\begin{aligned} R_1 &= 800\Omega \\ R_2 &= 200\Omega \\ C &= 5nF \end{aligned}$$

$$\hat{Z} = R_1 + \frac{R_2 \cdot \frac{1}{j\omega \cdot C}}{R_2 + \frac{1}{j\omega \cdot C}}$$

$$\begin{aligned} \hat{Z} &= (993,1377 - 36,4056j)\Omega = 993,8047 \cdot e^{j2,1^\circ}\Omega \\ \hat{Y} &= (1,00556 + 0,03686j) \cdot 10^{-3}S = 1,006234 \cdot e^{-j2,1^\circ}mS \end{aligned}$$

Př.7.9: Vypočítejte admitanci $\hat{Y}$ paralelního zapojení rezistoru a kondenzátoru. Vypočítejte $\hat{I}_R, \hat{I}_C, \hat{I}$ a ztrátový úhel. $C = 20\mu F, R = 100\Omega, f = 500Hz, \hat{U} = 100V$





$$\hat{Y} = \frac{1}{R} + \frac{1}{\frac{1}{j\omega C}} = \frac{1}{R} + j\omega C = G + jB_C = \frac{1}{100} + 2 \cdot 2\pi \cdot 500 \cdot 20 \cdot 10^{-6} = (0,01 + 0,0628j)S$$

$$\hat{I}_R = \frac{U}{R} = \frac{100}{100} = 1A$$

$$\hat{I}_C = U \cdot j\omega C = 100 \cdot j \cdot 0,0628 = 6,28j\ A$$

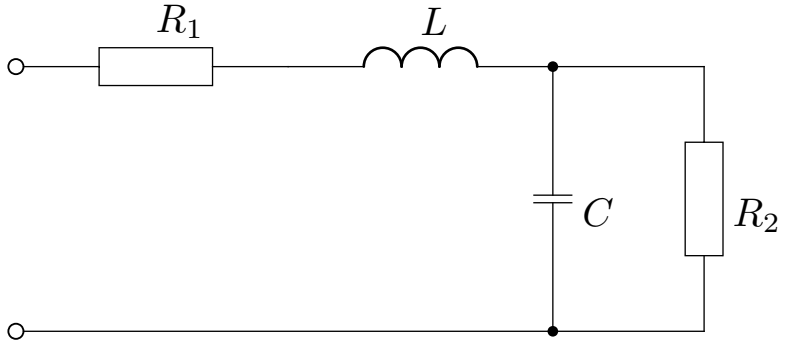
$$\hat{I} = 1 + 6,28j = \sqrt{1^2 + 6,28^2} \cdot e^{j \cdot arctg\frac{6,28}{1}} = 6,36 \cdot e^{j \cdot 80,95^\circ} A$$

$$\hat{Z} = \frac{1}{\hat{Y}} = \frac{\hat{U}}{\hat{I}} = \frac{100}{6,36\angle 80,95^\circ} = 15,7\angle -80,95^\circ\Omega$$

Ztrátový úhel:

$$\vartheta_C = 90^\circ - 80,95^\circ = 9,05^\circ$$

Př.7.10: Vypočítejte celkovou impedanci $\hat{Z}$ obvodu při kmitočtu $f = 250Hz$. $R_1 = 47\Omega, R_2 = 22\Omega, C = 20\mu F, L = 18mH$



$$\hat{Z} = R_1 + j\omega \cdot L + \frac{R_2 \cdot \frac{1}{j\omega \cdot C}}{R_2 \cdot \frac{1}{j\omega \cdot C}} = 47 + j \cdot 2\pi \cdot 250 \cdot 18 \cdot 10^{-3} + \frac{22 \cdot \frac{1}{j \cdot 2\pi \cdot 250 \cdot 20 \cdot 10^{-6}}}{22 + \frac{1}{j \cdot 2\pi \cdot 250 \cdot 20 \cdot 10^{-6}}} =$$

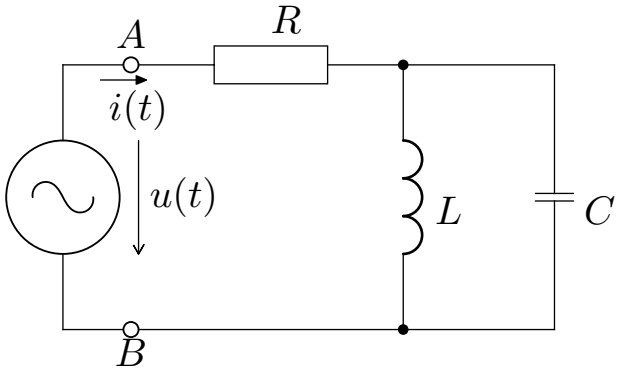
$$= (61,888 + 17,984j)\Omega = 64,448 \cdot e^{j \cdot 16,2^\circ}\Omega$$

Př.7.11: Pro uvedený obvod určete:

a) Celkovou impedanci zátěže Z_{ab} obecně i numericky

b) Amplitudu celkového proudu I_m a jeho fázy φ_i

c) Celkový výkon zátěže (komplexní, činný, jalový, zdánlivý)



$$\begin{aligned} u(t) &= U_m sin(\omega t + \varphi_m), U_m = 314V \\ \varphi_m &= 0, f = 50Hz \\ R &= 100\Omega \\ C &= 20\mu F \\ L &= 0,2H \end{aligned}$$

$$\begin{aligned} \hat{Z}_{ab} &= R + \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = R - j\frac{\frac{L}{C}}{\omega L - \frac{1}{\omega C}} = 100 - j\frac{\frac{0,2}{10 \cdot 10^{-6}}}{2\pi \cdot 50 \cdot 0,2 - \frac{1}{2\pi \cdot 50 \cdot 10 \cdot 10^{-6}}} \\ &= (100 + 78,2846j)\Omega = 126,99796 \cdot e^{j \cdot 38,05^\circ}\Omega \end{aligned}$$

$$\hat{I}_m = \frac{\hat{U}_m}{\hat{Z}_{ab}} = \frac{314}{100 + 78,2846j} = (1,9469 - 1,15241j)A = 2,4725 \cdot e^{-j38,05^\circ} A$$

$$\begin{aligned} \hat{S} &= \hat{U} \cdot \hat{I}^\bullet = 314 \cdot 1,7483 \cdot e^{-j38,05^\circ} = 549,0604 \cdot e^{-j38,05^\circ} \\ &= (432,3703 - 338,412779j)VA \end{aligned}$$

$$P = Re|\hat{S}| = 432,3703W$$

$$Q = Im|\hat{S}| = 338,412779VAr$$

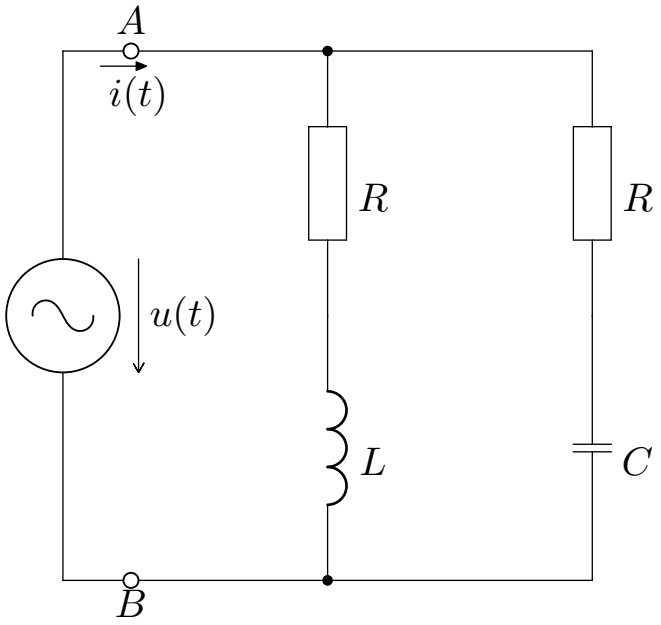
$$|\hat{S}| = 549,0604VA$$

Př.7.12: Pro uvedený obvod určete:

a) Celkovou impedanci zátěže Z_{ab} obecně i numericky

b) Amplitudu celkového proudu I_m a jeho fázy φ_i

c) Celkový výkon zátěže (komplexní, činný, jalový, zdánlivý)



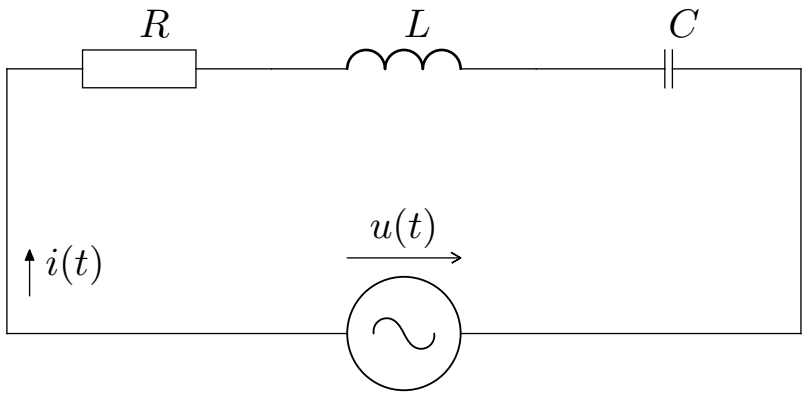
$$\begin{aligned} u(t) &= U_m sin(\omega t + \varphi_m), U_m = 314V \\ \varphi_m &= 0, f = 50Hz \\ R &= 60\Omega \\ C &= 100\mu F \\ L &= 0,36H \end{aligned}$$

$$\hat{Z}_{ab} = \frac{(R + j\omega L) \cdot (R + \frac{1}{j\omega C})}{R + j\omega L + R + \frac{1}{j\omega C}} = \frac{60 + j \cdot 2\pi \cdot 50 \cdot 0,36) \cdot (60 + \frac{1}{j \cdot 2\pi \cdot 50 \cdot 100 \cdot 10^{-6}})}{(60 + j \cdot 2\pi \cdot 50 \cdot 0,36 + 60 + \frac{1}{j \cdot 2\pi \cdot 50 \cdot 100 \cdot 10^{-6}}} =$$

$$= (67,3952 - 9,6548j)\Omega = 68,0833 \cdot e^{-j \cdot 8,15^\circ}\Omega$$

$$\begin{aligned}
\hat{I}_m &= \frac{\hat{U}_m}{\hat{Z}_{ab}} = \frac{314}{67,3952 - 9,6548j} = (4,5654 + 0,654j)A = 4,6119 \cdot e^{j \cdot 8,15^\circ} \\
\hat{S} &= \hat{U} \cdot \hat{I}^\bullet = 314 \cdot 3,2612 \cdot e^{j8,15^\circ} = 1024,6599 \cdot e^{j8,15^\circ} \\
&= (1013,6599 - 145,213j)VA \\
\\
P &= Re|\hat{S}| = 1013,6599W \\
\\
Q &= Im|\hat{S}| = 145,213VAr \\
\\
|\hat{S}| &= 1024,6599VA
\end{aligned}$$

Př.7.13: Vypočítejte proud tekoucí obvodem.



$$\begin{aligned} u(t) &= 14,1 sin\omega t \\ f &= 150 Hz \\ R &= 20\Omega \\ C &= 53\mu F \\ L &= 31,8 mH \end{aligned}$$

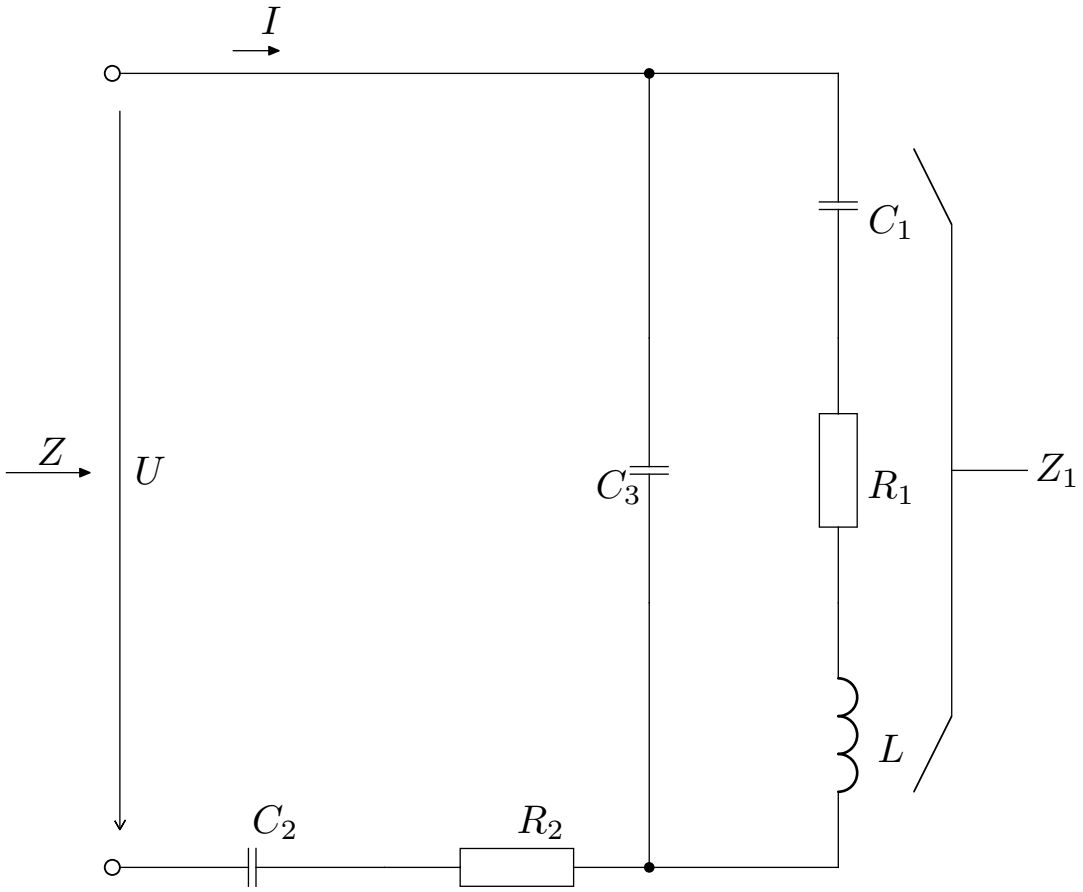
$$\hat{Z} = R + j\omega L + \frac{1}{j\omega C} = 20 + j \cdot 2\pi \cdot 150 \cdot 31,8 \cdot 10^{-3} + \frac{1}{j \cdot 2\pi \cdot 150 \cdot 53 \cdot 10^{-6}} =$$

$$= (20 + 9,9513j)\Omega = 22,3389 \cdot e^{-j \cdot 26,45^\circ}\Omega$$

$$\hat{I}_m = \frac{\hat{U}_m}{\hat{Z}} = \frac{14,1}{20 + 9,9513j} = (0,5651 - 0,2812j)A = 0,6312 \cdot e^{-j \cdot 26,45^\circ}$$

$$i(t) = 0,6312 \cdot sin(2 \cdot \pi \cdot 150 \cdot t - 26,45)$$

Př.7.14: Vypočítejte zjednodušováním obvodu vstupní impedanci $\hat{Z}$. Určete proud $\hat{I}$.



$$
\begin{aligned}
U &= 220V \\
f &= 50Hz \\
R_1 &= 300\Omega \\
R_2 &= 100\Omega \\
C_1 &= 5\mu F \\
C_2 &= 10\mu F \\
C_3 &= 2\mu F \\
L &= 1H
\end{aligned}
$$

$$\begin{aligned}\hat{Z}_1 &= R_1 + j\omega L + \frac{1}{j\omega C_1} = 300 + j(2\pi \cdot 50 \cdot 1 - \frac{1}{2\pi \cdot 50 \cdot 5 \cdot 10^{-6}}) = \\ &= (300 - 322,46j)\Omega = 440,43\angle -47,07°\Omega\end{aligned}$$

$$\begin{aligned}\hat{Z}_2 &= R_2 + \frac{1}{j\omega C_2} = 100 + \frac{1}{j \cdot 2\pi \cdot 50 \cdot 10 \cdot 10^{-6}}) = \\ &= (100 - 318,31j)\Omega = 333,648\angle 72,56°\Omega\end{aligned}$$

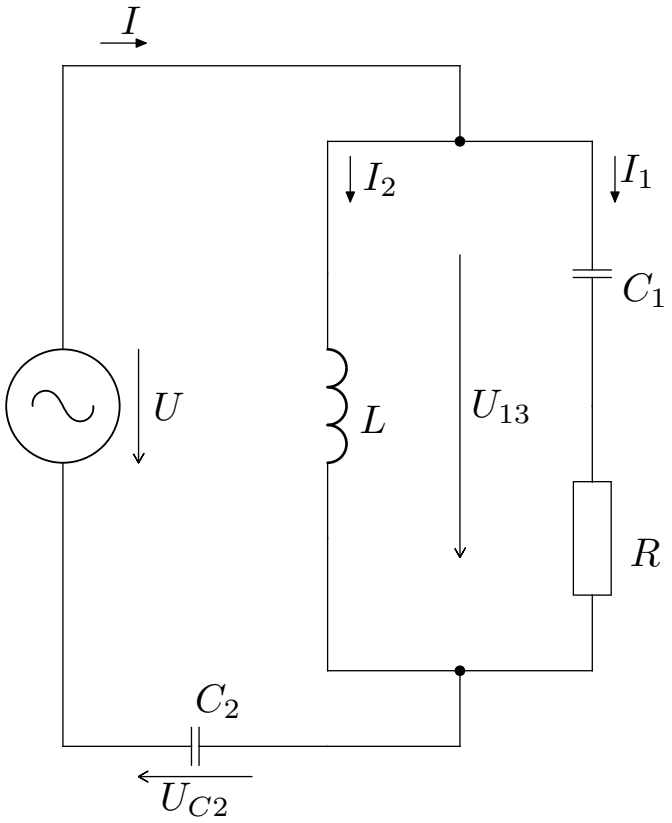
$$\hat{Z}_3 = \frac{1}{j\omega C_3} = \frac{1}{j \cdot 2\pi \cdot 50 \cdot 2 \cdot 10^{-6}}) = -1591,55j\Omega = 1591,55\angle -90°\Omega$$

$$\begin{aligned}\hat{Z}_{13} &= \frac{\hat{Z}_1 \cdot \hat{Z}_3}{\hat{Z}_1 + \hat{Z}_3} = \frac{440,43\angle -47,07^\circ \cdot 1591,55\angle -90^\circ}{440,43\angle -47,07^\circ + 1591,55\angle -90^\circ} = \\ &= 361,8148\angle 55,98^\circ\Omega = (202,44 - 299,88j)\Omega\end{aligned}$$

$$\begin{aligned}\hat{Z} &= \hat{Z}_2 + \hat{Z}_{13} = 100 - 318,31j + 202,44 - 299,88j = \\ &= (302,44 - 618,19j)\Omega = 688,21\angle -63,13^\circ\Omega\end{aligned}$$

$$\hat{I} = \frac{\hat{U}}{\hat{Z}} = \frac{220}{688,21\angle -63,13^\circ} = 0,3197\angle 63,93^\circ A = (0,14 + 0,287j)A$$

Př.7.15: Určete proudy $\hat{I}_1, \hat{I}_5, \hat{I}$ a napětí$\hat{I}_{C2}, \hat{I}_{13}$, je-li dáno:



$$\begin{aligned} \hat{U} &= 10V \\ f &= 50Hz \\ R &= 2k\Omega \\ C_1 &= 1,59\mu F \\ C_2 &= 3,18\mu F \\ L &= 6,36H \end{aligned}$$

$$\begin{aligned}
\hat{Z}_1 &= R - j\frac{1}{\omega C_1} = 2 \cdot 10^3 - j\frac{1}{2\pi \cdot 50 \cdot 1,59 \cdot 10^{-6}} = \\
&= (2000 - 2001j)\Omega = 2829,8 \cdot e^{-j45^\circ}
\end{aligned}$$

$$\begin{aligned}
\hat{Z}_{13} &= \frac{\hat{Z}_1 \cdot j\omega L}{\hat{Z}_1 + j\omega L} = \frac{(2000 - 2001j) \cdot j \cdot c\pi \cdot 50 \cdot 6,36}{(2000 - 2001j) + j \cdot c\pi \cdot 50 \cdot 6,36} = \\
&= (1996 + 2001,94j)\Omega = 2827 \cdot e^{j45^\circ}\Omega
\end{aligned}$$

$$\begin{aligned}
\hat{Z} &= \hat{Z}_{13} - j\frac{1}{\omega C_2} = (1996 + 2001,94j) - j\frac{1}{2\pi \cdot 50 \cdot 3,18 \cdot 10^{-6}} = \\
&= (1996 + 1000,967j)\Omega = 2233 \cdot e^{j26,6^\circ}\Omega
\end{aligned}$$

$$\hat{I} = \frac{\hat{U}}{\hat{Z}} = \frac{10}{2233 \cdot e^{j26,6^\circ}} = 4,478 \cdot e^{-j26,6^\circ} A$$

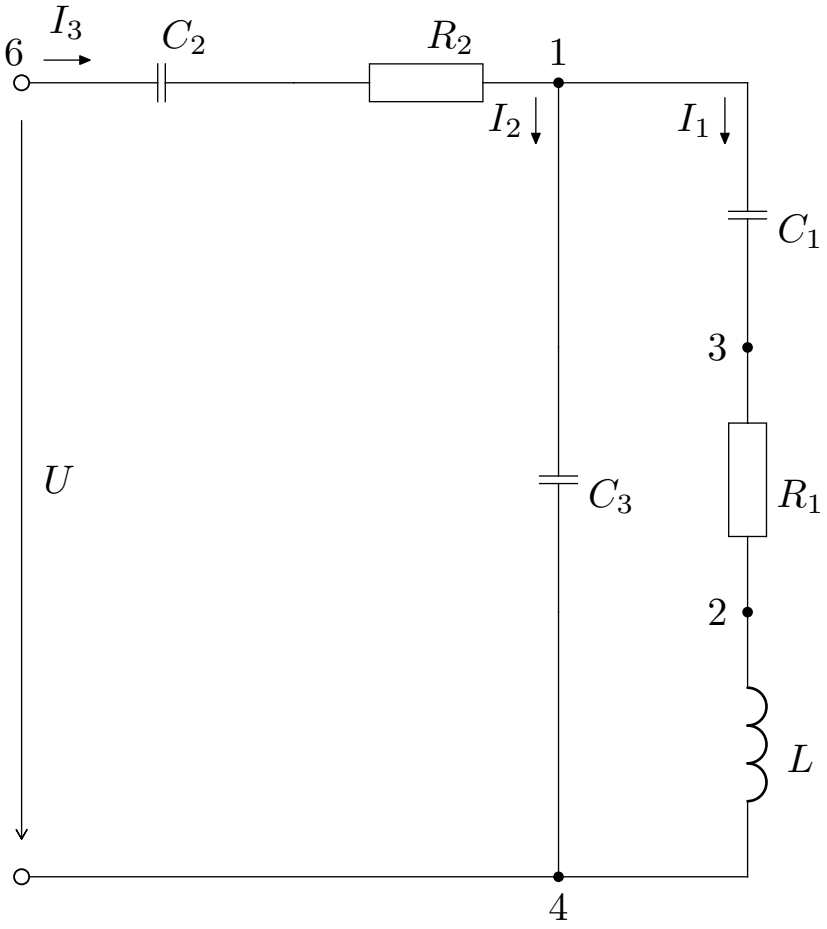
$$\begin{aligned} \hat{U}_{C2} &= \hat{I} \cdot \frac{1}{j\omega C_2} = 4,478 \cdot e^{-j26,6^\circ} \cdot \frac{1}{j \cdot 2\pi \cdot 50 \cdot 3,18 \cdot 10^{-6}} = \\ &= 4,55 \cdot e^{-j116,6^\circ} V = (-2,04 - 4,07j)V \end{aligned}$$

$$\hat{U}_{13} = \hat{U} - \hat{U}_{C2} = 10 - (-2,04 - 4,07j) = (12 + 4,07j)V = 12,7 \cdot e^{j18,67^\circ} V$$

$$\hat{I}_1 = \frac{\hat{U}_{13}}{\hat{Z}_1} = \frac{12,7 \cdot e^{j18,67^\circ}}{2829,8 \cdot e^{-j45^\circ}} = 4,49 \cdot e^{j63,7^\circ} mA$$

$$\hat{I}_2 = \frac{\hat{U}_{13}}{j\omega L} = \frac{12,7 \cdot e^{j18,67^\circ}}{j \cdot c\pi \cdot 50 \cdot 6,36} = 6,36 \cdot e^{-j71^\circ} mA$$

Př.8.1: Vypočítejte metodou úměrných veličin proudy $\hat{I}_1, \hat{I}_5, \hat{I}_3$ a určete celkovou impedanci obvodu $\hat{Z}$.



$$\begin{aligned} \hat{U} &= 220V \\ f &= 50Hz \\ R_1 &= 300\Omega \\ R_2 &= 100\Omega \\ C_1 &= 5\mu F \\ C_2 &= 10\mu F \\ C_3 &= 2\mu F \\ L &= 1H \end{aligned}$$

Předpokládáme, že známe proud $\hat{I}_1$

$$\begin{aligned}
\hat{U}_{C1} &= \frac{\hat{I}_1}{j\omega C_1} = \frac{\hat{I}_1}{j \cdot 2\pi \cdot 50 \cdot 5 \cdot 10^{-6}} = -636j \cdot \hat{I}_1 \\
\hat{U}_{R1} &= R_1 \cdot \hat{I}_1 = 300 \cdot \hat{I}_1 \\
\hat{U}_L &= j\omega L \cdot \hat{I}_1 = j \cdot 2\pi \cdot 50 \cdot 1 \cdot \hat{I}_1 = 314j \cdot \hat{I}_1
\end{aligned}$$

Napětí mezi uzly 1 a 4, zároveň napětí na C_3:

$$\hat{U}_{C3} = \hat{U}_{C1} + \hat{U}_{R1} + \hat{U}_L = -636j \cdot \hat{I}_1 + 300 \cdot \hat{I}_1 + 314j \cdot \hat{I}_1 = (300 - 323j) \cdot \hat{I}_1$$

$$\hat{I}_2 = j\omega C_3 \cdot \hat{U}_{C3} = j \cdot 2\pi \cdot 50 \cdot 2 \cdot 10^{-6} \cdot (300 - 323j) \cdot \hat{I}_1 = (0,203 + 0,189j) \cdot \hat{I}_1$$

$$\hat{I}_3 = \hat{I}_1 + \hat{I}_2 = (0,203 + 0189j) \cdot \hat{I}_1 + \hat{I}_1 = (1,203 + 0,189j) \cdot \hat{I}_1$$

$$\hat{U}_{C2} = \frac{\hat{I}_3}{j\omega C_2} = \frac{(1,203 + 0189j) \cdot \hat{I}_1}{j \cdot 2\pi \cdot 50 \cdot 10 \cdot 10^{-6}} = (60,2 + 382j) \cdot \hat{I}_1$$

Napětí zdroje:

$$\begin{aligned} \hat{U} &= \hat{U}_{C2} + \hat{U}_{R2} + \hat{U}_{C3} = \\ &= (60,2 + 382j) \cdot \hat{I}_1 + (120,3 + 18,9j) \cdot \hat{I}_1 + (300 - 323j) \cdot \hat{I}_1 = \\ &= (480,2 - 687,2j) \cdot \hat{I}_1 \end{aligned}$$

Z toho:

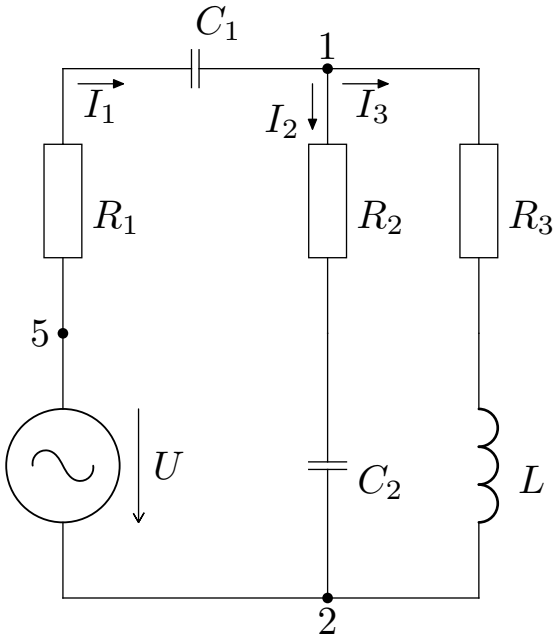
$$\hat{I}_1 = \frac{\hat{U}}{480,2 - 687,2j} = \frac{220}{480,2 - 687,2j} = 0,262\angle 55° A$$

$$\hat{I}_2 = (0,203 + 0,189j) \cdot \hat{I}_1 = 0,073\angle 98° A$$

$$\hat{I}_3 = (1,203 + 0,189j) \cdot \hat{I}_1 = 0,32\angle 64° A$$

$$\hat{Z} = \frac{\hat{U}}{\hat{I}_3} = \frac{220}{0,32\angle 64°} = 688,6\angle -64°\Omega$$

Př.8.2: Jaká je hodnota proudu $\hat{I}_3$ a jeho počáteční fáze Ψ_i?



$$\begin{aligned}
\hat{U} &= 100\angle 30°V \\
R_1 &= 27\Omega \\
R_2 &= 30\Omega \\
R_3 &= 20\Omega \\
\frac{1}{\omega C_1} &= 25\Omega \\
\frac{1}{\omega C_2} &= 18\Omega \\
\omega L &= 30\Omega
\end{aligned}$$

Předpokládejme proud $\hat{I}_{\bullet 3} = 1A$.

$$\hat{U}_{\bullet R3} = R_3 \cdot \hat{I}_{\bullet 3} = 20 \cdot 1 = 20V$$

$$\hat{U}_{\bullet L} = j\omega L \cdot \hat{I}_{\bullet 3} = j \cdot 30 \cdot 1 = j30V$$

$$\hat{U}_{\bullet 12} = \hat{U}_{\bullet R3} + \hat{U}_{\bullet L} = (20 + j30)V$$

$$\hat{I}_{\bullet 2} = \frac{\hat{U}_{\bullet 12}}{R_2 - \frac{j}{\omega C_2}} = \frac{20 + j30}{30 - j18} = 1,0306\angle 86,76°A$$

$$\hat{I}_{\bullet 1} = \hat{I}_{\bullet 2} + \hat{I}_{\bullet 3} = 0,0582 + j1,0295 + 1 = 1,4764\angle 44,209°A$$

$$\begin{aligned}\hat{U}_{\bullet 51} &= \hat{I}_{\bullet 1} \cdot (R_1 - \frac{j}{\omega C_1}) = 1,4764\angle 44,209° \cdot (27 - j25) = \\ &= 1,4764\angle 44,209° \cdot 36,8\angle -42,8° = 54,3152\angle 1,409°V = (54,3 + j1,336)V\end{aligned}$$

$$\hat{U}_{\bullet} = \hat{U}_{\bullet 51} + \hat{U}_{\bullet 12} = 54,3 + j1,336 + 20 + j3 = (74,3 + j31,336)V = 80,638\angle 22,868°V$$

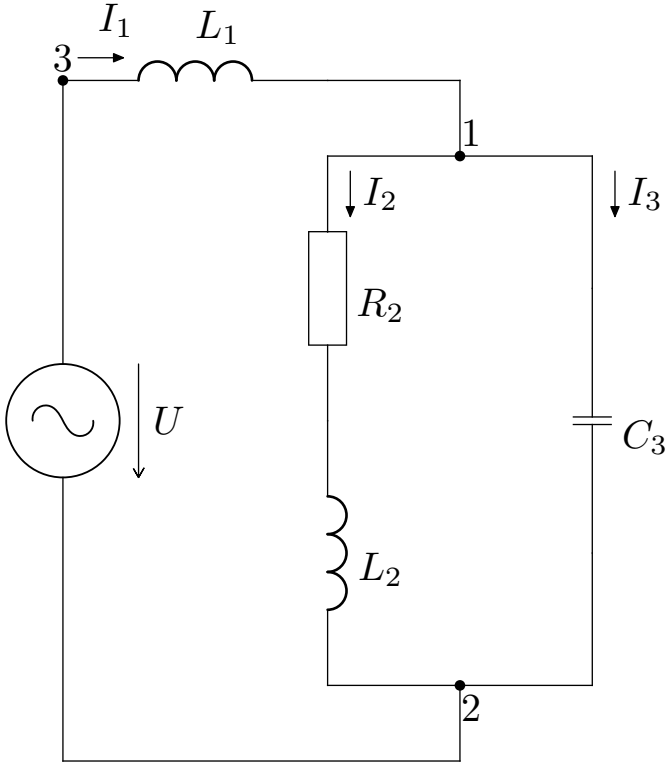
Skutečné napětí zdroje je $\hat{U} = 100\angle 30°V$

Poměr

$$\hat{p} = \frac{\hat{U}}{\hat{U}_\bullet} = \frac{100\angle 30°}{80,638\angle 22,868°} = 1,24\angle 7,132°$$

$$\hat{I}_3 = 1 \cdot 1,24\angle 7,132° = 1,24\angle 7,132°A$$

Př.8.3: Jaká je hodnota proudu $\hat{I}_3$ a jeho počáteční fáze Ψ_i?



$$\begin{aligned} \hat{U} &= 30V \\ R_2 &= 40\Omega \\ \frac{1}{\omega C_3} &= 20\Omega \\ \omega L_1 &= 25,5\Omega \\ \omega L_2 &= 10\Omega \end{aligned}$$

Předpokládejme proud $\hat{I}_{\bullet 3} = 1A$.

$$\hat{U}_{\bullet 12} = \hat{I}_{\bullet 3} \cdot j\frac{1}{\omega C_3} = 1 \cdot j20 = j20V$$

$$\hat{I}_{\bullet 2} = \frac{\hat{U}_{\bullet 12}}{R_2 + j\omega L_2} = \frac{j20}{40 + j100} = 0,1857\angle 21,8°A$$

$$\hat{I}_{\bullet 1} = \hat{I}_{\bullet 2} + \hat{I}_{\bullet 3} = 0,1724 + j0,06896 + 1 = 1,1724 + j0,06896 = 1,1744\angle 3,366°A$$

$$\hat{U}_{\bullet 31} = \hat{I}_{\bullet 1} \cdot j\omega L_1 = 1,1744\angle 3,366° \cdot j25,5 = 29,948\angle 93,366°$$

$$\begin{aligned}\hat{U}_{\bullet} &= \hat{U}_{\bullet 31} + \hat{U}_{\bullet 12} = -1,7586 + j29,8965 + j20 = \\ &= (-1,7586 + j49,8965)V = 49,9275\angle 92°V\end{aligned}$$

Skutečné napětí zdroje je $\hat{U} = 30V$

Poměr

$$\hat{p} = \frac{\hat{U}}{\hat{U}_\bullet} = \frac{30}{49,9275\angle 92^\circ} = 0,6008\angle -92^\circ$$

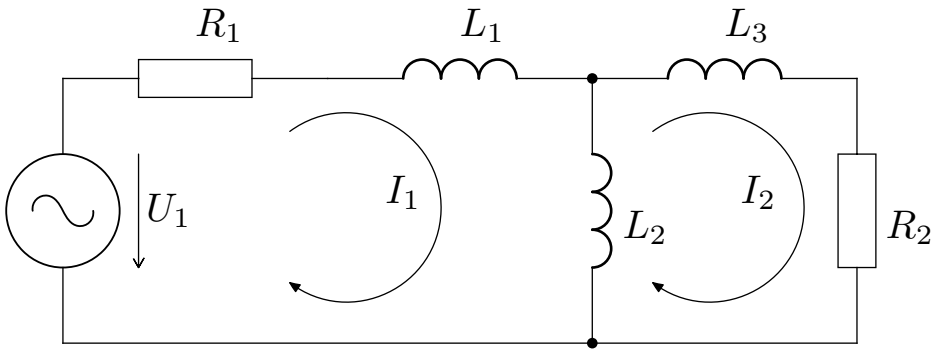
$$\hat{I}_3 = 1 \cdot 0,6008\angle -92^\circ = 0,6008\angle -92^\circ A$$

$$\hat{I}_2 = 0,1857\angle 21,8^\circ \cdot 0,6008\angle -92^\circ = 0,1109\angle -69,8^\circ A$$

$$\hat{I}_1 = 1,1744\angle 3,366^\circ \cdot 0,6008\angle -92^\circ = 0,7018\angle -88,2^\circ A$$

2.3 Univerzální metody

Př.8.4: Proudy ve větvích obvodu vypočítejte metodou smyčkových proudů.



$$\begin{aligned} \hat{U}_1 &= 10V \\ f &= 20kHz \\ R_1 &= 5\Omega \\ R_2 &= 20\Omega \\ L_1 &= 0,1mH \\ L_2 &= 2mH \\ L_3 &= 0,25mH \end{aligned}$$

$$R_1\hat{I}_1 + j\omega L_1 \cdot \left(\hat{I}_1 - \hat{I}_2\right) - \hat{U}_1 = 0$$

$$j\omega L_1 \cdot \left(\hat{I}_2 - \hat{I}_1\right) + (j\omega L_2 - R_2) \cdot \hat{I}_2 = 0$$

Rovnice smyčkových proudů můžeme zapsat přímo v maticovém tvaru:

$$\begin{pmatrix} R_1 + j\omega(L_1 + L_2) & -j\omega L_2 \\ -j\omega L_2 & R_2 + j\omega(L_2 + L_3) \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{U}_1 \\ 0 \end{pmatrix}$$

Dosadíme zadané hodnoty:

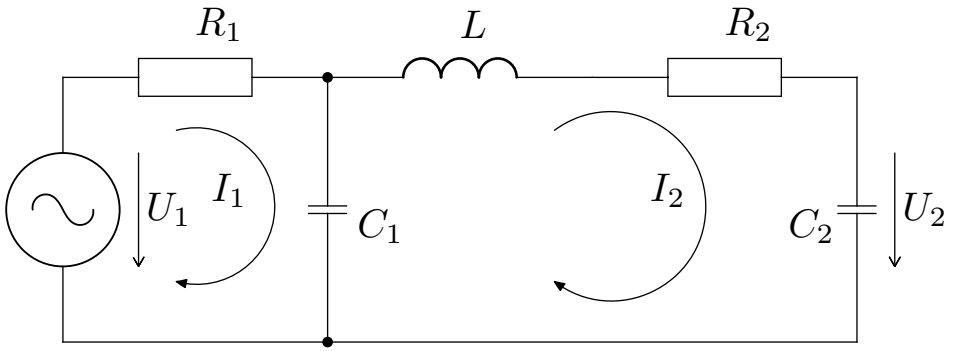
$$\begin{pmatrix} 5 + j251,89 & -j251,33 \\ -j251,33 & 20 + j282,74 \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$$

Proud zdroje: $\hat{I}_Z = \hat{I}_1 = 0,21519\angle -63,52°A$

Proud prvkem L_2: $\hat{I}_{L2} = \hat{I}_1 - \hat{I}_2 = 0,028276\angle -91,95°$

Proud rezistorem R_2: $\hat{I}_{R_2} = \hat{I}_2 = 0,19081\angle -59,47°$

Př.8.5: Vypočítejte napětí U_2 na výstupu nezatíženého filtru pomocí metody smyčkových proudů.



$$\begin{aligned} \hat{U}_1 &= 1V \\ f &= 100kHz \\ R_1 &= 200\Omega \\ R_2 &= 50\Omega \\ L &= 5H \\ C_1 &= C_2 = 50\mu F \end{aligned}$$

$$R_1\hat{I}_1 + \frac{1}{j\omega C_1}\cdot\left(\hat{I}_1 - \hat{I}_2\right) - \hat{U}_1 = 0$$

$$\underline{\frac{1}{j\omega C_1}\cdot\left(\hat{I}_2 - \hat{I}_1\right) + \left(j\omega L + R_2 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right)\cdot\hat{I}_2 = 0}$$

$$\frac{1}{j\omega C_1} = \frac{1}{j \cdot 2\pi \cdot 100 \cdot 50 \cdot 10^{-6}} = -j31,82\Omega = (0 - j31,83)\Omega = \frac{1}{j\omega C_2}$$

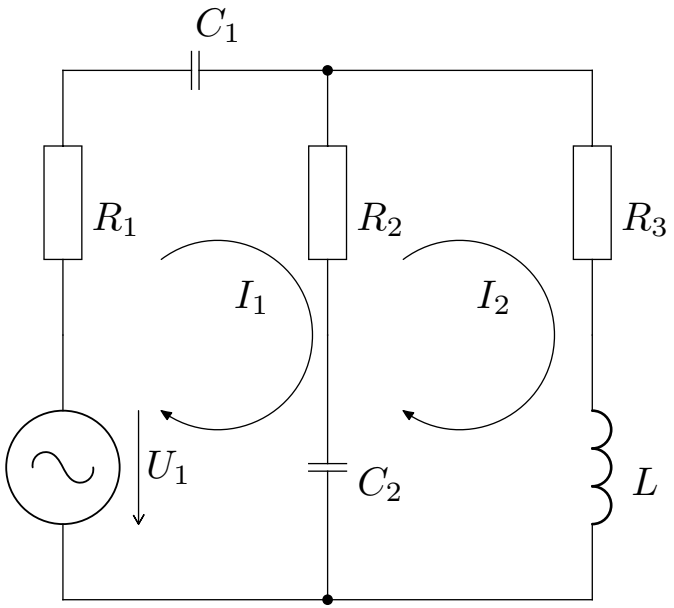
$$\hat{I}_1 \cdot (200 - j31,83) + j31,83 \cdot \hat{I}_1 = \hat{U}_1$$

$$\underline{\hat{I}_1 \cdot j31,83 + \hat{I}_2(50 + 43077,9) = 0}$$

$$\hat{I}_2 = 5,104 \cdot 10^{-5} \cdot e^{-j169,9^\circ} A$$

$$\hat{U}_2 = \frac{\hat{I}_2}{j\omega C_2} = \frac{5,104 \cdot 10^{-5} \cdot e^{-j169,9^\circ}}{j \cdot 2\pi \cdot 100 \cdot 50 \cdot 10^{-6}} = 1,625 \cdot 10^{-3} \cdot e^{j100,1^\circ} V$$

Př.8.6: Vypočítejte v uvedeném obvodu proudy jednotlivých větví pomocí metody smyčkových proudů.



$$\begin{aligned}
\hat{U}_1 &= 100\angle 30^\circ = (86,6 + j50)V \\
R_1 &= 27\Omega \\
R_2 &= 30\Omega \\
R_3 &= 20\Omega \\
\frac{1}{\omega C_1} &= 25\Omega \\
\frac{1}{\omega C_2} &= 18\Omega \\
\omega L &= 30\Omega
\end{aligned}$$

$$\begin{pmatrix} R_1 + \frac{1}{j\omega C_1} + R_2 + \frac{1}{j\omega C_2} & -R_2 - \frac{1}{j\omega C_2} \\ -R_2 - \frac{1}{j\omega C_2} & \frac{1}{j\omega C_2} + R_2 + R_3 + j\omega L \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{U}_1 \\ 0 \end{pmatrix}$$

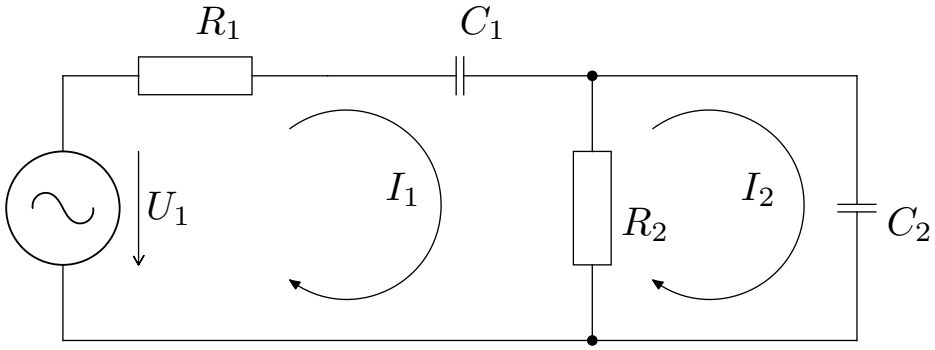
$$\begin{pmatrix} 57-j43 & -30+j18 \\ -30+j18 & 50+j12 \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} 86,6+j50 \\ 0 \end{pmatrix}$$

$$\hat{I}_1 = (1,14+j1,426)A = 1,826\angle 51,37°A$$

$$\hat{I}_2 = (1,233+j0,1495)A = 1,242\angle 6,913°A$$

$$\begin{aligned} \hat{I}_{R2} &= \hat{I}_2 - \hat{I}_2 = 1,14+j1,426-1,233-j0,1495 = \\ &= (-0,093+j1,2765)A = 1,28\angle 94,77°A \end{aligned}$$

Př.8.7: Vypočítejte v uvedeném obvodu proudy jednotlivých větví pomocí metody smyčkových proudů.



$$\begin{aligned} \hat{U}_1 &= 10V \\ R_1 &= R_2 = 10k\Omega \\ C_1 &= C_2 = 0,1\mu F \end{aligned}$$

Řešte pro frekvence

1. $f = 100Hz$
2. $f = 160Hz$
3. $f = 250Hz$

$$\begin{pmatrix} R_1 + \frac{1}{j\omega C_1} + R_2 & -R_2 \\ -R_2 & R_2 + \frac{1}{j\omega C_2} \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{U}_1 \\ 0 \end{pmatrix}$$

ad 1.) $f = 100Hz$

$$\begin{pmatrix} R_1 + \frac{1}{j\omega C_1} + R_2 & -R_2 \\ -R_2 & R_2 + \frac{1}{j\omega C_2} \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{U}_1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 10 \cdot 10^3 + \frac{1}{j \cdot 2\pi \cdot 100 \cdot 0{,}1 \cdot 10^{-6}} + 10 \cdot 10^3 & -10 \cdot 10^3 \\ -10 \cdot 10^3 & 10 \cdot 10^3 + \frac{1}{j \cdot 2\pi \cdot 100 \cdot 0{,}1 \cdot 10^{-6}} \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$$

$$\hat{U}_R 2 = R_2 \cdot (\hat{I}_1 - \hat{I}_2) = 3,1737 \angle 17,80° V$$

ad 2.) $f = 160Hz$

$$\begin{pmatrix} R_1 + \frac{1}{j\omega C_1} + R_2 & -R_2 \\ -R_2 & R_2 + \frac{1}{j\omega C_2} \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{U}_1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 10 \cdot 10^3 + \frac{1}{j \cdot 2\pi \cdot 160 \cdot 0{,}1 \cdot 10^{-6}} + 10 \cdot 10^3 & -10 \cdot 10^3 \\ -10 \cdot 10^3 & 10 \cdot 10^3 + \frac{1}{j \cdot 2\pi \cdot 160 \cdot 0{,}1 \cdot 10^{-6}} \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$$

$$\hat{U}_R 2 = R_2 \cdot (\hat{I}_1 - \hat{I}_2) = 3{,}333 \angle -0{,}202° V$$

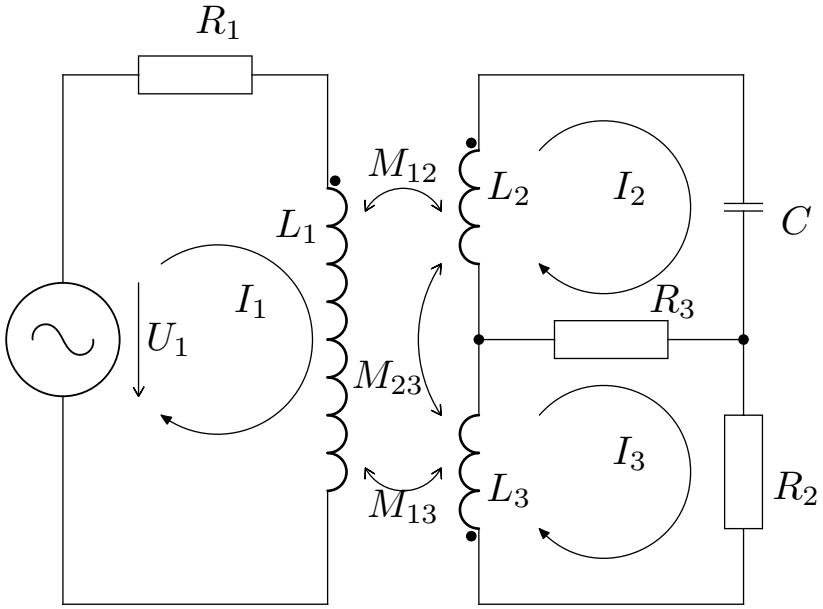
ad 2.) $f = 250Hz$

$$\begin{pmatrix} R_1 + \frac{1}{j\omega C_1} + R_2 & -R_2 \\ -R_2 & R_2 + \frac{1}{j\omega C_2} \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} \hat{U}_1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 10 \cdot 10^3 + \frac{1}{j \cdot 2\pi \cdot 250 \cdot 0{,}1 \cdot 10^{-6}} + 10 \cdot 10^3 & -10 \cdot 10^3 \\ -10 \cdot 10^3 & 10 \cdot 10^3 + \frac{1}{j \cdot 2\pi \cdot 250 \cdot 0{,}1 \cdot 10^{-6}} \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$$

$$\hat{U}_R 2 = R_2 \cdot (\hat{I}_1 - \hat{I}_2) = 3{,}183 \angle -17{,}30° V$$

Př.8.8: Obvod na obrázku řešte metodou smyčkových proudů.



$$\begin{aligned}
\hat{U}_1 &= 10V \\
f &= 10kHz \\
R_1 &= 50\Omega \\
R_2 &= 100\Omega \\
R_3 &= 200\Omega \\
L_1 &= 2,5mH \\
L_2 &= 1,8mH \\
L_3 &= 1,8mH \\
M_{12} &= 2mH \\
M_{23} &= 1,6mH \\
M_{13} &= 2mH \\
C &= 0,3\mu F
\end{aligned}$$

$$\begin{pmatrix} R_1 + j\omega L_1 & -j\omega M_{12} & +j\omega M_{13} \\ -j\omega M_{12} & R_3 + j\omega L_2 - \frac{j}{\omega C} & -(R_3 + j\omega M_{23}) \\ +j\omega M_{13} & -(R_3 + j\omega M_{23}) & R_2 + R_3 + j\omega L_3 \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{I}_3 \end{pmatrix} = \begin{pmatrix} \hat{U}_1 \\ 0 \\ 0 \end{pmatrix}$$

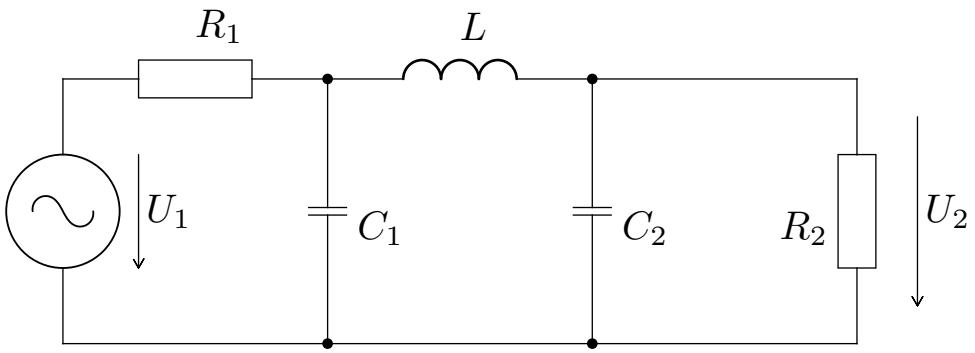
$$\begin{pmatrix} 50 + j157,08 & -j125,66 & +j125,66 \\ -j125,66 & 200 + j47,478 & -200 - j100,53 \\ +j125,66 & -200 - j100,53 & 300 + j113,097 \end{pmatrix} \cdot \begin{pmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{I}_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}$$

$$\hat{I}_1 = 0,050960\angle -53,72°A$$

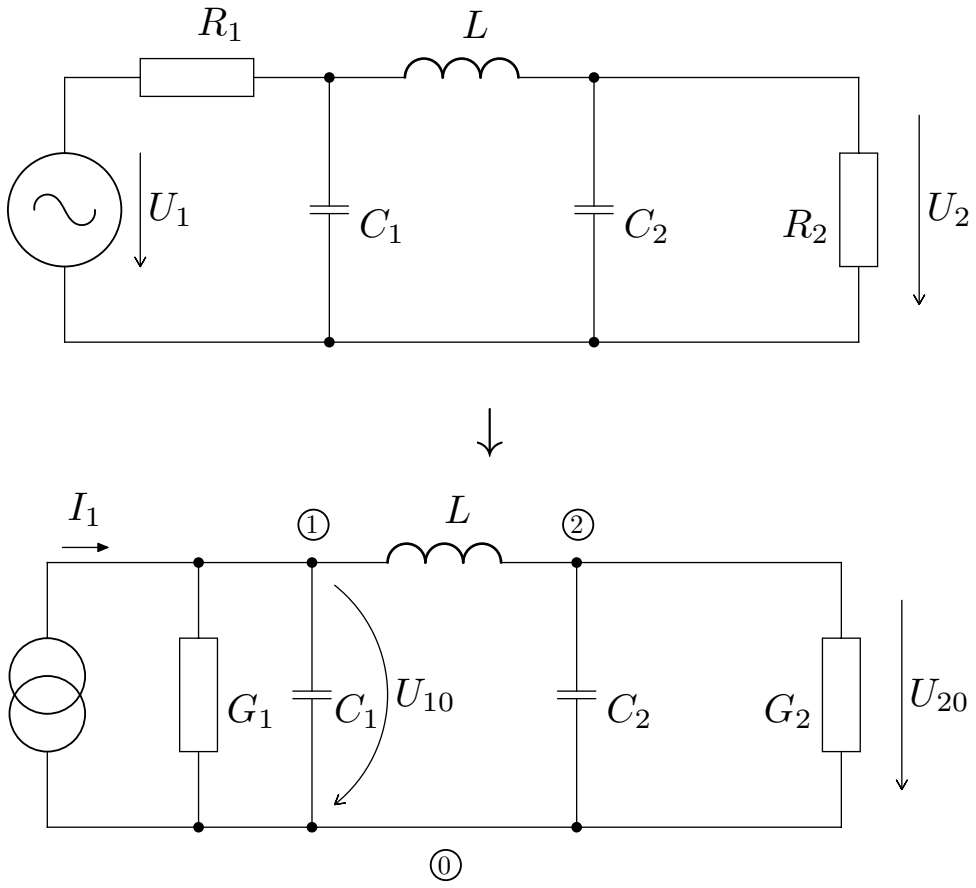
$$\hat{I}_2 = 0,025855\angle 51,15°A$$

$$\hat{I}_3 = 0,013609\angle 133,99°A$$

Př.8.9: Vypočítejte napětí U_2 na výstupu filtru pomocí metody uzlových napětí (MUN).

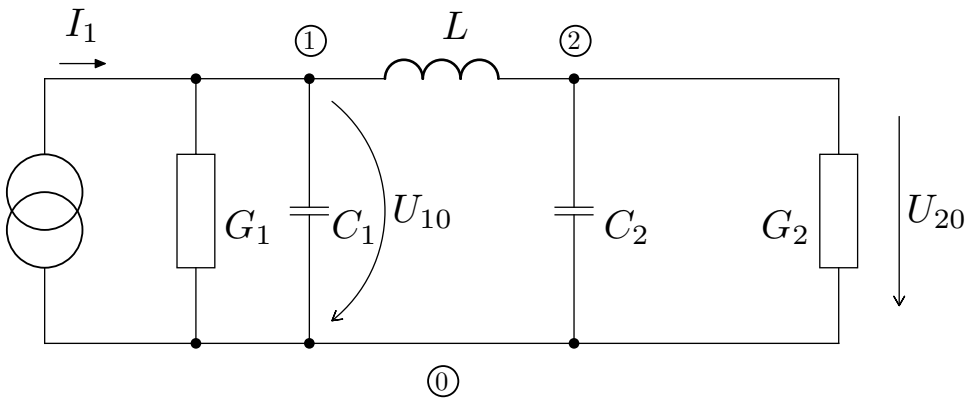


$$\begin{aligned} \hat{U}_1 &= 10V \\ f &= 100Hz \\ R_1 &= 200\Omega \\ R_2 &= 50\Omega \\ L &= 5H \\ C_1 &= C_2 = 50\mu F \end{aligned}$$



Napěťový zdroj přepočteme na proudový:

$$\begin{aligned}
\hat{I}_1 &= \frac{\hat{U}_1}{R_1} = \frac{10}{200} = 0,05A \\
G_1 &= \frac{1}{R_1} = \frac{1}{200} = 0,005S \\
G_2 &= \frac{1}{R_2} = \frac{1}{50} = 0,02S \\
j\omega C_1 &= j\omega C_2 = \\
&= j \cdot 2\pi \cdot 100 \cdot 50 \cdot 10^{-6} = \\
&= j0,031416S \\
\frac{1}{j\omega L} &= \frac{1}{j \cdot 2\pi \cdot 100 \cdot 5} = \\
&= -0,000318309S
\end{aligned}$$



Napíšeme soustavu rovnic:

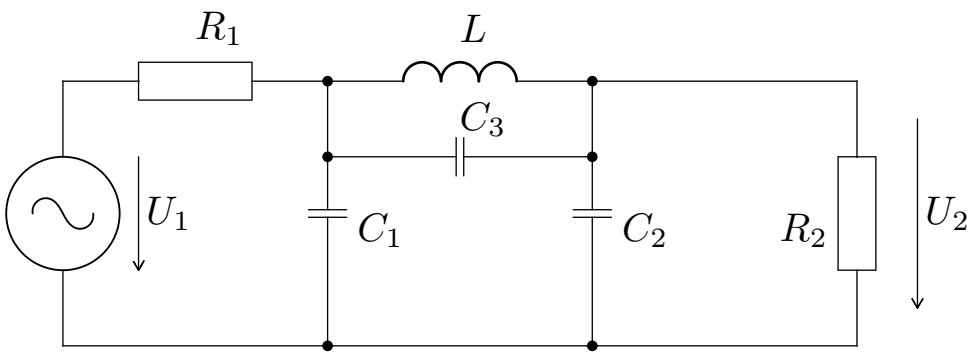
$$-\hat{I}_1 + \hat{U}_{10} \cdot (G_1 + j\omega C_1) + \frac{1}{j\omega L}\left(\hat{U}_{10} - \hat{U}_{20}\right) = 0$$

$$\hat{U}_{20}\,(G_2 + j\omega C_2) + \frac{1}{j\omega L}\left(\hat{U}_{20} - \hat{U}_{10}\right) = 0$$

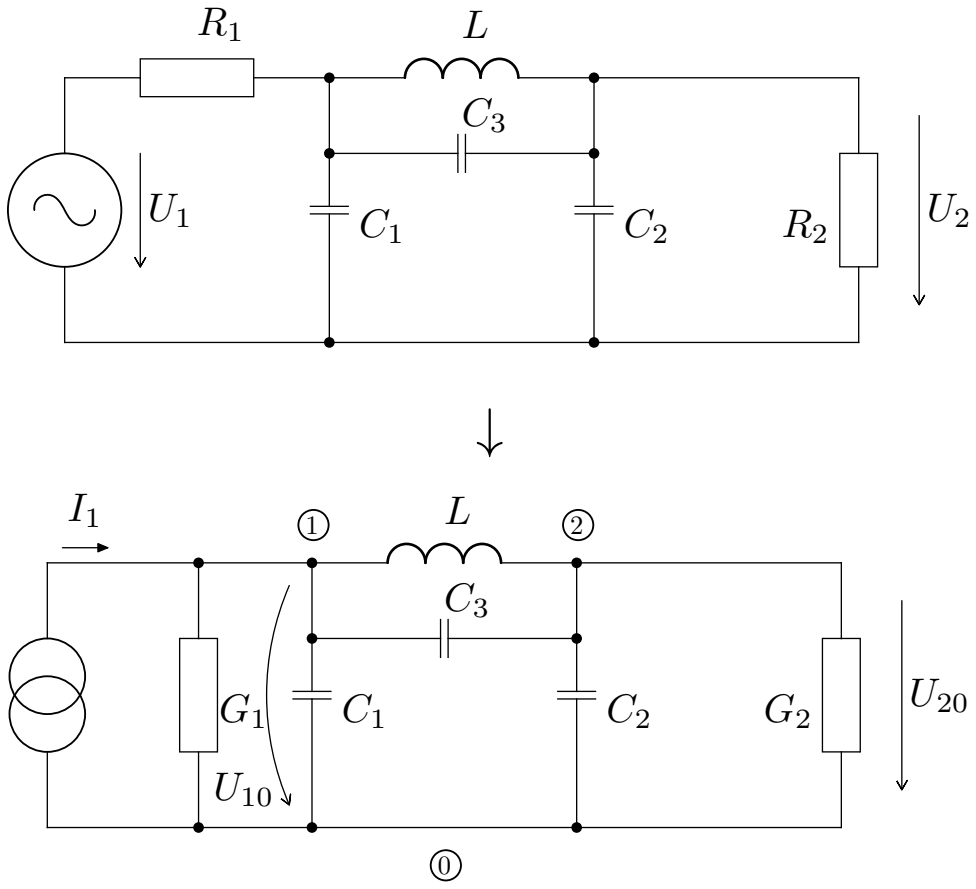
$$\begin{pmatrix} G_1 + j\omega C_1 + \frac{1}{j\omega L} & -\frac{1}{j\omega L} \\ -\frac{1}{j\omega L} & G_2 + j\omega C_2 + \frac{1}{j\omega L} \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_{10} \\ \hat{U}_{20} \end{pmatrix} = \begin{pmatrix} \hat{I}_1 \\ 0 \end{pmatrix}$$

$$\hat{U}_2 = \hat{U}_{20} = 1,367 \cdot 10^{-2} \cdot e^{j131,9^\circ} V$$

Př.8.10: Vypočítejte napětí U_2 na výstupu filtru pomocí metody uzlových napětí (MUN).

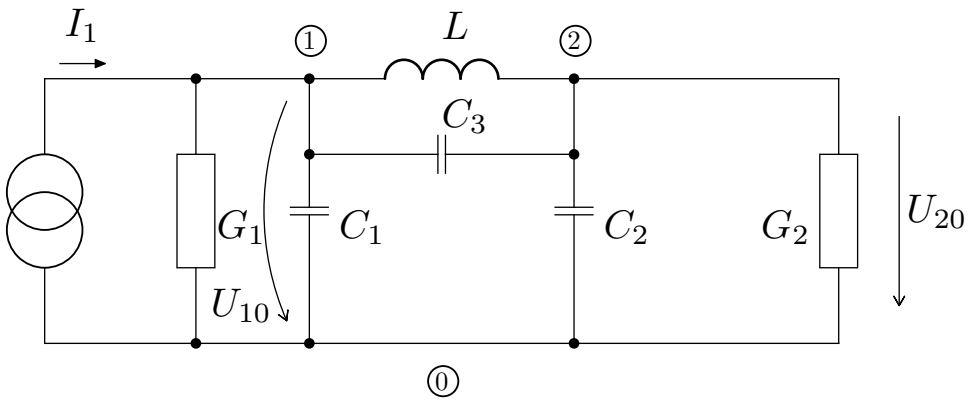


$$\begin{aligned}
\hat{U}_1 &= 10V \\
f &= 100Hz \\
R_1 &= 200\Omega \\
R_2 &= 50\Omega \\
L &= 1H \\
C_1 &= C_2 = 50\mu F \\
C_3 &= 2,5\mu F
\end{aligned}$$



Napěťový zdroj přepočteme na proudový:

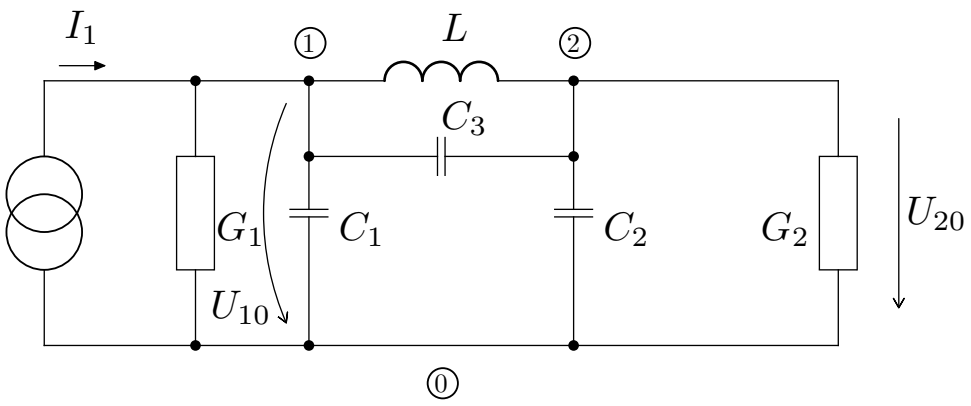
$$\begin{aligned}
\hat{I}_1 &= \frac{\hat{U}_1}{R_1} = \frac{10}{200} = 0,05A \\
G_1 &= \frac{1}{R_1} = \frac{1}{200} = 0,005S \\
G_2 &= \frac{1}{R_2} = \frac{1}{50} = 0,02S \\
j\omega C_1 &= j\omega C_2 = \\
&= j \cdot 2\pi \cdot 100 \cdot 50 \cdot 10^{-6} = \\
&= j0,031416S \\
j\omega C_3 &= j \cdot 2\pi \cdot 100 \cdot 2,5 \cdot 10^{-6} = \\
&= j1,570796 \cdot 10^{-3}S \\
\frac{1}{j\omega L} &= \frac{1}{j \cdot 2\pi \cdot 100 \cdot 1} = \\
&= -1,591549 \cdot 10^{-3}S
\end{aligned}$$



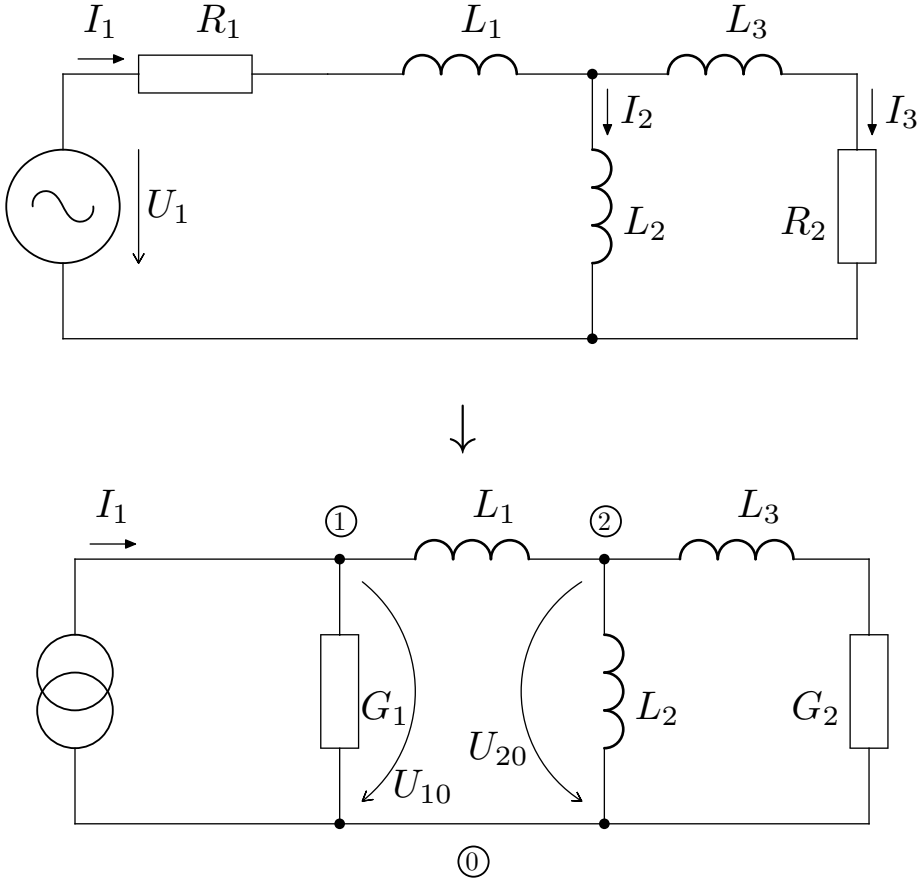
$$\begin{pmatrix} G_1 + j\omega C_1 + j\omega C_3 + \frac{1}{j\omega L} & -j\omega C_3 - \frac{1}{j\omega L} \\ -j\omega C_3 - \frac{1}{j\omega L} & G_2 + j\omega C_2 + j\omega C_3 + \frac{1}{j\omega L} \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_{10} \\ \hat{U}_{20} \end{pmatrix} = \begin{pmatrix} \hat{I}_1 \\ 0 \end{pmatrix}$$

$$\hat{U}_2 = \hat{U}_{20} = 8,77 \cdot 10^{-4} \cdot e^{j131,6^\circ} V$$

Př.8.11: Vypočítejte proudy ve větvích obvodu metodou uzlových napětí.



$$\begin{aligned}
\hat{U}_1 &= 10V \\
f &= 20kHz \\
R_1 &= 5\Omega \\
R_2 &= 20\Omega \\
L_1 &= 0,1mH \\
L_2 &= 2mH \\
L_3 &= 0,25mH
\end{aligned}$$



$$\hat{I}_1 = \frac{\hat{U}_1}{R_1} = \frac{10}{5} = 2A$$

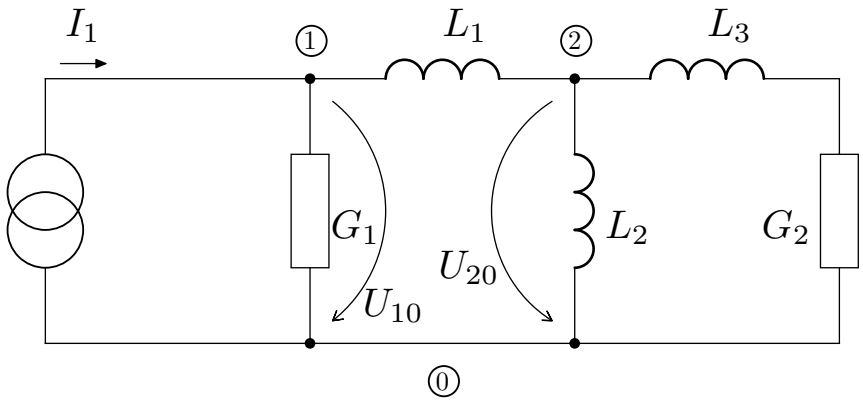
$$G_1 = \frac{1}{R_1} = \frac{1}{5} = 0,2S$$

$$G_2 = \frac{1}{R_2} = \frac{1}{20} = 0,05S$$

$$\frac{1}{j\omega L_1} = \frac{1}{j \cdot 2\pi \cdot 20 \cdot 10^3 \cdot 0,1 \cdot 10^{-3}} = -0,07957S$$

$$\frac{1}{j\omega L_2} = \frac{1}{j \cdot 2\pi \cdot 20 \cdot 10^3 \cdot 2 \cdot 10^{-3}} = -0,0039788S$$

$$Z_{L3G2} = \frac{G_2 \cdot \frac{1}{1\omega L_3}}{G_2 + \frac{1}{1\omega L_3}} = \frac{0,05 \cdot \frac{1}{j \cdot 2\pi \cdot 20 \cdot 10^3 \cdot 0,25 \cdot 10^{-3}}}{0,05 + \frac{1}{j \cdot 2\pi \cdot 20 \cdot 10^3 \cdot 0,25 \cdot 10^{-3}}} = (0,01442 - j0,02265)\Omega$$



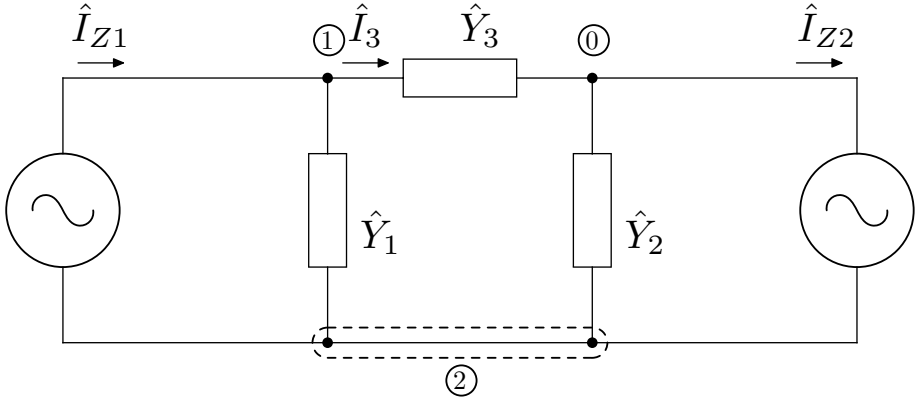
$$\begin{pmatrix} G_1 + \frac{1}{j\omega L_1} & -\frac{1}{j\omega L_1} \\ -\frac{1}{j\omega L_1} & \frac{1}{j\omega L_1} + \frac{1}{j\omega L_2} + Z_{L3G2} \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_{10} \\ \hat{U}_{20} \end{pmatrix} = \begin{pmatrix} \hat{I}_1 \\ 0 \end{pmatrix}$$

$$\hat{I}_1 = 0,21519\angle -63,52°A$$

$$\hat{I}_2 = 0,028276\angle -91,95°A$$

$$\hat{I}_3 = 0,19081\angle -59,47°A$$

Př.8.12: Vypočítejte proud $\hat{I}_3$ v obvodu metodou uzlových napětí.



$$\begin{aligned}
\hat{I}_{Z1} &= 2,2 \cdot e^{j15^\circ} A \\
\hat{I}_{Z2} &= (4 - j1,5) A \\
\hat{Y}_1 &= 0,15 S \\
\hat{Y}_2 &= j0,3 S \\
\hat{Y}_3 &= (0,2 - j0,1) S
\end{aligned}$$

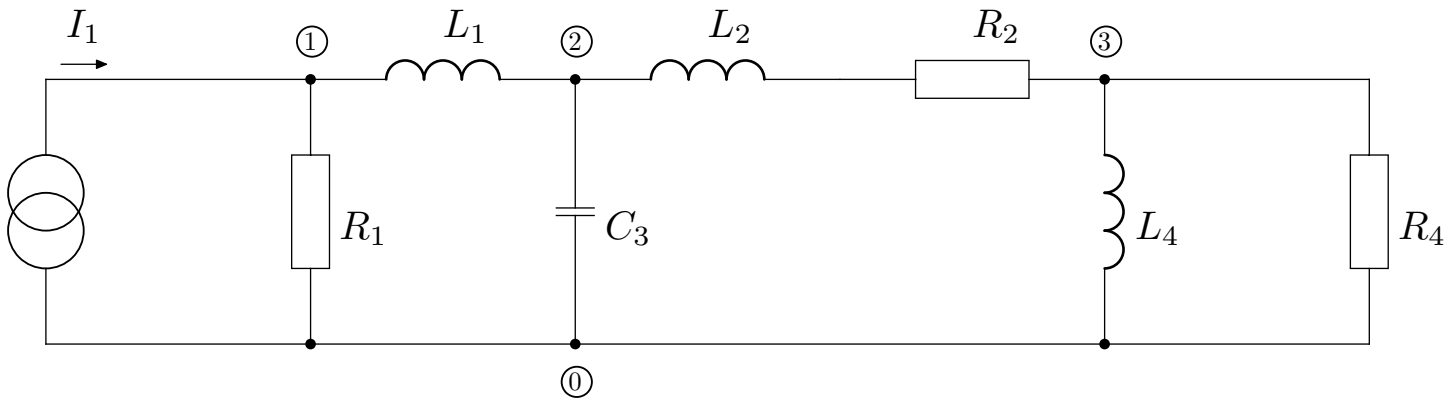
$$\begin{pmatrix} \hat{Y}_1 + \hat{Y}_3 & -\hat{Y}_1 \\ -\hat{Y}_1 & \hat{Y}_1 + \hat{Y}_2 \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_{10} \\ \hat{U}_{20} \end{pmatrix} = \begin{pmatrix} \hat{I}_{Z1} \\ \hat{I}_{Z2} - \hat{I}_{Z1} \end{pmatrix}$$

$$\begin{pmatrix} 0,35 - 0,1j & -0,15 \\ -0,15 & 0,15 + 0,3j \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_{10} \\ \hat{U}_{20} \end{pmatrix} = \begin{pmatrix} 2,125 + 0,569j \\ 1,8749 - 2,0694 \end{pmatrix}$$

$$\hat{U}_{10} = (5,37408 - 1,185938j) V$$

$$\hat{I}_3 = \hat{U}_{10} \cdot \hat{Y}_3 = (5,37408 - 1,185938j) \cdot (0,2 - j0,1) = 1,23 \cdot e^{-j39^\circ} A$$

Př.8.13: Obvod na obrázku řešte metodou uzlových napětí a vypočtěte proud $\hat{I}_2$ tekoucí rezistorem R_2.



$$\hat{I}_1 = (26,5 + 58,5j)A$$

$$R_1 = 1\Omega$$

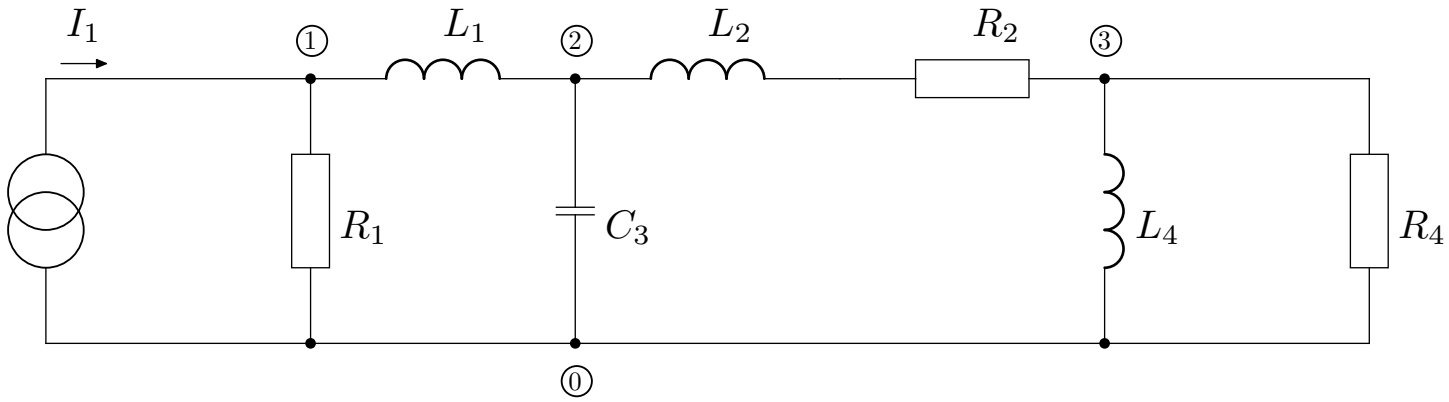
$$R_2 = 2,5\Omega$$

$$\omega L_1 = 1\Omega$$

$$\omega L_2 = 5\Omega$$

$$\frac{1}{\omega C_3} = 2\Omega$$

$$\omega L_4 = 5\Omega$$



$$\hat{Z}_2 = R_2 + j\omega L_2 = (2,5 + 5j)\Omega$$

$$\hat{Y}_2 = \frac{1}{\hat{Z}_2} = (0,08 - 0,16j)S$$

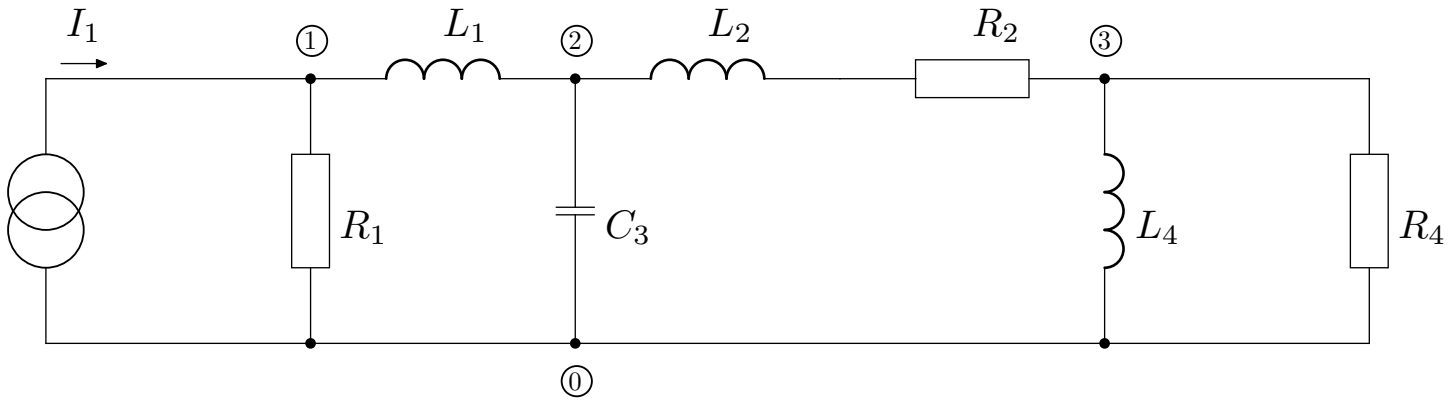
$$G_1 = \frac{1}{R_1} = 1S$$

$$\frac{1}{j\omega L_1} = -1j\ S$$

$$j\omega C_3 = 0,5j\ S$$

$$\frac{1}{j\omega L_4} = -0,2j\ S$$

$$G_4 = \frac{1}{R_4} = 0,2S$$



Uzlové rovnice zapíšeme v maticovém tvaru:

$$\begin{pmatrix} 1-j & j & 0 \\ j & 0,08-0,66j & -0,08+0,16 \\ 0 & -0,08+0,16j & 0,28-0,36j \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_{10} \\ \hat{U}_{20} \\ \hat{U}_{30} \end{pmatrix} = \begin{pmatrix} 26,5+58,5j \\ 0 \\ 0 \end{pmatrix}$$

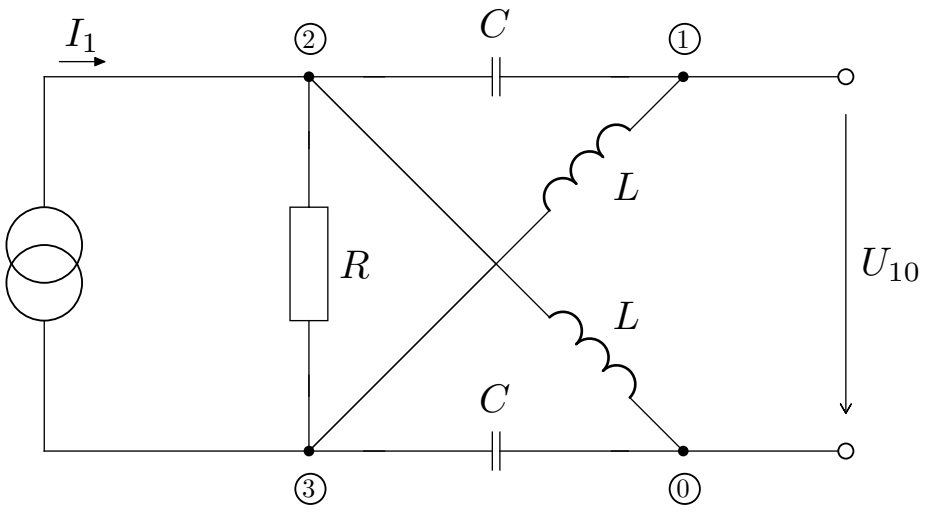
Řešením této soustavy rovnic obdržíme uzlová napětí:

$$\hat{U}_1 = 38,495+27,862j = 47,520\angle 35,90°V$$
$$\hat{U}_2 = 69,133+39,857j = 79,799\angle 29.96°V$$
$$\hat{U}_3 = 29,656+10,012j = 31,300\angle 18,65°V$$

$$\hat{U}_{23} = \hat{U}_2 - \hat{U}_3 = (39,477 + 29,845j)V = 49,489\angle 37,09°V$$

$$\hat{I}_2 = \frac{\hat{U}_{23}}{\hat{Z}_2} = \frac{39,477 + 29,845j}{2,5 + 5j} = 7,933 - 3,929j = 8,853\angle -26,35°A$$

Př.8.14: Metodou uzlových napětí určete výstupní napětí $\hat{U}_{10}$ v obvodu dle obrázku.



$$\hat{I}_1 = 20A$$

$$\omega L = 5\Omega$$

$$\frac{1}{\omega C} = 20\Omega$$

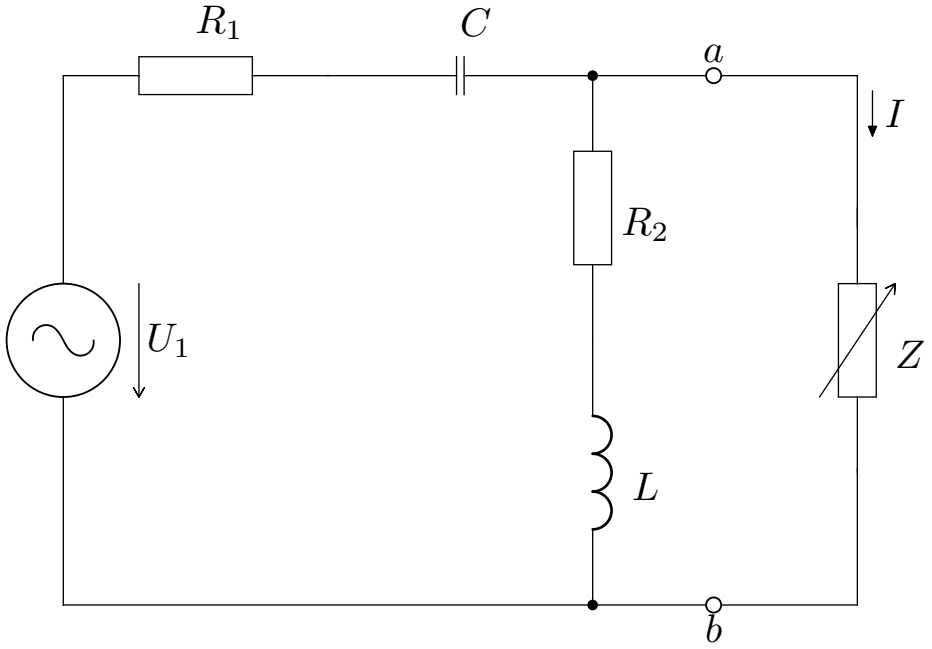
$$R = 10\Omega$$

$$\begin{pmatrix} \frac{1}{j\omega L} + j\omega C & -j\omega C & -\frac{1}{j\omega L} \\ -j\omega C & j\omega C + \frac{1}{j\omega L} + G & -G \\ -\frac{1}{j\omega L} & -G & j\omega C + \frac{1}{j\omega L} + G \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_{10} \\ \hat{U}_{20} \\ \hat{U}_{30} \end{pmatrix} = \begin{pmatrix} 0 \\ \hat{I}_1 \\ -\hat{I}_1 \end{pmatrix}$$

$$\begin{pmatrix} -0,15j & -0,05j & 0,2j \\ -0,05j & 0,1-1,5j & -0,1 \\ 0,2j & -0,1 & 0,1-1,15j \end{pmatrix} \cdot \begin{pmatrix} \hat{U}_{10} \\ \hat{U}_{20} \\ \hat{U}_{30} \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ -20 \end{pmatrix}$$

$$\hat{U}_{10} = -120 + 160j = 200\angle 126,87°V$$

Př.9.1: Vypočítejte proud $\hat{I}$ v závislosti na impedanci $\hat{Z}$.



$$u_1(t) = 33 \cdot sin\left(100t + \frac{\pi \cdot 18^c irc}{180}\right)$$

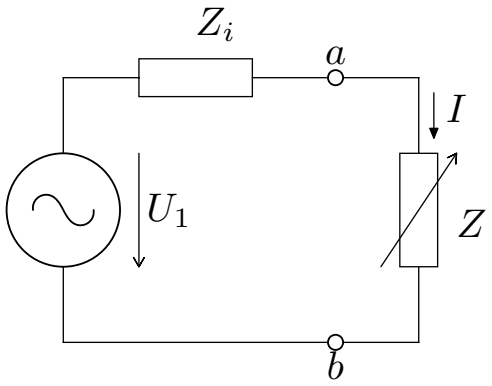
$$R_1 = 20\Omega$$

$$R_2 = 15\Omega$$

$$C = 1000\mu F$$

$$L = 250mH$$

Podle Theveninovy věty nahradíme obvod vzhledem ke svorkám a-b náhradním napěťovým zdrojem.



$$\hat{U}_1 = U_{1ef}\angle\alpha = \frac{33}{12}\angle 18° = 23,33\angle 18°$$

$$\hat{Z}_1 = R_1 + \frac{1}{j\omega C_1} = 20 - \frac{1}{100\cdot 10^3\cdot 10^{-6}}j = (20-10j)\Omega$$

$$\hat{Z}_2 = R_2 + j\omega L = 15 + 100\cdot 250\cdot 10^{-3}j = (15+25j)\Omega$$

$\hat{U}_i = \hat{U}_{ab0}$ napětí mezi uzly a-b naprázdno

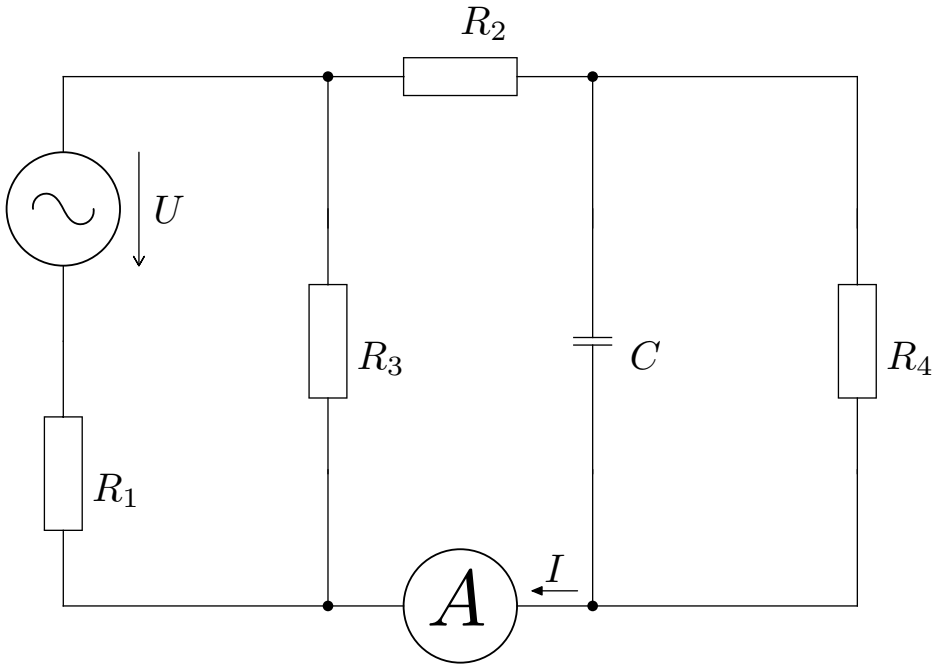
$$\hat{U}_i = \hat{U}_{ab0} = \hat{U}_1\cdot\frac{\hat{Z}_2}{\hat{Z}_1+\hat{Z}_2} = 23,33\angle 18°\cdot\frac{15+25j}{35+15j} = 17,86\angle 53,84°V$$

Vnitřní impedance náhradního zdroje:

$$\hat{Z}_i = \hat{Z}_{zb} = \frac{\hat{Z}_1 \cdot \hat{Z}_2}{\hat{Z}_1 + \hat{Z}_2} = \frac{(20 - 10j)(15 + 25j)}{(35 + 15j)} = 17,12\angle 9,27°\Omega$$

$$\hat{I} = \frac{\hat{U}_i}{\hat{Z}_i + \hat{Z}_1} = \frac{17,86\angle 53,84°}{17,12\angle 9,27° + \hat{Z}} = \frac{(10,54 + 14,42j)}{(16,9 + 2,76j) + \hat{Z}} A$$

Př.9.2: Vypočítejte metodou náhradního napěťového zdroje proud tekoucí ampérmetrem A.



$$\hat{U} = 40V$$

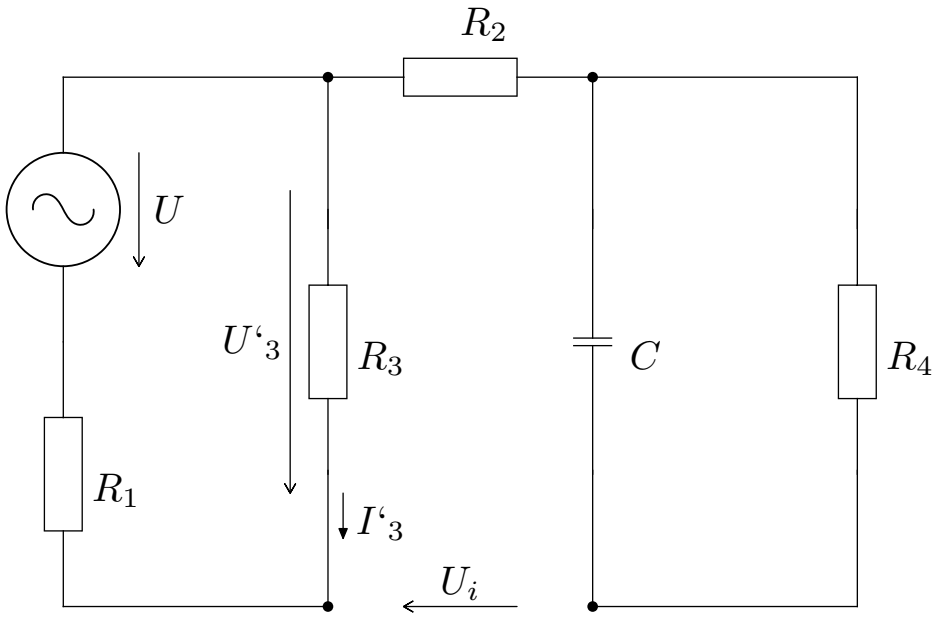
$$R_1 = 200\Omega$$

$$R_2 = 160\Omega$$

$$R_3 = 120\Omega$$

$$R_4 = 80\Omega$$

$$\frac{1}{\omega C} = 60\Omega$$

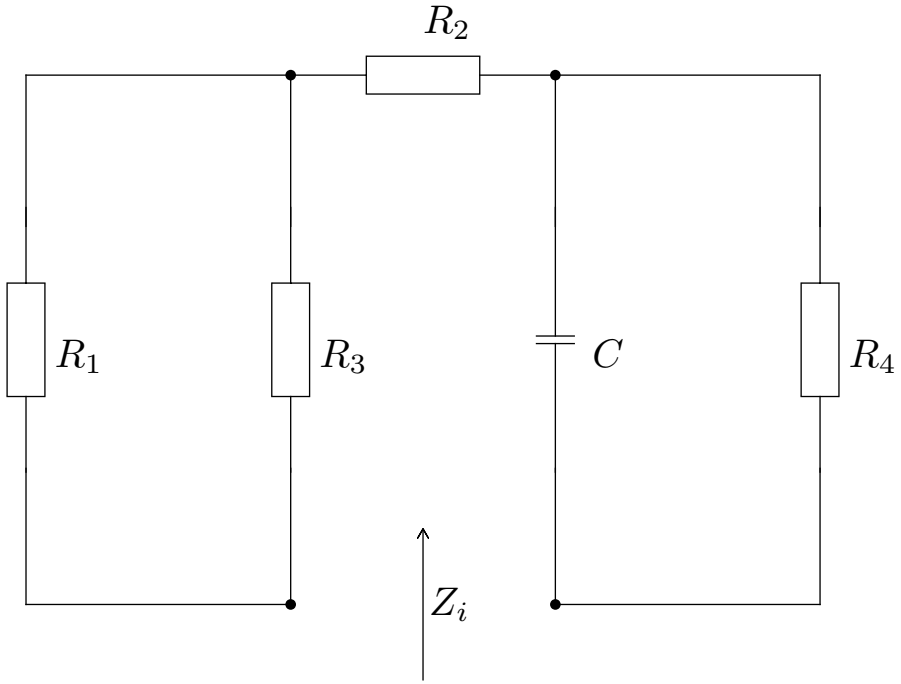


$$\hat{I}‘_3 = \frac{\hat{U}}{R_1 + R_2} = \frac{40}{200 + 120} = 0,125A$$

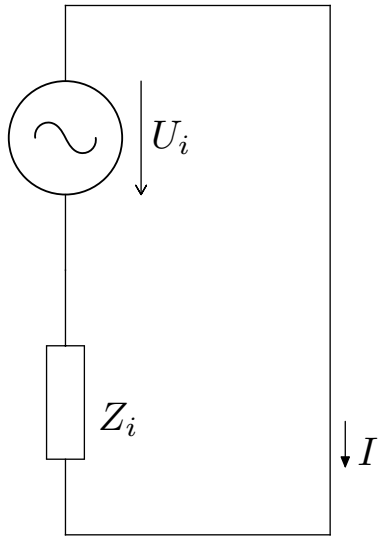
$$\hat{U}‘_3 = R_3 \cdot \hat{I}‘_3 = 120 \cdot 0,125 = 15V$$

$$-\hat{U}‘_3 + \hat{U}_i = 0$$

$$\hat{U}_i = \hat{U}‘_3 = 15V$$

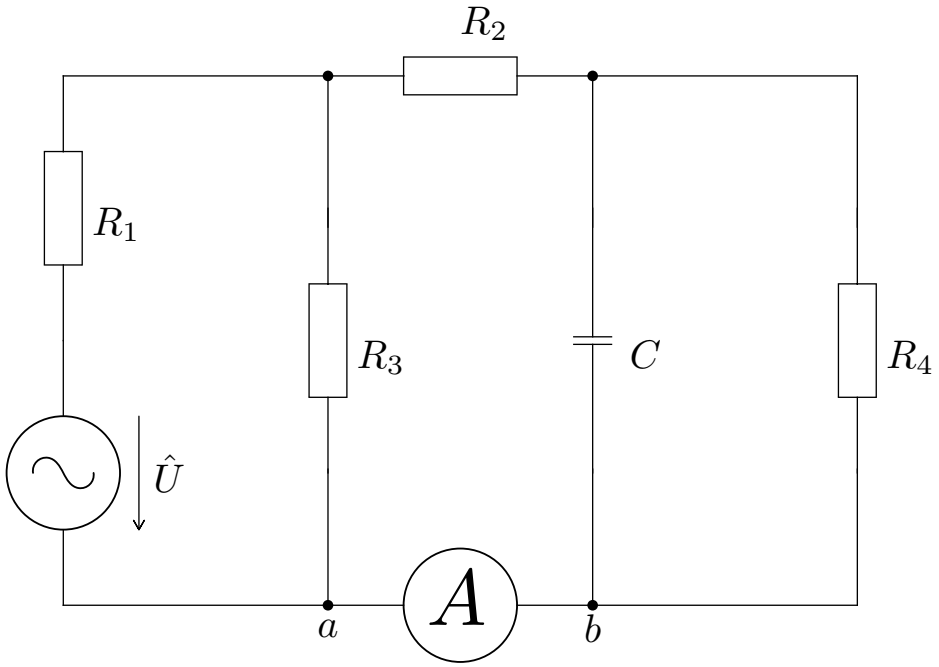


$$\hat{Z}_i = \frac{R_1 \cdot R_3}{R_1 + R_3} + R_2 + \frac{-R_4 \cdot \frac{j}{\omega C}}{R_4 - \frac{j}{\omega C}} = \frac{200 \cdot 120}{320} + 160 - \frac{80 \cdot 60j}{80 - 60j} = (263,8 - 38,4j)\Omega$$



$$\hat{I} = \frac{\hat{U}_i}{\hat{Z}_i} = \frac{15}{263,8 - 38,4j} = 0,0563\angle 8,28° A$$

Př.9.3: Určete parametry náhradního zdroje a proud, který ukazuje am=érmetr, v obvodu na obrázku.



$$\hat{U} = 40V$$

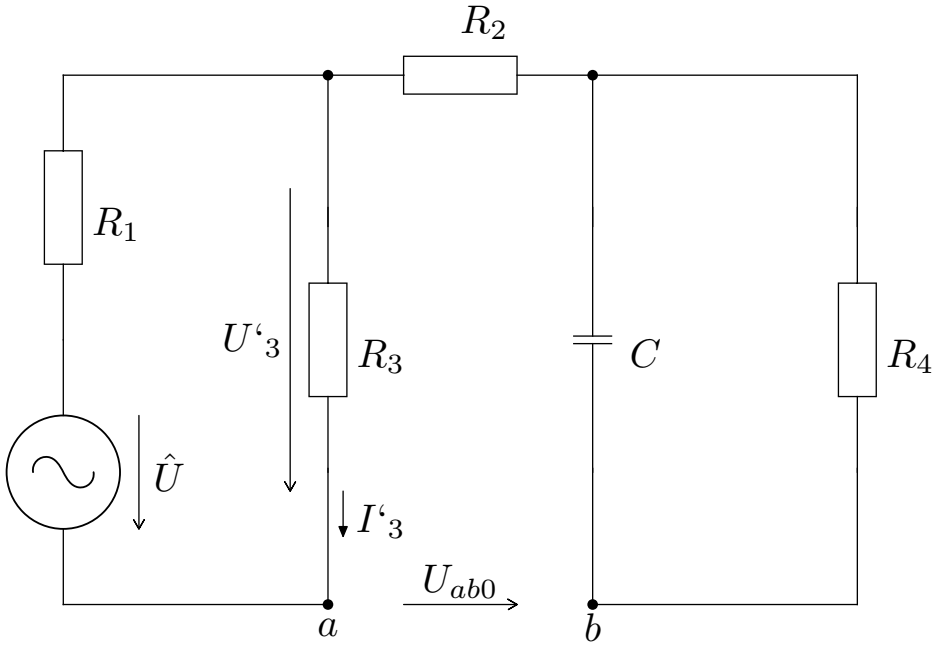
$$R_1 = 200\Omega$$

$$R_2 = 160\Omega$$

$$R_3 = 120\Omega$$

$$R_4 = 80\Omega$$

$$\frac{1}{\omega C} = 60\Omega$$

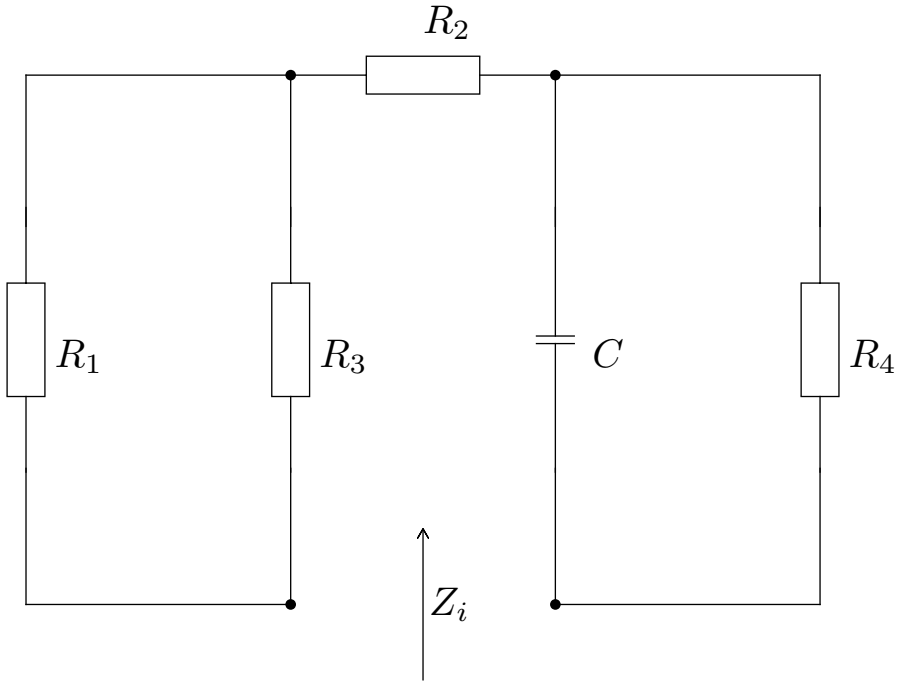


$$\hat{I}'_3 = \frac{\hat{U}}{R_1 + R_2} = \frac{40}{200 + 120} = 0,125A$$

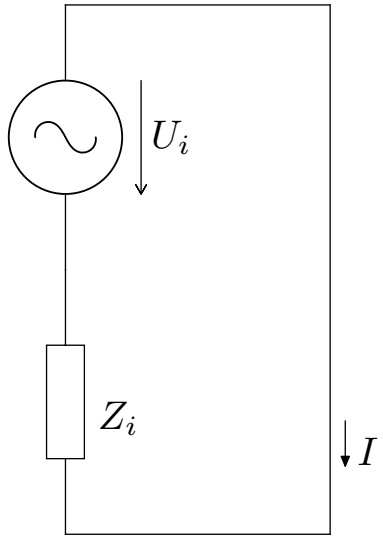
$$\hat{U}'_3 = R_3 \cdot \hat{I}'_3 = 120 \cdot 0,125 = 15V$$

$$\hat{U}'_3 + \hat{U}_{ab0} = 0$$

$$\hat{U}_{ab0} = -\hat{U}'_3 = -15V$$



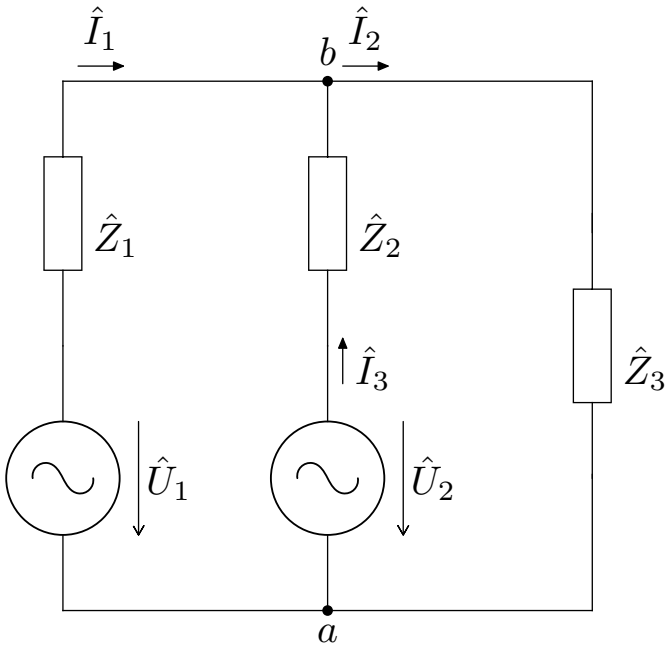
$$\hat{Z}_i = \frac{R_1 \cdot R_3}{R_1 + R_3} + R_2 + \frac{-R_4 \cdot \frac{j}{\omega C}}{R_4 - \frac{j}{\omega C}} = \frac{200 \cdot 120}{320} + 160 - \frac{80 \cdot 60j}{80 - 60j} = (263,8 - 38,4j)\Omega$$



$$\hat{I} = \frac{\hat{U}_i}{\hat{Z}_i} = \frac{-15}{263,8 - 38,4j} = 0,0563\angle -171,7° A$$

$$I_A = 0,0563 A$$

Př.9.4: Určete proud impedancí $\hat{Z}_2$ metodou náhradního zdroje.



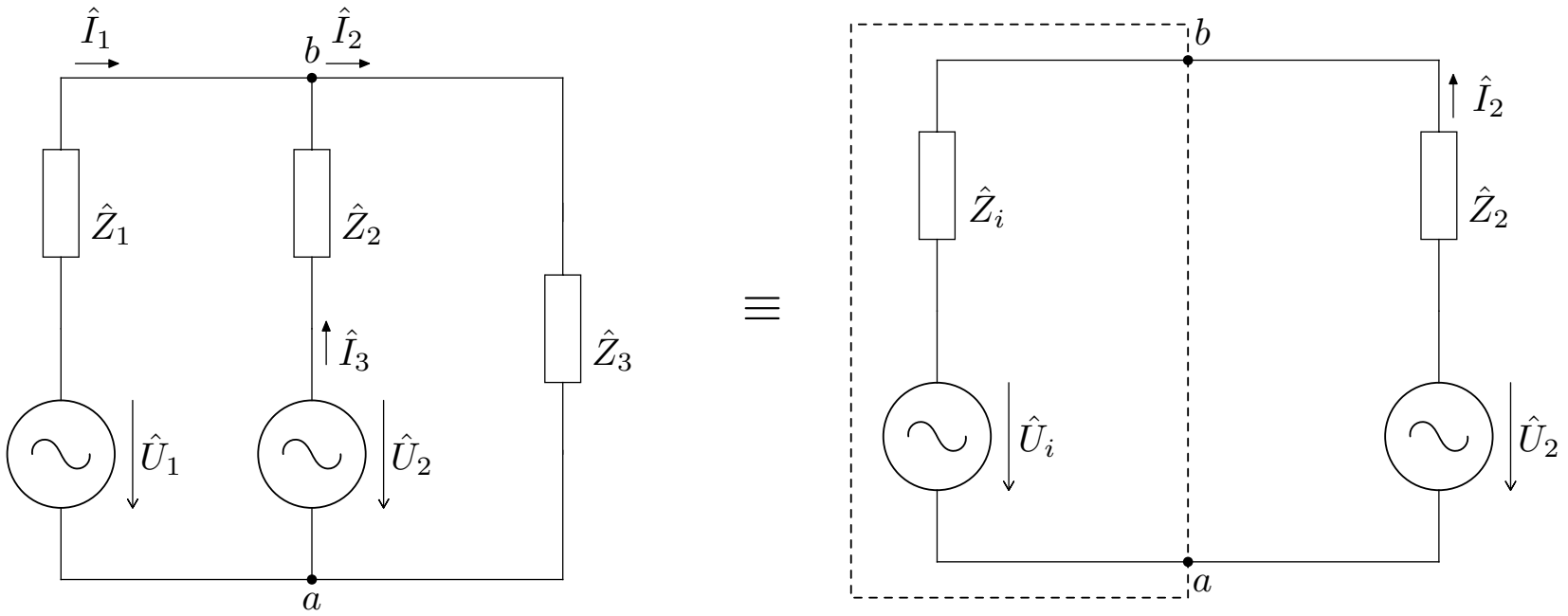
$$\hat{Z}_1 = (50 + 30j)\Omega$$

$$\hat{Z}_2 = (50 + 30j)\Omega$$

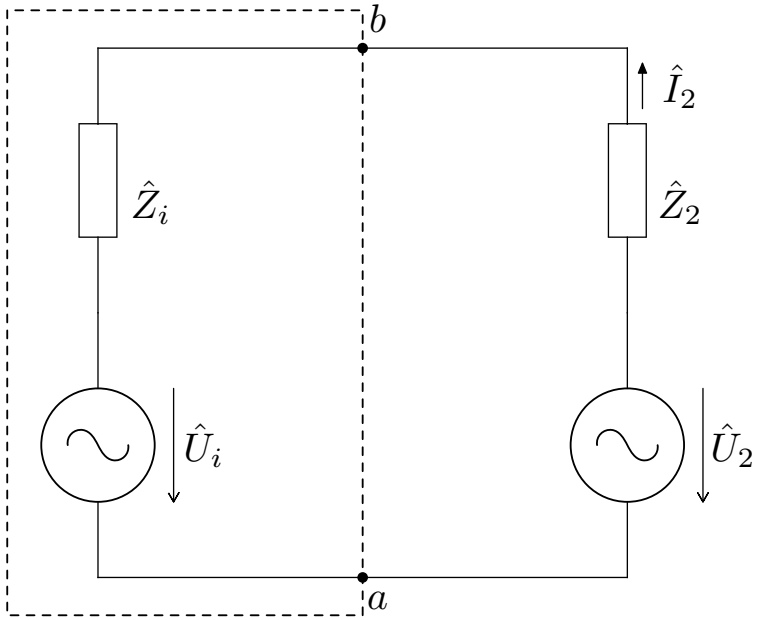
$$\hat{Z}_3 = 100\Omega$$

$$\hat{U}_1 = 100V$$

$$\hat{U}_2 = 100\angle -30°V$$

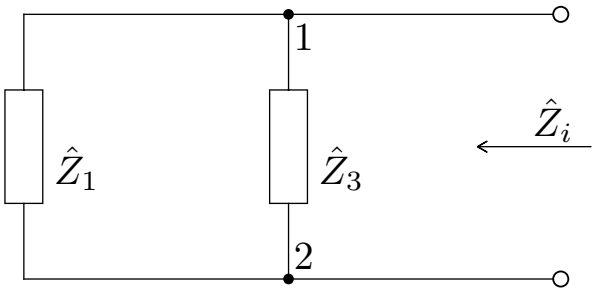


Vnitřní napětí náhradního zdroje $\hat{U}_{12} = \hat{U}_{13}$



$$\hat{I} = \frac{\hat{U}_1}{\hat{Z}_1 + \hat{Z}_3}$$

$$\hat{U}_{12} = \hat{I}_i = \hat{I}` \cdot \hat{Z}_3 = \frac{\hat{U}_1 \cdot \hat{Z}_3}{\hat{Z}_1 + \hat{Z}_3} = \frac{100 \cdot 100}{50 + 30j + 100} = (64,2 - 12,8j)V$$



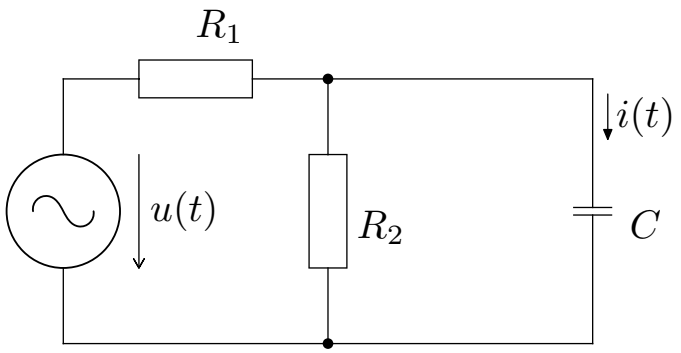
Vnitřní impedance náhradního zdroje:

$$\hat{Z}_i = \frac{\hat{Z}_1 \cdot \hat{Z}_3}{\hat{Z}_1 + \hat{Z}_3} = \frac{(50 + 30j) \cdot 100}{150 + 30j} = (35,9 + 12,8j)\Omega$$

Hledaný proud $\hat{I}_2$:

$$\hat{I}_2 = \frac{\hat{U}_2 - \hat{U}_i}{\hat{Z}_i + \hat{Z}_2} = \frac{100\angle -30° - (64,2 - 12,8j)}{85,9 + 42,8j} = 0,452\angle -85,5°A$$

Př.9.5: Pomocí Theveninovy věty vypočítejte proud $i(t)$.



$$\begin{aligned} u(t) &= U_m \cdot sin(\omega t) = 12 \cdot sin(2 \cdot \pi \cdot 50 \cdot t) \\ R_1 &= 1k\Omega \\ R_2 &= 4k\Omega \\ C &= 10\mu F \end{aligned}$$

$$\hat{U}_m = 12 \cdot e^{j0} = 12V$$

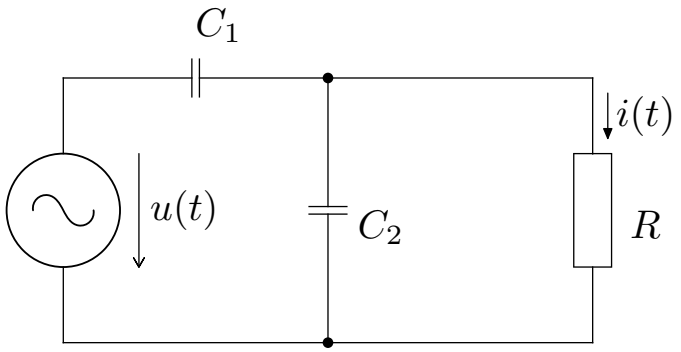
$$\hat{Z}_i = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{10^3 \cdot 4 \cdot 10^3}{5 \cdot 10^3} = 800\Omega$$

$$\hat{U}_{im} = \hat{U}_m \cdot \frac{R_2}{R_1 + R_2} = 12 \cdot \frac{4 \cdot 10^3}{5 \cdot 10^3} = 9,6V$$

$$\hat{I}_m = \frac{\hat{U}_{im}}{\hat{Z}_i + \frac{1}{j\omega C}} = \frac{9,6}{800 - j\frac{1}{2\pi \cdot 50 \cdot 10 \cdot 10^{-6}}} = 11,15 \cdot e^{j21,7^\circ} mA$$

$$i(t) = I_m \cdot sin(\omega t) = 11,15 \cdot sin(2 \cdot \pi \cdot 50 \cdot t + 21,7^\circ)\ mA$$

Př.9.6: Pomocí Theveninovy věty vypočítejte proud $i(t)$.



$$\begin{aligned} u(t) &= U_m \cdot sin(\omega t) = 50 \cdot sin(2 \cdot \pi \cdot 50 \cdot t) \\ R_1 &= 1k\Omega \\ C_1 &= 10\mu F \\ C_2 &= 20\mu F \end{aligned}$$

$$\hat{U}_m = 50 \cdot e^{j0} = 50V$$

$$\hat{Z}_i = \frac{\frac{1}{j\omega C_1} \cdot \frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = \frac{\frac{1}{j \cdot 2\pi \cdot 50 \cdot 10 \cdot 10^{-6}} \cdot \frac{1}{j \cdot 2\pi \cdot 50 \cdot 20 \cdot 10^{-6}}}{\frac{1}{j \cdot 2\pi \cdot 50 \cdot 10 \cdot 10^{-6}} + \frac{1}{j \cdot 2\pi \cdot 50 \cdot 20 \cdot 10^{-6}}} = -106,103j\ \Omega$$

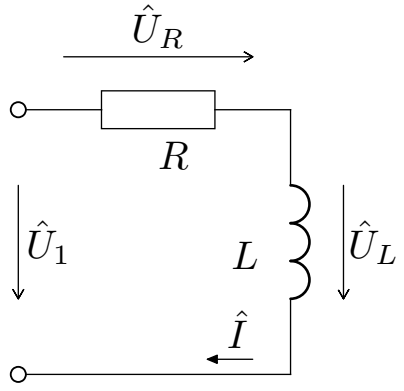
$$\hat{U}_{im} = \hat{U}_m \cdot \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = 50 \cdot \frac{\frac{1}{j \cdot 2\pi \cdot 50 \cdot 20 \cdot 10^{-6}}}{\frac{1}{j \cdot 2\pi \cdot 50 \cdot 10 \cdot 10^{-6}} + \frac{1}{j \cdot 2\pi \cdot 50 \cdot 20 \cdot 10^{-6}}} = 16,667V$$

$$\hat{I}_m = \frac{\hat{U}_{im}}{\hat{Z}_i + R} = \frac{16,667}{-106,103j + 1000} = 16,57 \cdot e^{j6,057^\circ} mA$$

$$i(t) = I_m \cdot sin(\omega t) = 16,57 \cdot sin(2 \cdot \pi \cdot 50 \cdot t + 6,057^\circ)\ mA$$

2.4 Přenosové funkce RC, RL a RLC obvodů

Př.9.7: Vypočítejte napětí $\hat{U}_L$ a přenos napětí $\hat{K}_U$ obvodu RL.

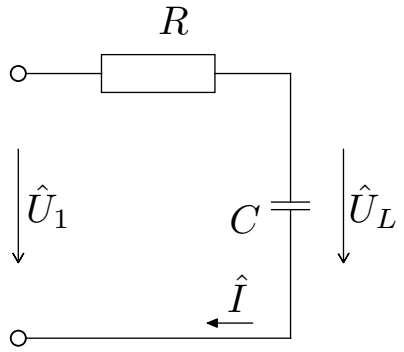


$$\begin{aligned}
\hat{U}_1 &= 10V \\
R &= 2k\Omega \\
L &= jmH \\
\omega &= 5 \cdot 10^4 s^{-1}
\end{aligned}$$

$$\omega L = 5 \cdot 10^4 \cdot 10^{-3} = 50\Omega$$

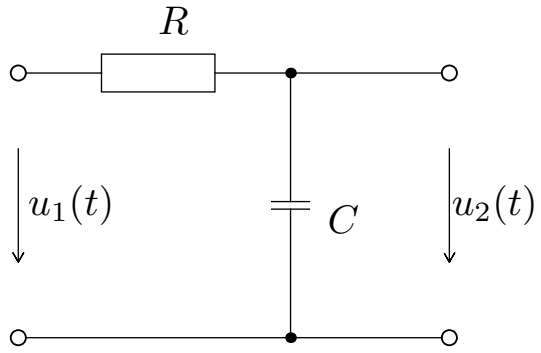
$$\begin{aligned}
\hat{K}_U &= \frac{\hat{U}_L}{\hat{U}_1} = \frac{j\omega L}{R + j\omega L} = \frac{j\omega L \cdot (R - j\omega L)}{(R + j\omega L) \cdot (R - j\omega L)} = \frac{\omega^2 L^2}{R^2 + \omega^2 L^2} + j \cdot \frac{\omega L R}{R^2 + \omega^2 L^2} = \\
&= \frac{2500}{4 \cdot 10^6 + 2,5 \cdot 10^3} + j \cdot \frac{50 \cdot 2 \cdot 10^3}{4 \cdot 10^6 + 2,5 \cdot 10^3} = 6,2461 \cdot 10^{-4} + j249,84 \cdot 10^{-4} = \\
&= (6,2461 + j249,84) \cdot 10^{-4} \\
\hat{U}_L &= \hat{U}_1 \cdot \hat{K}_U = 10 \cdot (6,2461 + j249,84) \cdot 10^{-4} = 0,24992 \angle 88,568° V.
\end{aligned}$$

Př.9.8: Vypočítejte obecně přenos napětí $\hat{K}_U$ v obvodu RC, je-li $u(t) = U_{1m} \cdot sin\omega t$.



$$\hat{K}_{Uc} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega\tau_C}$$

Př.9.9: U přenosového článku z obrázku vypočítejte hodnotu výstupního napětí a určete, v které oblasti článek pracuje, je-li kmitočet vstupního harmonického napětí $f = 100Hz$.



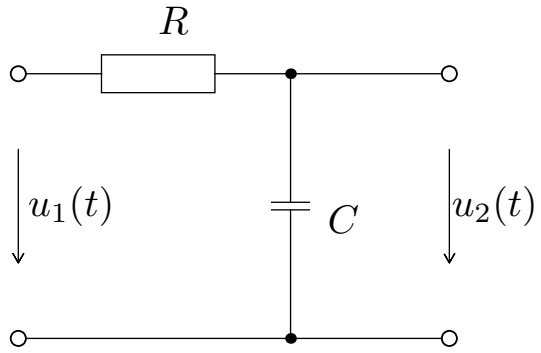
$$\begin{aligned} u_1(t) &= U_{1m} \cdot sin\omega t = 10 \cdot \sqrt{2} \cdot sin\omega t \\ R &= 1k\Omega \\ C &= 1\mu F \end{aligned}$$

$$\begin{aligned} \hat{U}_1 &= 10 \cdot e^{j0^\circ} \\ \hat{U}_2 &= \hat{U}_1 \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \hat{U}_1 \cdot \frac{1}{1 + j\omega RC} = 10 \cdot \frac{1}{1 + j \cdot 2\pi \cdot 100 \cdot 1000 \cdot 2 \cdot 10^{-6}} = \\ &= 7,9577 \cdot 10^{-3} \cdot e^{-j89,95^\circ} V \end{aligned}$$

$$u_2(t) = 7,9577 \cdot 10^{-3} \cdot \sqrt{2} \cdot sin(\omega t - 89,95^\circ)V = 0,011254 \cdot sin(\omega t - 89,95^\circ)V$$

$\Longrightarrow$ Přenosový článek pracuje v oblasti integrace.

Př.9.10: U přenosového článku z obrázku vypočítejte hodnotu výstupního napětí a určete, v které oblasti článek pracuje, je-li kmitočet vstupního harmonického napětí $f = 20Hz$.

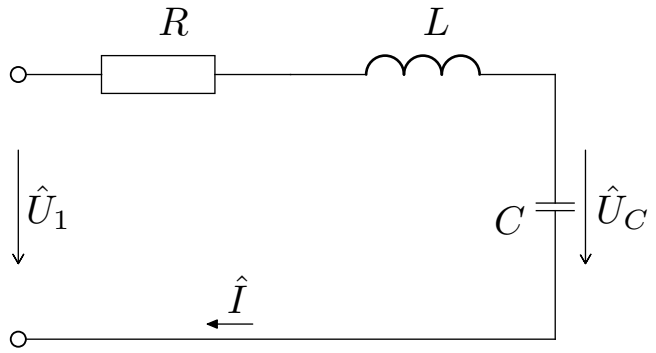


$$\begin{aligned} u_1(t) &= U_{1m} \cdot sin\omega t = 10 \cdot \sqrt{2} \cdot sin\omega t \\ R &= 1k\Omega \\ C &= 1\mu F \end{aligned}$$

$$\begin{aligned} \hat{U}_1 &= 10 \cdot e^{j0^\circ} \\ \hat{U}_2 &= \hat{U}_1 \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \hat{U}_1 \cdot \frac{1}{1 + j\omega RC} = 10 \cdot \frac{1}{1 + j \cdot 2\pi \cdot 20 \cdot 1000 \cdot 2 \cdot 10^{-6}} = \\ &= 2,4375 \cdot e^{j75,89^\circ} V \end{aligned}$$

$$u_2(t) = 2,4375 \cdot \sqrt{2} \cdot sin(\omega t + 75,89^\circ)V = 3,4471 \cdot sin(\omega t + 75,89^\circ)V$$

Př.9.11: Vypočítejte obecně přenos napětí $\hat{K}_{UC}$ v obvodu RLC.



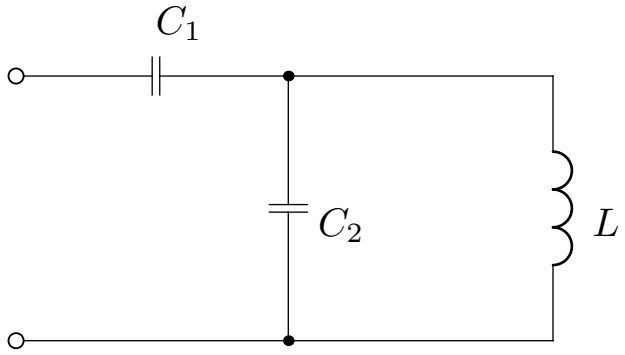
$$\hat{I} = \frac{\hat{U}_1}{R + j\omega L + \frac{1}{j\omega C}}$$

$$\hat{U}_C = \frac{\hat{I}}{j\omega C} = \hat{U}_1 \cdot \frac{1}{1 - \omega^2 LC + j\omega RC}$$

$$\hat{K}_{UC} = \frac{\hat{U}_C}{\hat{U}_1} = \frac{1}{1 - \frac{\omega L}{R} \cdot \omega RC + j\omega RC} = \frac{1}{1 - \omega^2 \tau_L \tau_C + j\omega \tau_C}$$

2.5 Rezonanční obvody

Př.10.1: Pro obvod na obrázku určete všechny kmitočty, při nichž nastává napěťová, popř. proudová rezonance.



Podmínka napěťové rezonance $\hat{Z} = 0$

Podmínka proudové rezonance $\hat{Y} = 0$

$$\hat{Z} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2 + \frac{1}{j\omega L}} = \frac{1}{j\omega C_1} + \frac{j\omega L}{1 - \omega^2 L C_2} = j\frac{\omega^2 L C_1 - (1 - \omega^2 L C_2)}{\omega C_1 (1 - \omega^2 L C_2)}$$

Podmínka napěťové rezonance $\hat{Z} = 0$

$$\omega^2 LC_1 - (1 - \omega^2 LC_2) = 0$$

$$\omega_{rn} = \frac{1}{\sqrt{L(C_1 - C_2)}}$$

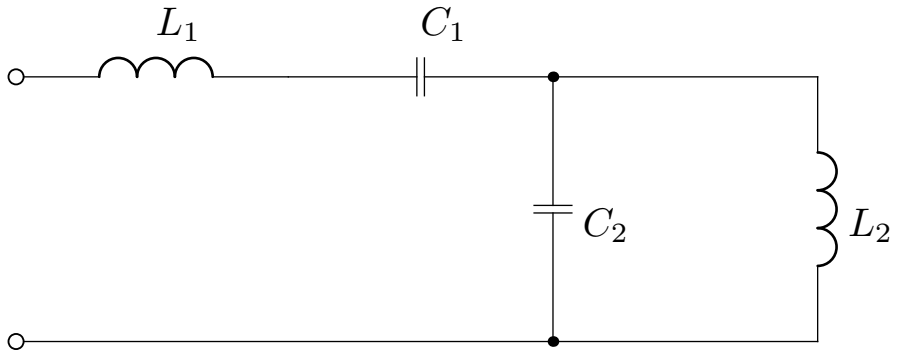
Podmínka proudové rezonance $\hat{Y} = 0$

$$\hat{Y} = \frac{1}{\hat{Z}} = \frac{\omega C_1(1 - \omega^2 LC_2)}{\omega^2 LC_1 - (1 - \omega^2 LC_2)}$$

$$\omega C_1(1 - \omega^2 LC_2) = 0$$

$$\omega_{rp} = \frac{1}{\sqrt{LC_2}}$$

Př.10.2: Určete všechny rezonanční kmitočty reaktančního dvojpólu z obrázku.



$$\begin{aligned} L_1 &= 10mH \\ L_1 &= 1mH \\ C_1 &= 1\mu F \\ C_1 &= 5\mu F \end{aligned}$$

$$\hat{Z} = j\omega L_1 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2 + \frac{1}{j\omega L_2}} = \frac{1 - \omega^2(L_1C_1 + L_2C_2 + L_2C_1) + \omega^4 L_1L_2C_1C_2}{j\omega C_1(1 - \omega^2 L_2C_2)} =$$

$$= \frac{1 - 1,6 \cdot 10^{-8}\omega^2 + 5 \cdot 10^{-17}\omega^4}{j\omega \cdot 10^{-6} \cdot (1 - 5 \cdot 10^{-9}\omega^2)}$$

Kmitočty napěťové rezonance:

$$1 - 1,6 \cdot 10^{-8}\omega^2 + 5 \cdot 10^{-17}\omega^4 = 0$$

$$\omega_1 = 9,24 \cdot 10^3 s^{-1}$$
$$\omega_2 = 15,3 \cdot 10^3 s^{-1}$$

Kmitočty proudové rezonance:

$$j\omega \cdot 10^{-6} \cdot (1 - 5 \cdot 10^{-9}\omega^2) = 0$$

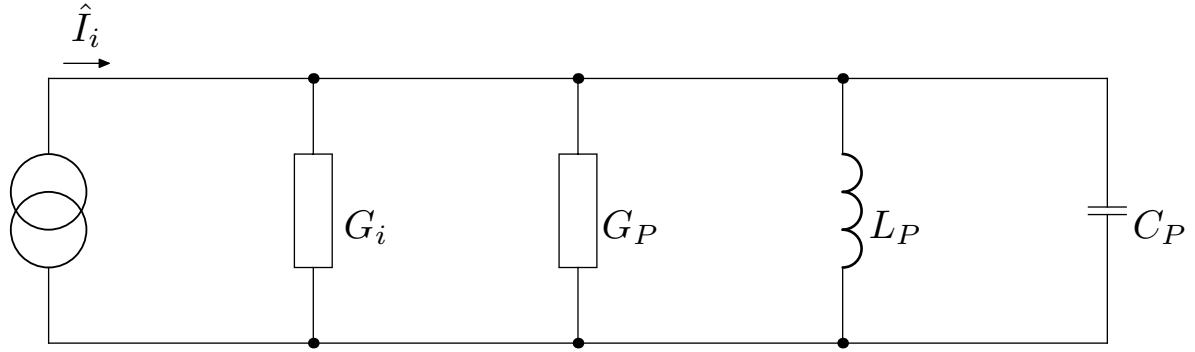
$$\omega_3 = 14,14 \cdot 10^3 s^{-1}$$

Př.10.3: Sériový rezonanční obvod RLC má parametry: $R = 10\Omega, L = 100\mu H$, $C = 100pF$. Určete rezonanční kmitočet ω_r, charakteristickou impedanci Z_{ch}, útlum δ a činitel jakosti Q. Určete proud I_r, výkon v obvodu P_r, napětí na cívce U_{Lr} a kondenzátoru U_{Cr}, při rezonanci, je-li obvod připojen na zdroj o napětí $U = 1V$.

$$\begin{aligned}
\omega_r &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100 \cdot 10^{-6} \cdot 100 \cdot 10^{-12}}} = 10^7 s^{-1} \\
f_r &= \frac{\omega_r}{2\pi} = 1,6 MHz \\
Z_{ch} &= \sqrt{\frac{L}{C}} = 1000\Omega \\
\delta &= \frac{10}{1000} = 0,01 \\
Q &= \frac{Z_{ch}}{R} = \frac{1000}{10} = 100 \\
I_r &= \frac{U}{R} = 0,1A = 100mA \\
P_r &= R \cdot I_r^2 = 0,1W \\
U_{Lr} &= U_{Cr} = Z_{ch} \cdot I_r = 1000 \cdot 0,1 = 100V
\end{aligned}$$

Př.10.4: Paralelní rezonanční obvod se skládá z kamacitoru $C = 139pF$ a cívky s indukčností $L = 20\mu H$ a sériového ztrátového odporu $R = 5\Omega$. Obvod je připojen na zdroj o napětí $\hat{U}_i = 10V$ s vnitřním odporem $R_i = 100\Omega$. Vypočítejte rezonanční kmitočet ω_r, činitel jakosti Q, proud odebíraný obvodem a napětí na rezonančním obvodu při rezonanci.

Sériový obvod převedeme na obvod s proudovým zdrojem.



$$\hat{I}_i = \frac{\hat{U}_i}{R_i} \qquad G_P = \frac{R}{R^2 + (\omega L)^2}$$

$$G_i = \frac{1}{R_i} \qquad L_P = \frac{1}{R^2 + (\omega L)^2} \cdot \frac{1}{(\omega L)^2}$$

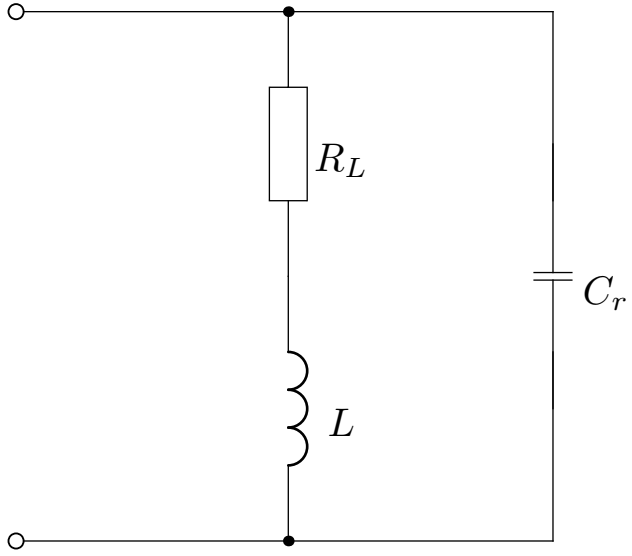
Podmínka rezonance $Im\left[\hat{Y}\right] = 0$

$$\begin{aligned}
\hat{Y} &= G_i + G_P + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right) \\
\omega_r &= \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = 1,88 \cdot 10^7 s^{-1} \\
f_r &= 3MHz
\end{aligned}$$

$$\begin{aligned}
G_{Pr} &= 3,5 \cdot 10^{-5} S \\
L_{Pr} &= 2 \cdot 10^{-5} H \\
Q &= \frac{1}{\omega_r L_{Pr}(G_i + G_{Pr})} = \frac{1}{120\pi \cdot 4,48 \cdot 10^{-5}} = 59,5 \\
\hat{I}_r &= \hat{I}_i \cdot \frac{G_{Pr}}{G_i + G_{Pr}} = \hat{U}_i \cdot \frac{G_i \cdot G_{Pr}}{G_i + G_{Pr}} = 7,8 \cdot 10^{-5} A \\
\hat{U}_r &= \frac{\hat{I}_r}{G_{Pr}} = 2,22V
\end{aligned}$$

$$
\begin{aligned}
j\omega L &= 314,16\Omega \\
j\omega C_1 &= j \cdot 314,16 \cdot 5 \cdot 10^{-6} = 0,0015708j\ S \\
\frac{1}{j\omega C_1} &= -636,62j\ \Omega \\
\\
\hat{Z}_1 &= R_1 + j\omega L + \frac{1}{j\omega C_1} = 300 + j(314,16 - 636,62) = (300 - 322,46j)\Omega \\
j\omega C_3 &= j \cdot 314,16 \cdot 2 \cdot 10^{-6} = 0,00062832j\ S \\
\hat{Z}_2 &= \frac{1}{j\omega C_3} = -1591,55j\ \Omega;; \\
\hat{Z}_{13} &= \frac{\hat{Z}_1 \cdot \hat{Z}_3}{\hat{Z}_1 + \hat{Z}_3} = \frac{(300 - 322,46j) \cdot (-1591,55j)}{300 - 1914,01j} = (202,54 - 300,04j)\Omega \\
\hat{Z}_2 &= R_2 + \frac{1}{j\omega C_2} = 100 - j \cdot \frac{1}{314,16 \cdot 10 \cdot 10^{-6}} = (100 - 318,31j)\Omega \\
\\
\hat{Z} &= \hat{Z}_2 + \hat{Z}_{13} = (100 - 318,31j) + (202,54 - 300,04j) = (302,54 - 618,35j)\Omega
\end{aligned}
$$

Př.10.5: Paralelní rezonanční obvod je složen z induktoru $L = 0,4mH$, rezistoru $R_L = 20\Omega$ a kapacitoru C, je napájen zdrojem o frekvenci $f = 600kHz$. Určete hodnoty kapacity C_r, aby nastala rezonance. Určete činitel jakosti Q a impedanci obvodu při rezonanci.



Podmínka rezonance $Im\left[\hat{Y}\right] = 0$

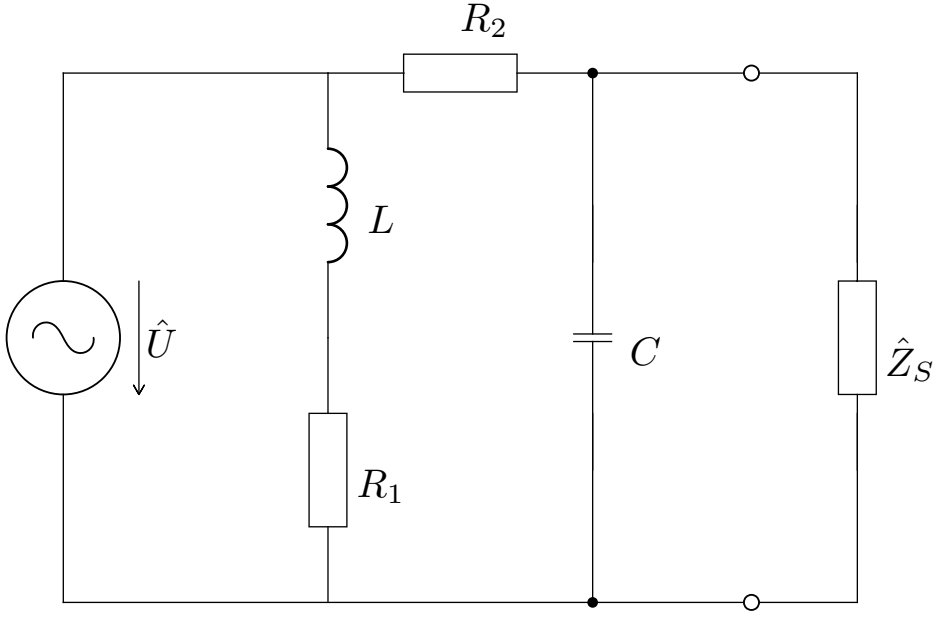
$$\begin{aligned}
\hat{Y} &= \frac{1}{R_L + j\omega L} + j\omega C = \\
&= \frac{R_L}{R_L^2 + (\omega L)^2} - j \cdot \left[\frac{\omega L}{R_L^2 + (\omega L)^2} - \omega C\right] \\
C_r &= \frac{L}{R_L^2 + (\omega L)^2} = \\
&= \frac{4 \cdot 10^{-4}}{20^2 + (2\pi \cdot 6 \cdot 10^5 \cdot 4 \cdot 10^{-4})^2} = 177pF
\end{aligned}$$

$$Q = \frac{\omega L}{R_L} = \frac{2\pi \cdot 6 \cdot 10^5 \cdot 4 \cdot 10^{-4}}{20} = 75,5$$

$$\hat{Z}_r = \frac{1}{Re\left[\hat{Y}\right]} = \frac{R_L^2 + (\omega L)^2}{R_L} = \frac{20^2 + 2,66 \cdot 10^6}{20} = 113k\Omega$$

2.6 Výkonové přizpůsobení

Př.10.6: Obvod podle obrázku je napájen ze zdroje harmonického napětí $\hat{U} = 100V$, s kmitočtem $f = 100Hz$. Vypočtěte zatěžovací impedanci $\hat{Z}_S$ tak, aby výkon na zátěži byl maximální a určete jeho velikost.



$$R_1 = 250\Omega$$

$$R_2 = 1,5k\Omega$$

$$L = 2,5H$$

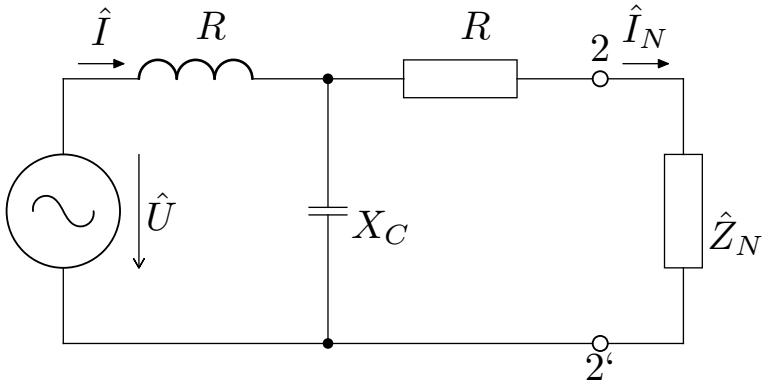
$$C = 1,06pF$$

$$\hat{Z}_i = \frac{R_i}{1 + j\omega C R_2} = \frac{100}{1 + j} = (750 - 750j)\Omega$$

$$\hat{Z}_S = \hat{Z}_i^{\bullet} = (750 + 750j)\Omega$$

$$P_S = 0,5P = 0,5 \cdot \frac{U^2}{2R_2} = 1,67W$$

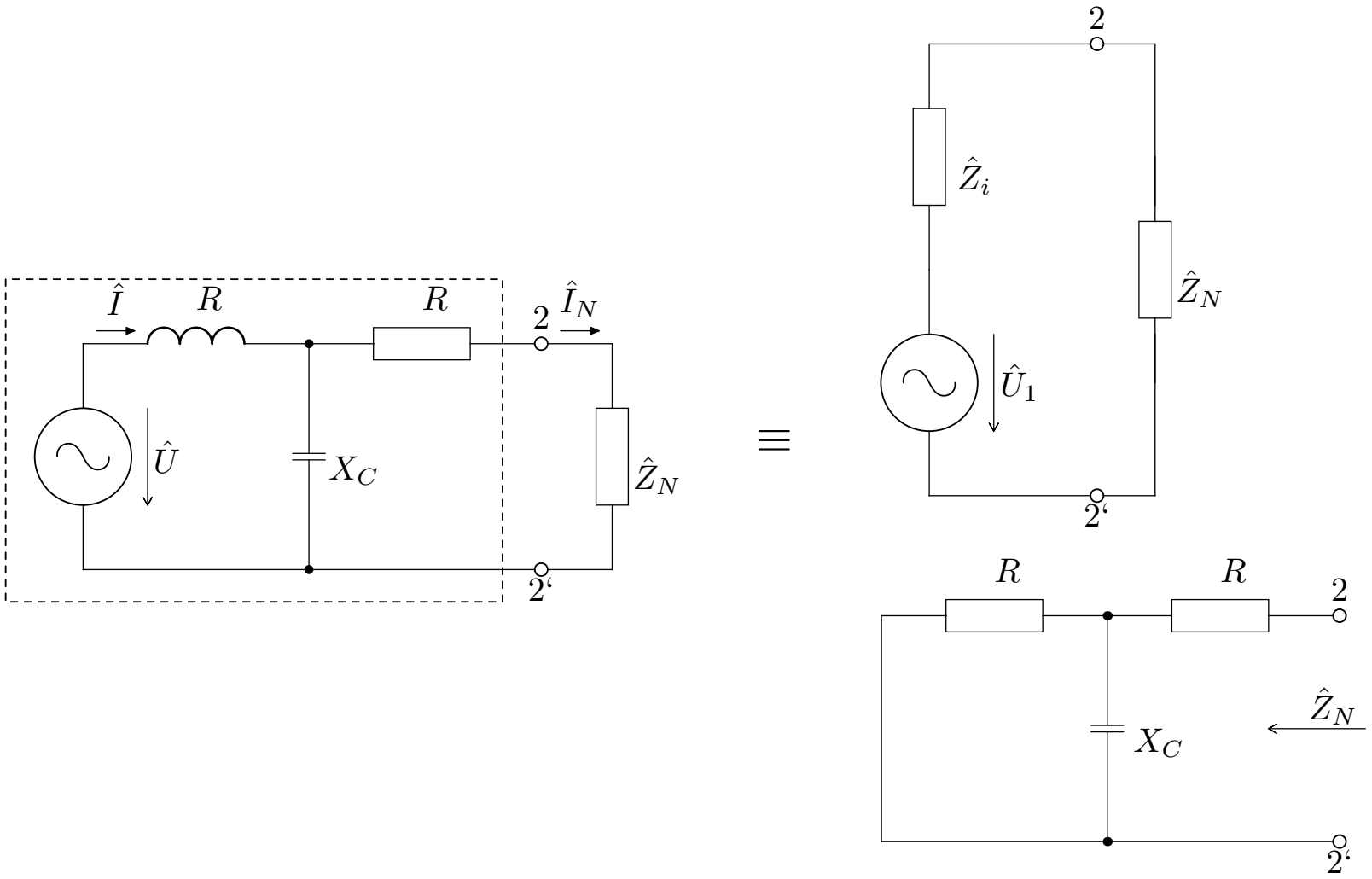
Př.10.6: Vypočítejte impedanci $\hat{Z}_N$ tak, aby výkon jí dodaný byl maximální, vypočtěte jeho velikost.



$$R = 10\Omega$$

$$\frac{1}{\omega C} = 30\Omega$$

$$U = 100V$$


$\hat{I}$
R
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2
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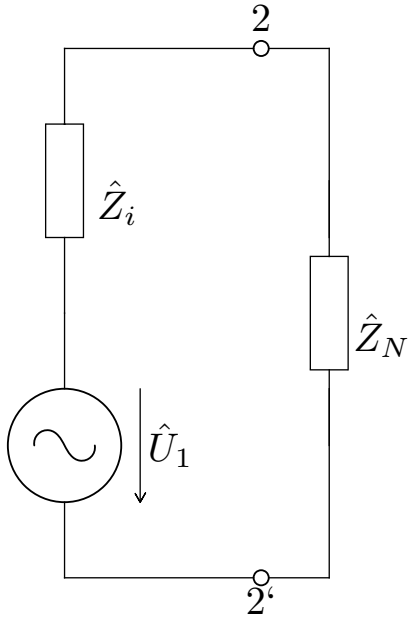
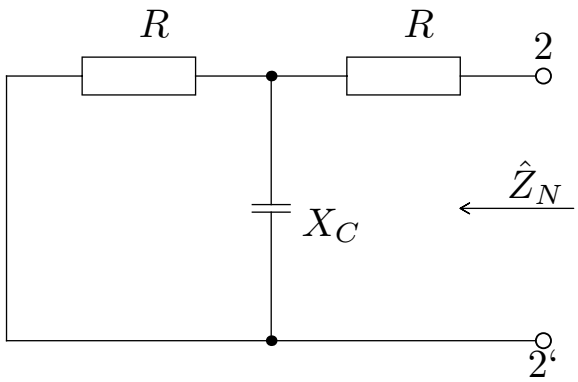


$$\hat{U}_i = \frac{\hat{U}}{R - \frac{j}{\omega C}} \cdot \frac{1}{j\omega C} = \frac{100}{10 - 30j} \cdot (-30j) = 90 - 30j = 95\angle - 18,25°V$$



$$\hat{Z}_i = R + \frac{-j \cdot \frac{1}{\omega C} \cdot R}{R - \frac{j}{\omega C}} = 10 + \frac{-30j \cdot 10}{10 - 30j} = (19 - 3j)\Omega$$

Max. výkon bude při $\hat{Z}_N = \hat{Z}_i^{\bullet} = (19 + 3j)\Omega$

$$P_{Nmax} = \frac{\hat{U}_i^2}{4 \cdot R_N} = \frac{95^2}{4 \cdot 19} = 118W$$

Výkon dodaný zdrojem:

$$\hat{I} = \frac{\hat{U}}{R_1 + \frac{-\frac{j}{\omega C}\left(R_1 + \hat{Z}_N\right)}{R_1 + \hat{Z}_N - \frac{j}{\omega C}}} = \frac{100}{10 - \frac{30 \cdot (29 + 3j)}{29 - 27j}} = 33\angle 28,37°A$$

$$P_Z = Re\left[\hat{U} \cdot \hat{I}^{\bullet}\right] = Re\left[100 \cdot 3,3\angle -28,37°\right] = 100 \cdot 3,3 \cdot cos28,37° = 290W$$

Účinnost $\eta = \frac{P_{Nmax}}{P_Z} = \frac{118}{290} = 0,407 \approx 40,7\%$