Teams of Pushdown Automata

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Team of Pushdown Automata

A team of pushdown automata of degree n (n–team pda), $n \geq 2$, is a construct

$$\tau = (M_1, \ldots, M_n)$$

where for all $i = 1, \ldots, n$

$M_i = (Q_i, \Sigma_i, \Gamma_i, R_i, q_i^0, Z_i^0, F_i)$ is a pushdown automaton and

Q_i is a set of states,
R_i is a set of transition rules of the form $zpa \rightarrow \gamma q$:

- $z \in \Gamma_i$
- $p, q \in Q_i$
- $a \in \Sigma_i \cup \{\varepsilon\}$

q_i^0 is an initial state,
Γ_i is a pushdown alphabet,
Σ_i is an input alphabet,
Z_i^0 is a start pushdown symbol,
F_i is a set of states.

- $\Sigma = \bigcup_{i=1}^{n} \Sigma_i$

Configuration

A configuration of team pushdown automata is a triple

$$((p_1, \dots, p_n), a\omega, (Z_1\beta_1, \dots, Z_n\beta_n))$$

where

- $(p_1, \dots, p_n) \in Q$,
- $a \in \Sigma \cup \{\varepsilon\}$ and $\omega \in \Sigma^*$,
- $(Z_1\beta_1, \dots, Z_n\beta_n) \in \Gamma$.

Move

Let

- $\chi = ((p_1, \dots, p_n), a\omega, (Z_1\beta_1, \dots, Z_n\beta_n))$ and
- $\chi' = ((q_1, \dots, q_n), \omega, (\alpha_1\beta_1, \dots, \alpha_n\beta_n))$

be two configurations. $\chi \vdash \chi'$ if there exists a $j \in \{1, \dots, n\}$ such that:

- $Z_j p_j a \rightarrow \alpha_j q_1 \in R_j$ and
- for all $i \in \{1, \dots, n\} \setminus \{j\}$, $q_i = p_i$ and $\alpha_i = Z_i$

Language of Team of Pushdown Automata

$$L_*(\tau) = \{\omega \in \Sigma^* : (q^0, \omega, Z^0) \vdash^* (q, \varepsilon, \gamma)\}$$

where

- $q^0 = (q_1^0, \ldots, q_n^0)$ is the start state,
- $q \in Q$,
- $\omega \in \Sigma^*$,
- $\gamma = (\varepsilon, \ldots, \varepsilon)$.

Denotation of Language Family

$$\mathscr{L}(n\text{–PDA}x, * -\text{comp})$$

where

x
- $[-\varepsilon]$ – ε–moves are not allowed
- nothing – ε–moves are allowed

Example

- $M_1 = (\{p_0, p_1\}, \{a, b\}, \{a, \#\}, R_1, p_0, \#)$
  $R_1: \ \#p_0 a \rightarrow ap_0,\ ap_0 a \rightarrow aap_0,\ ap_0 b \rightarrow p_1,\ ap_1 b \rightarrow p_1$
- $M_2 = (\{p_0, p_1\}, \{c, d\}, \{c, \#\}, R_2, p_0, \#)$
  $R_2: \ \#p_0 c \rightarrow cp_0,\ cp_0 c \rightarrow ccp_0,\ cp_0 d \rightarrow p_1,\ cp_1 d \rightarrow p_1$
- $L(M_1) = \{a^n b^n : \ n \geq 1\}$
- $L(M_2) = \{c^n d^n : \ n \geq 1\}$
- $\tau = (M_1, M_2)$
- $L(\tau) =$

Example

- $M_1 = (\{p_0, p_1\}, \{a, b\}, \{a, \#\}, R_1, p_0, \#)$
  $R_1 :\ \#p_0a \rightarrow ap_0,\ ap_0a \rightarrow aap_0,\ ap_0b \rightarrow p_1,\ ap_1b \rightarrow p_1$
- $M_2 = (\{p_0, p_1\}, \{c, d\}, \{c, \#\}, R_2, p_0, \#)$
  $R_2 :\ \#p_0c \rightarrow cp_0,\ cp_0c \rightarrow ccp_0,\ cp_0d \rightarrow p_1,\ cp_1d \rightarrow p_1$
- $L(M_1) = \{a^nb^n :\ n \geq 1\}$
- $L(M_2) = \{c^nd^n :\ n \geq 1\}$
- $\tau = (M_1, M_2)$
- $L(\tau) = \{x :$
  - $x \in (a + c)^+(a + b + c + d)^*(b + d)^+$,
  - $occure(x, a) = occure(x, b)$ and
  - $occure(x, c) = occure(x, d)$

  $\} = L_1 \parallel L_2$

- $u \parallel v = u_1v_1 \dots u_mv_m$: $u = u_1 \dots u_m$, $v = v_1 \dots v_m$ and $u, v \in \Sigma^*$
- $L_1 \parallel L_2 = \{\omega :\ \omega \in u \parallel v,\ u \in L_1$ and $v \in L_2\}$

Computation Modes

Competence in Teams of Pushdown Automata

Let $f \in \{\leq k : \ k \geq 2\} \cup \{= k, \geq k : \ k \geq 1\}$.
The number n satisfies f iff nf.

Example

7 satisfies ≤ 10 because $7 \leq 10$.

f–Comp–Mode Move

Let $J \subseteq \{1, \ldots, n\}$ where $|J|$ satisfies f, such that for all $j \in J$,

- $Z_j = X$ for some $X \in \bigcap_{t \in J} \Gamma_t$,
- $Xp_j a \rightarrow \alpha q_j \in R$

and for all $i \in \{1, \ldots, n\} \backslash J$ either

- $Z_i \neq X$ or
- no rule with $Xp_i a$ on the left is in R_i.

Then

$$((p_1, \ldots, p_n), a\omega, (Z_1\beta_1, \ldots, Z_n\beta_n)) \vdash^f ((q_1, \ldots, q_n), \omega, (\alpha_1\beta_1, \ldots, \alpha_n\beta_n))$$

Languages

$L_f(\tau) = \{\omega : (q^0, \omega, Z^0) \vdash^f \ldots \vdash^f (q, \varepsilon, Z\}$ for some $q \in Q\}$ where $f \in \{\leq k : k \geq 2\} \cup \{= k, \geq k : k \geq 1\}$

Language Families

$$\mathscr{L}(n\text{–PDAx}, f - \text{comp})$$

where

- x
  - $[-\varepsilon]$ – ε–moves is not allowed
  - nothing – ε–moves is allowed
- f
  - computation mode, $f \in \{\leq k : k \geq 2\} \cup \{= k, \geq k : k \geq 1\}$

Example

- $M_1 = (\{p_0, p_1\}, \{a, b\}, \{a, \#\}, R_1, p_0, \#)$
- $M_2 = (\{p_0, p_1\}, \{c, d\}, \{c, \#\}, R_2, p_0, \#)$

$$
\begin{array}{llll}
R_1: & ap_0A \to p_1AA & R_2: & aq_0A \to AAq_1 \\
 & ap_1A \to p_1AA & & aq_1A \to AAq_1 \\
 & bp_1A \to p_2 & & bq_1A \to q_2 \\
 & bp_2A \to p_2A & & bq_2A \to q_2 \\
 & cp_2A \to p_3 & & cq_2A \to Aq_2 \\
 & cp_3A \to p_3 & &
\end{array}
$$

- $L(M_1) = \{a^m b^n c^m : m, n \geq 1\}$
- $L(M_2) = \{a^k b^{k+1} : k \geq 1\} \cup \{a^k b^k c^m : k, m \geq 1\}$
- $\tau = (M_1, M_2)$
- $L(\tau)_{=1} = \emptyset$
- $L(\tau)_{=2} = L(\tau)_{\geq 2} = \{a^n b^n c^n : n \geq 1\}$
- $L(\tau)_{\leq 2} = \{a^m b^n c^m : 1 \leq m \leq n\} \cup \{a^k b^k c^m : 1 \leq k \leq m\}$
- $L(\tau)_{\geq 1} = L(\tau)_{\leq 2}$

Accepting Power

Theorem

- $\mathscr{L}(n\text{–PDA}[-\varepsilon], * -\text{comp}) \subset \mathscr{L}((n+1)\text{–PDA}[-\varepsilon], * -\text{comp})$

Theorem

- $\mathscr{L}((kn)\text{–PDA}[-\varepsilon], * -\text{comp}) \subset \mathscr{L}(n\text{–PDA}[-\varepsilon], g_1 - \text{comp})$
- $\mathscr{L}(n\text{–PDA}[-\varepsilon], * -\text{comp}) \subset \mathscr{L}(n\text{–PDA}[-\varepsilon], g_2 - \text{comp})$

where

- $g_1 \in \{= k, \geq k : k \geq 2\}$ and
- $g_2 \in \{= 1, \geq 1\} \cup \{\leq k : k \geq 2\}$

Theorem

- $\mathscr{L}(3\text{–PDA}, = 2 - \text{comp}) = \mathscr{L}(3\text{–PDA}, \geq 2 - \text{comp}) = \mathscr{L}(ER)$

Maurice H. ter Beek Erzsébet Csuhaj-Varjú and Victor Mitrana.
Teams of Pushdown Automata .
Lecture Notes in Computer Science, 2003.

Thank you for your attention!