

# Cooperating Distributed Grammar Systems

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## CD Grammar System

A **cooperating distributed (CD) grammar system** of degree  $n$ ,  $n \geq 1$ , is a construct

$$\Gamma = (N, T, S, P_1, \dots, P_n)$$

where

$N, T, S$  are defined as usual

$P_i$  is a finite set of context-free productions, called **component** of  $\Gamma$ ,  
for each  $i \in \{1, \dots, n\}$

## Notation

- the  $i$ th grammar  $G_i = (N, T, P_i, S)$
- $y \not\Rightarrow z$  –  $y$  does not directly derive  $z$

## Terminating Mode

For each  $i = 1, \dots, n$ , **terminating derivation** by the  $i$ th component

$$x \xRightarrow{i}^t y$$

iff

- 1  $x \Rightarrow^* y$  in  $G_i = (N, T, P_i, S)$  and
- 2  $y \not\Rightarrow z$  for all  $z \in (N \cup T)^*$

## Note

$y \in T^*$  is not required

# CD Grammar Systems – Derivation Modes II

## $k$ -Step Derivation in $G_i$

$$x \Rightarrow^k y$$

iff  $x \Rightarrow^k y$  in  $G_i$

## At Most $k$ -Step Derivation in $G_i$

$$x \Rightarrow^{\leq k} y$$

iff  $x \Rightarrow^j y$  in  $G_i$  for some  $j \leq k$

## At Least $k$ -Step Derivation in $G_i$

$$x \Rightarrow^{\geq k} y$$

iff  $x \Rightarrow^j y$  in  $G_i$  for some  $j \geq k$

# CD Grammar Systems – Generated Language

## Set of Derivation Modes

$$D = \{*, t\} \cup \{\leq k, = k, \geq k : k = 1, 2, \dots\}$$

## Set of Possible Derivations

$$F(G_j, u, f) = \{v : u \overset{f}{j} v\}, \text{ where } j \in \{1, \dots, n\}, f \in D, u \in V^*$$

## Generated Language

$$L_f(\Gamma) = \{ w \in T^* : \text{there are } v_0, v_1, \dots, v_m \text{ such that} \\ v_i \in F(G_{j_i}, v_{i-1}, f), i = 1, \dots, m, j_i \in \{1, \dots, n\}, \\ v_0 = S, v_m = w, \text{ for some } m \geq 1 \}$$

## Example

$$\Gamma = (\{S, A, A', B, B'\}, \{a, b, c\}, S, P_1, P_2)$$

where

$$P_1 = \{S \rightarrow S, S \rightarrow AB, A' \rightarrow A, B' \rightarrow B\}$$

$$P_2 = \{A \rightarrow aA'b, B \rightarrow cB', A \rightarrow ab, B \rightarrow c\}$$

$$L_f(\Gamma) = \{a^n b^n c^m : m, n \geq 1\}, f \in \{= 1, \geq 1, *, t\} \cup \{\leq k : k \geq 1\}$$

$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) = \{a^n b^n c^n : n \geq 1\}$$

$$L_{=k}(\Gamma) = L_{\geq k}(\Gamma) = \emptyset, k \geq 3$$

## Example

$$\Gamma = (\{S, A\}, \{a\}, S, P_1, P_2, P_3)$$

where

$$P_1 = \{S \rightarrow AA\}$$

$$P_2 = \{A \rightarrow S\}$$

$$P_3 = \{A \rightarrow a\}$$

$$L_{\textcolor{red}{t}}(\Gamma) = \{a^{2^n} : n \geq 1\}$$

# Denotation of CD Language Families

## Denotation of CD Language Families

$$CD_x^y(f)$$

where

$f$  derivation mode,  $f \in D$

$y$     ■ **nothing** – no  $\varepsilon$ -productions  
      ■  $\varepsilon$  –  $\varepsilon$ -productions allowed

$x$     ■  **$n$**  – degree at most  $n$ ,  $n \geq 1$   
      ■  $\infty$  – the number of components is not limited

## Note

$CD_\infty(=)$  the union of all families  $CD_\infty(=k)$  for  $k = 1, 2, \dots$

$CD_\infty(\geq)$  the union of all families  $CD_\infty(\geq k)$  for  $k = 1, 2, \dots$



## Theorem

- $CD_{\infty}^y(f) = \mathcal{L}(CF)$ , for all  $f \in \{= 1, \geq 1, *\} \cup \{\leq k : k \geq 1\}$
- $\mathcal{L}(CF) = CD_1^y(f) \subset CD_2^y(f) \subseteq CD_r^y(f) \subseteq CD_{\infty}^y(f) \subseteq \mathcal{L}(M)$ , for all  $f \in \{= k, \geq k : k \geq 2\}$ ,  $r \geq 3$
- $CD_r^y(\geq k) \subseteq CD_r^y(\geq k + 1)$
- $CD_{\infty}^y(\geq) \subseteq CD_{\infty}^y(=)$
- $\mathcal{L}(CF) = CD_1^y(t) = CD_2^y(t) \subset CD_3^y(t) = CD_{\infty}^y(t) = \mathcal{L}(ET0L)$

# Hybrid CD Grammar Systems

- combination of different modes is possible

## Hybrid CD Grammar System

$$\Gamma = (N, T, S, (P_1, f_1), \dots, (P_n, f_n))$$

where

$N, T, S, P_i$  are defined as in the case of CD grammar systems

$f_i$  is the mode of the  $i$ th component,  $f_i \in D$  for all  $i \in \{1, \dots, n\}$

## Generated Language

$$L(\Gamma) = \{w \in T^* : \text{there are } v_0, v_1, \dots, v_m \text{ such that} \\ v_i \in F(G_{j_i}, v_{i-1}, f_{j_i}), i = 1, \dots, m, j_i \in \{1, \dots, n\}, \\ v_0 = S, v_m = w, \text{ for some } m \geq 1\}$$

# Denotation of Hybrid CD Language Families

## Denotation of Hybrid CD Language Families

$$XCD_{x,v}^y(f)$$

where

$x, y, f$  are defined as in the case of CD grammar systems

- $v$ 
  - $m$  – each  $P_i$  contains at most  $m$  productions,  $m \geq 1$
  - $\infty$ , **nothing** – any number of productions
- $X$ 
  - **nothing** – nondeterministic
  - $D$  – deterministic (for each  $P_i$ ,  $A \rightarrow u, A \rightarrow w \in P_i$  satisfy  $u = w$ )
  - $H$  – hybrid (then,  $(f)$  is not written)

## Example


$HCD_n^\varepsilon$  is a hybrid CD grammar system with at most  $n$  components


## Theorem


- $CD_{\infty,\infty}(f) = CD_{\infty,1}(f) = \mathcal{L}(CF)$ , for all  $f \in \{= 1, \geq 1, *\} \cup \{\leq k : k \geq 1\}$
- $\mathcal{L}(CF) \subset CD_{\infty,1}^{\varepsilon}(t) \subset CD_{\infty,2}^{\varepsilon}(t) \subseteq CD_{\infty,3}^{\varepsilon}(t) \subseteq CD_{\infty,4}^{\varepsilon}(t) \subseteq CD_{\infty,5}^{\varepsilon}(t) = CD_{\infty,\infty}^{\varepsilon}(t) = \mathcal{L}(ET0L)$
- $CD_{n,m}(f) \subset CD_{n+1,m}(f)$ ,  $f \in \{*, t\}$
- $CD_{n,m}(f) \subset CD_{n,m+1}(f)$ ,  $f \in \{*, t\}$

## Theorem

- $\mathcal{L}(CF) \subset HCD_1 \subset HCD_2 \subseteq HCD_3 \subseteq HCD_4 = HCD_{\infty} = \mathcal{L}(M, ac)$
- $\mathcal{L}(ET0L) \subset HCD_4$
- $CD_{\infty}(=) \subset HCD_3$

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