VNVe 2019/2020 - Final project

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Solve all examples and send your final solution (pdf file) and all source codes (LaTex, MATLAB, C, C++, etc.) to e-mail address satek@fit.vutbr.cz **until May 17, 2020**. Everything will be packed in one zip file 'surname.zip'.

1 Problem - Derivation reduction

Solve linear differential equation of the 4^{th} order

$$y'''' + a_3 y''' + a_2 y'' + a_1 y' + a_0 y = b_4 z'''' + b_3 z''' + b_2 z'' + b_1 z' + b_0 z \quad (1)$$

with zero initial conditions, constant forcing function $z = 1$ and constants

$$a_3 = 12,\ a_2 = 24,\ a_1 = 2,\ a_0 = 3,$$

$$b_4 = 1,\ b_3 = 13,\ b_2 = 8,\ b_1 = 13,\ b_0 = 1\,.$$

Use Method of Derivative Order Reduction with Auxiliary Variable (MDORwAV) and the Method of Successive Integration (MSI).

1.1 Tasks:

1. Write the system of first order differential equations for the both methods (MDORwAV and MSI), use symbolic operator p and $1/p$ for operation of derivation and integration, respectively.
2. Draw (e.g. in Dia software) the scheme with inverting blocks (inverting - integrators and summators).
3. Show the stability of the system (use Routh-Hurwitz stability criterion).
4. Implement both schemes (MDORwAV and MSI) in MATLAB-Simulink. Plot the graphs of the solutions $y_{MDORwAV}$ and y_{MSI}. Plot the absolute error $|y_{MDORwAV} - y_{MSI}|$ in the separate figure.

2 Problem - Discharging of Capacitance

Let's find a solution of the voltage on capacitance $u_C(t)$ and the current $i(t)$ in RC circuit Fig. 1. Parameters of the RC circuit are $R = 10\Omega$, $C = 10^{-2}$F, $u_C(0) = 10$V.

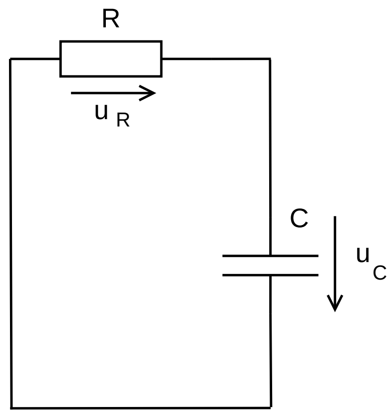

Figure 1: RC Circuit

2.1 Tasks:

1. Set ordinary differential equation (initial value problem) for voltage on capacitance $u_C(t)$ in the RC circuit Fig. 1 .

2. Find analytic solution of $u_C(t)$.

3. Find numerical solution of $u_C(t)$: use the Euler method, the Runge-Kutta of 2nd and 4th order. Implement it using your own code (e.g. MATLAB, C, C++, Java, etc. it is on your choice, what you prefer). Compare your numerical solutions with the Taylor series based numerical solution (in TKSL software).

4. Compare analytic solution with numerical solutions - plot figures with absolute error between numerical solutions and analytical solution.

5. Prepare a block scheme of this problem and solve it numerically in MATLAB-Simulink. Compare the numerical solution from Simulink with the previous numerical solutions.

6. Change the parameter of capacitance in the RC circuit Fig. 1 to the new value $C = 10^{-6}$F and compare the total number of integration

steps used in different numerical methods (Euler method, Runge-Kutta methods and Taylor series based method - TKSL). Which numerical method allow you to make a larger integration step size? NOTE: Use smaller time of simulation.

3 Problem - Charging of Capacitance

Let's find a solution of the voltage on capacitance $u_C(t)$ and the current $i(t)$ in RC circuit in Fig. 2. Parameters of the RC circuit are $R = 10\Omega$, $C = 10^{-2}$F, $U = 10$V, $u_C(0) = 1$V.

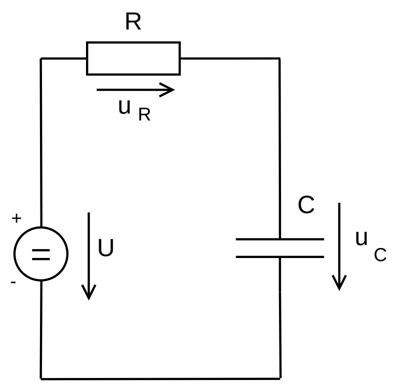

Figure 2: RC Circuit with Supply Voltage U

3.1 Tasks:

1. Set ordinary differential equation (initial value problem) for voltage on capacitance $u_C(t)$ in the RC circuit Fig. 1 .

2. Find analytic solution of $u_C(t)$.

3. Find numerical solution of $u_C(t)$: use the Euler method, the Runge-Kutta of 2nd and 4th order. Implement it using your own code (e.g. MATLAB, C, C++, Java, etc. it is on your choice, what you prefer). Compare your numerical solutions with the Taylor series based numerical solution (in TKSL software).

4. Compare analytic solution with numerical solutions - plot figures with absolute error between numerical solutions and analytical solution.

5. Prepare a block scheme of this problem and solve it numerically in MATLAB-Simulink. Compare the numerical solution from Simulink with the previous numerical solutions.

6. Change the parameter of capacitance in the RC circuit Fig. 2 to the new value $C = 10^{-6}$F and compare the total number of integration steps used in different numerical methods (Euler method, Runge-Kutta methods and Taylor series based method - TKSL). Which numerical method allow you to make a larger integration step size? NOTE: Use smaller time of simulation.

4 Problem - RLC circuit

Let's find a solution of the voltage on capacitance $u_C(t)$ and the current $i(t)$ in RLC circuit Fig. 3. Parameters of the RLC circuit are $R = 200\Omega$, $L = 10^{-2}$H, $C = 10^{-6}$F, $u_C(0) = 0$V, $i(0) = 0$A and input voltage $u(t) = U \sin(\omega t)$V, where $U = 1$V and $\omega = 1000$rad/s.

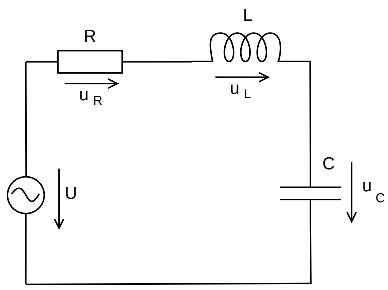

Figure 3: RLC Circuit with AC Supply Voltage U

4.1 Tasks:

1. Set ordinary differential equation (initial value problem) for voltage on capacitance $u_C(t)$ in the RLC circuit Fig. 3 .

2. Find analytic solution of $u_C(t)$ and $i(t)$.

3. Find numerical solution of $u_C(t)$: use the Euler method, the Runge-Kutta of 2nd and 4th order. Implement it using your own code (e.g. MATLAB, C, C++, Java, etc. it is on your choice, what you prefer). Compare your numerical solutions with the Taylor series based numerical solution (in TKSL software).

4. Compare analytic solution with numerical solutions - plot figures with absolute error between numerical solutions and analytical solution.

5. Prepare a block scheme of this problem and solve it numerically in MATLAB-Simulink. Compare the numerical solution from Simulink with the previous numerical solutions. NOTE: Use auxiliary differential equations to generate the input voltage - the function $u(t) = \sin(1000t)$.