*A Testing Theory for Real-Time Systems

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*I. INTRODUCTION

- The aim of this paper is to develop a semantic theory for real-time system specification based on timed transition systems modeling the behaviour of real-time processes.

*II. PRELIMINARIES AND NOTATIONS

- Our constructions are based on some alphabet A represent- ing a set of actions excluding the internal action τ and on a time alphabet L which contains some kind of positive numbers (such as N or R+). A set of clocks C is a set of variables over L. We use A, L, and C in sans-serif face exclusively for this purpose, so that their purpose is often consider understood throughout the paper.

II. PRELIMINARIES AND NOTATIONS

- *A. Timed Transition Systems*

Definition 1: TIMED TRACE LANGUAGES. For a timed transition system (state) p the timed finite-trace language $L_f(p)$, maximal-trace (complete-trace) language $L_m(p)$, infinite-trace language $L_I(p)$, and divergence language $L_D(p)$ of p are

$$
\begin{aligned}
L_f(p) &= \{\text{trace}(\pi) : \pi \in \Pi_f(p)\} \\
L_m(p) &= \{\text{trace}(\pi) : \pi \in \Pi_m(p)\} \\
L_I(p) &= \{\text{trace}(\pi) : \pi \in \Pi_I(p)\} \\
L_D(p) &= \{w \in (A \times L \times \mathbb{T}(C) \times 2^C)^{\sqcup} \\
&\qquad \cup (A \times L \times \mathbb{T}(C) \times 2^C)^{\omega} : p \,\square\, w\}
\end{aligned}
$$

Definition 2: TIMED ω-REGULAR TRACE LANGUAGE. The timed ω-regular trace language of some timed transition system p is $L_\omega(p) = \{\text{trace}(\pi) : \pi \in \Pi_\omega(p)\} \subseteq (A \times L \times \mathbb{T}(C) \times 2^C)^\omega$, where $\Pi_\omega(p)$ contains exactly all the *ω-regular timed paths*; that is, ω-final states must occur infinitely often in any $\pi \in \Pi_\omega(p)$.

- *B. Timed Propositional Temporal Logic*

II. PRELIMINARIES AND NOTATIONS

- Timed Propositional Temporal Logic (TPTL) is one of the most general temporal logics with time constraints. TPTL extends linear-time temporal logic (LTL), by adding time constraints, so that its semantics is given with respect to timed traces, that is, timed words in $(A \times L)^* \cup (A \times L)^\omega$. We use TPTL without congruence, but we just call it TPTL for short.

*III. A TESTING THEORY FOR REAL TIME

Definition 3: TIMED PROCESSES AND TESTS. A timed process $((A \times L) \cup \{\tau\}, S, \rightarrow, p_0)$ is a timed transition system $((A \times L) \cup \{\tau\}, \square, S, \rightarrow, p_0)$ with an empty set of clocks (and thus with no time constraints). It follows that all the traces of any timed process are in the set $(A \times L)^{\square} \cup (A \times L)^{\omega}$.

A timed test $(A \cup \{\tau\}, C, T, \rightarrow_t, \Sigma, \Omega, t_0)$ is a timed transition system $((A \times \square) \cup \{\tau\}, C, T, \rightarrow, t_0)$ with the addition of $\Sigma \subseteq T$ of success states and $\Omega \subseteq T$ of ω-final states. Note that $L = \square$ for tests and therefore $\rightarrow_t \subseteq (T \times A \times \mathbb{T}(C) \times 2^C \times T) \cup (T \times \{\tau\} \times T)$.

Definition 4: PARTIAL COMPUTATION. A partial computation c of a timed process p and a timed test t is a potentially infinite sequence $(\langle p_{i-1}, t_{i-1}\rangle \xmapsto[\mathbb{t}_i, C_i]{(a_i, o_i)}_R \langle p_i, t_i\rangle)_{0< i\leq k}$, where $k \in N \cup \{\omega\}$, such that $p_i \in P$ and $t_i \in T$ for all $0 < i \leq k$; and $\delta_i \in L$ is taken from p, $\mathbb{t}_i$ and C_i are taken from t, and $R \in \{1, 2, 3\}$ for all $0 < i \leq k$. The relation ↦ is defined by the following rules:

- $\langle p_{i-1}, t_{i-1}\rangle \xmapsto{\tau}_1 \langle p_i, t_i\rangle$ if $a_i = \tau$, $p_{i-1} \xrightarrow{\tau}_p p_i$, $t_{i-1} = t_i$, and $t_{i-1} \notin \Sigma$,
- $\langle p_{i-1}, t_{i-1}\rangle \xmapsto{\tau}_2 \langle p_i, t_i\rangle$ if $a_i = \tau$, $p_{i-1} = p_i$, $t_{i-1} \xrightarrow{\tau}_t t_i$, and $t_{i-1} \notin \Sigma$,
- $\langle p_{i-1}, t_{i-1}\rangle \xmapsto[\mathbb{t}_i, C_i]{(a_i, \delta_i)}_3 \langle p_i, t_i\rangle$ if $(a_i, \delta_i) \in A \times L$, $p_{i-1} \xrightarrow{(a_i, \delta_i)}_p p_i$, $t_{i-1} \xrightarrow[\mathbb{t}_i, C_i]{(a_i, \delta_i)}_t t_i$, and $t_{i-1} \notin \Sigma$.

COMPUTATION. A partial cor
enever: If $k \in N$ then c is m
$\rightarrow_t$, and $\mathrm{init}_p(p_k) \cap \mathrm{init}_t(t_k)$ =
s not satisfy the time constraint
$\sqcap_l(p)$. $C(p, t)$ is the set of a

c is *successful* if $t_{|c|} \in \Sigma$ wh
$\sqcap_\omega(t)$ whenever $|c| = \omega$.

Definition 6: TIMED MAY AND MUST PREORDERS. p *may pass* t (written p $\text{may}_{\mathbb{T}}$ t), if and only if there exists at least one successful computation $c \in C(p,t)$; p *must pass* t (written p $\text{must}_{\mathbb{T}}$ t) if and only if every computation $c \in C(p,t)$ is successful.

$p \sqsubseteq_{\mathbb{T}}^{may} q$ if and only if $\forall t \in T$: p $\text{may}_{\mathbb{T}}$ t $\Longrightarrow$ q $\text{may}_{\mathbb{T}}$ t; and $p \sqsubseteq_{\mathbb{T}}^{must} q$ if and only if $\forall t \in T$: p $\text{must}_{\mathbb{T}}$ t $\Longrightarrow$ q $\text{must}_{\mathbb{T}}$ t.

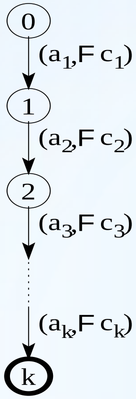
*IV. ALTERNATIVE CHARACTERIZATIONS OF TIMED PREORDERS

Theorem 1:

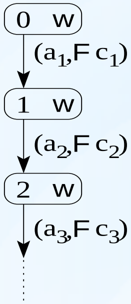
1) $p \sqsubseteq_{\mathbb{T}}^{may} q$ if and only if $L_f(p) \subseteq L_f(q)$ and $L_\omega(p) \subseteq L_\omega(q)$.
2) $p \sqsubseteq_{\mathbb{T}}^{must} q$ if and only if for all $w \in (A \times L)^{\square} \cup (A \times L)^{\omega}$ such that $p \square w$ it holds that:
   a) $q \square w$,
   b) if $|w| < \omega$ then $\forall q' : q \overset{w}{\Longrightarrow} q'$ implies $\exists p' : p \overset{w}{\Longrightarrow} p'$ and $init_p(p') \subseteq init_q(q')$, and
   c) if $|w| = \omega$ then $w \in L_\omega(p)$ implies $w \in L_\omega(q)$.

Theorem 2: Let p and q be timed processes such that p is purely nondeterministic. Then $p \sqsubseteq_{\mathbb{T}}^{must} q$ if and only if all of the following hold:

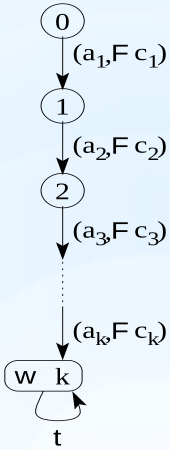
$$
\begin{aligned}
L_D(q) &\subseteq L_D(p) && (1)\\
L_f(q) \setminus L_D(q) &\subseteq L_f(p) && (2)\\
L_m(q) \setminus L_D(q) &\subseteq L_m(p) && (3)\\
L_\omega(q) \setminus L_D(q) &\subseteq L_\omega(p). && (4)
\end{aligned}
$$



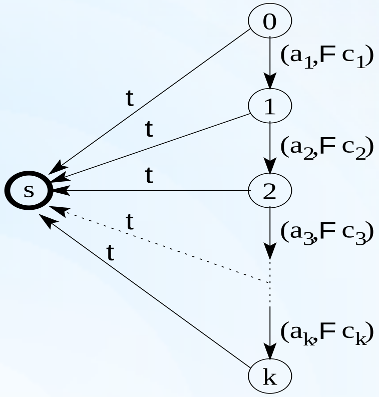
$t_w^{May,*}$



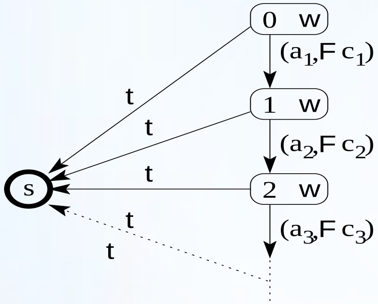
$t_w^{May,\omega}$



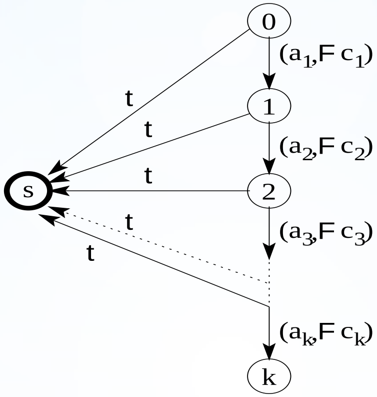
$t_w^{May,div}$



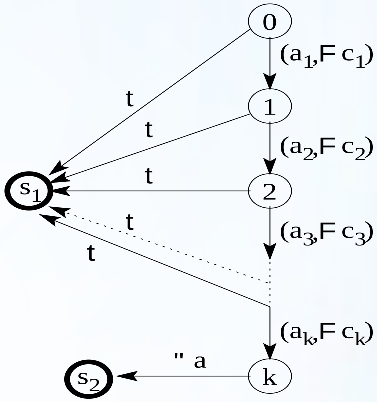
$t_w^{\Downarrow,*}$



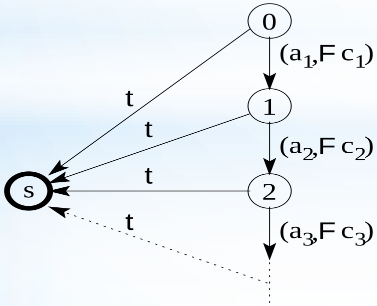
$t_w^{\Downarrow,\omega}$



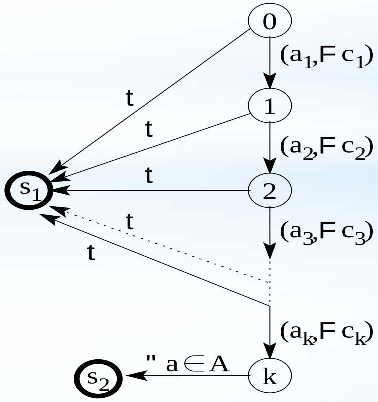
$t_w^{Must,*}$



$t_w^{Must,max}$



$t_w^{Must,\omega}$



$t_{w,A}^{Must}$

Lemma 3:

1) Let $w \in (A \times L)^{\square}$. Then, $w \in L_f(p)$ if and only if $p\ \mathrm{may}_{\mathbb{T}}\ t_w^{May,\square}$.
2) Let $w \in (A \times L)^{\omega}$. Then $w \in L_{\omega}(p)$ if and only if $p\ \mathrm{may}_{\mathbb{T}}\ t_w^{May,\omega}$.
3) Let $w \in (A \times L)^{\square}$. Then, $w \in L_{\omega}(p)$ if and only if $p\ \mathrm{may}_{\mathbb{T}}\ t_w^{May,div}$.
4) Let $w \in (A \times L)^{\square}$. Then, $p \sqcup w$ if and only if $p\ \mathrm{must}_{\mathbb{T}}\ t_w^{\square,\square}$.
5) Let $w \in (A \times L)^{\square} \cup (A \times L)^{\omega}$. Then, $p \sqcup w$ if and only if $p\ \mathrm{must}_{\mathbb{T}}\ t_w^{\square,\omega}$.
6) Let $w \in (A \times L)^{\square}$ such that $p \sqcup w$. Then, $w \notin L_f(p)$ if and only if $p\ \mathrm{must}_{\mathbb{T}}\ t_w^{Must,\square}$.
7) Let $w \in (A \times L)^{\square}$ such that $p \sqcup w$. Then, $w \notin L_m(p)$ if and only if $p\ \mathrm{must}_{\mathbb{T}}\ t_w^{Must,max}$.
8) Let $w \in (A \times L)^{\omega}$ such that $p \sqcup w$. Then, $w \notin L_{\omega}(p)$ if and only if $p\ \mathrm{must}_{\mathbb{T}}\ t_w^{Must,\omega}$.

- The proof of Theorem 1 relies extensively on these intuitive properties of timed tests. Notice that the usage of ω-state tests (that is, tests that accept based on an acceptance family, not only on Suc)—even when discussing finite-state timed processes—is justified by our view that timed tests represent the arbitrary, potentially irregular behaviour of the unknown real-time environment.
- The proof of Theorem 2 also relies on the properties of the timed tests introduced in Lemma 3.

*V. TIMED TEST GENERATION

Theorem 4: Given a TPTL formula φ there exists a test T_{φ} such that $p \models_{\gamma} \varphi$ for any suitable γ if and only if p $\text{must}_{\mathbb{T}}$ T_{φ} for any timed process p. T_{φ} can be algorithmically constructed starting from φ.

*VI. CONCLUSIONS

Autors proposed in this paper a model of timed tests based on timed transition systems. Group of autors addressed the problem of characterizing infinite behaviours of timed processes by developing a theory of timed ω-final states. This theory is inspired by the acceptance family of B¨uchi automata. We also extended the testing theory of De Nicola and Hennessy to timed testing. We then studied the derived timed may and must preorders and developed an alternative characterization for them. This characterization is very similar to the characterization of De Nicola and Hennessy's testing preorders, which shows that our preorders are fully back compatible: they extend the existing preorders as mentioned, but they do not take anything away. Further into the characterization process we also showed that the timed must preorder is equivalent to a variant of reverse timed trace inclusion when its first argument is purely nondeterministic.

*Thank You for atention

*REFERENCES

1. Stefan D. Bruda and Chun Dai,” A Testing Theory for Real-Time Systems”, INTERNATIONAL JOURNAL OF COMPUTERS Issue 3, Volume 4, 2010

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