

# Multi-Grammars

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# Multisequential Grammar

## Multisequential Grammar

$$G = (V, T, P, S, K)$$

where

$V, T, S$  are defined as usual

$K$  is a finite set of **selectors** of the form

$$X_1 \text{ act}(Y_1) \dots X_n \text{ act}(Y_n) X_{n+1}$$

where  $n \geq 1$ ,

$$X_i \in \{Z^* : Z \subseteq V\}, Y_j \in \{Z : Z \subseteq V, Z \neq \emptyset\},$$

$i = 1, \dots, n+1$ , and  $j = 1, \dots, n$

$P$  is a finite set of productions of the form  $a \rightarrow x$ , where  $a \in V$ ,  
 $x \in V^*$

# Multisequential Grammar – Derivation Step

## Direct Derivation

If there is

$$X_1 \text{ act}(Y_1) \dots X_n \text{ act}(Y_n) X_{n+1} \in K$$

satisfying

1  $u_i \in X_i$ , for all  $i = 1, \dots, n + 1$ ,

2  $a_j \in Y_j$  and

3  $a_j \rightarrow x_j \in P$ ,

for all  $j = 1, \dots, n$ , then

$$u_1 a_1 \dots u_n a_n u_{n+1} \Rightarrow u_1 x_1 \dots u_n x_n u_{n+1}$$

## Note

The generated language and  $\Rightarrow^*$  are defined as usual

# Multisequential Grammar – Example

## Example

$$G = (\{S, a, b, c\}, \{a, b, c\}, P, S, K)$$

where

$$P = \{S \rightarrow abc, \\ a \rightarrow aa, \\ b \rightarrow bb, \\ c \rightarrow cc\}$$

and

$$K = \{\text{act}(\{S\}), \\ \{a\}^* \text{act}(\{a\}) \{b\}^* \text{act}(\{b\}) \{c\}^* \text{act}(\{c\})\}$$

$$L(G) = \{a^n b^n c^n : n \geq 1\}$$

# Multisequential Grammar – Generative Power

## Generative Power

$$\mathcal{L}(MS) = \mathcal{L}(RE)$$

## Descriptive Complexity

Every recursively enumerable language is generated by a multisequential grammar containing **2 nonterminals** and **2 selectors**.

# Multicontinuous Grammar

## Multicontinuous Grammar

$$G = (V, T, P, S, K)$$

where

$V, T, S$  are defined as usual

$K$  is a finite set of selectors of the form

$$X_1 \text{ act}(Y_1)X_2 \dots X_n \text{ act}(Y_n)X_{n+1}$$

where  $n \geq 1$ ,

$$X_i \in \{Z^* : Z \subseteq V\}, Y_j \in \{\textcolor{red}{Z}^+ : Z \subseteq V, Z \neq \emptyset\},$$

$i = 1, \dots, n+1$ , and  $j = 1, \dots, n$

$P$  is a finite set of productions of the form  $a \rightarrow x$ , where  $a \in V$ ,  
 $x \in V^*$

# Multicontinuous Grammar – Derivation Step

## Direct Derivation

If there is

$$X_1 \text{ act}(Y_1) \dots X_n \text{ act}(Y_n) X_{n+1} \in K$$

satisfying

- 1  $u_i \in X_i$ , for all  $i = 1, \dots, n + 1$ ,
- 2  $y_j \in Y_j$  and
- 3  $y_j = y_{j_1} \dots y_{j_{m_j}}$ ,  $z_j = z_{j_1} \dots z_{j_{m_j}}$ , where  $y_{j_1} \rightarrow z_{j_1}, \dots, y_{j_{m_j}} \rightarrow z_{j_{m_j}} \in P$  for some  $m_j \geq 1$ ,

for all  $j = 1, \dots, n$ , then

$$u_1 y_1 \dots u_n y_n u_{n+1} \Rightarrow u_1 z_1 \dots u_n z_n u_{n+1}$$

## Note

The generated language and  $\Rightarrow^*$  are defined as usual

## Example

$$G = (\{S, a, b, c\}, \{a, b, c\}, P, S, K)$$

where

$$P = \{S \rightarrow abc, \\ a \rightarrow aa, \\ b \rightarrow bb, \\ c \rightarrow cc\}$$

and

$$K = \{\text{act}(\{S\}^+), \\ \text{act}(\{a\}^+) \text{act}(\{b\}^+) \text{act}(\{c\}^+)\}$$

$$L(G) = \{a^{2^n} b^{2^n} c^{2^n} : n \geq 0\}$$



# Multicontinuous Grammar – Generative Power

## Generative Power

$$\mathcal{L}(MC) = \mathcal{L}(RE)$$

## Descriptive Complexity

Every recursively enumerable language is generated by a multicontinuous grammar containing **3 nonterminals** and **2 selectors**.

# Multiparallel Grammar

## Multiparallel Grammar

$$G = (V, T, P, S, K)$$

where

$V, T, S$  are defined as usual

$K$  is a finite set of selectors of the form

$$F_1 F_2 \dots F_m$$

where

$$F_j \in \{W^+ : W \subseteq V, W \neq \emptyset\},$$

$j = 1, \dots, m$ , for some  $m \geq 1$

$P$  is a finite set of productions of the form  $a \rightarrow x$ , where  $a \in V$ ,  
 $x \in V^*$

# Multiparallel Grammar – Derivation Step

## Direct Derivation

If there is

$$\pi \in K$$

satisfying

- 1 **either**  $u = S$  and  $S \rightarrow v \in P$ ,
- 2 **or** there is  $k \geq 1$  so that
  - $u = a_1 \dots a_k$ , where  $a_i \in V$ ,
  - $u \in \pi$ ,
  - $v = x_1 \dots x_k$  and  $a_i \rightarrow x_i \in P$

for all  $i = 1, \dots, k$ , then

$$u \Rightarrow v$$

## Note

The generated language and  $\Rightarrow^*$  are defined as usual

## Example

$$G = (\{S, A, B, C, a, b, c\}, \{a, b, c\}, P, S, K)$$

where

$$P = \{S \rightarrow aAbBcC, \quad S \rightarrow abc, \\ A \rightarrow aA, \quad A \rightarrow a, \quad a \rightarrow a, \\ B \rightarrow bB, \quad B \rightarrow b, \quad b \rightarrow b, \\ C \rightarrow cC, \quad C \rightarrow c, \quad c \rightarrow c\}$$

and

$$K = \{a\}^+ \{A\}^+ \{b\}^+ \{B\}^+ \{c\}^+ \{C\}^+$$

$$L(G) = \{a^n b^n c^n : n \geq 1\}$$

## Generative Power

$$\mathcal{L}(MP) = \mathcal{L}(RE)$$

## Descriptive Complexity

Every recursively enumerable language is generated by a multiparallel grammar containing **7 nonterminals** and **4 selectors** of the **length 5** ( $m = 5$ ).



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