

# Regulated Rewriting – The Hierarchy

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# Hierarchy of Language Families – Description

## Notation

*ac* appearance checking

$\varepsilon$  with erasing productions

*M* matrix grammars

*P* programmed grammars

*RC* random context grammars

*PER* permitting grammars

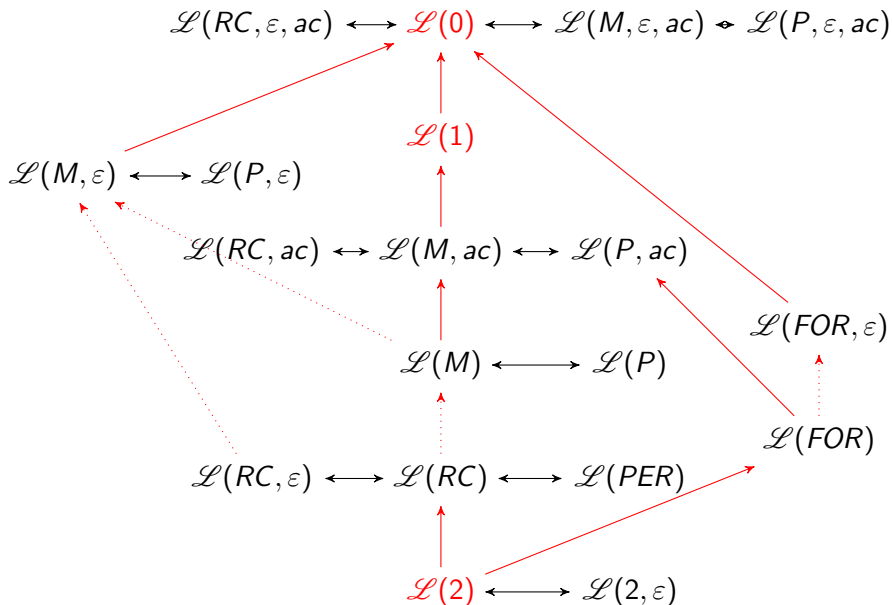
*FOR* forbidding grammars

0, 1, 2 type-0, type-1, type-2 grammars, respectively

## Example

$\mathcal{L}(RC, \varepsilon, ac)$  – the family of languages generated by **random context grammars** with **erasing productions** in **appearance checking** mode

# Regulated Rewriting – The Hierarchy



# Proof of $\mathcal{L}(M) = \mathcal{L}(P)$

## Theorem

$$\mathcal{L}(M) = \mathcal{L}(P).$$

## Proof–Basic Idea

- 1  $\mathcal{L}(M) \subseteq \mathcal{L}(P)$ : Transform any matrix grammar to an equivalent programmed grammar
- 2  $\mathcal{L}(P) \subseteq \mathcal{L}(M)$ : Transform any programmed grammar to an equivalent matrix grammar
- 3  $\mathcal{L}(M) = \mathcal{L}(P)$

# Proof of $\mathcal{L}(M) \subseteq \mathcal{L}(P)$ I

Let

$$H = (G, M)$$

be a matrix grammar, where

- $G = (N, T, P, S)$  is a context-free grammar
- $M$  is a finite language over  $P$

Express  $M$  as

$$M = \{m_1, \dots, m_r\}$$

for some  $r \geq 1$  and

$$m_i = p_{i_1} \dots p_{i_{k_i}}$$

with  $k_i = |m_i|$ . Set

$$N' = \{A' : A \in N\}$$

Define the homomorphism  $h$  from  $(T \cup N)^*$  to  $(T \cup N')^*$  as

- $h(a) = a$  for every  $a \in T$
- $h(A) = A'$  for every  $A \in N$

# Proof of $\mathcal{L}(M) \subseteq \mathcal{L}(P)$ II

- 1 For every  $p_{i_j} : A_{i_j} \rightarrow x_{i_j}$  such that  $\text{alph}(x_{i_j}) \cap N \neq \emptyset$  (some nonterminals occur in  $x_{i_j}$ ) and  $j < |m_i|$  (not the last production in  $m_i$ ), introduce this programmed production:

$$(\langle i, j \rangle : A_{i_j} \rightarrow x_{i_j}, \{\langle i, j+1 \rangle\})$$

- 2 For every  $p_{i_j} : A_{i_j} \rightarrow x_{i_j}$  such that  $\text{alph}(x_{i_j}) \cap N = \emptyset$  (no nonterminal occurs in  $x_{i_j}$ ) and  $j < |m_i|$  (not the last production in  $m_i$ ), introduce
- a  $(\langle i, j \rangle : A_{i_j} \rightarrow h(A_{i_j}), \{\langle i, j, 1 \rangle, \dots, \langle i, j, m \rangle\})$
  - b  $(\langle i, j, q \rangle : A_q \rightarrow A_q, \{\langle i, j \rangle'\})$  for  $q = 1, \dots, m$  (make sure there is a nonterminal)
  - c  $(\langle i, j \rangle' : h(A_{i_j}) \rightarrow x_{i_j}, \{\langle i, j+1 \rangle\})$
- provided that  $N = \{A_1, \dots, A_m\}$
- 3 For every  $p_{i_{k_i}} : A_{i_{k_i}} \rightarrow x_{i_{k_i}}$  (simulate the last production of  $m_i$  and start simulating a new matrix), introduce

$$(\langle i, k_i \rangle : A_{i_{k_i}} \rightarrow x_{i_{k_i}}, \{\langle 1, 1 \rangle, \dots, \langle r, 1 \rangle\})$$

# Proof of $\mathcal{L}(M) \subseteq \mathcal{L}(P)$ III

Let  $P'$  consist of all rules constructed above and

$$(\$ : S' \rightarrow S, \{\langle 1, 1 \rangle, \dots, \langle r, 1 \rangle\})$$

Define the programmed grammar

$$G' = (N \cup N' \cup \{S'\}, T, P', S')$$

Observe that

$$L(H) = L(G')$$



# Proof of $\mathcal{L}(P) \subseteq \mathcal{L}(M)$ I

Let

$$H = (G, R)$$

be a programmed grammar. Consider the simplified description of  $H$  which uses productions of the form

$$(p : A \rightarrow x, R(p))$$

- 1 If  $(p : A \rightarrow x, Q), (r : B \rightarrow y, R) \in P$  and  $r \in Q$ , then introduce the matrix

$$(\langle A \rightarrow x \rangle \rightarrow A, A \rightarrow x, B \rightarrow \langle B \rightarrow y \rangle)$$

(simulation will continue)

- 2 If  $(p : A \rightarrow x, Q) \in P$ , then introduce

$$(\langle A \rightarrow x \rangle \rightarrow A, A \rightarrow x)$$

(simulation ends)



# Proof of $\mathcal{L}(P) \subseteq \mathcal{L}(M)$ II

3 If  $(p : S \rightarrow x, Q) \in P$ , introduce

$$(S' \rightarrow \langle S \rightarrow x \rangle)$$

(simulation starts)

Set

$$\begin{aligned} N' &= N \\ &\cup \{ \langle A \rightarrow x \rangle : A \rightarrow x \in P \} \\ &\cup \{ S' \} \end{aligned}$$

Define the matrix grammar

$$G' = (N', T, P', S')$$

where  $P'$  consists of all the above matrices. Observe that

$$L(H) = L(G')$$





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