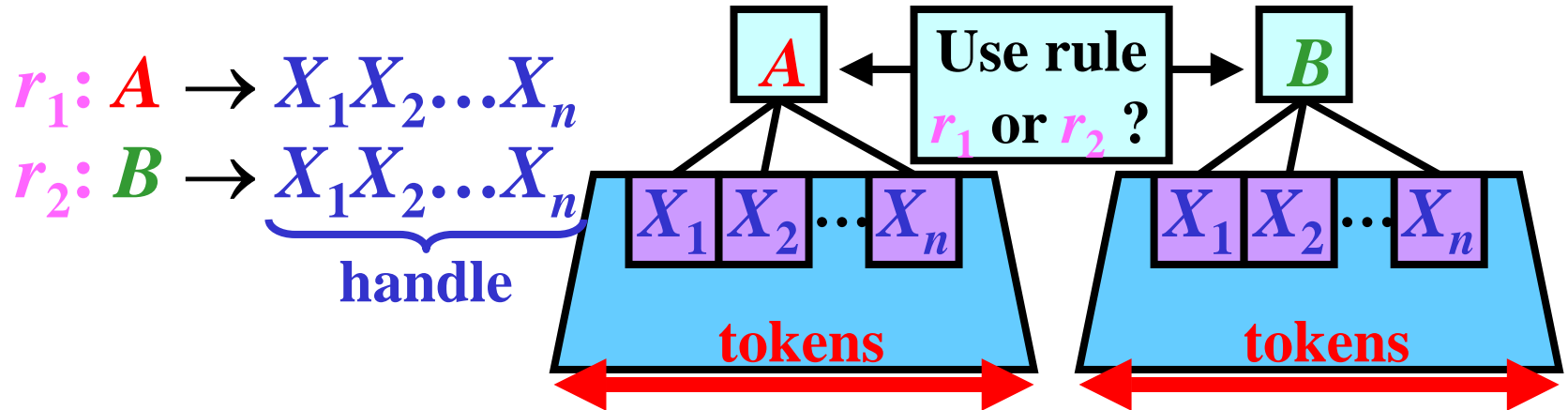


Part VI.

Bottom-Up Parsing

Bottom-Up Parsing: Problems

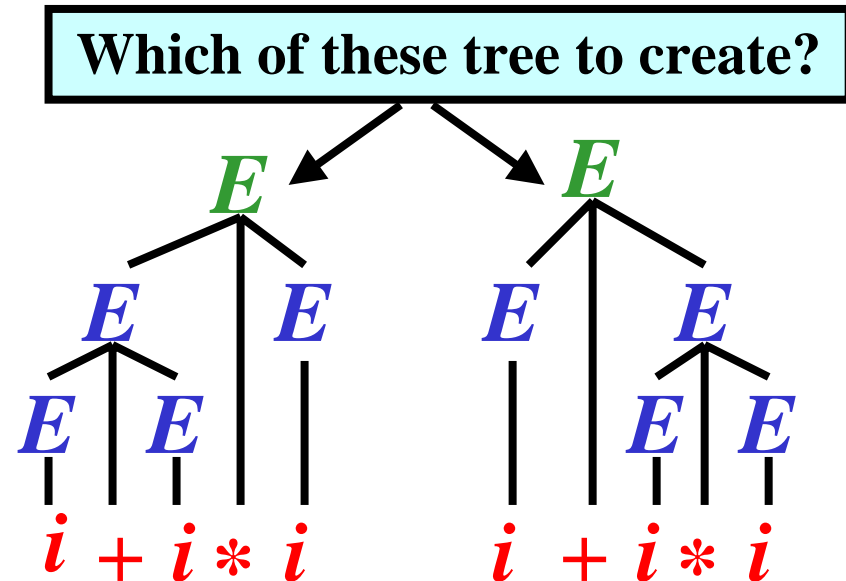
1) Two or more rules have the same *handle*



Note: A *handle* is the right-hand side of a rule.

2) Ambiguous grammars

$G_{expr2} = (N, T, P, E)$, where
 $N = \{E\}$, $T = \{i, +, *, (,)\}$,
 $P = \{$
 $\quad 1: E \rightarrow E+E, \quad 2: E \rightarrow E * E,$
 $\quad 3: E \rightarrow (E), \quad 4: E \rightarrow i \}$



Bottom-Up Parsers

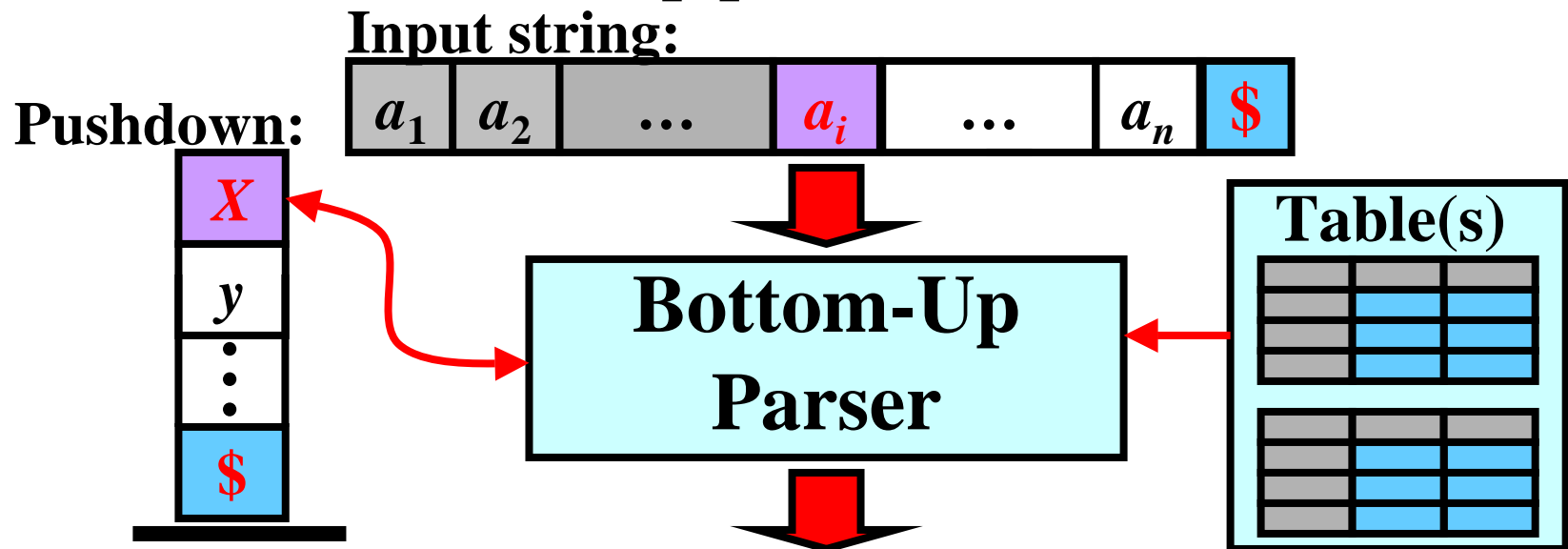
1) Operator-precedence parser

- the least powerful, but simple & easy-to-make

2) LR parser

- the most powerful

• Model of Bottom-Up parser:



Right parse = **reverse** sequence of rules used in the **rightmost derivation** of the tokenized source program

Operator-Precedence Parser

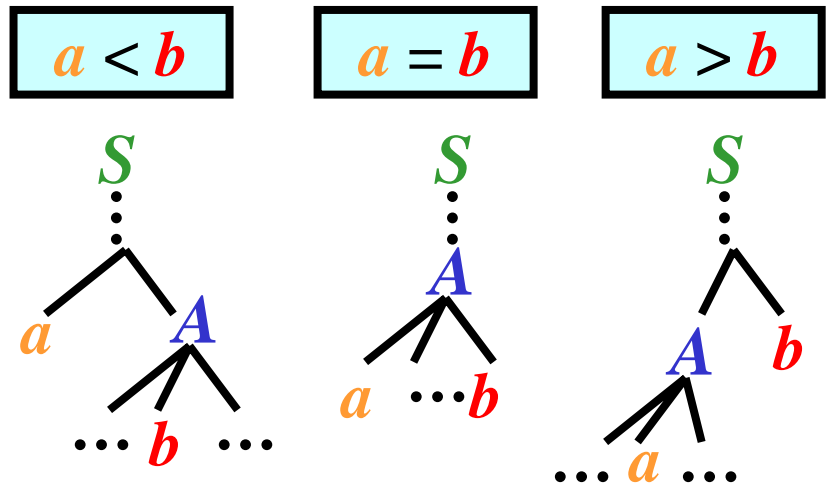
- No two distinct nonterminals have the same handle
 - No ϵ -rules.
-
- Let $G = (N, T, P, S)$ be CFG, where $T = \{a_1, a_2, \dots, a_n\}$

Precedence-table:

	a_1	...	a_j	...	a_n	\$
a_1						
...						
a_i						
...						
a_n						
\$						

Table[a_i, a_j] $\in \{<, =, >, blank\}$

Illustration of meaning of $<, =, >$:



Operator-Precedence Parser: Algorithm

- **Input:** Precedence-table for $G = (N, T, P, S)$; $x \in T^*$
 - **Output:** Right parse of x if $x \in L(G)$; otherwise, error
-
- **Method:**
 - Push $\$$ onto the pushdown;
 - **repeat**
 - let a = the current token and
 b = the topmost **terminal** on the pushdown
 - **case** Table[b, a] **of:**
 - = : push(a) & read next a from input string
 - < : replace b with $b<$ on the pushdown & push(a) & read next a from input string
 - > : **if** $\langle y$ is the pushdown top string **and** $r: A \rightarrow y \in P$ **then** replace $\langle y$ with A & write r to output **else error**
 - **blank : error**
 - **until** $a = \$$ and $b = \$$
 - **success**

Operator-Precedence Parser: Example

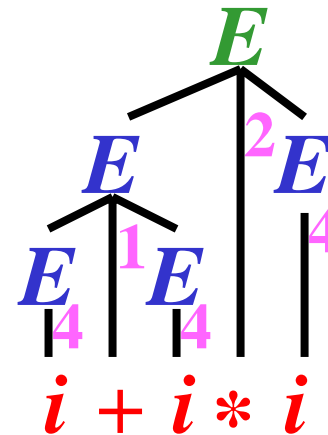
$G_{expr2} = (N, T, P, E)$, where $N = \{E\}$, $T = \{i, +, *, (,)\}$,
 $P = \{1: E \rightarrow E+E, 2: E \rightarrow E * E, 3: E \rightarrow (E), 4: E \rightarrow i\}$

Precedence-table for G_{expr2} :

		Input token					
		+	*	()	i	\$
Pushdown topmost token	+	>	<	<	>	<	>
	*	>	>	<	>	<	>
	(<	<	<	=	<	
)	>	>		>	>	
	i	>	>		>	>	
	\$	<	<	<		<	

Note: Operator associativity and precedence rules underlie the precedence table:

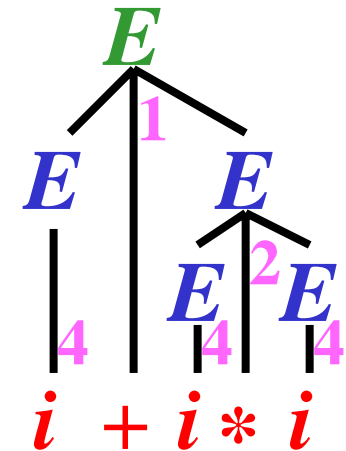
☹ Wrong tree:



Right parse:

44142
←

☺ Right tree:



Right parse:

44421
←

Operator-Precedence Parsing: Example

	+	*	()	<i>i</i>	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>	>	
<i>i</i>	>	>		>	>	
\$	<	<	<		<	

Rules:

1: $E \rightarrow E + E$

2: $E \rightarrow E * E$

3: $E \rightarrow (E)$

4: $E \rightarrow i$

Input string: $i + i * i \$$

Pushdown	Op	Input	Rule
\$	<	$i + i * i \$$	
$\$ < i$	>	$+ i * i \$$	4: $E \rightarrow i$
$\$ E$	<	$+ i * i \$$	
$\$ < E +$	<	$i * i \$$	
$\$ < E + < i$	>	$* i \$$	4: $E \rightarrow i$
$\$ < E + E$	<	$* i \$$	
$\$ < E + < E *$	<	$i \$$	
$\$ < E + < E * < i$	>	$\$$	4: $E \rightarrow i$
$\$ < E + < E * E$	>	$\$$	2: $E \rightarrow E * E$
$\$ < E + E$	>	$\$$	1: $E \rightarrow E + E$
$\$ E$		$\$$	

Success

Right parse: 44421

Construction of Precedence Table 1/5

- Let $G_{expr} = (N, T, P, E)$, where $N = \{E\}$,
 $T = \{ (,), id_1, id_2, \dots, id_m, op_1, op_2, \dots, op_n \}$,
 $P = \{ E \rightarrow (E), E \rightarrow id_1, E \rightarrow id_2, \dots, E \rightarrow id_m,$
 $E \rightarrow E op_1 E, E \rightarrow E op_2 E, \dots, E \rightarrow E op_n E \}$

Note: id_1, id_2, \dots, id_m are identifiers,

op_1, op_2, \dots, op_n are different operators

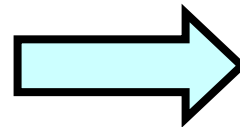
1) Precedence of operators:

- If op_i has higher precedence than op_j then

$$op_i > op_j \text{ and } op_j < op_i$$

Example: Precedence-table part derived from the precedence of operators in G_{expr2}

$$\begin{array}{l} * > + \\ + < * \end{array}$$



	+	*
+		<
*	>	

Construction of Precedence Table 2/5

2) Associativity:

Note:

- op_i is left-associative $\Leftrightarrow a \text{ op}_i b \text{ op}_i c = (a \text{ op}_i b) \text{ op}_i c$
- op_i is right-associative $\Leftrightarrow a \text{ op}_i b \text{ op}_i c = a \text{ op}_i (b \text{ op}_i c)$

- Let op_i and op_j have equal precedence

- If op_i and op_j are left associative then

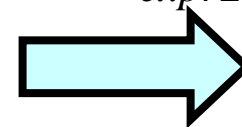
$$\text{op}_i > \text{op}_j \text{ and } \text{op}_j > \text{op}_i$$

- If op_i and op_j are right associative then

$$\text{op}_i < \text{op}_j \text{ and } \text{op}_j < \text{op}_i$$

Example: Precedence-table part derived from the associativity of operators in G_{expr2}

+ is left-associative
* is left-associative



	+	*
+	>	
*		>

Construction of Precedence Table 3/5

3) Identifiers:

- If $a \in T$ may precede id_i , then $a < id_i$
- If $a \in T$ may follow id_i , then $id_i > a$

Example: Precedence-table part for identifiers

$\$i * (i + i) * i$
 $\downarrow \quad \downarrow \downarrow \downarrow \quad \downarrow$
 $\$, (, +, *$ may precede i

$i * (i + i) * i \$$
 $\downarrow \quad \downarrow \downarrow \downarrow \quad \downarrow$
 $*, +,), \$$ may follow i

	+	*	()	i	\$
+					<	
*					<	
(<	
)						
i	>	>	>			>
\$					<	

Construction of Precedence Table 4/5

4) Parentheses:

- A pair of parentheses:
- Let $a \in T - \{), \$\}$. Then,
- Let $a \in T - \{(\, \$\}$. Then,
- Let $a \in T$ and a may **precede** (. Then,
- Let $a \in T$ and a may **follow**). Then,

(=)
(< a
a >)

a < (
) > a

Example: Precedence-table part for parentheses.

$\$(i + ((i * (i + (i + i))))))$

↓ ↓ ↓ ↓
\$, (, *, + may precede (

$(((((i + i) + i) * i)) + i)\$$

↓ ↓ ↓ ↓
+, *,), \$ may follow)

	+	*	()	i	\$
+			<	>		
*			<	>		
(<	<	<	=	<	
)	>	>		>		>
i				>		
\$			<			

Construction of Precedence Table 5/5

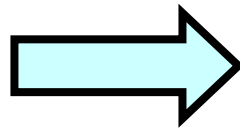
5) End Marker \$

- Let op_i be any operator. Then:

$$\$ < op_i \text{ and } op_i > \$$$

Example: Precedence-table part for end-markers.

\$	<	+
\$	<	*
+	>	\$
*	>	\$



		+	*	\$
+				>
*				>
\$	<	<		

Summary:

	+	*	()	i	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	
)	>	>		>		>
i	>	>		>		>
\$	<	<	<		<	


LR-Parser

- Let $G = (N, T, P, S)$ be a CFG,
where $N = \{A_1, A_2, \dots, A_n\}$, $T = \{a_1, a_2, \dots, a_m\}$
- LR-parser is a EPDA, M , with states
 $Q = \{q_0, q_1, \dots, q_k\}$, where q_0 is the start state.
- M is based on LR table that has these two parts
 - 1) **Action part**
 - 2) **Go-to part**

Action Part & Go-to Part

Action Part:

α	a_1	...	a_j	...	a_p	\$
q_0						
...						
q_i						
...						
q_k						




$\alpha[q_i, a_j] = 1$ or 2 or 3 or 4

- 1) sq : $s = \text{shift}$, $q \in Q$
- 2) rp : $r = \text{reduce}$, $p \in P$
- 3) 😊 : success
- 4) **blank**: error

Go-to Part:

β	A_1	...	A_j	...	A_q
q_0					
...					
q_i					
...					
q_k					



$\beta[q_i, A_j] = 1$ or 2

- 1) q : $q \in Q$
- 2) **blank**

LR-Parser: Algorithm

- **Input:** LR-table for $G=(N, T, P, S)$; $x \in T^*$
 - **Output:** Right parse of x if $x \in L(G)$; otherwise, error
-
- **Method:**
 - push($\langle \$, q_0 \rangle$) onto pushdown; $state := q_0$;
 - **repeat**
 - let a = the current token
 - case** $\alpha[state, a]$ **of:**
 - **sq:** push($\langle a, q \rangle$) & read next a from input string & $state := q$;
 - **rp:** if $p: A \rightarrow X_1X_2\dots X_n \in P$ and $\langle ?, q \rangle \langle X_1, ? \rangle \langle X_2, ? \rangle \dots \langle X_n, ? \rangle$ is pushdown top then $state := \beta[q, A]$ & replace $\langle X_1, ? \rangle \langle X_2, ? \rangle \dots \langle X_n, ? \rangle$ with $\langle A, state \rangle$ on the pushdown & write r to output
 - else error**
 - 😊: **success**
 - blank: **error**
 - until success or error**



LR-Parser: Example 1/2

$G_{expr1} = (N, T, P, E)$, where $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,
 $P = \{$
1: $E \rightarrow E+T$,
2: $E \rightarrow T$,
3: $T \rightarrow T*F$,
4: $T \rightarrow F$,
5: $F \rightarrow (E)$,
6: $F \rightarrow i$
}

LR-table for G_{expr1} :

α	i	$+$	$*$	$($	$)$	$\$$
0	s5			s4		
1		s6				☺
2		r2	s7		r2	r2
3		r4	r4		r4	r4
4	s5			s4		
5		r6	r6		r6	r6
6	s5			s4		
7	s5			s4		
8		s6			s11	
9		r1	s7		r1	r1
10		r3	r3		r3	r3
11		r5	r5		r5	r5

Action part
for G_{expr1}

Go-to part
for G_{expr1}

β	E	T	F
0	1	2	3
1			
2			
3			
4	8	2	3
5			
6		9	3
7			10
8			
9			
10			
11			

LR-Parser: Example 2/2

Rules: 1: $E \rightarrow E+T$, 2: $E \rightarrow T$, 3: $T \rightarrow T*F$,
 4: $T \rightarrow F$, 5: $F \rightarrow (E)$, 6: $F \rightarrow i$

Input string: $i * i \$$

Pushdown	St.	Input	Enter	Rule
$\langle \$, 0 \rangle$	0	$i * i \$$	$\alpha[0, i] = s5$	
$\langle \$, 0 \rangle \langle i, 5 \rangle$	5	$* i \$$	$\alpha[5, *] = r6$	6: $F \rightarrow i$
			$\beta[0, F] = 3$	
$\langle \$, 0 \rangle \langle F, 3 \rangle$	3	$* i \$$	$\alpha[3, *] = r4$	4: $T \rightarrow F$
			$\beta[0, T] = 2$	
$\langle \$, 0 \rangle \langle T, 2 \rangle$	2	$* i \$$	$\alpha[2, *] = s7$	
$\langle \$, 0 \rangle \langle T, 2 \rangle \langle *, 7 \rangle$	7	$i \$$	$\alpha[2, i] = s5$	
$\langle \$, 0 \rangle \langle T, 2 \rangle \langle *, 7 \rangle \langle i, 5 \rangle$	5	$\$$	$\alpha[5, \$] = r6$	6: $F \rightarrow i$
			$\beta[7, F] = 10$	
$\langle \$, 0 \rangle \langle T, 2 \rangle \langle *, 7 \rangle \langle F, 10 \rangle$	10	$\$$	$\alpha[10, \$] = r3$	3: $T \rightarrow T*F$
			$\beta[0, T] = 2$	
$\langle \$, 0 \rangle \langle T, 2 \rangle$	2	$\$$	$\alpha[2, \$] = r2$	2: $E \rightarrow T$
			$\beta[0, E] = 1$	
$\langle \$, 0 \rangle \langle E, 1 \rangle$	1	$\$$	$\alpha[1, \$] = \text{☺}$	Success Right parse: 64632

Construction of LR Table: Introduction

- **One parsing algorithm but many algorithms for the construction of LR table.**
-

Basic algorithms for the construction of LR table:

- 1) **Simple LR (SLR)**: the least powerful, but simple and few states
 - 2) **Canonical LR**: more powerful, but many states
 - 3) **Lookahead LR (LALR)**: the best because the most powerful and the same number of states as SLR
-

Extended Grammar with a “Dummy Rule”

Gist: Grammar with special “starting rule”

Definition: Let $G = (N, T, P, S)$ be a CFG, $S' \notin N$.
Extended grammar for G is grammar
 $G' = (N \cup \{S'\}, T, P \cup \{S' \rightarrow S\}, S')$.

Why a dummy rule? When $S' \rightarrow S$ is used and the input token is endmarker, then **syntax analysis is successfully completed.**

Example:

$G_{\text{expr1}} = (N, T, P, E)$, where $N = \{E, F, T\}$, $T = \{i, +, *, (,)\}$,
 $P = \{$

1: $E \rightarrow E+T$,	2: $E \rightarrow T$,	3: $T \rightarrow T*F$,
4: $T \rightarrow F$,	5: $F \rightarrow (E)$,	6: $F \rightarrow i$

 $\}$

Extended grammar for G_{expr1} :

$G'_{\text{expr1}} = (N, T, P, E')$, where $N = \{E', E, F, T\}$, $T = \{i, +, *, (,)\}$,
 $P = \{$

0: $E' \rightarrow E$,	1: $E \rightarrow E+T$,	2: $E \rightarrow T$,	3: $T \rightarrow T*F$,
4: $T \rightarrow F$,	5: $F \rightarrow (E)$,	6: $F \rightarrow i$	

 $\}$

Construction of LR Table: Items

Gist: Item is a rule of CFG with \bullet in the right side of rule.

Definition: Let $G = (N, T, P, S)$ be a CFG, $A \rightarrow x \in P$, $x = yz$. Then, $A \rightarrow y\bullet z$ is an *item*.

Example: Consider $E \rightarrow E+T$

All items for $E \rightarrow E+T$ are:

$E \rightarrow \bullet E+T$, $E \rightarrow E\bullet+T$, $E \rightarrow E+\bullet T$, $E \rightarrow E+T\bullet$

Meaning: $A \rightarrow y\bullet z$ means that if y appears on the pushdown top and a prefix of the input is eventually reduced to z , then yz ($= x$) as a handle can be reduced to A according to $A \rightarrow x$.

Closure of Item: Algorithm

Note: $\text{Closure}(I)$ is the set of items defined by the following algorithm:

- **Input:** $G = (N, T, P, S)$; item I
 - **Output:** $\text{Closure}(I)$
-
- **Method:**
 - $\text{Closure}(I) := \{I\}$;
 - **Apply the following rule until $\text{Closure}(I)$ cannot be changed:**
 - **if $A \rightarrow y \bullet Bz \in \text{Closure}(I)$ and $B \rightarrow x \in P$ then add $B \rightarrow \bullet x$ to $\text{Closure}(I)$**

Closure of Item: Example 1/2

$G'_{expr1} = (N, T, P, E')$, where $N = \{E', E, F, T\}$, $T = \{i, +, *, (,)\}$,
 $P = \{$

0 : $E' \rightarrow E$,	1 : $E \rightarrow E+T$,	2 : $E \rightarrow T$,	3 : $T \rightarrow T*F$,
4 : $T \rightarrow F$,	5 : $F \rightarrow (E)$,	6 : $F \rightarrow i$	}

Task: $Closure(I)$ for $I = E' \rightarrow \bullet E$

$Closure(I) := \{E' \rightarrow \bullet E\}$

1) $E' \rightarrow \bullet E \in Closure(I)$ and $E \rightarrow E+T \in P$:
 add $E \rightarrow \bullet E+T$ to $Closure(I)$

$Closure(I) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T\}$

2) $E' \rightarrow \bullet E \in Closure(I)$ and $E \rightarrow T \in P$:
 add $E \rightarrow \bullet T$ to $Closure(I)$

$Closure(I) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T\}$

3) $E \rightarrow \bullet T \in Closure(I)$ and $T \rightarrow T*F \in P$:
 add $T \rightarrow \bullet T*F$ to $Closure(I)$

$Closure(I) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T*F\}$

Closure of Item: Example 2/2

$G'_{expr1} = (N, T, P, E')$, where $N = \{E', E, F, T\}$, $T = \{i, +, *, (,)\}$,
 $P = \{$ **0**: $E' \rightarrow E$, **1**: $E \rightarrow E+T$, **2**: $E \rightarrow T$, **3**: $T \rightarrow T*F$,
4: $T \rightarrow F$, **5**: $F \rightarrow (E)$, **6**: $F \rightarrow i$ $\}$

4) $E \rightarrow \bullet T \in \text{Closure}(I)$ and $T \rightarrow F \in P$:
 add $T \rightarrow \bullet F$ to $\text{Closure}(I)$

$\text{Closure}(I) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T*F,$
 $T \rightarrow \bullet F\}$

5) $T \rightarrow \bullet F \in \text{Closure}(I)$ and $F \rightarrow (E) \in P$:
 add $F \rightarrow \bullet (E)$ to $\text{Closure}(I)$

$\text{Closure}(I) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T*F,$
 $T \rightarrow \bullet F, F \rightarrow \bullet (E)\}$

6) $T \rightarrow \bullet F \in \text{Closure}(I)$ and $F \rightarrow i \in P$:
 add $F \rightarrow \bullet i$ to $\text{Closure}(I)$

Summary:

$\text{Closure}(I) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T*F,$
 $T \rightarrow \bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i\}$

Set Θ_G for Grammar G 1/2

Gist: Θ_G is the set of all prefixes of the right-hand sides of rules from G .

Definition: Let $G = (N, T, P, S)$ be CFG.

$$\Theta_G = \{ \langle y \rangle : A \rightarrow y \bullet z \text{ is an item in } G \}$$

Example:

$G'_{\text{expr1}} = (N, T, P, E')$, where $N = \{E', E, F, T\}$, $T = \{i, +, *, (,)\}$,
 $P = \{ \begin{array}{l} 0: E' \rightarrow E, \quad 1: E \rightarrow E+T, \quad 2: E \rightarrow T, \quad 3: T \rightarrow T*F, \\ 4: T \rightarrow F, \quad 5: F \rightarrow (E), \quad 6: F \rightarrow i \end{array} \}$

Task: $\Theta_{G'_{\text{expr1}}}$

1) Members of $\Theta_{G'_{\text{expr1}}}$ of length 0: $\langle \epsilon \rangle \in \Theta_{G'_{\text{expr1}}}$

2) Members of $\Theta_{G'_{\text{expr1}}}$ of length 1:

$E' \rightarrow \underline{E}, E \rightarrow \underline{E}+T, E \rightarrow \underline{T}, T \rightarrow \underline{T}*F, T \rightarrow \underline{E}, F \rightarrow \underline{(E)}, F \rightarrow \underline{i}$

$\langle E \rangle \in \Theta_{G'_{\text{expr1}}}$ $\langle T \rangle \in \Theta_{G'_{\text{expr1}}}$ $\langle F \rangle, \langle (\rangle, \langle i \rangle \in \Theta_{G'_{\text{expr1}}}$

Set Θ_G for Grammar G 2/2

$$G'_{\text{expr1}} = (N, T, P, \mathbf{E}'), \text{ where } N = \{\mathbf{E}', \mathbf{E}, \mathbf{F}, \mathbf{T}\}, T = \{\mathbf{i}, +, *, (,)\},$$

$$P = \left\{ \begin{array}{llll} \mathbf{0}: \mathbf{E}' \rightarrow \mathbf{E}, & \mathbf{1}: \mathbf{E} \rightarrow \mathbf{E}+\mathbf{T}, & \mathbf{2}: \mathbf{E} \rightarrow \mathbf{T}, & \mathbf{3}: \mathbf{T} \rightarrow \mathbf{T}*\mathbf{F}, \\ \mathbf{4}: \mathbf{T} \rightarrow \mathbf{F}, & \mathbf{5}: \mathbf{F} \rightarrow (\mathbf{E}), & \mathbf{6}: \mathbf{F} \rightarrow \mathbf{i} & \end{array} \right\}$$

3) Members of $\Theta_{G'_{\text{expr1}}}$ of length 2:

$$\mathbf{E}' \rightarrow \mathbf{E}, \underbrace{\mathbf{E} \rightarrow \mathbf{E}+\mathbf{T}}_{\langle \mathbf{E}+ \rangle \in \Theta_{G'_{\text{expr1}}}}, \mathbf{E} \rightarrow \mathbf{T}, \underbrace{\mathbf{T} \rightarrow \mathbf{T}*\mathbf{F}}_{\langle \mathbf{T}* \rangle \in \Theta_{G'_{\text{expr1}}}}, \mathbf{T} \rightarrow \mathbf{F}, \underbrace{\mathbf{F} \rightarrow (\mathbf{E})}_{\langle (\mathbf{E}) \rangle \in \Theta_{G'_{\text{expr1}}}}, \mathbf{F} \rightarrow \mathbf{i}$$

4) Members of $\Theta_{G'_{\text{expr1}}}$ of length 3:

$$\mathbf{E}' \rightarrow \mathbf{E}, \underbrace{\mathbf{E} \rightarrow \mathbf{E}+\mathbf{T}}_{\langle \mathbf{E}+\mathbf{T} \rangle \in \Theta_{G'_{\text{expr1}}}}, \mathbf{E} \rightarrow \mathbf{T}, \underbrace{\mathbf{T} \rightarrow \mathbf{T}*\mathbf{F}}_{\langle \mathbf{T}*\mathbf{F} \rangle \in \Theta_{G'_{\text{expr1}}}}, \mathbf{T} \rightarrow \mathbf{F}, \underbrace{\mathbf{F} \rightarrow (\mathbf{E})}_{\langle (\mathbf{E}) \rangle \in \Theta_{G'_{\text{expr1}}}}, \mathbf{F} \rightarrow \mathbf{i}$$

Summary:

$$\Theta_{G'_{\text{expr1}}} = \left\{ \begin{array}{l} \langle \varepsilon \rangle, \langle \mathbf{E} \rangle, \langle \mathbf{T} \rangle, \langle \mathbf{F} \rangle, \langle (\rangle, \langle \mathbf{i} \rangle, \langle \mathbf{E}+ \rangle, \\ \langle \mathbf{T}* \rangle, \langle (\mathbf{E}) \rangle, \langle \mathbf{E}+\mathbf{T} \rangle, \langle \mathbf{T}*\mathbf{F} \rangle, \langle (\mathbf{E}) \rangle \end{array} \right\}$$

Contents(x): Algorithm

Note: For all $x \in \Theta_G$, $Contents(x)$ is the set of items defined by the following algorithm:

- **Input:** Extended $G = (N, T, P, S')$; Θ_G
- **Output:** $Contents(x)$ for all $x \in \Theta_G$

• **Method:**

- $Contents(\langle \varepsilon \rangle) := Closure(S' \rightarrow \bullet S)$;
- for each $x \in \Theta_G - \{\langle \varepsilon \rangle\}$: $Contents(x) := \emptyset$
- **Apply the following rule until no $Contents$ set can be changed:**
 - if $A \rightarrow y \bullet X z \in Contents(\langle x \rangle)$, where $X \in N \cup T$ and $\langle x X \rangle \in \Theta_G$ then
 - add** $Closure(A \rightarrow y X \bullet z)$ to $Contents(\langle x X \rangle)$

Contents(x): Example 1/9

$$G'_{\text{expr1}} = (N, T, P, \mathbf{E}'), \text{ where } N = \{\mathbf{E}', \mathbf{E}, \mathbf{F}, \mathbf{T}\}, T = \{\mathbf{i}, +, *, (,)\},$$

$$P = \left\{ \begin{array}{llll} \mathbf{0}: \mathbf{E}' \rightarrow \mathbf{E}, & \mathbf{1}: \mathbf{E} \rightarrow \mathbf{E}+\mathbf{T}, & \mathbf{2}: \mathbf{E} \rightarrow \mathbf{T}, & \mathbf{3}: \mathbf{T} \rightarrow \mathbf{T}*\mathbf{F}, \\ & \mathbf{4}: \mathbf{T} \rightarrow \mathbf{F}, & \mathbf{5}: \mathbf{F} \rightarrow (\mathbf{E}), & \mathbf{6}: \mathbf{F} \rightarrow \mathbf{i} \end{array} \right\}$$

$$\Theta_{G'_{\text{expr1}}} = \left\{ \begin{array}{l} \langle \varepsilon \rangle, \langle \mathbf{E} \rangle, \langle \mathbf{T} \rangle, \langle \mathbf{F} \rangle, \langle (\rangle, \langle \mathbf{i} \rangle, \langle \mathbf{E}+ \rangle, \\ \langle \mathbf{T}* \rangle, \langle (\mathbf{E} \rangle, \langle \mathbf{E}+\mathbf{T} \rangle, \langle \mathbf{T}*\mathbf{F} \rangle, \langle (\mathbf{E}) \rangle \end{array} \right\}$$

0) $\text{Contents}(\langle \varepsilon \rangle) := \text{Closure}(\mathbf{E}' \rightarrow \bullet \mathbf{E}) =$

$$\left\{ \mathbf{E}' \rightarrow \bullet \mathbf{E}, \mathbf{E}' \rightarrow \bullet \mathbf{E}+\mathbf{T}, \mathbf{E}' \rightarrow \bullet \mathbf{T}, \mathbf{T} \rightarrow \bullet \mathbf{T}*\mathbf{F}, \mathbf{T} \rightarrow \bullet \mathbf{F}, \mathbf{F} \rightarrow \bullet (\mathbf{E}), \mathbf{F} \rightarrow \bullet \mathbf{i} \right\}$$

$\mathbf{E}' \rightarrow \bullet \mathbf{E} \in \text{Contents}(\langle \varepsilon \rangle) \ \& \ \langle \varepsilon \mathbf{E} \rangle = \langle \mathbf{E} \rangle \in \Theta_{G'_{\text{expr1}}}$:

add $\text{Closure}(\mathbf{E}' \rightarrow \mathbf{E} \bullet) = \{\mathbf{E}' \rightarrow \mathbf{E} \bullet\}$ to $\text{Contents}(\langle \mathbf{E} \rangle)$

$\mathbf{E} \rightarrow \bullet \mathbf{E}+\mathbf{T} \in \text{Contents}(\langle \varepsilon \rangle) \ \& \ \langle \varepsilon \mathbf{E} \rangle = \langle \mathbf{E} \rangle \in \Theta_{G'_{\text{expr1}}}$:

add $\text{Closure}(\mathbf{E} \rightarrow \mathbf{E} \bullet + \mathbf{T}) = \{\mathbf{E} \rightarrow \mathbf{E} \bullet + \mathbf{T}\}$ to $\text{Contents}(\langle \mathbf{E} \rangle)$

$\mathbf{E} \rightarrow \bullet \mathbf{T} \in \text{Contents}(\langle \varepsilon \rangle) \ \& \ \langle \varepsilon \mathbf{T} \rangle = \langle \mathbf{T} \rangle \in \Theta_{G'_{\text{expr1}}}$:

add $\text{Closure}(\mathbf{E} \rightarrow \mathbf{T} \bullet) = \{\mathbf{E} \rightarrow \mathbf{T} \bullet\}$ to $\text{Contents}(\langle \mathbf{T} \rangle)$

Contents(x): Example 2/9

⋮

$Contents(\langle \epsilon \rangle) =$

$\{ \checkmark E \rightarrow \bullet E, \checkmark E \rightarrow \bullet E+T, \checkmark E \rightarrow \bullet T, \checkmark T \rightarrow \bullet T^*F, \checkmark T \rightarrow \bullet F, \checkmark F \rightarrow \bullet (E), \checkmark F \rightarrow \bullet i \}$

$T \rightarrow \bullet T^*F \in Contents(\langle \epsilon \rangle) \ \& \ \langle \epsilon T \rangle = \langle T \rangle \in \Theta_{G'expr1}:$

add $Closure(T \rightarrow T \bullet^*F) = \{ T \rightarrow T \bullet^*F \}$ to $Contents(\langle T \rangle)$

$T \rightarrow \bullet F \in Contents(\langle \epsilon \rangle) \ \& \ \langle \epsilon F \rangle = \langle F \rangle \in \Theta_{G'expr1}:$

add $Closure(T \rightarrow F \bullet) = \{ T \rightarrow F \bullet \}$ to $Contents(\langle F \rangle)$

$F \rightarrow \bullet (E) \in Contents(\langle \epsilon \rangle) \ \& \ \langle \epsilon (\rangle = \langle (\rangle \in \Theta_{G'expr1}:$

add $Closure(F \rightarrow (\bullet E)) = \{ F \rightarrow (\bullet E), E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T^*F, T \rightarrow \bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i \}$ to $Contents(\langle (\rangle)$

$F \rightarrow \bullet i \in Contents(\langle \epsilon \rangle) \ \& \ \langle \epsilon i \rangle = \langle i \rangle \in \Theta_{G'expr1}:$

add $Closure(F \rightarrow i \bullet) = \{ F \rightarrow i \bullet \}$ to $Contents(\langle i \rangle)$

Contents(x): Example 3/9

$$\checkmark \text{Contents}(\langle \epsilon \rangle) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T^*F, \\ T \rightarrow \bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i\}$$

$$\text{Contents}(\langle E \rangle) = \{E' \rightarrow E\bullet, E \rightarrow E\bullet+T\}$$

$$\text{Contents}(\langle T \rangle) = \{E \rightarrow T\bullet, T \rightarrow T\bullet^*F\}$$

$$\text{Contents}(\langle F \rangle) = \{T \rightarrow F\bullet\}$$

$$\text{Contents}(\langle (\rangle) = \{F \rightarrow (\bullet E), E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T^*F, \\ T \rightarrow \bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i\}$$

$$\text{Contents}(\langle i \rangle) = \{F \rightarrow i\bullet\}$$

$$\text{Contents}(\langle E+ \rangle) = \emptyset$$

$$\text{Contents}(\langle T^* \rangle) = \emptyset$$

$$\text{Contents}(\langle (E \rangle) = \emptyset$$

$$\text{Contents}(\langle E+T \rangle) = \emptyset$$

$$\text{Contents}(\langle T^*F \rangle) = \emptyset$$

$$\text{Contents}(\langle (E) \rangle) = \emptyset$$

Contents(x): Example 4/9

1) $Contents(\langle E \rangle) = \{E \checkmark \rightarrow E\bullet, E \checkmark \rightarrow E\bullet+T\}$:

$E \rightarrow E\bullet$: nothing

$E \rightarrow E\bullet+T \in Contents(\langle E \rangle) \ \& \ \langle E+ \rangle \in \Theta_{G'expr1}$:

add $Closure(E \rightarrow E+\bullet T) = \{E \rightarrow E+\bullet T, T \rightarrow \bullet T^*F, T \rightarrow \bullet F, F \rightarrow \bullet(E), F \rightarrow \bullet i\}$ to $Contents(\langle E+ \rangle)$

2) $Contents(\langle T \rangle) = \{E \checkmark \rightarrow T\bullet, T \checkmark \rightarrow T\bullet^*F\}$:

$E \rightarrow T\bullet$: nothing

$T \rightarrow T\bullet^*F \in Contents(\langle T \rangle) \ \& \ \langle T^* \rangle \in \Theta_{G'expr1}$:

add $Closure(T \rightarrow T^*\bullet F) = \{T \rightarrow T^*\bullet F, F \rightarrow \bullet(E), F \rightarrow \bullet i\}$ to $Contents(\langle T^* \rangle)$

3) $Contents(\langle F \rangle) = \{T \checkmark \rightarrow F\bullet\}$:

$T \rightarrow F\bullet$: nothing

Contents(x): Example 5/9

4) $Contents(< (>) =$

$\{ \checkmark F \rightarrow (\bullet E), \checkmark E \rightarrow \bullet E+T, \checkmark E \rightarrow \bullet T, \checkmark T \rightarrow \bullet T*F, \checkmark T \rightarrow \bullet F, \checkmark F \rightarrow \bullet (E), \checkmark F \rightarrow \bullet i \}$

$F \rightarrow (\bullet E) \in Contents(< (>) \ \& \ < (E > \in \Theta_{G'expr1}:$

add $Closure(F \rightarrow (E\bullet)) = \{ F \rightarrow (E\bullet) \}$ to $Contents(< (E >)$

$F \rightarrow \bullet E+T \in Contents(< (>) \ \& \ < (E > \in \Theta_{G'expr1}:$

add $Closure(F \rightarrow E\bullet+T) = \{ F \rightarrow E\bullet+T \}$ to $Contents(< (E >)$

$E \rightarrow \bullet T \in Contents(< (>) \quad \text{but } < (T > \notin \Theta_{G'expr1}:$ nothing

$T \rightarrow \bullet T*F \in Contents(< (>) \quad \text{but } < (T > \notin \Theta_{G'expr1}:$ nothing

$T \rightarrow \bullet F \in Contents(< (>) \quad \text{but } < (F > \notin \Theta_{G'expr1}:$ nothing

$F \rightarrow \bullet (E) \in Contents(< (>) \quad \text{but } < ((> \notin \Theta_{G'expr1}:$ nothing

$T \rightarrow \bullet i \in Contents(< (>) \quad \text{but } < (i > \notin \Theta_{G'expr1}:$ nothing

5) $Contents(< i >) = \{ \checkmark F \rightarrow \bullet i \}:$

$F \rightarrow i\bullet :$ nothing

Contents(x): Example 6/9

$$\checkmark \text{Contents}(\langle \epsilon \rangle) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T^*F, \\ T \rightarrow \bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i\}$$

$$\checkmark \text{Contents}(\langle E \rangle) = \{E' \rightarrow E\bullet, E \rightarrow E\bullet+T\}$$

$$\checkmark \text{Contents}(\langle T \rangle) = \{E \rightarrow T\bullet, T \rightarrow T\bullet^*F\}$$

$$\checkmark \text{Contents}(\langle F \rangle) = \{T \rightarrow F\bullet\}$$

$$\checkmark \text{Contents}(\langle (\rangle) = \{F \rightarrow (\bullet E), E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T^*F, \\ T \rightarrow \bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i\}$$

$$\checkmark \text{Contents}(\langle i \rangle) = \{F \rightarrow i\bullet\}$$

$$\text{Contents}(\langle E+ \rangle) = \{E \rightarrow E+\bullet T, T \rightarrow \bullet T^*F, T \rightarrow \bullet F, F \rightarrow \bullet (E), \\ F \rightarrow \bullet i\}$$

$$\text{Contents}(\langle T^* \rangle) = \{T \rightarrow T^*\bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i\}$$

$$\text{Contents}(\langle (E \rangle) = \{F \rightarrow (E\bullet), E \rightarrow E\bullet+T\}$$

$$\text{Contents}(\langle E+T \rangle) = \emptyset$$

$$\text{Contents}(\langle T^*F \rangle) = \emptyset$$

$$\text{Contents}(\langle (E) \rangle) = \emptyset$$

Contents(x): Example 7/9

6) $Contents(\langle E+ \rangle) =$

$\{ \checkmark E \rightarrow E+ \bullet T, \checkmark T \rightarrow \bullet T * F, \checkmark T \rightarrow \bullet F, \checkmark F \rightarrow \bullet (E), \checkmark F \rightarrow \bullet i \}$

$E \rightarrow E+ \bullet T \in Contents(\langle E+ \rangle) \ \& \ \langle E+T \rangle \in \Theta_{G'expr1}$:

add $Closure(E \rightarrow E+T \bullet) = \{ E \rightarrow E+T \bullet \}$ to $Contents(\langle E+T \rangle)$

$T \rightarrow \bullet T * F \in Contents(\langle E+ \rangle) \ \& \ \langle E+T \rangle \in \Theta_{G'expr1}$:

add $Closure(T \rightarrow T \bullet * F) = \{ T \rightarrow T \bullet * F \}$ to $Contents(\langle E+T \rangle)$

$T \rightarrow \bullet F \in Contents(\langle E+ \rangle)$ but $\langle E+F \rangle \notin \Theta_{G'expr1}$: nothing

$F \rightarrow \bullet (E) \in Contents(\langle E+ \rangle)$ but $\langle E+ (\rangle \notin \Theta_{G'expr1}$: nothing

$T \rightarrow \bullet i \in Contents(\langle E+ \rangle)$ but $\langle E+ i \rangle \notin \Theta_{G'expr1}$: nothing

7) $Contents(\langle T* \rangle) = \{ \checkmark T \rightarrow T* \bullet F, \checkmark F \rightarrow \bullet (E), \checkmark F \rightarrow \bullet i \}$

$T \rightarrow T* \bullet F \in Contents(\langle T* \rangle) \ \& \ \langle T*F \rangle \in \Theta_{G'expr1}$:

add $Closure(T \rightarrow T*F \bullet) = \{ T \rightarrow T*F \bullet \}$ to $Contents(\langle T*F \rangle)$

$F \rightarrow \bullet (E) \in Contents(\langle T* \rangle)$ but $\langle T* (\rangle \notin \Theta_{G'expr1}$: nothing

$T \rightarrow \bullet i \in Contents(\langle T* \rangle)$ but $\langle T* i \rangle \notin \Theta_{G'expr1}$: nothing

Contents(x): Example 8/9

8) $Contents(\langle (E) \rangle) = \{ F \checkmark \rightarrow (E) \bullet, E \checkmark \rightarrow E \bullet + T \}$

$F \rightarrow (E) \bullet \in Contents(\langle (E) \rangle) \ \& \ \langle (E) \rangle \in \Theta_{G'_{expr1}}$:

add $Closure(E \rightarrow (E) \bullet) = \{ F \rightarrow (E) \bullet \}$ to $Contents(\langle (E) \rangle)$

$E \rightarrow E \bullet + T \in Contents(\langle (E) \rangle)$ but $\langle (E +) \rangle \notin \Theta_{G'_{expr1}}$: nothing

9) $Contents(\langle E + T \rangle) = \{ E \checkmark \rightarrow E + T \bullet, T \checkmark \rightarrow T \bullet * F \}$

$E \rightarrow E + T \bullet$: nothing

$T \rightarrow T \bullet * F \in Contents(\langle E + T \rangle)$ but $\langle E + T * \rangle \notin \Theta_{G'_{expr1}}$: nothing

10) $Contents(\langle E + T \rangle) = \{ T \checkmark \rightarrow T * F \bullet \}$

$T \rightarrow T * F \bullet$: nothing

11) $Contents(\langle (E) \rangle) = \{ F \checkmark \rightarrow (E) \bullet \}$

$F \rightarrow (E) \bullet$: nothing

Contents(x): Example 9/9

$$\checkmark \text{Contents}(\langle \epsilon \rangle) = \{E' \rightarrow \bullet E, E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T^*F, \\ T \rightarrow \bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i\}$$

$$\checkmark \text{Contents}(\langle E \rangle) = \{E' \rightarrow E\bullet, E \rightarrow E\bullet+T\}$$

$$\checkmark \text{Contents}(\langle T \rangle) = \{E \rightarrow T\bullet, T \rightarrow T\bullet^*F\}$$

$$\checkmark \text{Contents}(\langle F \rangle) = \{T \rightarrow F\bullet\}$$

$$\checkmark \text{Contents}(\langle (\rangle) = \{F \rightarrow (\bullet E), E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T^*F, \\ T \rightarrow \bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i\}$$

$$\checkmark \text{Contents}(\langle i \rangle) = \{F \rightarrow i\bullet\}$$

$$\checkmark \text{Contents}(\langle E+ \rangle) = \{E \rightarrow E+\bullet T, T \rightarrow \bullet T^*F, T \rightarrow \bullet F, F \rightarrow \bullet (E), \\ F \rightarrow \bullet i\}$$

$$\checkmark \text{Contents}(\langle T^* \rangle) = \{T \rightarrow T^*\bullet F, F \rightarrow \bullet (E), F \rightarrow \bullet i\}$$

$$\checkmark \text{Contents}(\langle (E \rangle) = \{F \rightarrow (E\bullet), E \rightarrow E\bullet+T\}$$

$$\checkmark \text{Contents}(\langle E+T \rangle) = \{E \rightarrow E+T\bullet, T \rightarrow T\bullet^*F\}$$

$$\checkmark \text{Contents}(\langle T^*F \rangle) = \{T \rightarrow T^*F\bullet\}$$

$$\checkmark \text{Contents}(\langle (E) \rangle) = \{F \rightarrow (E)\bullet\}$$

Construction of LR-table: Algorithm

- **Input:** Extended $G = (N, T, P, S')$; Θ_G ;
 $Contents(x)$ for all $x \in \Theta_G$; $Follow(A)$ for all $x \in A$
 - **Output:** LR-table for G (α = Action part, β = Go-to part)
-
- **Method:**
 - $StatesOfTable := \Theta_G$; $StartingState := \langle \epsilon \rangle$
 - for each $\langle x \rangle \in \Theta_G$ do
 - for each $I \in Contents(\langle x \rangle)$ do
 - case I of
 - $I = A \rightarrow y \bullet X z$, where $X \in N$:
 - if $A \rightarrow y X \bullet z \in Contents(\langle q \rangle)$ then $\beta[\langle x \rangle, X] := \langle q \rangle$
 - $I = A \rightarrow y \bullet X z$, where $X \in T$:
 - if $A \rightarrow y X \bullet z \in Contents(\langle q \rangle)$ then $\alpha[\langle x \rangle, X] := s \langle q \rangle$
 - $I = S' \rightarrow S \bullet$: $\alpha[\langle x \rangle, \$] := \text{☺}$
 - $I = A \rightarrow y \bullet$ ($A \neq S'$):
 - for each $a \in Follow(A)$ do $\alpha[\langle x \rangle, a] := rp$,
 - where p is a label of rule $A \rightarrow y$

Construction of LR-table: Example 1/5

Task: LR-table for G_{expr1}

	α						β			
	i	$+$	$*$	$($	$)$	$\$$	E	T	F	
$\langle \epsilon \rangle$							$\langle E \rangle$	$\langle T \rangle$	$\langle F \rangle$	
$\langle E \rangle$	$Contents(\langle \epsilon \rangle):$									
$\langle T \rangle$	$I = E' \rightarrow \bullet E \in Contents(\langle \epsilon \rangle):$									
$\langle F \rangle$	$E' \rightarrow E \bullet \in Contents(\langle E \rangle): \beta[\langle \epsilon \rangle, E] := \langle E \rangle$									
$\langle (\rangle$										
$\langle i \rangle$	$I = E \rightarrow \bullet E + T \in Contents(\langle \epsilon \rangle):$									
$\langle E + \rangle$	$E \rightarrow E \bullet + T \in Contents(\langle E \rangle): \beta[\langle \epsilon \rangle, E] := \langle E \rangle$									
$\langle T * \rangle$										
$\langle (E \rangle$	$I = E \rightarrow \bullet T \in Contents(\langle \epsilon \rangle):$									
$\langle E + T \rangle$	$E \rightarrow T \bullet \in Contents(\langle T \rangle): \beta[\langle \epsilon \rangle, T] := \langle T \rangle$									
$\langle T * F \rangle$										
$\langle (E) \rangle$	$I = E \rightarrow \bullet T * F \in Contents(\langle \epsilon \rangle):$									
	$E \rightarrow T \bullet * F \in Contents(\langle T \rangle): \beta[\langle \epsilon \rangle, T] := \langle T \rangle$									
	$I = E \rightarrow \bullet F \in Contents(\langle \epsilon \rangle):$									
	$E \rightarrow F \bullet \in Contents(\langle F \rangle): \beta[\langle \epsilon \rangle, F] := \langle F \rangle$									

Construction of LR-table: Example 2/5

Task: LR-table for G_{expr1}

	α						β		
	i	$+$	$*$	$($	$)$	$\$$	E	T	F
$\langle \epsilon \rangle$	$s\langle i \rangle$			$s\langle (\rangle$			$\langle E \rangle$	$\langle T \rangle$	$\langle F \rangle$
$\langle E \rangle$									
$\langle T \rangle$									
$\langle F \rangle$									
$\langle (\rangle$									
$\langle i \rangle$									
$\langle E+ \rangle$									
$\langle T^* \rangle$									
$\langle (E \rangle$									
$\langle E+T \rangle$									
$\langle T^*F \rangle$									
$\langle (E) \rangle$									

$Contents(\langle \epsilon \rangle)$:
 $I = F \rightarrow \bullet(E) \in Contents(\langle \epsilon \rangle)$:
 $F \rightarrow (\bullet E) \in Contents(\langle (\rangle)$: $\alpha[\langle \epsilon \rangle, (] := s\langle (\rangle$

$Contents(\langle i \rangle)$:
 $I = F \rightarrow \bullet i \in Contents(\langle i \rangle)$:
 $F \rightarrow i\bullet \in Contents(\langle i \rangle)$: $\alpha[\langle \epsilon \rangle, E] := s\langle i \rangle$

Construction of LR-table: Example 3/5

Task: LR-table for G_{expr1}

	α						β		
	i	$+$	$*$	$($	$)$	$\$$	E	T	F
$\langle \epsilon \rangle$	$s\langle i \rangle$			$s\langle (\rangle$			$\langle E \rangle$	$\langle T \rangle$	$\langle F \rangle$
$\langle E \rangle$		$s\langle E+ \rangle$				☺			
$\langle T \rangle$									
$\langle F \rangle$									
$\langle (\rangle$									
$\langle i \rangle$									
$\langle E+ \rangle$									
$\langle T* \rangle$									
$\langle (E \rangle$									
$\langle E+T \rangle$									
$\langle T*F \rangle$									
$\langle (E \rangle$									

$Contents(\langle E \rangle)$:
 $I = E' \rightarrow E \bullet \in Contents(\langle E \rangle): \alpha[\langle E \rangle, \$] := ☺$
 $I = E \rightarrow E \bullet + T \in Contents(\langle E \rangle)$:
 $E \rightarrow E + \bullet T \in Contents(\langle E+ \rangle): \alpha[\langle E+ \rangle, +] = s\langle E+ \rangle$

Construction of LR-table: Example 4/5

Task: LR-table for G_{expr1}

	α					β			
	i	$+$	$*$	$($	$)$	$\$$	E	T	F
$\langle \epsilon \rangle$	$s\langle i \rangle$			$s\langle (\rangle$			$\langle E \rangle$	$\langle T \rangle$	$\langle F \rangle$
$\langle E \rangle$		$s\langle E+ \rangle$				☺			
$\langle T \rangle$		$r2$	$s\langle T^* \rangle$		$r2$	$r2$			
$\langle F \rangle$									
$\langle (\rangle$									
$\langle i \rangle$									
$\langle E+ \rangle$									
$\langle T^* \rangle$									
$\langle E \rangle$									
$\langle E+T \rangle$									
$\langle T^*F \rangle$									
$\langle E \rangle$									

$Contents(\langle T \rangle)$:

$I = E \rightarrow T \bullet \in Contents(\langle T \rangle)$: $Follow(E) = \{+,), \$\}$

$\alpha[\langle T \rangle, +] = \alpha[\langle T \rangle,)] = \alpha[\langle T \rangle, \$] := r2$

Note: $E \rightarrow T$ is rule with label 2

$I = T \rightarrow T \bullet * F \in Contents(\langle T \rangle)$:

$T \rightarrow T^* \bullet F \in Contents(\langle T^* \rangle)$: $\alpha[\langle E \rangle, +] = s\langle T^* \rangle$

Construct the rest analogically.

Construction of LR-table: Example 5/5

Final LR-table for G_{expr1}

	α						β		
	i	$+$	$*$	$($	$)$	$\$$	E	T	F
$\langle \epsilon \rangle$	$s\langle i \rangle$			$s\langle (\rangle$			$\langle E \rangle$	$\langle T \rangle$	$\langle F \rangle$
$\langle E \rangle$		$s\langle E+ \rangle$				☺			
$\langle T \rangle$		$r2$	$s\langle T* \rangle$		$r2$	$r2$			
$\langle F \rangle$		$r4$	$r4$		$r4$	$r4$			
$\langle (\rangle$	$s\langle i \rangle$			$s\langle (\rangle$			$\langle (E \rangle$	$\langle T \rangle$	$\langle F \rangle$
$\langle i \rangle$		$r6$	$r6$		$r6$	$r6$			
$\langle E+ \rangle$	$s\langle i \rangle$			$s\langle (\rangle$				$\langle E+T \rangle$	$\langle F \rangle$
$\langle T* \rangle$	$s\langle i \rangle$			$s\langle (\rangle$					$\langle T*F \rangle$
$\langle (E \rangle$		$s\langle E+ \rangle$			$s\langle (E \rangle$				
$\langle E+T \rangle$		$r1$	$s\langle T* \rangle$		$r1$	$r1$			
$\langle T*F \rangle$		$r3$	$r3$		$r3$	$r3$			
$\langle (E \rangle$		$r5$	$r5$		$r5$	$r5$			

Renaming the states

**Rename
the states:**

Old	New
$\langle \epsilon \rangle$	0
$\langle E \rangle$	1
$\langle T \rangle$	2
$\langle F \rangle$	3
$\langle (\rangle$	4
$\langle i \rangle$	5
$\langle E+ \rangle$	6
$\langle T^* \rangle$	7
$\langle (E \rangle$	8
$\langle E+T \rangle$	9
$\langle T^*F \rangle$	10
$\langle (E) \rangle$	11

LR table for G_{expr1} with the renamed states:

α	i	$+$	$*$	$($	$)$	$\$$
0	s5			s4		
1		s6				☺
2		r2	s7		r2	r2
3		r4	r4		r4	r4
4	s5			s4		
5		r6	r6		r6	r6
6	s5			s4		
7	s5			s4		
8		s6			s11	
9		r1	s7		r1	r1
10		r3	r3		r3	r3
11		r5	r5		r5	r5

β	E	T	F
0	1	2	3
1			
2			
3			
4	8	2	3
5			
6		9	3
7			10
8			
9			
10			
11			