Modern Numerical Methods

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Abstract

The aim of the paper is to describe our new modern numerical methods and to show how to implement fully original- highly accurate and fast algorithms in up-to-date computer architectures to be effective. It is a fact that the accuracy and stability of the algorithms we have designed significantly exceeds the presently known systems from abroad. In particular, the paper wants to concentrate, using the previous results and latest development trends, on the simulation of dynamic systems and on extremely exact mathematical computations. The idea of implementation of our fully original - highly accurate and fast algorithms into advanced programs, the creation of application programs and an analysis of problems requiring parallel processing will be a part of our research.

Definite integrals and integral equations, due to the number of applications, are very important mathematical tools. Their solution using the Modern Taylor Series Method is the subject of this paper.

1. Definite Integrals

The problem of solving a definite one-dimensional integral taken as a function of the upper boundary can be transformed to solving a system of differential equations.

1.1 Illustrative Examples

Example 1: Let a definite integral

$$y = \int_0^1 e^t dt$$

be given.

The definite integral can be rewritten in the form:

$$y' = z$$

$$z = e^t.$$

The initial conditions can be obtained by substituting the value of the lower boundary for the variable t of the function z.

The numerical solution of the integral is obtained at the point corresponding to the upper boundary of the integral ($t_{max} = 1s$).

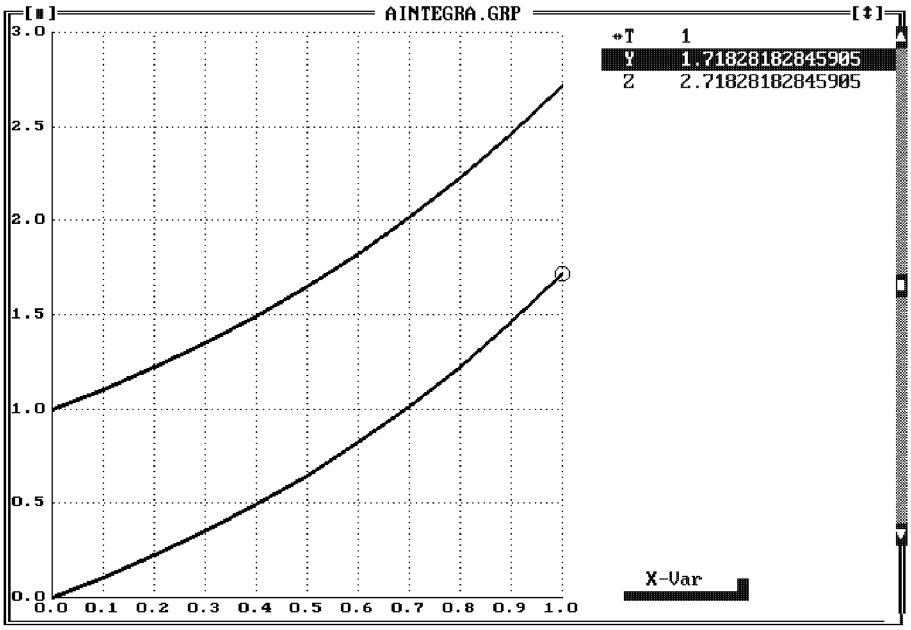

Figure 1:

The time functions of the value of the integral y and the function z being integrated are in Fig.1. A particular calculation of the integral for T=1s is in the right-hand part of Fig.1.

The corresponding source text in TKSL/486 is

```
var       y,z;
const     tmax=1,eps=1e-20;
system
          y'=z   &0;
          z=exp(t);
sysend.
```

Example 2: Let a definite integral

$$y = \int_{0}^{9.853} e^{2sint} dt$$

be given.

The definite integral can again be rewritten in the form:

$$y' = z$$

$$z = e^{2sint}.$$

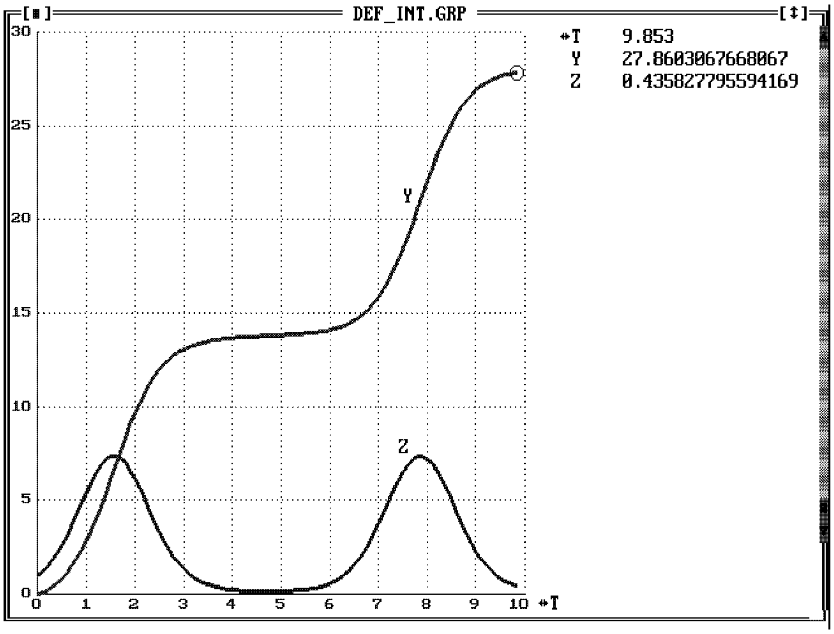

Figure 2:

The numerical solution of the integral is obtained at the point corresponding to the upper boundary of the integral ($t_{max} = 9.853s$).

The time functions of the value of the integral y and the function z being integrated are in Fig.2. A particular calculation of the integral for T=9.853s is in the right-hand part of Fig.2.

2. Fourier Analysis

A function y(t) which is periodic of period 2π, is continuous except for a finite number of finite discontinuities in any interval of length 2π, and has a right-hand and left-hand derivative at every point may be expanded into a Fourier series

$$y(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k coskt + \sum_{k=1}^{\infty} b_k sinkt$$

where the Fourier coefficients are given by

$$a_k = \frac{1}{\pi}\int_0^{2\pi} y(t)cosktdt$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} y(t)sinktdt$$

The definite integrals can be rewritten in the form:

$$a'_k = \frac{1}{\pi}y(t)coskt$$

$$b'_k = \frac{1}{\pi}y(t)sinkt$$

The numerical solution of the integral is again obtained at the point corresponding to the upper boundary of the integral ($t_{max} = 2\pi$).

In the following example y(t) = sint + cos2t. Fourier coefficients a_1, b_1, a_2 and b_2 were calculated. It is very illustrative to have a look at the results in Fig.3.

The source text in TKSL/486 is

```
var y,a1,a2,b1,b2,b100;
const PI=3.1415926535897932385,eps=1e-20,tmax=2*PI,k=1/PI;
system
y=cos(t)+sin(2*t);
b1'=k*y*sin(t) &0;
a1'=k*y*cos(t) &0;
b2'=k*y*sin(2*t) &0;
a2'=k*y*cos(2*t) &0;
{b100'=k*y*sin(100*t) &0;}
sysend.
```

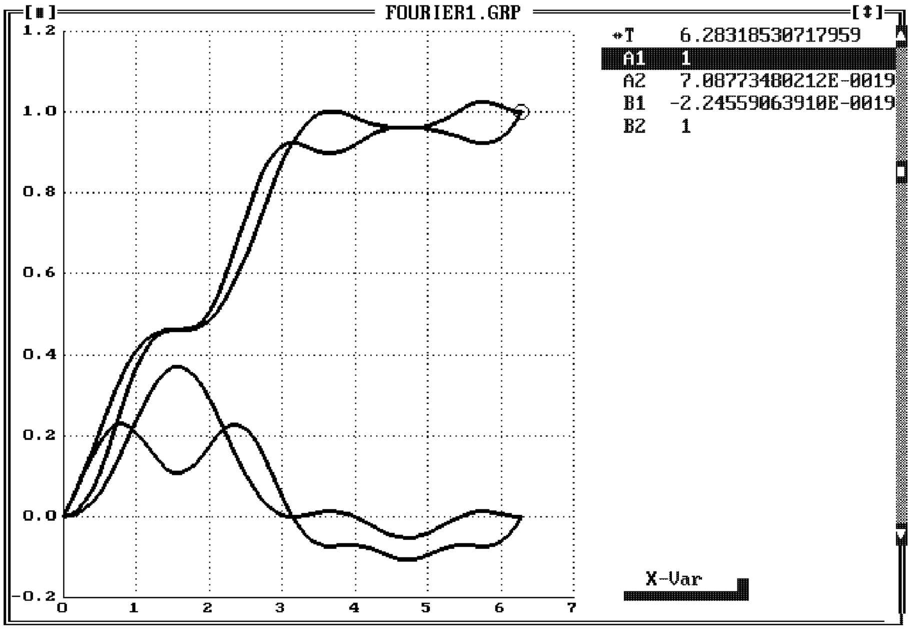

Figure 3:

3. Integral Equation

Applying the Modern Taylor Series Method to solving the Volterra integral equation, we get a following computation scheme.

Let an integral equation

$$y(t) = sint + \int_0^t y(s)ds$$

be given.
The integral equation can be rewritten in the form

$$y' = cost + y. \tag{1}$$

Note: The exact solution of (1) is

$$y = \frac{(e^t + sint - cost)}{2}.$$

The corresponding time functions of X and Y are shown in Fig.2. X is the exact solution. Y is the numerical solution of (1). It is very illustrative to compare both results.

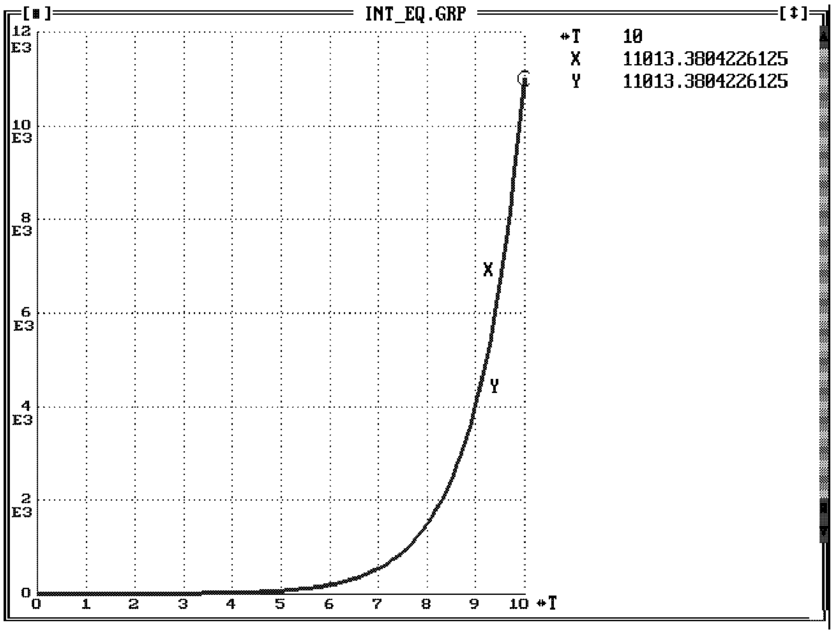

Figure 4:

The source text in TKSL/486 is

```
var y,x;
const tmax=10,eps=1e-20,dt=0.1;
system
y'=cos(t)+y    &0;
x=(exp(t)+sin(t)-cos(t))*1/2;
sysend.
```

4. Conclusion

The Modern Taylor Series Method proved to be extremely exact and fast when described numerical examples have been solved.

5. References

[1] Kunovský,J.: Modern Taylor Series Method. Habilitation theses. TU Brno, 1995