

VNVe 24.4. 2020

SOLUTION OF SYSTEMS OF LINEAR ALGEBRAIC EQUATIONS (SLAE)

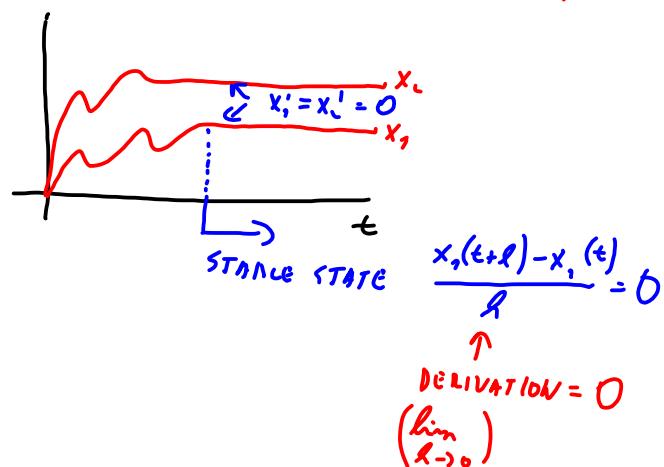
$$\begin{array}{l} x_1 + x_2 = 1 \\ x_1 - x_2 = 3 \end{array} \quad \begin{array}{l} \text{SOLUTION } (x_1 = 2) \\ \quad (x_2 = -1) \end{array}$$

→ TRANSFER TO THE SYSTEM OF DIFF. EQ.

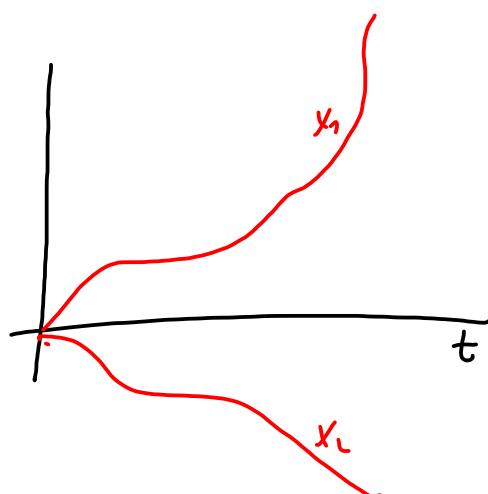
$$\begin{aligned} - (x_1 + x_2) + 1 &= 0 \quad \leftarrow x_1' \\ - (x_1 - x_2) + 3 &= 0 \quad \leftarrow x_2' \end{aligned} \quad \begin{array}{l} x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \vec{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \end{array}$$

We suppose STABLE SOLUTION

(Condition: MATRIX A IS
POSITIVE DEFINITE)



BUT WE CAN OBTAIN UNSTABLE
SOLUTION (e.g. MATRIX A IS
INDEFINITE)



POSITIVE DEFINITE SQUARE MATRIX A
(size $n \times n$)

$$\vec{x} \neq \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \vec{x}^T A \vec{x} > 0$$

EQUIVALENT PROPERTIES:

→ ALL EIGENVALUES ARE POSITIVE
 $|A - \lambda I| = 0$ $(\lambda_i > 0, i=1 \dots n)$

→ ALL SUBDETERMINANTS ARE POSITIVE

$$\begin{pmatrix} A_1 & & & \\ \vdots & \ddots & & \\ & & \ddots & \\ & & & A_m \end{pmatrix} \quad |A_1| > 0$$

$$|A_2| > 0$$

$$\vdots$$

$$|A_m| > 0$$

→ SOME SYMMETRIC MATRICES ARE
POSITIVE DEFINITE

$$A^T = A$$

OUR EXAMPLE:

$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ IS MATRIX A POSITIVE DEFINITE?

1) EIGENVALUES

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

$$\begin{aligned} & (1-\lambda)(-1-\lambda) - 1 \cdot 1 = \\ & = -1 - \cancel{\lambda} + \cancel{\lambda} + \lambda^2 - 1 \end{aligned}$$

$$\lambda^2 - 2 = 0 \rightarrow \lambda_1 = \sqrt{2} = 1,41$$

$$\lambda_2 = -\sqrt{2} = -1,41$$

MATRIX A IS INDEFINITE

$$(\exists \vec{x} \neq 0, \vec{y} \neq 0; \vec{x}^T A \vec{x} > 0 > \vec{y}^T A \vec{y})$$

2) SUBDETERMINANTS

$$|A_1| = 1 > 0 \quad \checkmark$$

$$A_2 = |A| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2 > 0$$

MATRIX A IS NOT POSITIVE DEFINITE

3) HOW TO CONSTRUCT POSITIVE DEFINITE A_{pos} MATRIX? :

$\underbrace{A^T A}_{A_{pos}} \dots$ IS SYMMETRIC POSITIVE DEFINITE MATRIX

STATIC TRANSFORMATION

$$\underbrace{A^T A \vec{x}}_{A_{\text{pos}}} = A^T \vec{b}$$

A ... INDEFINITE MATRIX

$$\vec{x}' = -A_{\text{pos}} \vec{x} + A^T \vec{b}$$

... SYN. POS. DEF. MATRIX

HOW TO IMPLEMENT E.G. IN TKSL ??

DEVIATIONS (RESIDUALS)

$$\downarrow$$

$$q_1 = x_1 + x_2 - 1$$

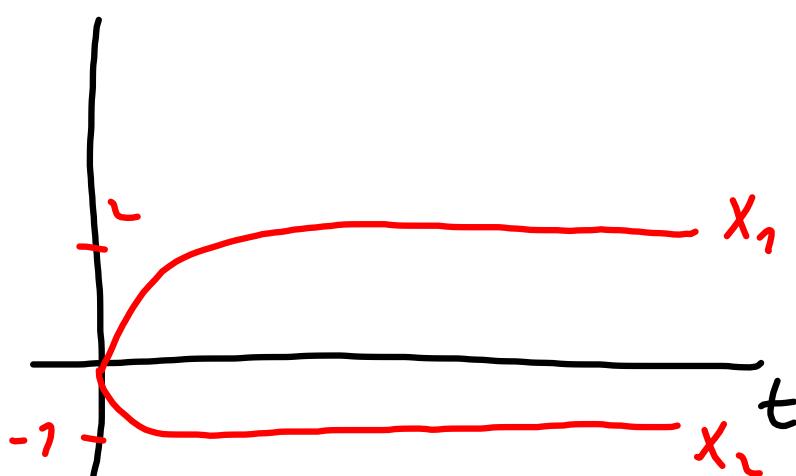
$$q_2 = x_1 - x_2 - 3$$

$$x'_1 = -(1 \cdot q_1 + 1 \cdot q_2) \quad \& \dots \rightarrow \vec{x}' = -A^T \vec{q}$$

$$x'_2 = -(1 \cdot q_1 - 1 \cdot q_2) \quad \& \dots$$

↑

$I_C = \text{ARBITRARY}$



Searching for real roots of polynomial

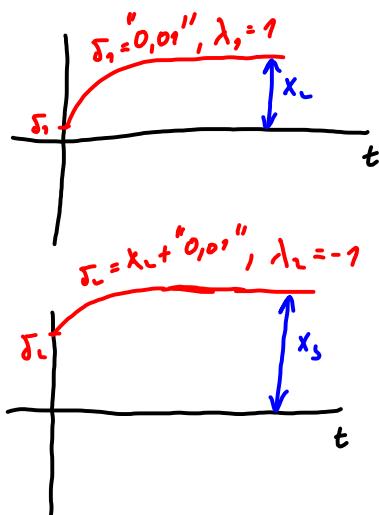
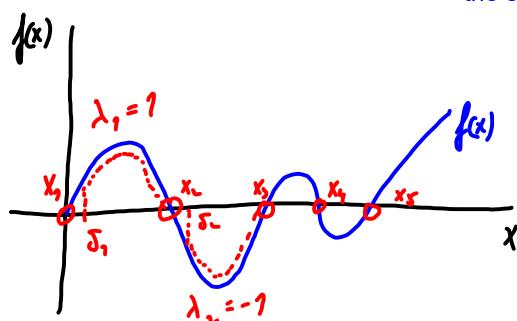
$$f(x) = 256x^9 - 576x^3 + 432x^5 - 120x^3 + 9x$$

$$f(x) = 0 \quad x \approx ?$$

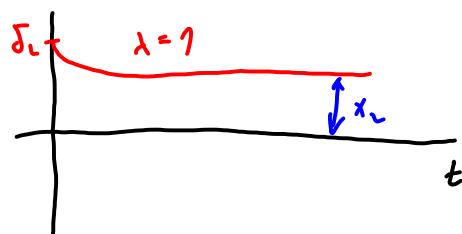
Diff. equation: $x' = 0$ "stable" state

$$x' = \lambda \cdot (256x^9 - 576x^3 + 432x^5 - 120x^3 + 9x) \quad \text{8 J !!!}$$

\uparrow value "1" or "-1" \downarrow "small" value
"close to" (behind)
the solution



Note: if we choose the same λ ,
we find the previous solution (root)



Example: Find the analytical solution of ODE

$$y'' - 5y''' - 7y'' + 29y' + 30y = 0,$$

$$y(0) = 1$$

$$y'(0) = 2$$

$$y''(0) = -2$$

$$y'''(0) = -4$$

1) Characteristic equation:

$$\underline{\lambda^4} - 5\underline{\lambda^3} - 7\underline{\lambda^2} + 29\underline{\lambda} + 30 = 0$$

→ Find solution of polynomial

$$\lambda_1 = 5, \lambda_2 = 3, \lambda_3 = -1, \lambda_4 = -2$$

... eigenvalues

2) General solution:

$$y = C_1 e^{5t} + C_2 e^{3t} + C_3 e^{-t} + C_4 e^{-2t}$$

integration constants

Find y' , y'' , y''' and substitute initial conditions

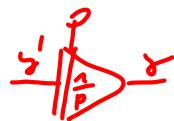
$$\begin{pmatrix} y(0) \\ y'(0) \\ y''(0) \\ y'''(0) \end{pmatrix} = \begin{pmatrix} C_1 & C_2 & C_3 & C_4 \\ 1 & 1 & 1 & 1 \\ 5 & 3 & -1 & -2 \\ 25 & 9 & -1 & -2 \\ 125 & 27 & -1 & -2 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix}$$

note: $e^0 = 1$

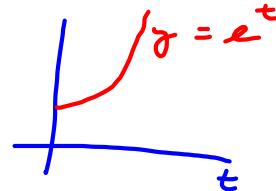
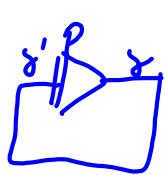
→ Solution e.g. in MATLAB...

EULER INTEGRATION

(HW REPRESENTATION)



$$\gamma' = \gamma, \quad \gamma(0) = 1 !!$$

NON-INTEGRATING
INTEGRATOR

- NUMERICAL CALCULATION

$$\gamma_1 = \gamma_0 + \frac{1}{\tau} \gamma'_0 + \dots$$

$$\boxed{\gamma_1 = \gamma_0 + \frac{1}{\tau} \cdot \gamma_0} \quad \text{NUMERIC OPERATIONS ?}$$

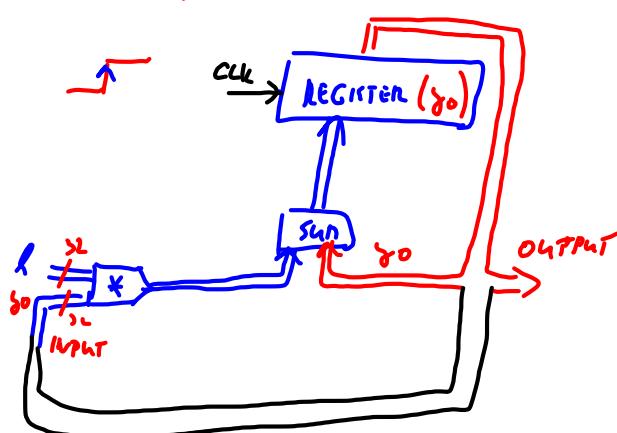
$$\gamma_1 = \gamma_0 + \frac{1}{\tau} \cdot \gamma_0 \quad \text{- MULTIPLICATION}$$

$$\gamma_2 = \gamma_1 + \frac{1}{\tau} \cdot \gamma_1 \quad \text{- ADDER}$$

⋮

PARALLEL-PARALLEL VERSION

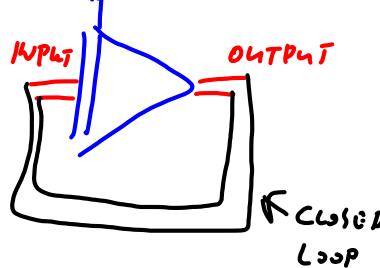
(PARALLEL CONNECTION LOGIC)



$$\gamma_1 = \gamma_0 + \frac{1}{\tau} \gamma_0$$

REGISTEN = D-TYPE FLIP-FLOP

$$\gamma_2 = \gamma_1 + \frac{1}{\tau} \gamma_1$$

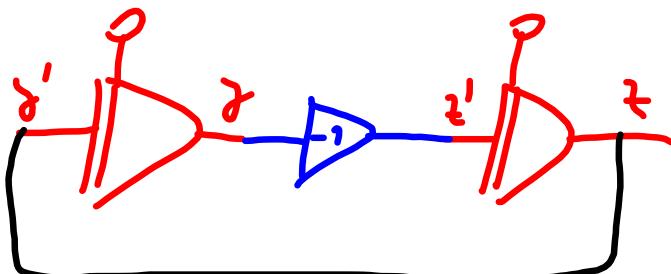


PARALLEL COOPERATION (Two INTEGRATORS)

EULER METHOD

$$\begin{aligned}\dot{\gamma}' &= z & \gamma(0) : \gamma_0 = \phi \\ \dot{z}' &= -\gamma & z(0) : z_0 = 1\end{aligned}$$

(NOTE: $\gamma = \sin(t)$,
 $z = \cos(t)$)



$$\gamma_1 = \gamma_0 + \lambda \cdot \gamma'(0)$$

$$z_1 = z_0 + \lambda \cdot z'(0)$$

(PROJECTION
 $\gamma' = t$
 $t' = -\gamma$)

$$\gamma_1 = \gamma_0 + \lambda \cdot z_0$$

$$z_1 = z_0 + \lambda(-\gamma_0)$$

