Net Reduction and the Analysis of Petri Nets

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Abstract: *This work defines and uses particular morphisms between Petri nets to reduce the graph of the net by agglomeration of place nodes and transition nodes. The reductions transform one net into another, preserving some dynamical properties of Petri nets such as liveness. Then the standard, computer-assisted analysis methods can be applied.*

1. Introduction

Petri nets were introduced by C. A. Petri in 1962. Since their introduction, Petri nets have been used to model many different systems and various classes of net have been developed. The standard class is the class of P/T Petri nets, but there are classes of coloured, timed and probabilistic net introduced by many authors as an extensions or restrictions of the standard class. The growing importance of Petri nets is related to their usefulness in modelling concurrent systems, distributed processing, communicating systems, all of whose dynamical behaviour can be easily expressed as movement of tokens in Petri nets.

2. Analysis methods

Petri nets can model an real system and provide an abstraction of the behaviour of the system without the encumbrance of irrelevant system detail. Then an analysis of the behaviour of the Petri net model will hopefully lead to important insights into the behaviour of the modelled system. Several techniques have been developed for the analysis of Petri nets. The basic method for investigating of the properties of Petri nets is using the state graph or coverability tree. The methods begin by constructing the state graph, which represents the dynamical behaviour of the Petri net as a transition system, in general infinite. The powerful and elegant analytical techniques for state graphs of Petri nets are based on linear algebra. These methods include the investigation of invariants, which reflect particular aspects of the repetitive behaviour of the net. Also included are methods based on net reduction.

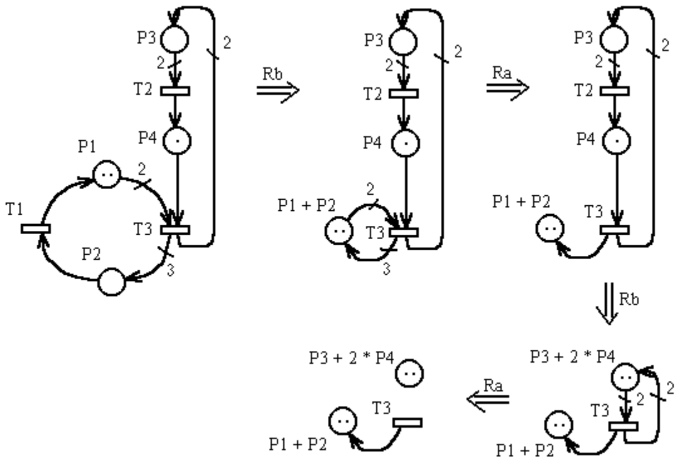
3. Reduction methods

Although the construction of the state graph is undeniably an efficient method for finding the properties of a small-sized Petri nets, it can be a lengthy process for a Petri nets which have a large number of reachable markings or states. The use of linear algebra also becomes difficult when the size of the Petri net increases.

Reduction methods enable a Petri net to be transformed into another simpler Petri net, while maintainig certain properties of the initial net. The simplified net reflects the behaviour of the original net and preserves some properties, but is not equivalent to the original net. If the original net had an interpretation in a system modelled by the net, this interpretation need not be applicable to the simplified net.

Reduction methods are divided into subclasses according to the properties which are preserved by the reductions. We shall present two main sublasses. The first is made up of methods preserving liveness and boundedness. The other class considered here contains reductions preserving invariants.

The reduction methods descibed in the literature for preserving of liveness are: *Substitution of a place*, *Supressing an implicit place*, *Suppressing a neutral transition*, *Suppressing one of two identical transitions*. There are two main methods for reducing preserving invariants. The one is applied to *pure transitions* in the net, the other to *self-loop transitions*. What the application of the reductions looks like is shown in the following example, where reductions preserving invariants are applied.



Simple reduction example

In this small example R_a means reduction applied to a self-loop transition, R_b reduction applied to a pure transition. As shown, reductions are repeated, until no further reduction can be applied. The result is a simplified net, whose properties can be investigated using any of standard methods.

In general the result of two reduction steps depends on the order in which the steps are applied, but every time the preserved properties are the same for both orders of application.

Reductions preserving invariants have an interesting property. Their application yields the invariants without using any other additional method. Their use is straightforward when the Petri net is described with using a change matrix and a algorithm for such use is available.

4. Conclusion

Reduction methods provide a means of reducing large Petri nets to a point where computer-assisted analysis methods developed for smaller Petri nets can be applied. Several types of reductions can be used for investigating basic properties of Petri nets – liveness & boundedness and invariants. Using of reduction methods precedes the application of standard methods. Standard methods are applied to simplified nets, so the analysis results are obtainable more easily and quickly.

Application of reduction methods is algorithmic, and all methods can be programmed. This allows inclusion of the reduction methods into tools for Petri net simulation and analysis, as e.g. in the PESIM system, developed in the Department of Computer Science and Engineering, Technical University of Brno.

Bibliography (selected)

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