Multi–Languages and Systems of Formal Models

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Motivation

Motivation to Study Systems

- Division of a large task into several smaller tasks.
- Systems can sometimes be more effective than the sum of the elements of the systems.
- Less descriptive complexity.

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Systems

- Serial: only one element is active in a computational step
- Parallel: more elements are active simultaneously
- Multi–systems: systems with more than one input or output

Multilanguage

Alphabet

An alphabet is a non–empty finite set of symbols.

String

Let Σ be an alphabet.

- ε is a string over Σ. (String with no symbols)
- If x is a string over Σ and $a \in \Sigma$, xa is string over Σ

Note: Σ^* is a set of all strings over the Σ

n–string

Let $\Sigma_1, \ldots, \Sigma_n$ be alphabets. An n–string is an n tuple $(\omega_1, \ldots, \omega_n)$, where for all $i \in \{1, \ldots, n\}$, ω_i is a string over alphabet Σ_i.

Note: n–$\Sigma^* = \Sigma_1^* \times \Sigma_2^* \times \ldots \times \Sigma_n^*$ where $\Sigma_1, \Sigma_2, \ldots, \Sigma_n$ are alphabets.

n-language

n–$L \subseteq n$–Σ^*

Multi–Generative Grammar System

n-Generative Nonterminal-Synchronized Grammar System: n-MGN

is $n+1$ tuple $\Gamma = (G_1, \ldots, G_n, Q)$, where:

- $\forall i \in \{1, \ldots, n\}$, $G_i = (N_i, T_i, P_i, S_i)$ is a context–free grammar
- Q is a set of n tuples of the form $(A_1, \ldots, A_n)$: $A_i \in N_i$

Sentential n-form

is any n tuple $\chi = (x_1, \ldots, x_n)$, where $x_i \in (N_i \cup T_i)^*$ for all $i = 1, \ldots, n$

Derivation Step

Let $\forall i = 1, \ldots, n,\ u_i \in T_i^*,\ A_i \in N_i,\ x_i, v_i \in (N_i \cup T_i)^*$ and:

- $\chi = (u_1 A_1 v_1, \ldots, u_n A_n v_n)$ and
- $\chi' = (u_1 x_1 v_1, \ldots, u_n x_n v_n)$ are sentential n-forms,
- $(A_1, \ldots, A_n) \in Q$ and $u_i A_i v_i \Rightarrow u_i x_i v_i$ in G_i for all $i = 1, \ldots, n$

Than $\chi \Rightarrow \chi'$ in Γ.

n-language generated by Γ: $n\text{-}L(\Gamma)$

$$n\text{-}L(\Gamma) = \{(w_1, \dots, w_n) \in T_1^* \times \dots \times T_n^* : (S_1, \dots, S_n) \Rightarrow^* (w_1, \dots, w_n)\}$$

Example

$\Gamma = (G_1, G_2, Q)$ where

$$G_1 = (\{S, A\}, \{a, b, c\}, \{1{:}S \to aS, 2{:}S \to aA, 3{:}A \to bAc, 4{:}A \to bc\}, S)$$

$$G_2 = (\{S, A\}, \{d\}, \{1{:}S \to SA, 2{:}S \to A, 3{:}A \to d\}, S)$$

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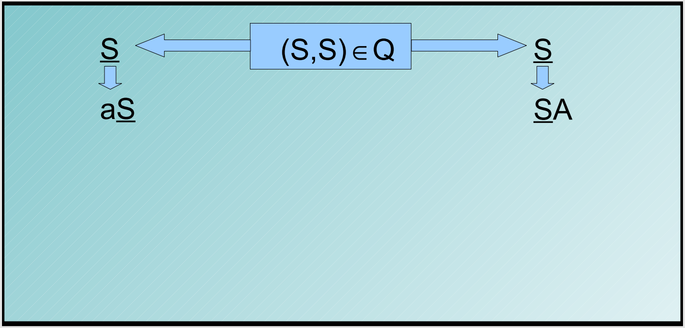

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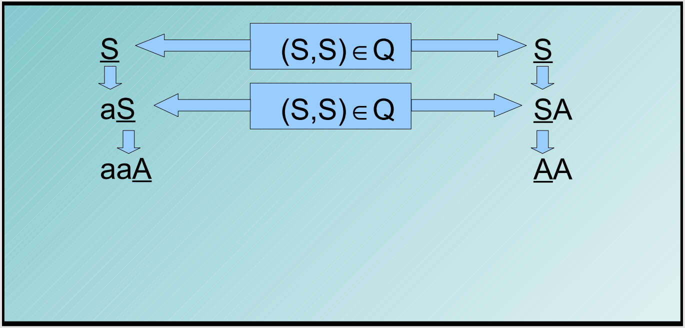

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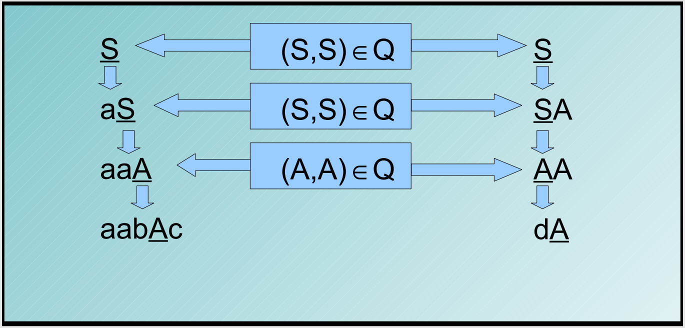

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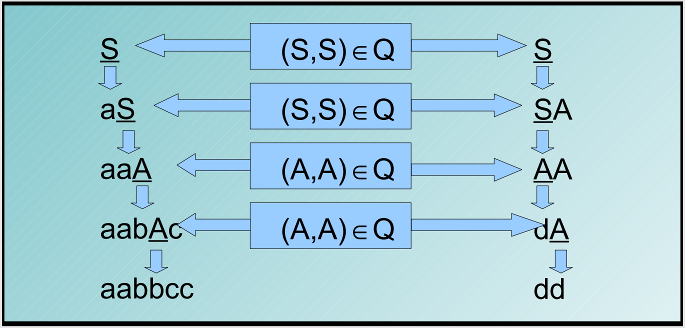

From *n*-languages to languages

Let K is a n-language:

- $L_{union}(K) = \{w_1, \dots, w_n : (w_1, \dots, w_n) \in K\}$
- $L_{concat}(K) = \{w_1 \dots w_n : (w_1, \dots, w_n) \in K\}$
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n-Languages from Previous Example

Let K be an n-language from the example. Then,

- $K = \{(a^m b^m c^m, d^m) : m \geq 1\}$
- $L_{union}(K) = \{a^m b^m c^m : m \geq 1\} \cup \{d^m : m \geq 1\}$
- $L_{concat}(K) = \{a^m b^m c^m d^m : m \geq 1\}$
- $L_{first}(K) = \{a^m b^m c^m : m \geq 1\}$

Let X$\in \{$MGN,MGR$\}$.

Families of n-languages

- n-$\mathcal{L}(n$-X$)$ is the family of n-languges generated by n-X

Let Y$\in \{concat, first, union\}$.

Families of languages

- ${}_{Y}\mathcal{L}(n\text{-}X) = \{{}_{Y}L(K) : \ K \in n\text{-}\mathcal{L}(n\text{-}X)\}$

Results

Let $n \geq 2$.

- ${}_{Y}\mathcal{L}(n\text{-}X) = RE$

Multiaccepting Automata System

n-accepting, state-synchronized AS (n-MAS)

is $(n+1)$–tuple $\vartheta = (M_1, \dots, M_n, \Psi)$, where:

- $M_i = (Q_i, \Sigma, \Gamma_i, \delta_i, s_i, z_{i,0}, F_i)$ is a PDA for every $i \in \{1, \dots, n\}$
- Ψ is a set of switch-rules of the form $(q_1, \dots, q_n) \rightarrow (\hbar_1, \dots, \hbar_n)$, where $\forall i \in \{1, \dots, n\}$:
  - $q_i \in Q_i$ and $\hbar_i \in \{e, d\}$

n-configuration

is n–tuple $\chi = (x_1^{\hbar_1}, \dots, x_n^{\hbar_n})$, where $\forall i = 1, \dots, n$:

- $x_i = (z_i q_i \omega_i) \in \Gamma_i^* Q_i \Sigma^*$ is an configuration of M_i,
- $\hbar_i \in \{d, e\}$ denotes nonactive and active component of M_i, respective,
- $\omega_i \in \Sigma^*$ is an unreaded input string

Move

$\chi = ((z_1 q_1 \omega_1)^{\hbar_1}, \dots, (z_n q_n \omega_n)^{\hbar_n}) \vdash \chi' = ((z'_1 q'_1 \omega'_1)^{\hbar'_1}, \dots, (z'_n q'_n \omega'_n)^{\hbar'_n})$, iff for all $i = 1, \dots, n$ hold:

- if $\hbar_i = e$ then $z_i q_i \omega_i \vdash z'_i q'_i \omega'_i$ in M_i
- if $\hbar_i = d$ then $z_i q_i \omega_i = z'_i q'_i \omega'_i$
- if $(q'_1, \dots, q'_n) \rightarrow (g_1, \dots, g_n) \in \Psi$ then $\hbar'_i = g_i$
- if $(q'_1, \dots, q'_n) \rightarrow (g_1, \dots, g_n) \notin \Psi$ then $\hbar'_i = \hbar_i$

n–language

n–lanugage of n-MAS is defined as

$$n\text{–}L(\vartheta) = \{(\omega_1, \dots, \omega_n) | \chi_0 \vdash^* \chi_f\},$$

where

- $\chi_0 = ((z_1 q_1 \omega_1)^e, \dots, (z_n q_n \omega_n)^e)$ be the start n-configuration and
- $\chi_f = ((q'_1)^{\hbar_1}, \dots, (q'_n)^{\hbar_n})$ be a finish n-configuration.

Example

M1:

1: $\#q_1a \rightarrow \#aq_1$
2: $aq_1a \rightarrow aaq_1$
3: $aq_1b \rightarrow q_2$
4: $aq_2b \rightarrow q_2$
5: $\#q_2c \rightarrow q_3$
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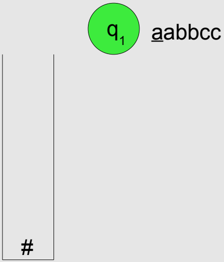

M2:

1: $\#q_1 \rightarrow \#q_2$
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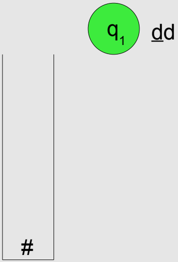

Ψ:

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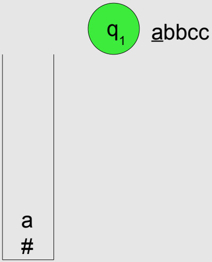

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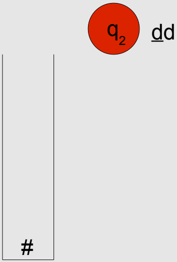

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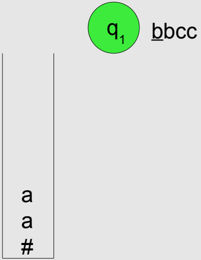

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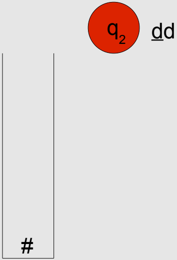

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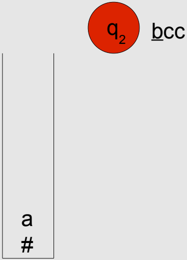

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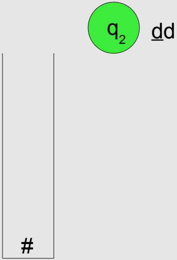

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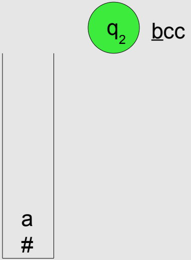

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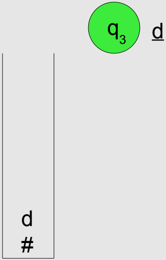

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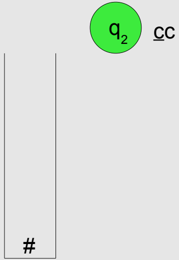

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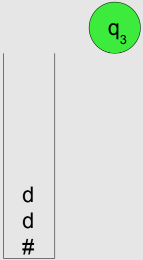

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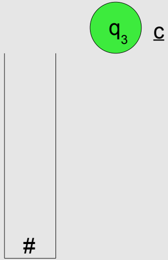

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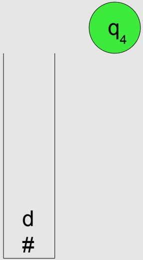

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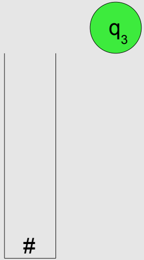

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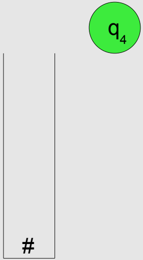

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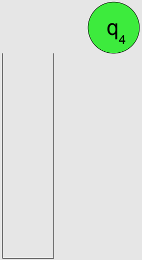
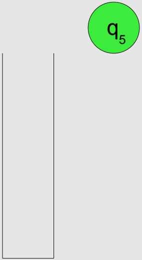


M2:

1: $\#q_1 \rightarrow \#q_2$
2: $\#q_2d \rightarrow \#dq_3$
3: $dq_3d \rightarrow ddq_3$
4: $dq_3 \rightarrow q_4$
5: $dq_4 \rightarrow q_4$
6: $\#q_4 \rightarrow q_5$

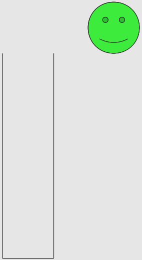


Ψ:
$(q_1,q_1) \rightarrow (e,e)$
$(q_1,q_2) \rightarrow (e,d)$
$(q_2,q_2) \rightarrow (d,e)$
$(q_2,q_3) \rightarrow (e,e)$
$(q_2,q_4) \rightarrow (d,d)$
$(q_3,q_5) \rightarrow (d,d)$
$(q_4,q_4) \rightarrow (d,d)$

Example

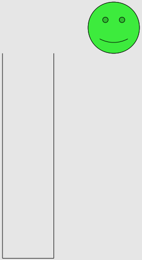
M1:

1: $\#q_1a \rightarrow \#aq_1$
2: $aq_1a \rightarrow aaq_1$
3: $aq_1b \rightarrow q_2$
4: $aq_2b \rightarrow q_2$
5: $\#q_2c \rightarrow q_3$
6: $\#q_3c \rightarrow q_3$
7: $\#q_3 \rightarrow q_4$

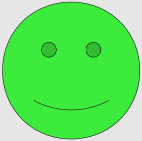


M2:

1: $\#q_1 \rightarrow \#q_2$
2: $\#q_2d \rightarrow \#dq_3$
3: $dq_3d \rightarrow ddq_3$
4: $dq_3 \rightarrow q_4$
5: $dq_4 \rightarrow q_4$
6: $\#q_4 \rightarrow q_5$



Ψ:
$(q_1,q_1) \rightarrow (e,e)$
$(q_1,q_2) \rightarrow (e,d)$
$(q_2,q_2) \rightarrow (d,e)$
$(q_2,q_3) \rightarrow (e,e)$
$(q_2,q_4) \rightarrow (d,d)$
$(q_3,q_5) \rightarrow (d,d)$
$(q_4,q_4) \rightarrow (d,d)$



Multiaccepting Automata System

n-Language from Previous Example

Let ϑ be an n-accepting automaton system from the example. Then,

- $n\text{-}L(\vartheta) = \{(a^m b^m c^m, d^m) : m \geq 1\}$

Let X $\in$ {MAS,MAT}.

Family of n-languages

- n-$\mathcal{L}$(n-X) is the family of n-languges described by n-X.

Theorem

The families

- n-$\mathcal{L}$(n-MGN),
- n-$\mathcal{L}$(n-MGR),
- n-$\mathcal{L}$(n-MAS) and
- n-$\mathcal{L}$(n-MAT)

coincide.

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Thank you for your attention!

End