Formal Models of Lindenmayer Systems

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Advanced Topics of Theoretical Computer Science

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Part I

Basic Definitions

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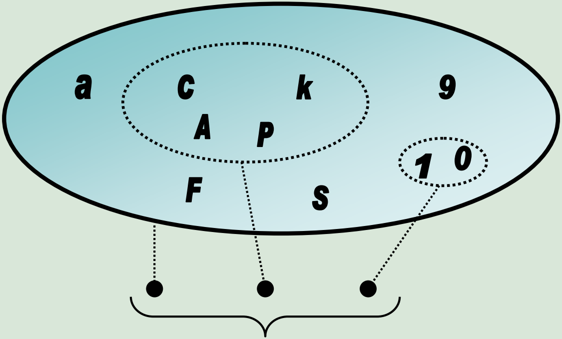

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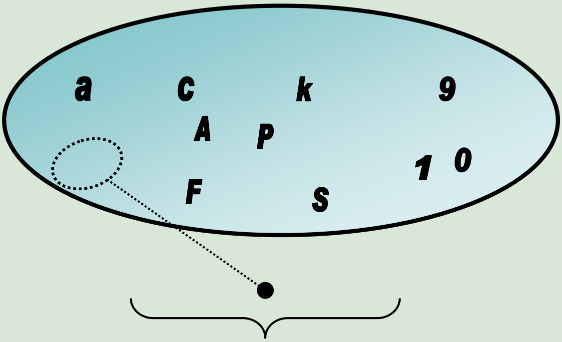

is not an alphabet.

Basic Definitions

String

Let Σ be an alphabet.

- ε is a string over Σ.
- If x is string over *Sigma* and $a \in \Sigma$, then xa is a string over Σ.

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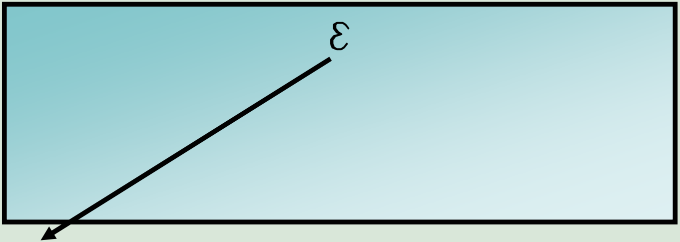

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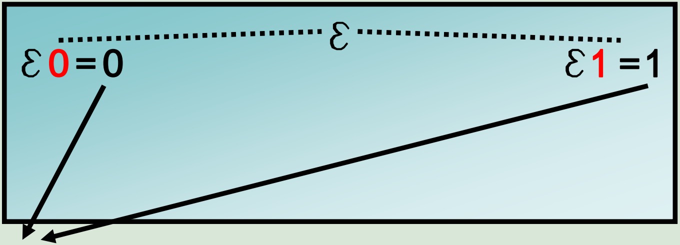

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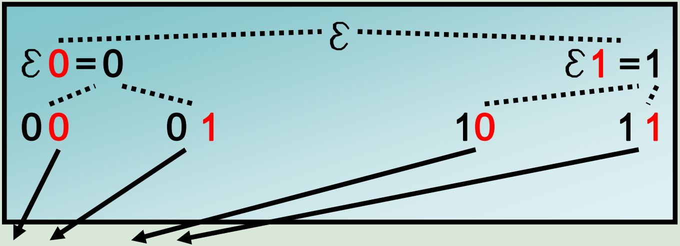

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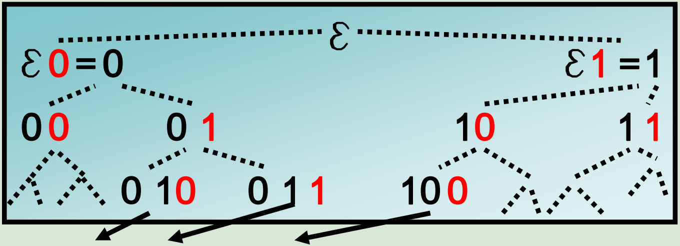

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Length of String

Let x be a string over Σ. The length of x, $|x|$, is defined as follows:

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- if $x = a_1 \ldots a_n$, then $|x| = n$.

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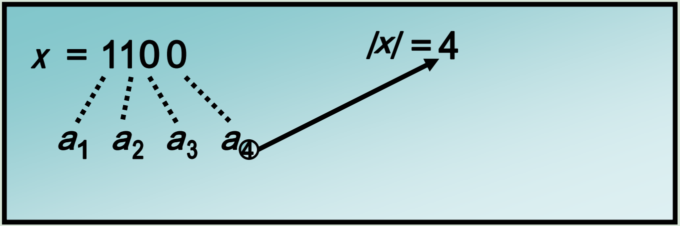

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Let x and y be two strings over Σ. The concatenation of x and y is a string xy.

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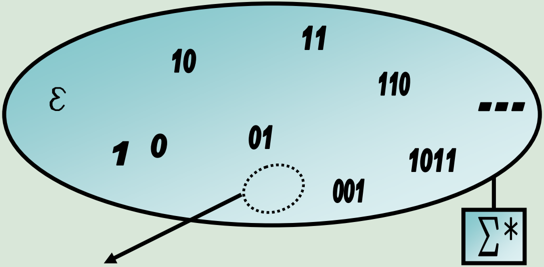

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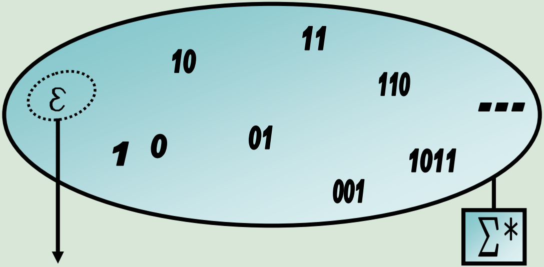

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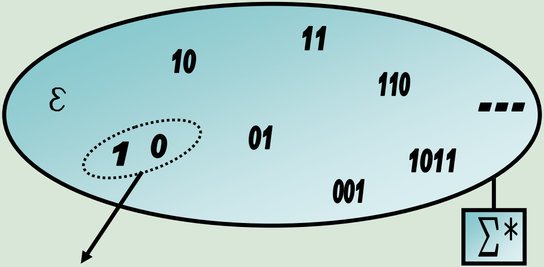

$L = \{x : |x| = 1\}$ is a language over Σ.

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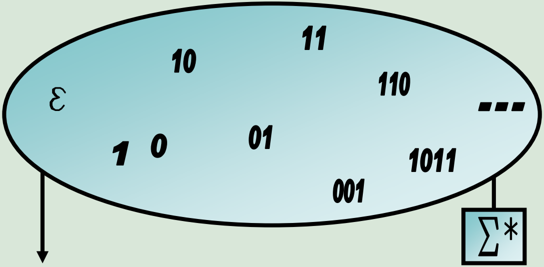

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Part II

0L Systems

0L–System

An 0L–System is a triple

$$G = (T, P, w)$$

where:

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P0L–System

Any 0L–system is a P0L–system (Propagating 0L–system), if for each $a \rightarrow x \in P$, $x \neq \varepsilon$.

Derivation step

Let $G = (T, P, w)$ be a 0L–system. Let n be a positive integer, $a_1, a_2, \dots, a_n \in T$, $x_1, x_2, \dots, x_n \in T^*$ and $a_i \to x_i \in P$ for each $i = 1, \dots, n$. Then, $a_1 a_2 \dots a_n$ directly derives $x_1 x_2 \dots x_n$ in G, written as $a_1 a_2 \dots a_n \Rightarrow x_1 x_2 \dots x_n$.

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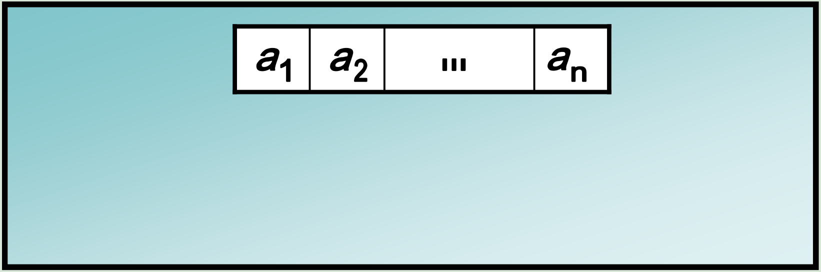
Rules: for every $i = 1, 2, \ldots, n$, $a_i \rightarrow x_i$



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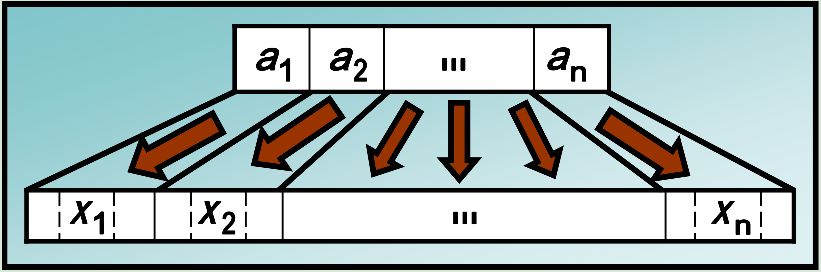

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Sequence of Derivation Steps

Let $G = (T, P, w)$ be a 0L–system and $x_0, x_1, x_2, \ldots, x_n \in T^*$ be $n + 1$ strings and $x_{i-1} \Rightarrow x_i$ in G for all $i = 1, 2, \ldots, n$. Then, G makes n defivation steps from x_0 to x_n; in symbols, $x_0 \Rightarrow^n x_n$.

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Properly Derivation

Let $G = (T, P, w)$ be a 0L–system and $x_0, x_n \in T^*$ be two strings, $x_0 \Rightarrow^n x_n$ in G for some $n \geq 1$. Then, x_0 properly defives x_n in G, written as $x_0 \Rightarrow^+ x_n$.

0L–Systems: Sequence of Derivation Steps

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Language of 0L–Systems

Generated Language

Let $G = (T, P, w)$ be a 0L–system. The language generated by G, $L(G)$, is defined as

$$L(G) = \{x : \ x \in T^* \text{ and } w \Rightarrow^* x\}$$

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Illustration:

- $G = (T, P, w)$,
- $w, x \in T^*$,
- **If** $w \Rightarrow \ldots \Rightarrow \ldots \Rightarrow x$
  - **then** $x \in L(G)$;
  - **otherwise**, $x \notin L(G)$

Length set of L

Let $G = (T, P, w)$ be a 0L–system and $L(G)$ be a language generated by G. Length set of $L(G)$,$|L(G)|$ is defined as

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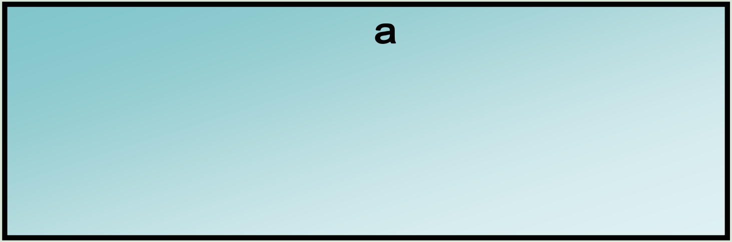

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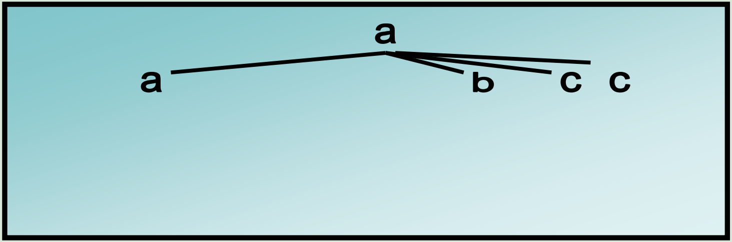

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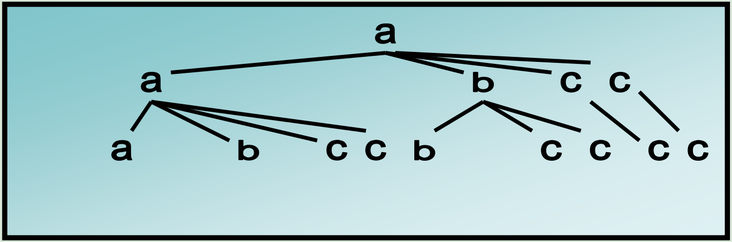

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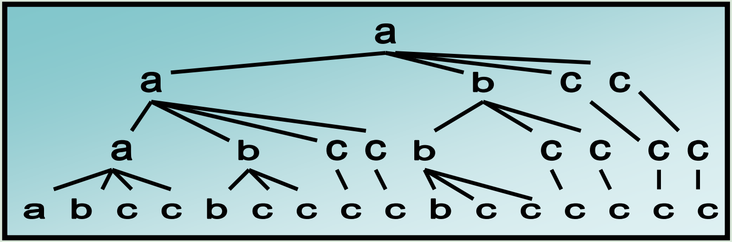

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$|L(G)| = \{i : i \geq 1 \text{ is a Fibonacci number}\}$

Example

$G = (\{1, 2, 3, 4, 5, 6, 7, 8, (,), \#, 0\}, P, 1)$ where P contains:

1	2	3	4	5	6	7	8	(	)	#	0
2#3	2	2#4	504	6	7	8(1)	8	(	)	#	0

1

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$(\ldots)$ branch

8 branch position

0 oblique wall

vertical wall

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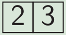

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2#2#504

2	2	5/4

$(\dots)$ branch

8 branch position

0 oblique wall

vertical wall

Example

$G = (\{1,2,3,4,5,6,7,8,(,),\#,0\}, P, 1)$ where P contains:

1	2	3	4	5	6	7	8	(	)	#	0
2#3	2	2#4	504	6	7	8(1)	8	(	)	#	0

2#2#60504

2 | 2 | 6 / 5 \ 4

(...) branch

8 branch position

0 oblique wall

vertical wall

Example

$G = (\{1, 2, 3, 4, 5, 6, 7, 8, (,), \#, 0\}, P, 1)$ where P contains:

1	2	3	4	5	6	7	8	(	)	#	0
2#3	2	2#4	504	6	7	8(1)	8	(	)	#	0

2#2#7060504

2 | 2 | 7 / 6 \ 5 / 4

(. . .) branch

8 branch position

0 oblique wall

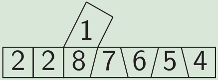
vertical wall

Example

$G = (\{1, 2, 3, 4, 5, 6, 7, 8, (,), \#, 0\}, P, 1)$ where P contains:

1	2	3	4	5	6	7	8	(	)	#	0
2#3	2	2#4	504	6	7	8(1)	8	(	)	#	0

2#2#8(1)07060504



(. . .) branch

8 branch position

0 oblique wall

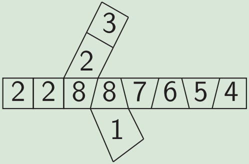
vertical wall

Example

$G = (\{1, 2, 3, 4, 5, 6, 7, 8, (,), \#, 0\}, P, 1)$ where P contains:

1	2	3	4	5	6	7	8	(	)	#	0
2#3	2	2#4	504	6	7	8(1)	8	(	)	#	0

2#2#8(2#3)08(1)07060504



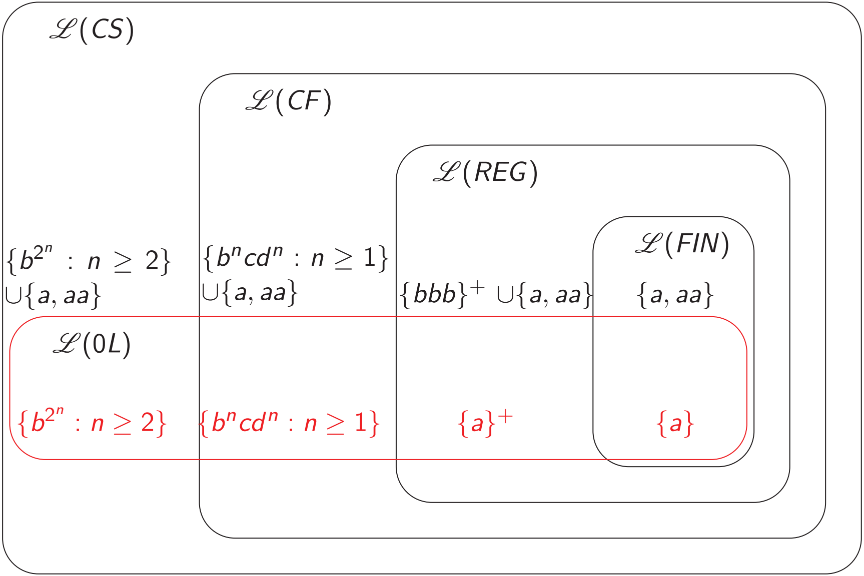
(. . .) branch

8 branch position

0 oblique wall

vertical wall

0L Systems in Chomsky Hierarchy



Part III

Alternatives of L–Systems

E0L–System

An E0L–System is a quadruple

$$G = (V, T, P, w)$$

where:

- V is an total alphabet,
- T is an alphabet and $V \subseteq T$,
- P is a set of the form $a \to x$ with $a \in V$ and $x \in V^*$,
- w is the start string ($w \in V^+$).

Derivation

$\Rightarrow$, $\Rightarrow^+$, $\Rightarrow^*$ – by analogy with 0L–systems

Generated Language

Let $G = (V, T, P, w)$ be a E0L–system. The language generated by G, $L(G)$, is defined as $L(G) = \{x : \ x \in T^* \text{ and } w \Rightarrow^* x\}$

Alternatives of L–Systems

T0L–System

An T0L–System is a $(n+2)$–tuple

$$G = (T, P_1, P_2, \ldots, P_n, w)$$

where:

- $n \geq 1$,
- for all $i = 1, 2, \ldots, n$, $G_i = (T, P_i, w)$ is an 0L–system.

Derivation

For $x, y \in T^*$,

- $x \Rightarrow y$ in G if $x \Rightarrow y$ in $G_i = (T, P_i, w)$ for some $i = 1, 2, \ldots, n$
- $\Rightarrow^+$, $\Rightarrow^*$ – by analogy with 0L–systems

Generated Language

Let $G = (T, P_1, P_2, \ldots, P_n, w)$ be a T0L–system. The language generated by G, $L(G)$, is defined as $L(G) = \{x : x \in T^* \text{ and } w \Rightarrow^* x\}$

ET0L–System

An ET0L–System is a $(n+3)$–tuple

$$G = (V, T, P_1, P_2, \ldots, P_n, w)$$

where:

- $n \geq 1$,
- for all $i = 1, 2, \ldots, n$, $G_i = (V, T, P_i, w)$ is an E0L–system.

Derivation

For $x, y \in T^*$,

- $x \Rightarrow y$ in G if $x \Rightarrow y$ in $G_i = (V, T, P_i, w)$ for some $i = 1, 2, \ldots, n$
- $\Rightarrow^+$, $\Rightarrow^*$ – by analogy with 0L–systems

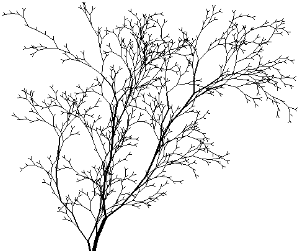
Generated Language

Let $G = (V, T, P_1, P_2, \ldots, P_n, w)$ be a ET0L–system. The language generated by G, $L(G)$, is defined as $L(G) = \{x : x \in T^* \text{ and } w \Rightarrow^* x\}$

Part IV

Conclusion

Conclusion



- 0L–system is natural rewriting system with geometrical property.
- Family of 0L–system languages are relatively small.
- There are some modifications of original 0L–systems, such that $E0L$, $T0L$ and $ET0L$ systems.
- $\mathscr{L}(\mathbf{CF}) \subset \mathscr{L}(\mathbf{E0L}) \subset \mathscr{L}(\mathbf{ET0L}) \subset \mathscr{L}(\mathbf{CS})$.

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Thank you for your attention!

End