Deep Pushdown Automata

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Abstract. Indisputably, context-free grammars and pushdown automata, which represent their fundamental automaton counterpart, fulfill a crucial role in the formal language theory. Over its history, this theory has modified the context-free grammars in many ways, including various regulated versions of these grammars. Many of these modified context-free grammars define a language family lying between the families of context-free and context-sensitive languages. To give a specific example, an infinite hierarchy of language families between the families of context-free and context-sensitive languages was established based on *n*-limited state grammars, which represent regulated grammars underlain by context-free grammars. As a matter of fact, most regulated context-free grammars without erasing productions are stronger than context-free grammars but no more powerful than context-sensitive grammars.

Compared to the number of grammatical modifications, there exist significantly fewer modifications of pushdown automata although the automata theory has constantly paid some attention to their investigation. Some of these modifications, such as finite-turn pushdown automata, define a proper subfamily of the family of context-free languages. On the other hand, some other modifications, such as two-pushdown automata, are as powerful as the Turing machines. As opposed to the language families generated by regulated context-free grammars without erasing productions, there are hardly any modifications of pushdown automata that define a language family between the families of context-free and context-sensitive languages. It thus comes as no surprise that most of these modified context-free grammars, including the *n*-limited state grammars, lack any automaton counterpart.

During this talk, we introduce deep pushdown automata, which represent a new modification of ordinary pushdown automata. However, as opposed of the previous modifications, the power of the deep pushdown automata is similar to the generative power of regulated context-free grammar without erasing productions because they are stronger than ordinary pushdown automata but less powerful than context-sensitive grammars. More precisely, these automata give rise to an infinite hierarchy of language families coinciding with the hierarchy resulting from the *n*-limited state grammars. In this sense, the deep pushdown automata represent the automaton counterpart to the state grammars and, in this sense, fill this gap.

The introduction of deep pushdown automata is inspired by the standard conversion of a context-free grammar to an equivalent pushdown automaton, M, frequently referred to as general top-down parser. Recall that during every move, M either *pops* or *expands* its pushdown depending on the symbol occurring on the pushdown top. If an input symbol, a, occurs on the pushdown top, M compares the pushdown top symbol with the current input symbol, and if they coincide, M pops the topmost symbol from the pushdown and proceeds to the next input symbol on the input tape. If a nonterminal occurs on the pushdown top, the parser expands its pushdown so it replaces the top nonterminal with a string. M accepts an input string, x, if it makes a sequence of moves so it completely reads x, empties its

pushdown, and enters a final state; the latter requirement of entering a final state is dropped in some books.

A deep pushdown automaton, $_{deep}M$, represents a slight generalization of M. Indeed, $_{deep}M$ works exactly as M except that it can make *expansions of depth m* so $_{deep}M$ replaces the mth topmost pushdown symbol with a string, for some $m \geq 1$. We demonstrate that the deep pushdown automata that make expansions of depth m or less, where $m \geq 1$, are equivalent to the m-limited state grammars, so these automata accept a proper language subfamily of the language accepted by the deep pushdown automata that make expansions of depth $m + 1$ or less. The resulting infinite hierarchy of language families obtained in this way occurs between the family of context-free and context-sensitive languages. For every positive number n, however, there exist some context-sensitive languages that cannot be accepted by any deep pushdown automata that make expansions of depth n or less.

In the conclusion of this talk, we formulate some open problem areas concerning the deep pushdown automata. Specifically, it suggests some deterministic and generalized versions of these automata to study.