Procedural Modeling and L-systems

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Advanced Topics of Theoretical Computer Science

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Modeling of Trees

Models of Trees

- Long history of computer simulation of branching.
- Interactions between elements and environment.
- Interactions leads to more complex models.

Assumptions

- Tree segments are straight and their girth is not considered.
- A mother segment produces two daughter segments through one branching process.
- The lengths of the two daughter segments are shortened by constant ratios, r_1 and r_2, with respect to the mother segment.
- The mother segment and its two daughter segments are contained in the same branch.

- Extended to capture impact of wind and gravity.
- Models rendeded by straight lines leads to "tree skeleton" images.
- Realistic synthetic images of trees are rendered by curves, surfaces and textures.

L-system Tree Example

Figure: Organic architecture by Greene

L-system Tree Example

Figure: Acer graphics by Bloomenthal

L-system Tree Example

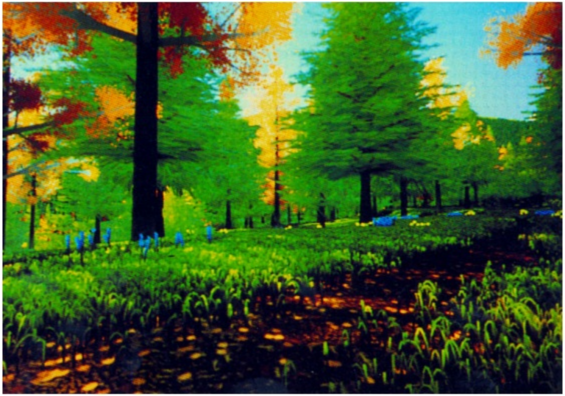


Figure: A forest scene by Reeves, 1984 Pixar

L-system Tree Example

Figure: Oil palm tree canopy from CIRAD Modelisation Laboratory

Impact of Discrete Time

For example, given a clock signal, a bud can either:

Transformation of a bud

- do nothing,
- become a flower,
- become an internode terminated by a new straight apex and one or more lateral apices subtended by leaves,
- die and disappear.

- These events occur according to stochastic laws.
- Geometric parameters (length, diameter, branching angles) are also calculated according to stochastic laws.
- Stochastic rules must describe the entire confiuration of plant that can be formed over a one-year period.

One L-system, Many Models

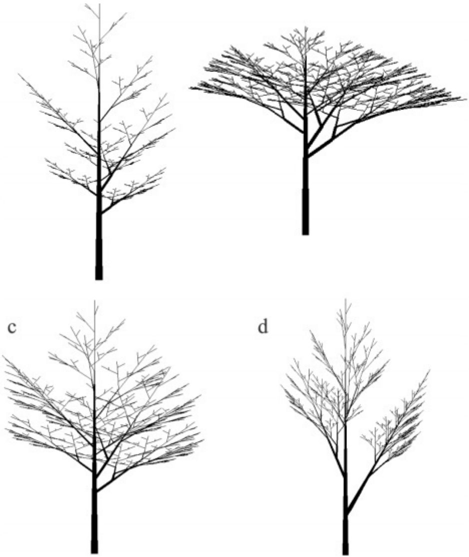


Figure: Tree-like structures of Honda generated by L-systems

```
n = 10
#define r1 0.9     /* contraction ratio for the trunk */
#define r2 0.6     /* contraction ratio for branches */
#define a0 45      /* branching angle from the trunk */
#define a2 45      /* branching angle for lateral axes */
#define d  137.5   /* divergence angle */
#define wr 0.707   /* width decrease rate */

ω  : A(1,10)
p1: A(l,w) : * → !(w)F(l)[&(a0)B(l*r2,w*wr)]/(d)A(l*r1,w*wr)
p2: B(l,w) : * → !(w)F(l)[-(a2)$C(l*r2,w*wr)]C(l*r1,w*wr)
p3: C(l,w) : * → !(w)F(l)[+(a2)$B(l*r2,w*wr)]B(l*r1,w*wr)
```

Figure: Tree-like structures of Honda generated by L-systems

One L-system, Many Models

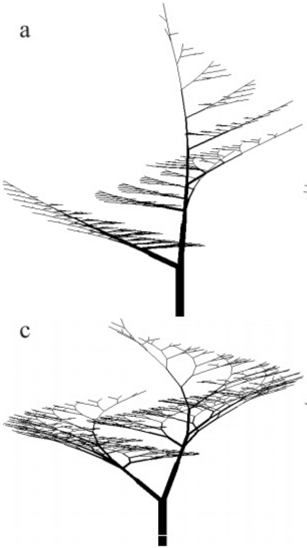


Figure: Tree-like structures of Aono and Kunii generated using L-systems

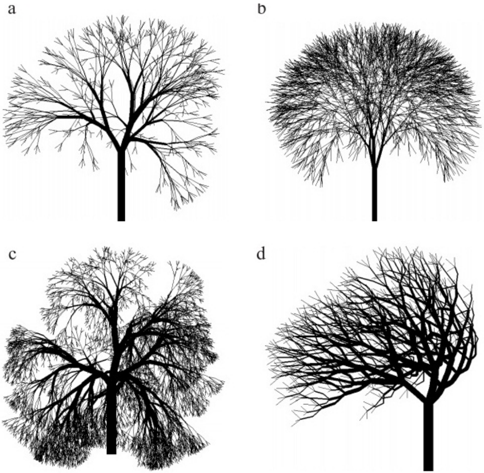
```
n = 10
#define r1 0.9   /* contraction ratio 1 */
#define r2 0.7   /* contraction ratio 2 */
#define a1 10    /* branching angle 1 */
#define a2 60    /* branching angle 2 */
#define wr 0.707 /* width decrease rate */

ω  :  A(1,10)
p1 :  A(l,w) : * → !(w)F(l)[&(a1)B(l*r1,w*wr)]
                   /(180)[&(a2)B(l*r2,w*wr)]
p2 :  B(l,w) : * → !(w)F(l)[+(a1)$B(l*r1,w*wr)]
                   [-(a2)$B(l*r2,w*wr)]
```

Figure: Tree-like structures of Aono and Kunii generated using L-systems

Ternary Branching

- So far: all segments are final length, no further elongation occurs.
- Now: new segments of constant length and increasing the lengths of previously created segments.



```
#define d1 94.74        /* divergence angle 1 */
#define d2 132.63       /* divergence angle 2 */
#define a 18.95         /* branching angle */
#define lr 1.109        /* elongation rate */
#define vr 1.732        /* width increase rate */

ω  :  !(1)F(200)/(45)A
p1 :  A : *      →  !(vr)F(50)[&(a)F(50)A]/(d1)
                    [&(a)F(50)A]/(d2)[&(a)F(50)A]
p2 :  F(l)  :  * →  F(l*lr)
p3 :  !(w)  :  * →  !(w*vr)
```

Figure: Tree-like structures of Aono and Kunii generated using L-systems

3D Tree-like Structure Example

Figure: Medicine Lake by Musgrave et al.

3D Tree-like Structure Example

Figure: A surrealistic elevator

Models of Plants

Models of Plants

- L-systems as a plant modeling tool.
- Generation of complex-looking objects from very small number of productions.
- Specification of L-systems is not a trivial task.
- General approach is needed to model the large variety of developmental patterns and structures found in nature.
- Methodology is based on the simulation of the development of real plants.

Developmental Process

- Emphasis on the space-time relation between plant parts.
  - Organs in various stages of development can be observed at the same time.
  - For example, some flowers may still be in the bud stage, others may be fully developed, and still others may have been transformed into fruits.
- Inherent capability of growth simulation.
  - The mathematical model can be used to generate biologically correct images of plants at diferent ages and to create sequences of images illustrating plant development in time.

Three Levels of Model Specification

1. Partial L-systems: The most abstract level, based on 0L-system, define the possibilities within which structures may develop.
2. L-system Schemata: Topology of individual plants and temporal aspects of their development. Interesting from a biological point of view.
3. Complete L-systems: Geometric apsects, growth rates of internodes, the values of branching angles, and the appearance of organs.

Growing Plant Example

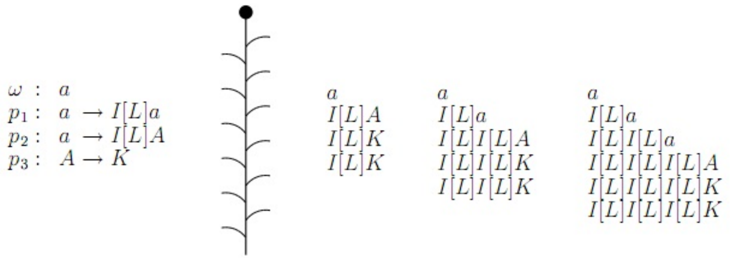


Figure: Growing plant

- Symbol *a* represents vegetative apex, *A* represents flowering apex.
- At each step *a* has a choice of forming either leaf *L*, internode *I* and new apex *a*, or transfroming into a flowering apex *A* with flower *K*.

- A partial L-system does not specify the moments in which developmental switches occur.
- The timing of these switches is specified at the level of L-system schemata, which incorporate mechanisms that control plant development.

Possibilities

- Stochastic mechanism
- The effect of environment
- Delay mechanism
- Accumulation of components
- Development controlled by a signal

Details can be found in the book *Algorimic Beauty of Plants*.

Complete models

```
#define T_a 7                 /* developmental switch time */
#define T_L 9                 /* leaf growth limit */
#define T_K 5                 /* flower growth limit */
#include L(0),L(1),...,L(T_L)  /* leaf shapes */
#include K(0),K(1),...,K(T_K)  /* flower shapes */

ω  : a(1)
p1 : a(t)  : t<T_a  →  F(1)[&(30)~L(0)]/(137.5)a(t+1)
p2 : a(t)  : t=T_a  →  F(20)A
p3 : A     : *      →  ~K(0)
p4 : L(t)  : t<T_L  →  L(t+1)
p5 : K(t)  : t<T_K  →  K(t+1)
p6 : F(l)  : l<2    →  F(l+0.2)
```

Figure: Crocuses

Complete models

Figure: Crocuses

Examples of Inorescences

$$
\begin{aligned}
\omega &: a \\
p_1 &: a \rightarrow I[L]a \\
p_2 &: a \rightarrow I[L]A \\
p_3 &: A \rightarrow I[K]A
\end{aligned}
$$

$$
\begin{aligned}
p_3' &: A \rightarrow I[IK_0]A \\
p_4 &: K_i \rightarrow K_{i+1},
\end{aligned}
$$

Figure: Lily-of-the-valley

More Examples

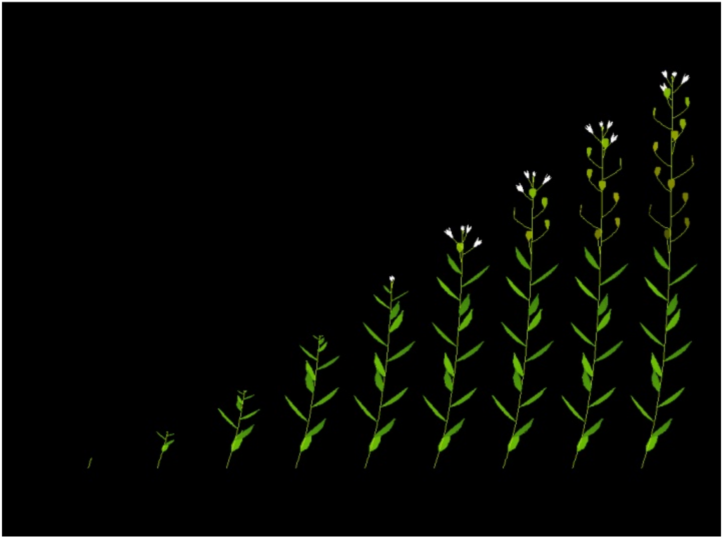


Figure: Development of Capsella bursa-pastoris.

More Examples

Figure: Apple twig

More Examples

$$
\begin{aligned}
\omega &: a \\
p_1 &: a \rightarrow I[L]a \\
p_2 &: a \rightarrow I[L]A \\
p_3 &: A \rightarrow I[L][b]A \\
p_4 &: A \rightarrow I[L][b]B \\
p_5 &: b \rightarrow I[L]b \\
p_6 &: b \rightarrow I[L]B \\
p_7 &: B \rightarrow I[K]B
\end{aligned}
$$

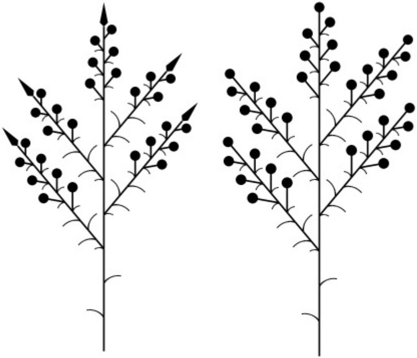


Figure: Dibotryoids

More Examples

Figure: A mint

More Examples

Figure: Development of Lychnis coronaria

More Examples

Figure: Mycelis model

More Examples

Figure: The garden of L

Models of Leaves

Models of Leaves

- Predefined surfaces
  - Standard computer graphics method.
  - But predefined surfaces do not grow.
- It is necessary to provide a mechanism for changing the shape as well as the size of surfaces in time.
- To trace surface boundaries using the turtle and fill the resulting polygons.

$$
\begin{aligned}
\omega &: L \\
p_1 &: L \rightarrow \{-FX + X - FX - | - FX + X + FX\} \\
p_2 &: X \rightarrow FX
\end{aligned}
$$

Production p_1 defines leaf L as a closed planar polygon. The braces { and } indicate that this polygon should be filled. Production p_2 increases the lengths of its edges linearly.

Example of filled polygon

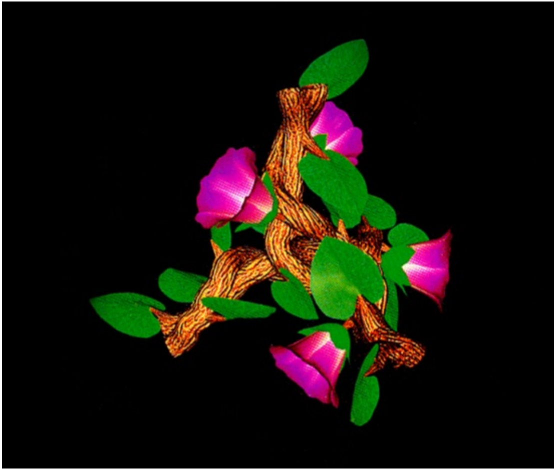


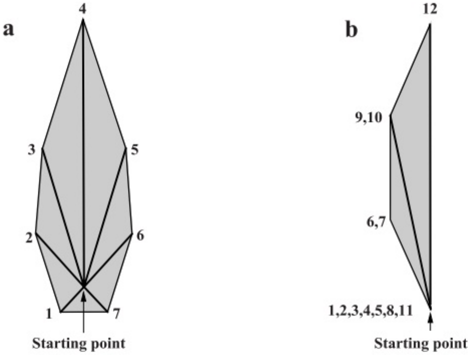
Figure: Maraldi figure by Greene

Example of filled polygon

Figure: The fern

Models of Leaves

- Filled polygons are efective only in the case of small, flat surfaces.
- In other cases it is more convenient to generate *framework*.
- Polygon vertices are specified by turtle positions marked by the dot symbol (.).
- The letter G indicates that the segments enclosed in the braces should not be interpreted as the edges.

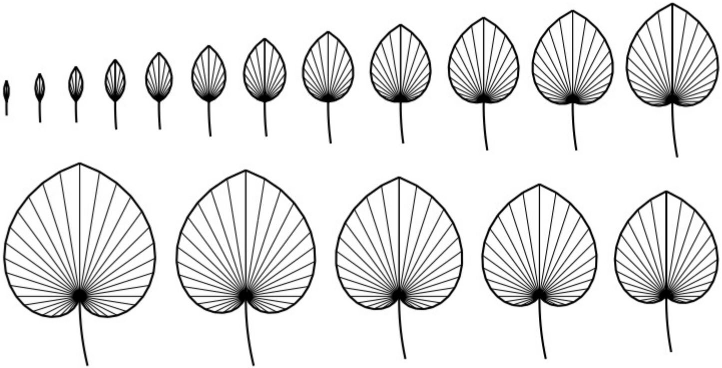


a{[++++G$._{1}$][++GG$._{2}$][+GGG$._{3}$][GGGGG$._{4}$][-GGG$._{5}$][--GG$._{6}$][----G$._{7}$]}

b [+[+[+[+A{$._{1}$]$._{2}$C$._{3}$}{$._{4}$]$._{5}$GC$._{6}$}{$._{7}$]$._{8}$GGC$._{9}$}{$._{10}$]$._{11}$GGGC$._{12}$}

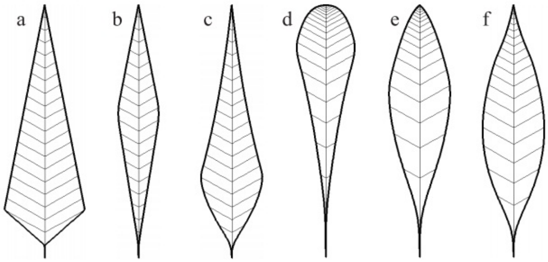
Figure: Surface specification using a tree structure as a framework

Surface as Framework Example



ω : `[A][B]`
p_1 : `A` $\rightarrow$ `[+A{.].C.}`
p_2 : `B` $\rightarrow$ `[-B{.].C.}`
p_3 : `C` $\rightarrow$ `GC`

Figure: Developmental sequence of a cordate leaf



n=20, δ=60°

```
#define LA 5    /* initial length - main segment */
#define RA 1    /* growth rate - main segment */
#define LB 1    /* initial length - lateral segment */
#define RB 1    /* growth rate - lateral segment */
#define PD 1    /* growth potential decrement */
```

ω : {.A(0)}
p_1 : A(t) : * → G(LA,RA)[-B(t).][A(t+1)][+B(t).]
p_2 : B(t) : t>0 → G(LB,RB)B(t-PD)
p_3 : G(s,r) : * → G(s*r,r)

Figure: A Family of simple leaves generated using a parametric L-system

Surface as Framework Example

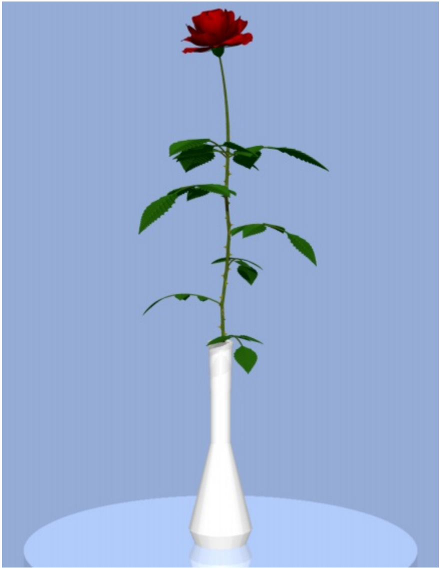


Figure: A rose in a vase

Models of Leaves

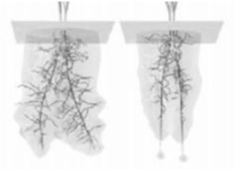
L-systems are used particularly for modeling plants and trees, river flows, seashells, buildings, road network in the virtual city, etc.

Benefits of L-systems

- As a general principle, L-systems based on a very small amount of input data creates relatively complex structures.
- The ability to choose the number of iteration causes different level of details of the model.
- Parallel rewriting of the strings simulates parallel development of the parts of object.
- Each sentential form represents the state of a object in certain discrete time.
- The state of an object in the following time can be obtain in one step, thus L-systems are useful for animation of development.

Aplications of L-systems

- Plant development modeling.
- Modeling of interaction between plant and an environment. The interaction is importatnt for modeling ecosystems, garden architecture, etc.



- Ecosystem modeling. Ecosystem is selfregulating society of plants and animals, which mulually interact. Models for garden architecture, simulation the impact of cut the trees, scenes for virtual reality, flight simulators, etc.
- Architecture modeling.
- Computer art.

Another plant development example

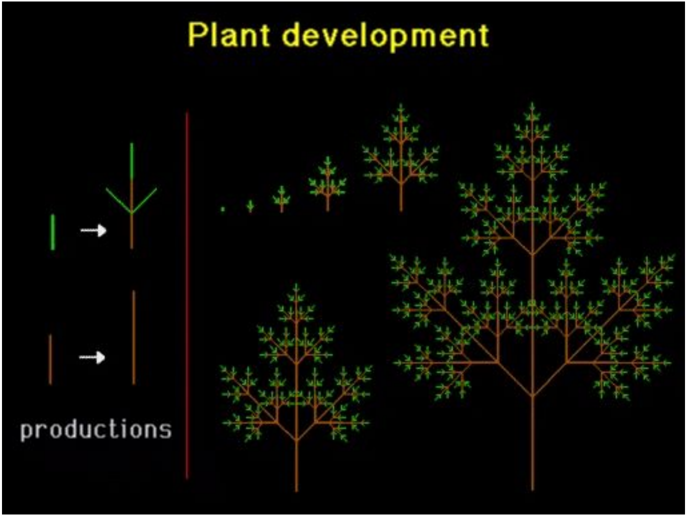


Figure: cobweb.ecn.purdue.edu

Simple ecosystem model

Simple ecosystem model

Computer Art Example

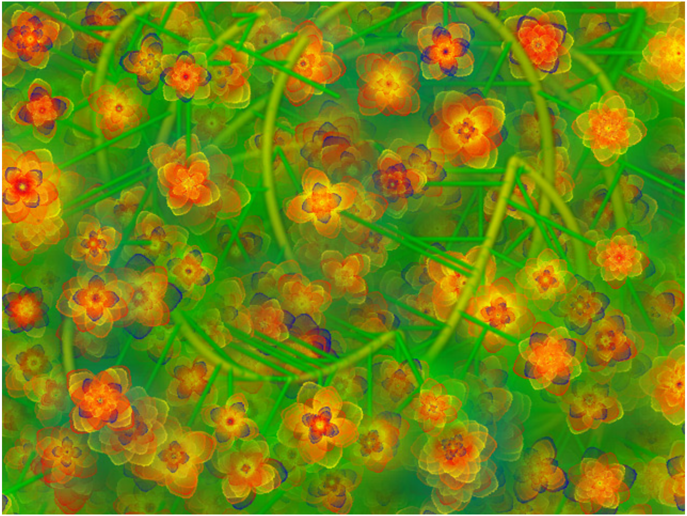


Figure: www.donrelyea.com

Various types of trees Example

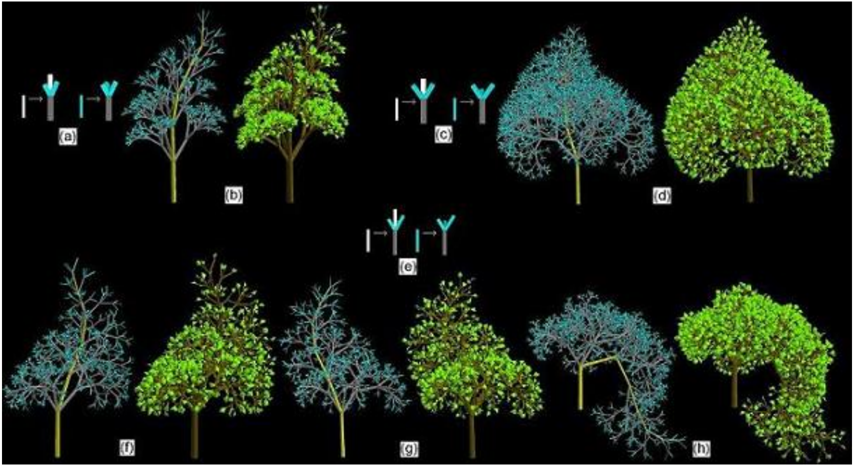


Figure: www-ui.is.s.u-tokyo.ac.jp

Examples of plant models

Figure: www.biota.org

Mathematical Garden Example

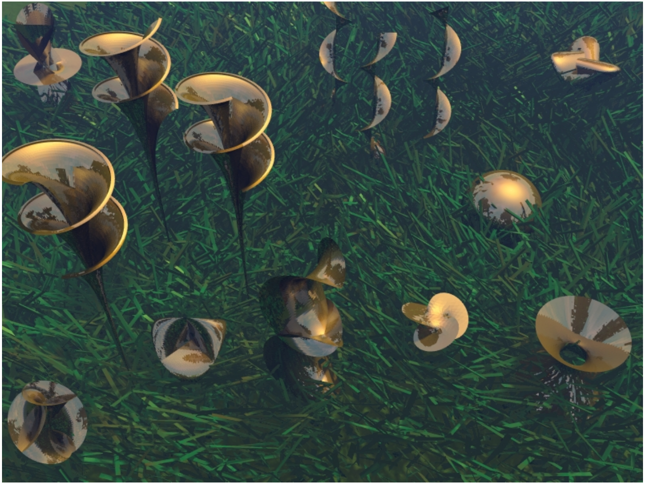


Figure: www.nada.kth.se

Figure: www.cs.washington.edu

Computer Art adn L-systems

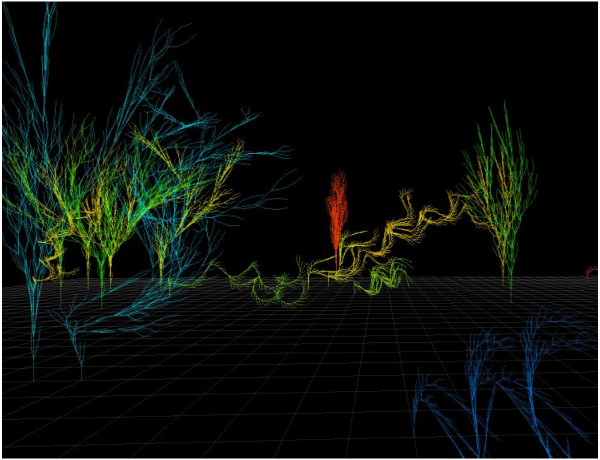


Figure: www.rockabit.com

Example of plant development

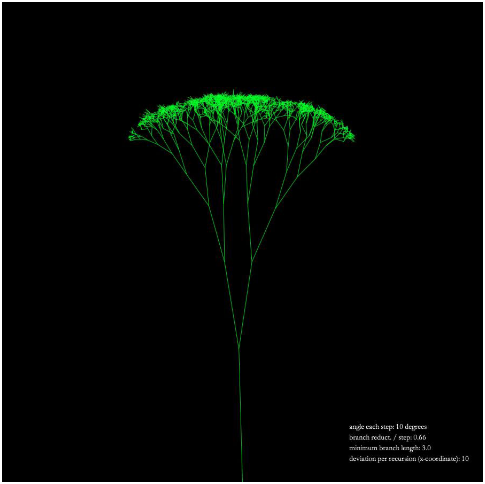
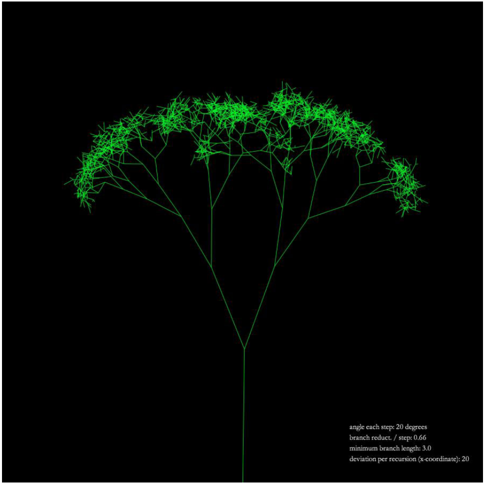


Figure: clausneergaard.files.wordpress.com

Example of plant development



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Example of plant development

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Example of plant development

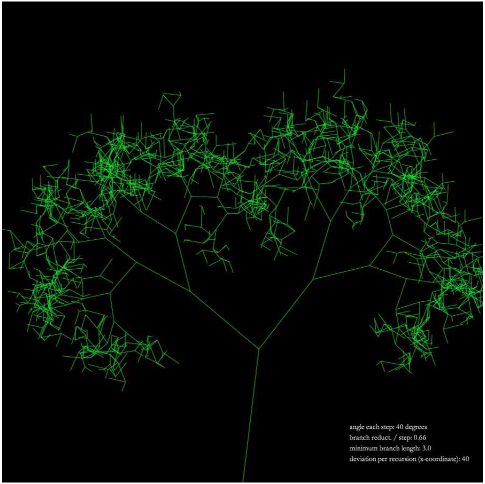


Figure: clausneergaard.files.wordpress.com

Procedural modeling of cities

Procedural modeling of cities

- Recently, L-systems are used for modeling of streets, buildings and cities.
- For this purposes, modification of L-system was introduced, so called *self-sensitive* L-systems.
- The main reason: to allow for spatial boundaries without increasing the complexity of L-systems.
- Note the differences between CAD modeling and Procedural modeling.

Main concepts for modeling the cities

- Spatial object division: $A \rightarrow div(axe, sizes)\{modules\}$
- Cyclic division: $A \rightarrow repeat(axe, size)\{module\}$

Scripting 3D Buildings

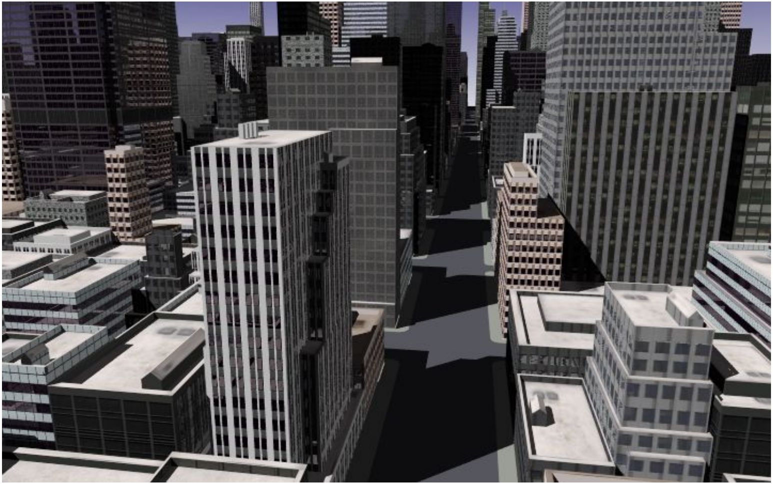


Figure: www.procedural.com

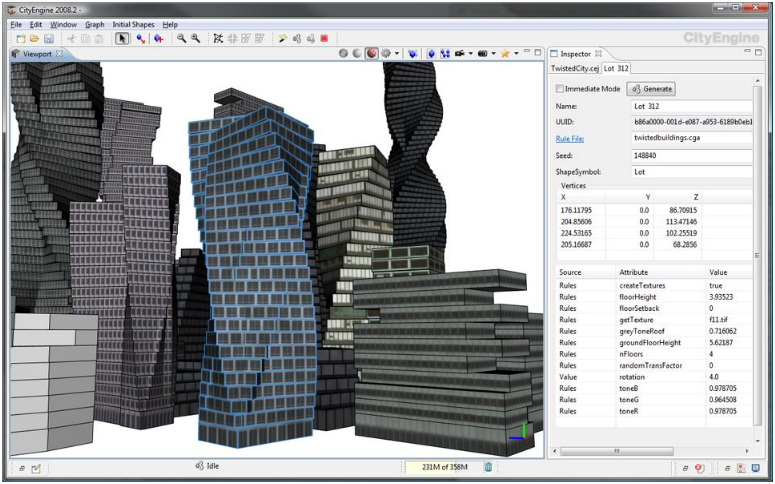


Figure: www.procedural.com

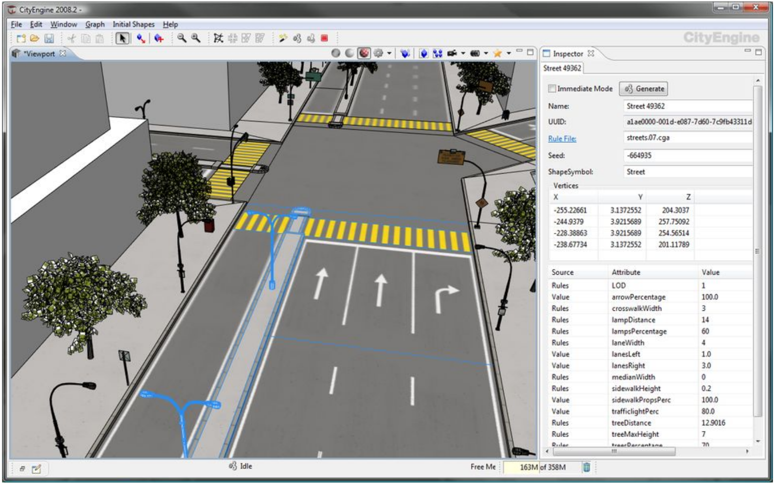


Figure: www.procedural.com

Map-Controlled City Modeling

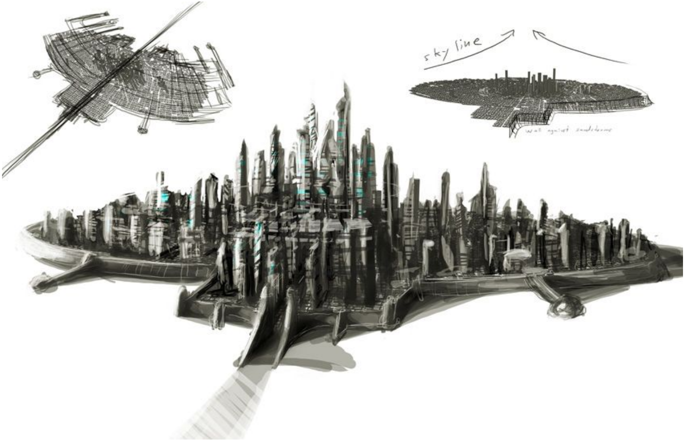


Figure: www.procedural.com

Batch Export of 3D Models

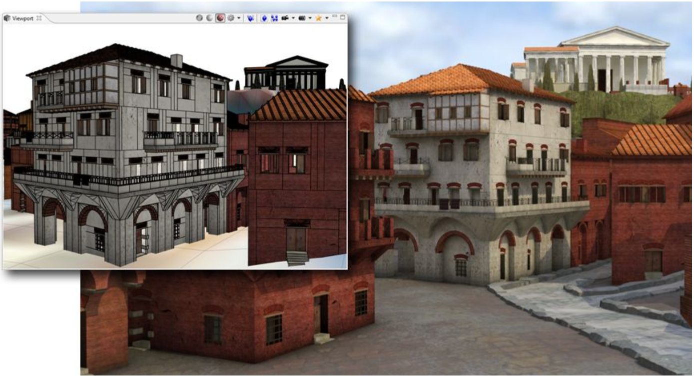


Figure: www.procedural.com

Building Models Example

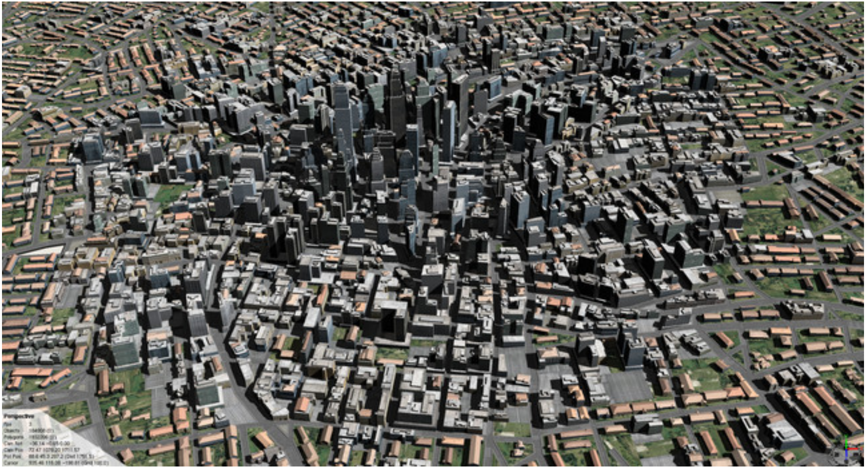
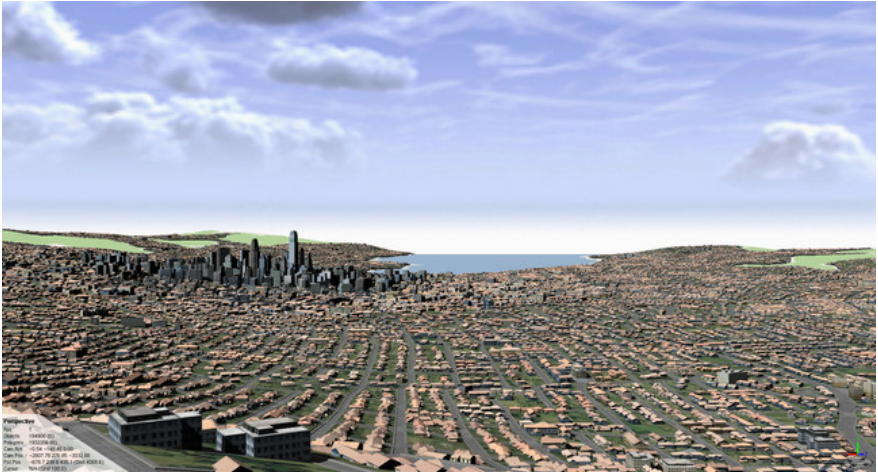


Figure: www.procedural.com/showcase

Building Models Example



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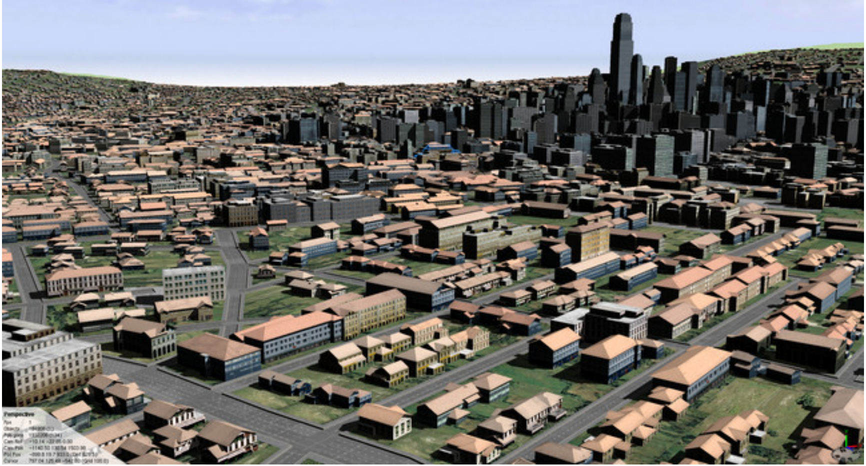

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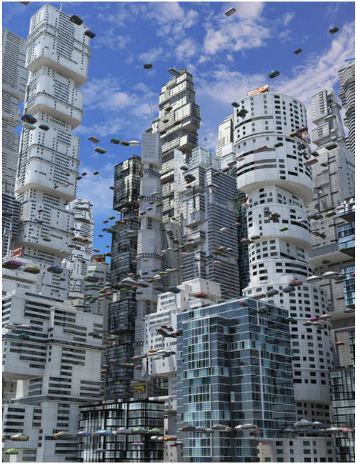

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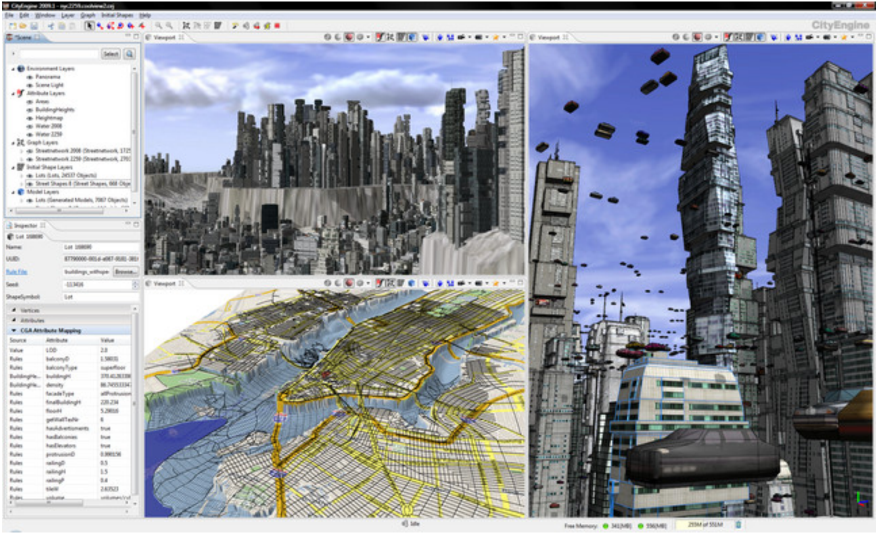

Figure: www.procedural.com/showcase

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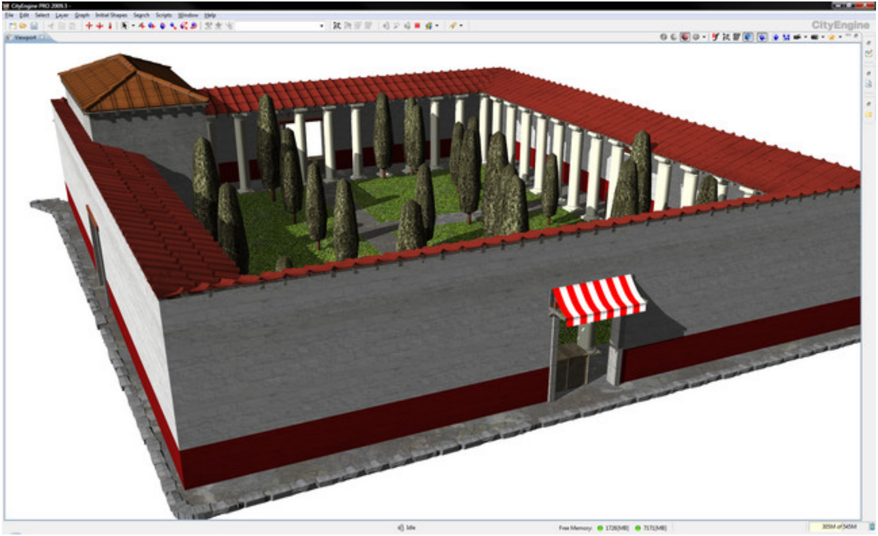

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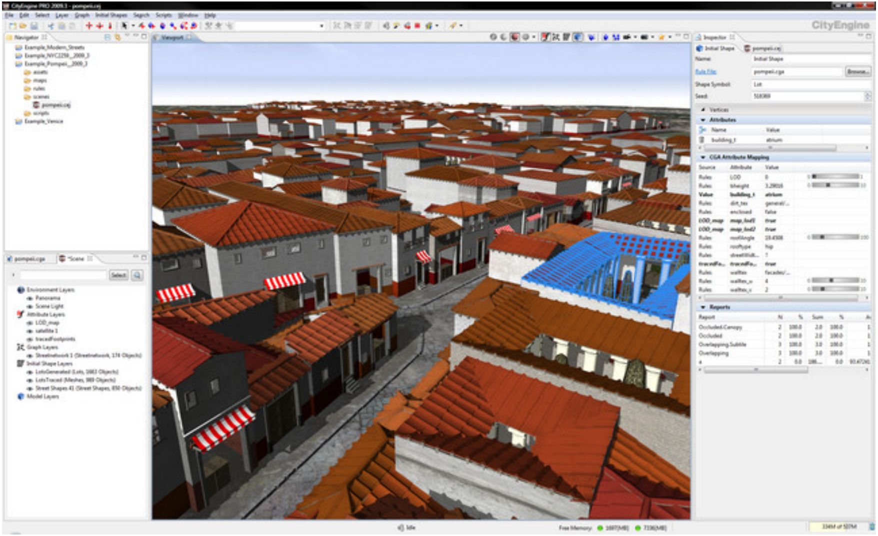

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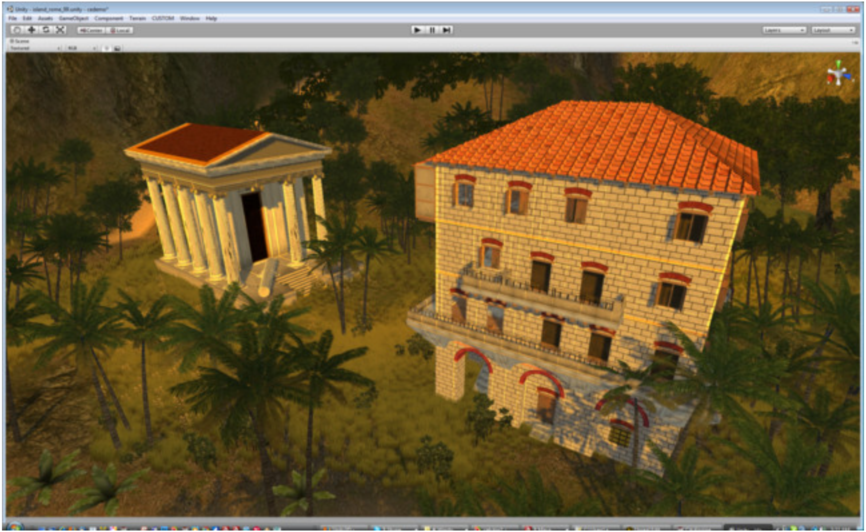

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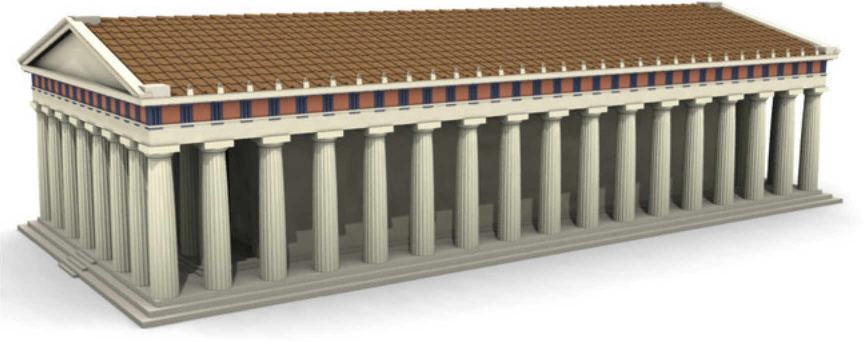

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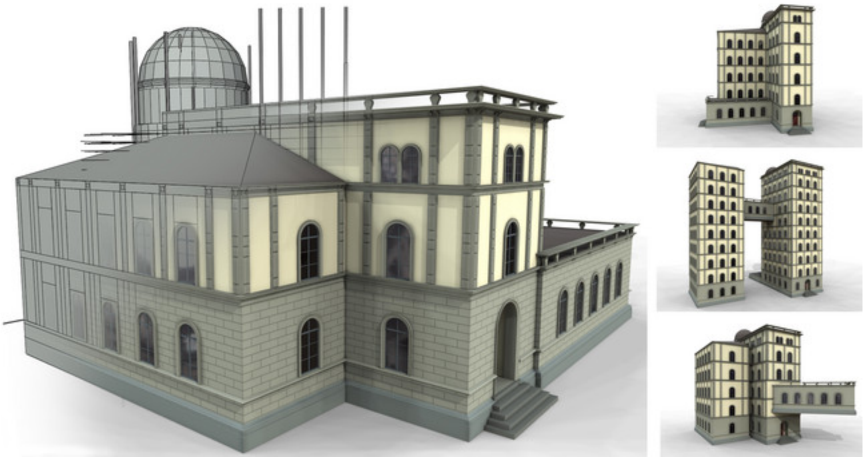

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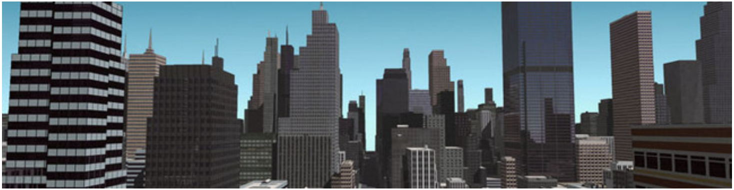

Figure: www.procedural.com/showcase

Building Models Example

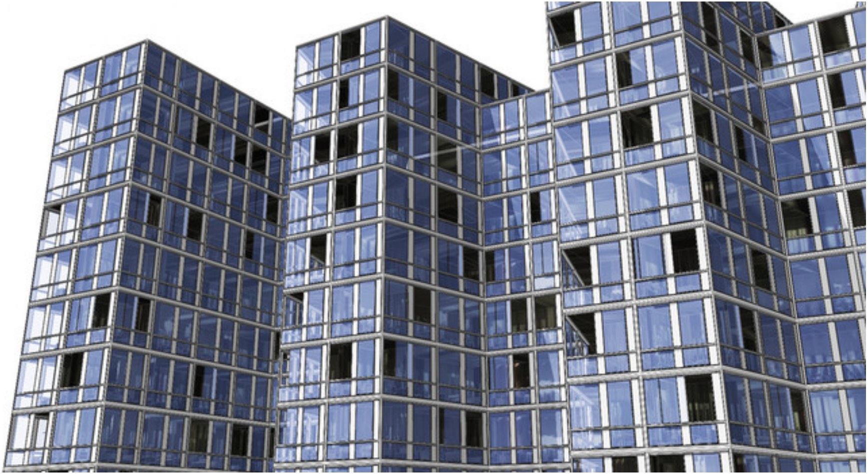

Figure: www.procedural.com/showcase

Building Models Example

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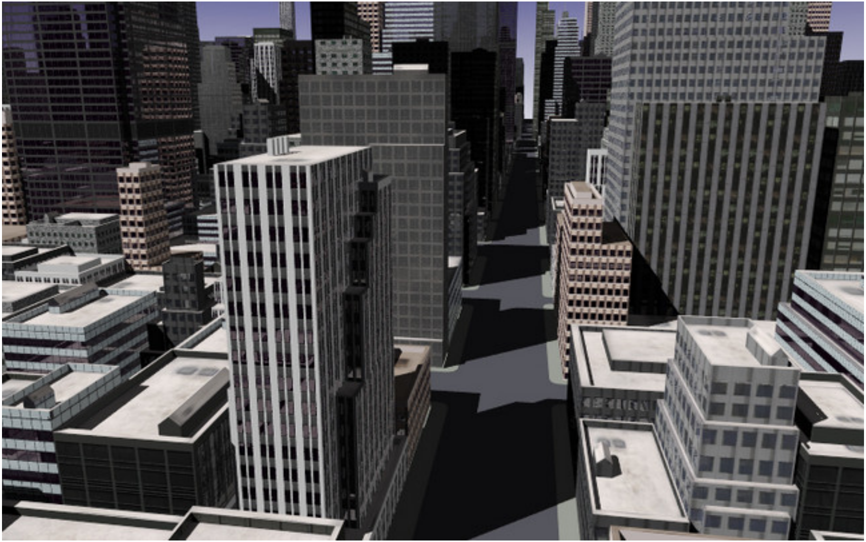

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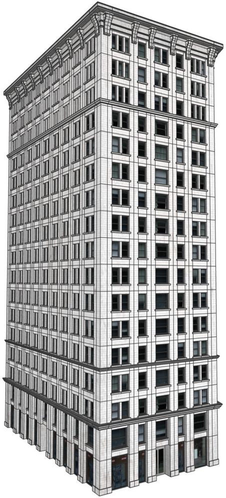
Building Models Example



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Building Models Example

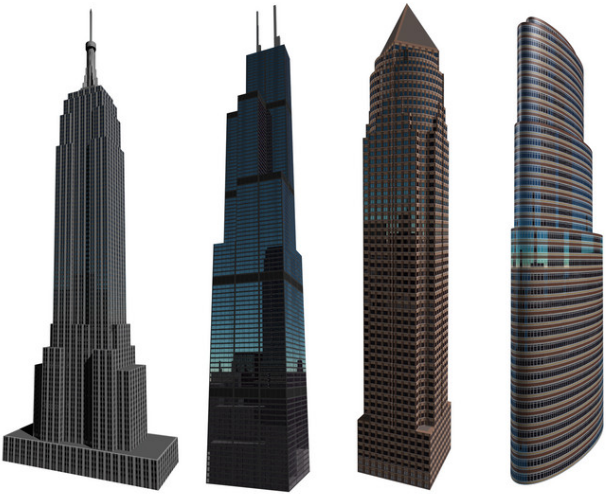
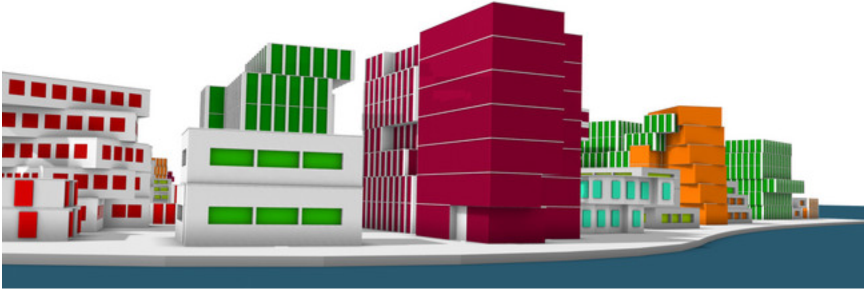


Figure: www.procedural.com/showcase

Building Models Example



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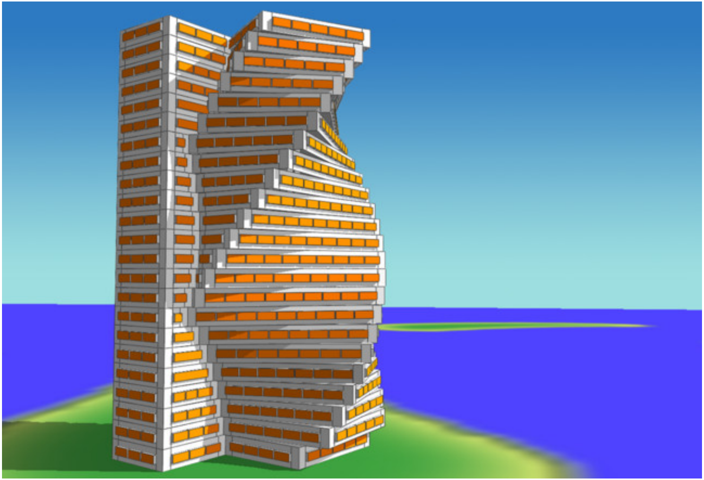
Building Models Example



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Building Models Example

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Thank you for your attention!

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