

Random Context Grammars

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Random Context Grammar

A (permitting) **random context grammar** is a pair

$$H = (G, R)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- R is a finite relation from P to N

Notation

If $p : A \rightarrow x \in P$, $R(p) = Q$, we write

$$(p : A \rightarrow x, Q)$$

where $Q \subseteq N$ is called a **permitting context**

Derivation Step

For $x, y \in V^*$, $p \in P$,

$$x \Rightarrow y [p] \text{ in } H$$

if

- 1 $x \Rightarrow y [p]$ in G and
- 2 $R(p) \subseteq \text{alph}(x)$

Random Context Grammar with Appearance Checking

Random Context Grammar with Appearance Checking

A **random context grammar with appearance checking** is a triple

$$H = (G, R, F)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- R, F are two finite relations from P to N

Notation

If $p : A \rightarrow x \in P$, $R(p) = Q$, and $F(p) = K$, we write

$$(p : A \rightarrow x, Q, K)$$

where Q and K are **permitting** and **forbidding contexts**, respectively

Derivation Step of Random Context Grammar with Appearance Checking

Forbidding Grammar

If every $(p : A \rightarrow x, Q, K)$ satisfies $Q = \emptyset$, then H is called a **forbidding grammar**.

Derivation Step

For $x, y \in V^*$, $p \in P$,

$$x \Rightarrow y [p] \text{ in } H$$

if

- 1 $x \Rightarrow y [p] \text{ in } G$
- 2 $R(p) \subseteq \text{alph}(x)$
- 3 $F(p) \cap \text{alph}(x) = \emptyset$

Example

1 $(S \rightarrow ABC, \emptyset)$

2 $(A \rightarrow aA', \{B\})$

$(B \rightarrow bB', \{C\})$

$(C \rightarrow cC', \{A'\})$

3 $(A' \rightarrow A, \{B'\})$

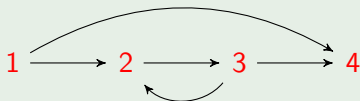
$(B' \rightarrow B, \{C'\})$

$(C' \rightarrow C, \{A\})$

4 $(A \rightarrow a, \{B\})$

$(B \rightarrow b, \{C\})$

$(C \rightarrow c, \emptyset)$



Example – Permitting Grammar II

Example

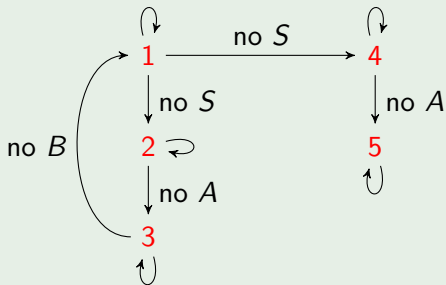
$$\begin{aligned} S &\Rightarrow ABC && 1 \\ &\Rightarrow^3 aA'bB'cC' && 2 \\ &\Rightarrow^3 aAbBcC && 3 \\ &\Rightarrow^3 aaA'bbB'ccC' && 2 \\ &\Rightarrow^3 aaAbbBccC && 3 \\ &\quad \vdots \\ &\Rightarrow^3 a^n Ab^n Bc^n C && 3 \\ &\Rightarrow^3 a^{n+1} b^{n+1} c^{n+1} && 4 \end{aligned}$$

$$L(H) = \{a^k b^k c^k : k \geq 1\}$$

Example – Forbidding Grammar I

Example

- (1 : $S \rightarrow AA, \emptyset, \{B, D\}$)
- (2 : $A \rightarrow B, \emptyset, \{S, D\}$)
- (3 : $B \rightarrow S, \emptyset, \{A, D\}$)
- (4 : $A \rightarrow D, \emptyset, \{S, B\}$)
- (5 : $D \rightarrow a, \emptyset, \{S, A, B\}$)



Example – Forbidding Grammar II

Example

| | | rules | |
|-----|-------------------------|---------------|---|
| S | \Rightarrow | AA | 1 |
| | \Rightarrow^2 | BB | 2 |
| | \Rightarrow^2 | SS | 3 |
| | \Rightarrow^2 | $AAAA$ | 1 |
| | \Rightarrow^4 | $BBBB$ | 2 |
| | \Rightarrow^4 | $SSSS$ | 3 |
| | | \vdots | |
| | \Rightarrow^{2^i} | $A^{2^{i+1}}$ | 1 |
| | $\Rightarrow^{2^{i+1}}$ | $D^{2^{i+1}}$ | 4 |
| | $\Rightarrow^{2^{i+1}}$ | $a^{2^{i+1}}$ | 5 |

$$L(H) = \{a^{2^n} : n \geq 1\}$$



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