

Programmed Grammars

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Programmed Grammar

A **programmed grammar** is a pair

$$H = (G, R)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- R is a finite relation on P

Notation

If $p : A \rightarrow x \in P$, $R(p) = Q$, we write

$$(p : A \rightarrow x, Q)$$

Derivation Step

For $(x, p), (y, q) \in V^* \times P$,

$$(x, p) \Rightarrow (y, q) \text{ in } H$$

if

- 1 $x \Rightarrow y [p]$ in G
- 2 $q \in R(p)$

Generated Language

$$L(H) = \{x \in T^* : (S, p) \Rightarrow^* (x, p') \text{ for some } p, p' \in P\}$$

Example

(1 : $S \rightarrow ABC, \{2, 5\}$)

(2 : $A \rightarrow aA, \{3\}$)

(3 : $B \rightarrow bB, \{4\}$)

(4 : $C \rightarrow cC, \{2, 5\}$)

(5 : $A \rightarrow a, \{6\}$)

(6 : $B \rightarrow b, \{7\}$)

(7 : $C \rightarrow c, \{7\}$)

($S, 1$) \Rightarrow ($ABC, 2$)

\Rightarrow ($aABC, 3$)

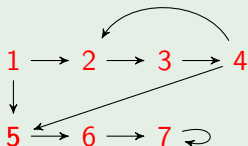
\Rightarrow ($aAbBC, 4$)

\Rightarrow ($aAbBcC, 5$)

\Rightarrow ($aabBcC, 6$)

\Rightarrow ($aabbC, 7$)

\Rightarrow ($aabbcc, 7$)



$$L(H) = \{a^n b^n c^n : n \geq 1\}$$

Programmed Grammar with Appearance Checking

Programmed Grammar with Appearance Checking

A **programmed grammar with appearance checking** is a triple

$$H = (G, R, F)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- R, F are finite relations on P

Notation

If $p : A \rightarrow x \in P$, $R(p) = U$, and $F(p) = V$, we write

$$(p : A \rightarrow x, U, V)$$

where U and V are **success** and **failure fields**, respectively

Derivation Step

For $(x, p), (y, q) \in V^* \times P$,

$$(x, p) \Rightarrow (y, q) \text{ in } H$$

if

- either $x \Rightarrow y [p]$ in G and $q \in R(p)$
- or $x = y$, $q \in F(p)$, p is not applicable to x

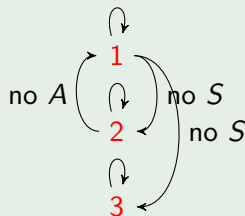
Example I

Example

(1 : $S \rightarrow AA, \{1\}, \{2, 3\}$)

(2 : $A \rightarrow S, \{2\}, \{1\}$)

(3 : $A \rightarrow a, \{3\}, \emptyset$)



$L(H) = \{a^{2^n} : n \geq 1\}$

$(S, 1) \Rightarrow (AA, 1) \Rightarrow (AA, 3)$

$\Rightarrow (Aa, 3)$

$\Rightarrow (aa, 3)$

$\Rightarrow (AA, 2)$

$\Rightarrow (AS, 2)$

$\Rightarrow (SS, 2)$

$\Rightarrow (SS, 1)$

$\Rightarrow (SAA, 1)$

$\Rightarrow (AAAA, 1) \Rightarrow (AAAA, 3)$

$\Rightarrow (AaAA, 3)$

$\Rightarrow (AaaA, 3)$

$\Rightarrow (aaaA, 3)$

$\Rightarrow (aaaa, 3)$

$\Rightarrow (AAAA, 2)$

$\Rightarrow (ASAA, 2) \dots$

Example

(1 : $S \rightarrow SC, \{1, 2\}, \emptyset$)

(2 : $S \rightarrow AA, \{3\}, \emptyset$)

(3 : $A \rightarrow B, \{4\}, \{5\}$)

(4 : $C \rightarrow D, \{3\}, \{7\}$)

(5 : $C \rightarrow C, \{6\}, \emptyset$)

(6 : $B \rightarrow A, \{6\}, \{3\}$)

(7 : $B \rightarrow A, \{7\}, \{8\}$)

(8 : $D \rightarrow A, \{9\}, \{10\}$)

(9 : $D \rightarrow C, \{9\}, \{3\}$)

(10 : $A \rightarrow a, \{10\}, \emptyset$)

Which language generates this grammar?



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